Quantum Electronics

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Module No. # 05
Lecture No. # 36
Quantum States of EM Field (Contd.)

We started looking at more than one single mode, that is multi-mode states and we started looking at two more states and took some examples. Do you have any questions?

Sir, we expressed general ket function psi phi as a product of 2 n 1 and n 2, does it hold a general case summation on n 1 and n 2 c n 1 and 2 and n 1 and n 2?

Actually, for example, when we quantize electromagnetic radiation, I find a discrete set of infinite number of oscillators, each oscillator can have energy of n plus half h cross omega, each oscillator is characterized by its annulation creation operators, each oscillator is decided by its ket state etcetera etcetera. So, the complete such state is given by, actually if I have n one photons in mode 1, n 2 photons in mode 2, n 3 photons in mode 3, n l photons in mode l infinite. This is 1 set 1 ket for example, where there is 1 state in which there are n 1 photons in mode 1, n 2 photons in mode 2 etcetera and this I written as n 1 n 2 n l. So, this product means, it is essentially this is a simplified form of writing the state. This is fine. What I am asking is that if we consider two mode states? A general 2 mode state will be summation over n 1 n 2 c n 1 n 2 and this. So, I have for example, have c n1 n 2 n 1 going from 0 to infinity and n 2 going from 2 to infinity.

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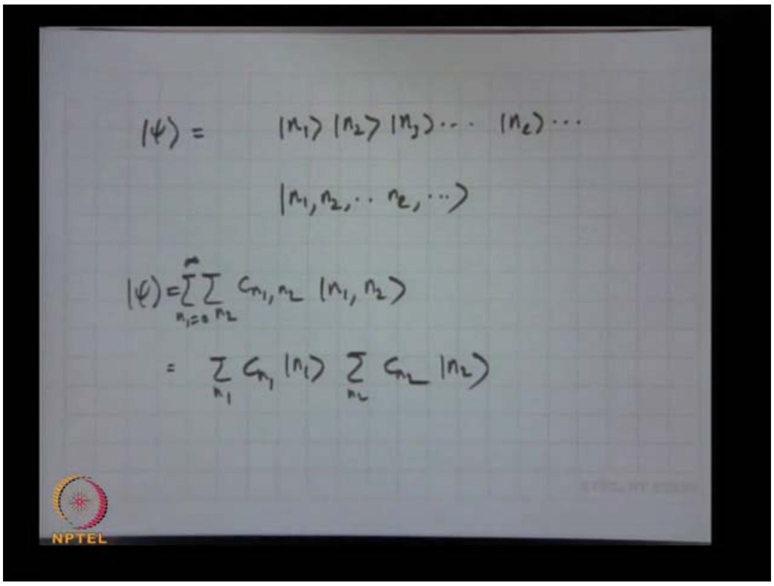

Now, and then what conditions can be separated into a form of a product of over summation of n 1 which are now the states n 1 and this thing? It depends on this expansion coefficient here. If you had for example, I would imagine that if this could be written as a product of a constant which is n 1 and n 2, then I can split this into 2, product of two sums, means I can write this as, sigma n 1 c n 1 n 1 sigma n 2.

Is there any physical significance of being able to split into these two parts? No, this is a subset of this. This is more general than this. This is more general. This is one set of states, in which, the overall state is represented as product of state one and state 2 mode 2 and mode 2. If you do any measurements of any observable on mode one, you do not touch mode 2. Though measurements on mode 1 will not influence any of our observations of mode 2, if you detect for example, whether there is suppose this is a horizontal and vertical polarization states of two modes, same frequency same propagation direction, if I measure, if I pass this entire state through a polarizer which is horizontally oriented corresponding to frequency omega 1. I split the omega 1 and omega 2 for example, then I will not influence any measurement of the second state, that does not happen in a state in which you cannot identify the product state.

What will be the identity of that? No, because what happens is any measurement done on one of the states influences the result of observation on the second state and those are the entangled states where this kind of correlation exists between measurements performed on mode 1 and mode 2 and this is the most general, this is a specific type of multi-mode state or 2 mode state.

So, before we look at beam splitter problem, I want to generalize this into a multi-mode state. So, for example, suppose I take in classical picture, a monochromatic wave, a monochromatic wave occupies time from t is equal to minus infinity plus infinity, because it is exponential I omega t or cos omega t or sin omega t forever for all times. The moment you make a finite duration wave, which means I say it is exponential I omega t or cos omega t from t is equal to 0 the capital t. It no more remains monochromatic, because you can take a fourier transform and show that this contains many frequencies.

So, it is also I can also build a wave which has a finite duration by adding waves of multiple frequencies meeting between various frequencies. So, that becomes a non-monochromatic wave then. I cannot have, I cannot say, I have a pulse, which is monochromatic, because the moment you make a pulse of a finite duration, it is no more monochromatic, it has a finite spectral width. Now, the single mode state that you have looked at, is a single frequency state, because single mode means single frequency single polarization state and one propagation direction k vector. This is this is state which exists for all times. If I take a single photon in that state, that single photon can be anywhere from t is equal to minus infinity or plus infinity, if I try to detect that photon, I can, it may be anywhere, I do not know.

Now, I can form what are called as wave packets by superposing modes of different frequencies. So, if I take for example, if I take two frequencies, I will have meeting between these 2 frequencies and I will have a distribution which is not uniform right across time, but it will be. So, if the limit what can I do is, I can write the state, a single photon state, what this implies is this is c 1110203 plus c 2011203 plus c 3010213.

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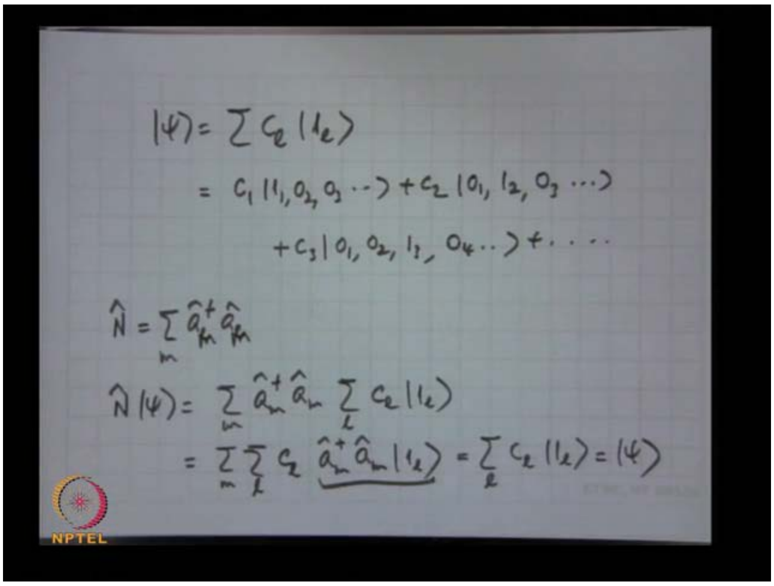

How many photons are there in this state? So, if I operate by the total number operator, you let me use some other index m a m. So, n psi, this is equal to sigma m sigma l c l a m dagger a m one l. I should be careful I used different indices for the state and the number operator here. The total number operator is consists of a sum of all the number operators corresponding to each mode. What is the value of this? If m is not equal to l, then the corresponding value of the number of photons in this. This is a short form. This is like this. So, if m is equal to 2 and this is first 1 it is 0 because 0 there is 02. So, if m is not equal to l, this gives me 0 and if m is equal to l, this gives me 11 l. So, what this gives me is, so, this is simply delta m l 1 l. So, what happens to this state sigma l? If m is not equal to l, a m dagger a m is the number operator for mth mode, and if this particular state corresponds to a value of l which is different from m and there are no photons in that mth mode and in that state and that it 0. Only m is equal to l terms will survive and if m is equal to l, then a m dagger a a l dagger a l 1 l is 1 l.

And. So, I get back c l. So, this means that this state is a single photon state, total number of photons is one and you can show that this is not an Eigen state of a m dagger a m. That means, let me try to find out whether this is an Eigen state of the number operator of one of the states one of the modes. So, a m dagger, a m dagger a m of psi is equal to sigma l c l am dagger a m 1 l, which is how much and this is not equal to psi.

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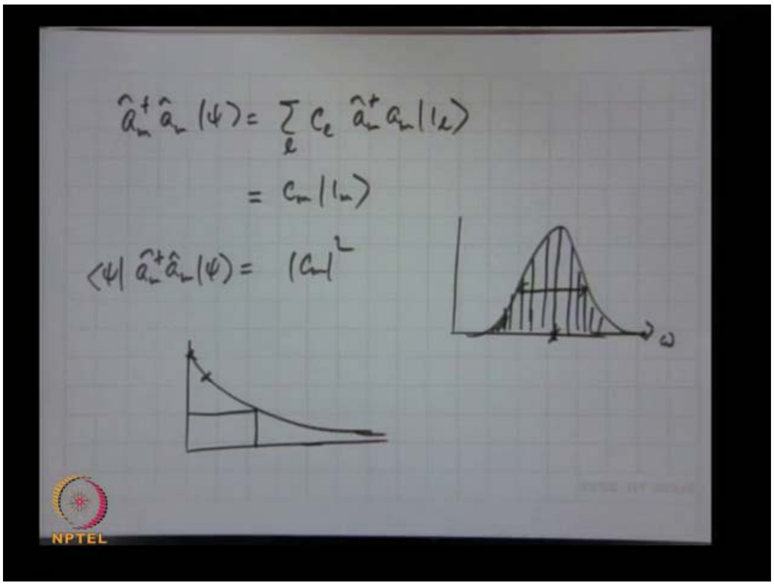

This is not an Eigen state. What it means is, if you were to measure the number of photons in the mth mode, you can not define. Every time there will be a different value, you can calculate expectation value. What will be the expectation value? So, mod c m square defines the probability of getting finding m photons in this state, in the m, that photon in the mth mode. That photon, there is only 1 photon, but that is in a superposition state. So, these modes could be different frequencies. These modes could correspond in this summation. These modes in the summation could correspond to this same propagation direction same polarization, but different frequencies.

So, by a choice of c l amplitudes, I can build a state, where just like in classical picture where I can have different frequencies adding up to give me a wave packet, I can build a single photon wave packet by choosing appropriate values of this coefficient c l. c l depends, if l represents various frequencies, because this l now corresponds to various frequencies. That is various modes corresponds to various frequencies, all propagating in the same direction having the same polarization state then by an appropriate choice of c l s mod c l square, I can add this up and it will become localized. It will become little more confined in time than in an infinite time space,

Just like in classical, if I have a single frequency wave, that wave extends from minus infinity to plus infinity, but if I add multiple frequencies appropriately, in fact, a continuous range of frequencies from omega 0 to omega 0 plus delta omega, I will get a wave packet, whose duration will be one by delta omega. If the spectral width is delta omega, I will have a duration of a pulse, which is one by delta omega, of the order of one by delta omega. So, depending on the amplitude c l that I choose here, the combination I choose here, I can actually build a a single photon wave packet. So, it is this kind of wave packet that emerges when an atom inside is excited and emits spontaneously. Because when an atom is an excited state and it emits by jumping down, it emits one photon. And that one photon is a wave packet like this it is coming out, its frequency is not well defined, because the energy levels for a finite width natural broadening etcetera etcetera we discussed.

So, what is going to happen is, this photon which comes out is a single photon, but in a form like this, it is not in a single mode, it is in a superposition of modes. And as you will be able to measure, you can find the probability of detecting it after a certain time. Once you've excited the atom, you can put a detector here and find out what is the probability of detecting after one nanosecond, what is the probability after detecting the photon after one point one nanosecond etcetera etcetera and you can form and you will get a certain distribution with respect to time which is exactly the spontaneous decay, that you can measure in spontaneous emission process.

So, these are multi modes single photon states, and when we say that, we have a photon incident on a beam splitter or on a device single photon incident, it is usually, this kind of a wave packet that comes in. There's only 1 photon, but it is in a state in which it is a superposition of many modes, many frequencies. If you choose a larger spectrum of frequencies, you will have a narrower wave packet. If you choose a smaller spectrum of frequencies, you will have a larger wave packet, which means, you have less uncertainty in where to find the photons within the wave packet, if you have a narrower spectrum. And finally, when into the monochromatic, it is everywhere.

So, this is what is meant by a. In fact, generalization of a 2 mode state in to multi-mode state and I can still have 1 photon in a multi-mode state, which means, it is occupying, it is in a superposition state of many frequencies if I take a photon propagating in one direction with a one polarization state, it would be essentially, it is this concept and, so this state does not have a single frequency, it has many frequencies. As you can see here the probability of detecting a photon in the mth mode, that means, your particular frequency is mod c m square and the appropriate choice of mod c m square I can build a wave packet yes.

When in a monochromatic medium that thing is that intensity in the time domain. Yes yes. That means I will not take make exactly at width. Yes yes, which means if I were to suppose I take a a quasi monochromatic single photon state, quasi monochromatic means it is almost monochromatic. So, suppose here it corresponds to a a delta p of say one millisecond duration. So, the corresponding frequency width is one by one millisecond which is one kilohertz or something like it approximately. So, one kilohertz bandwidth will give you one millisecond time. So, I prepare n large number of identical one millisecond duration single photon states and I do experiments on this.

So, what I would say, what I will say is, within this one millisecond you will not know when you will detect a photon. So, I let this gamma-logistics detector, I may first detect it right in the beginning, second detector may detect it after some time, third one third state, please remember one photon is detected only once, I am talking about multiple states, multiple one an ensemble of systems. So, this first one single photon state comes I measure, I find the duration within the one millisecond duration somewhere I find it, the next one I find another time.

Sir, that means, within this one millisecond duration any detector can detect? Will detect, if this is the size we define in the duration. Usually there is a wave packet which has a tail like this. So, there is also a probability of detecting much later also, but the probability is very very small. So, this mod c m square distribution will determine which detector will be taken this time with what property suppose I could choose, suppose in the limit of the quantization module. You see, what is the frequency spacing between the various modes? it depends on the value of l that I have chosen, when I quantize the electromagnetic field.

If I were to increase l, the frequency spacing become smaller and smaller and the frequency has become almost continuous. In that case, I can actually extend the sigma into an integral. Now, if I take mod c m square as a function of frequency to be like a Gaussian function, that means, there are various frequencies which are present with amplitude which depend on the Gaussian function , the corresponding wave packet in the time domain will also be a Gaussian function, because the Fourier transform of a

Gaussian is a Gaussian. So, I will have a Gaussian single photon wave packet generated by a mod c m square, which is again a Gaussian in frequency space. So, in that Gaussian for example, I would have, I could have this is some value of m. So, these are values of m which correspond to different values of frequencies, each value of m corresponds to one particular frequency.

Do we have something like a discrete step? If l is finite, as l tends to infinite values, as I increase my quantization volume, these frequencies get closer and closer. But then l over here refers to the different frequencies, I mean every l refers to the frequency? Yeah, in this summation, I am taking a combination of modes, for which each l corresponds to a different frequency. If we increase l let it, Not this l, I am taking about capital l, the volume ,the quantization volume, if I increase the frequency spacing becomes smaller and smaller and finally, I will have all the frequencies this is continuous almost becomes.

So, there will be a one particular value of m in this value, which will have its peak value, which corresponds to one particular frequency and there will be different values of m where the frequencies which corresponding frequencies where the amplitude will fall down. All it implies is the probability of detecting a photon with this frequency largest within this wave packet the probability of detecting a photon with this frequency is very very small. This will lead in the time domain to a wave packet which exactly looks like a Gaussian, whose width will be inversely dependent on this width in the frequency space. The larger this width is, the smaller is the time width. The smaller this width is, the larger is the wave packet width in the time domain.

So, in a spontaneous emission, the exponential decay we know, and the spectrum is what the Fourier transform of an exponentially decaying function, Lorentzian. So, this spectrum which you will get lorentzian spectrum, which also look like a Gaussian, it is much broader. It is a lorentzian spectrum and the probability of detecting a photon at this instant will be maximum and as the time delays, the probability will fall down to almost 0. And you will define a natural, a life time of a state etcetera etcetera. The life time of the state defines a spectrum width of the emission. So, there are all related to this. So, this is a very interesting state, because this is still a single photon state, but that photon is in a superposition of many frequencies. You can only define the probability of detecting a particular frequency for that photon.

Sir, with different probability, I will get different energy photons? Yes, Unknowingly uncertain here, because the frequencies are uncertain. If I have to precisely define the energy, it has to be a precise frequency. So, there is uncertainty in the frequency, there's uncertainty in the energy, there is uncertainty in the position, where when you will detect it when it comes to you. In fact, I will come to this detection a little later through another operator, but before that. So, I thought I will introduce this and this is a very interesting concept where I can generalize from a single mode state to a multi-mode state. In fact, I can further generalize to not having the same propagation direction or same polarization in to this sum could represent different propagation directions and different frequencies etcetera etcetera. It is very general.

Now, what I want to do today is because now I want to introduce this beam splitter. This is a very important component in many experiments which deal with quantum aspects of light. I can build a interferometer with a beam splitter I can do lot of things and beam splitter is also used as a model to incorporate losses or scattering etcetera in quantum optics discussions. So, what is a beam splitter? I have a device in which light is incident from here partially reflected and partially transmitted.

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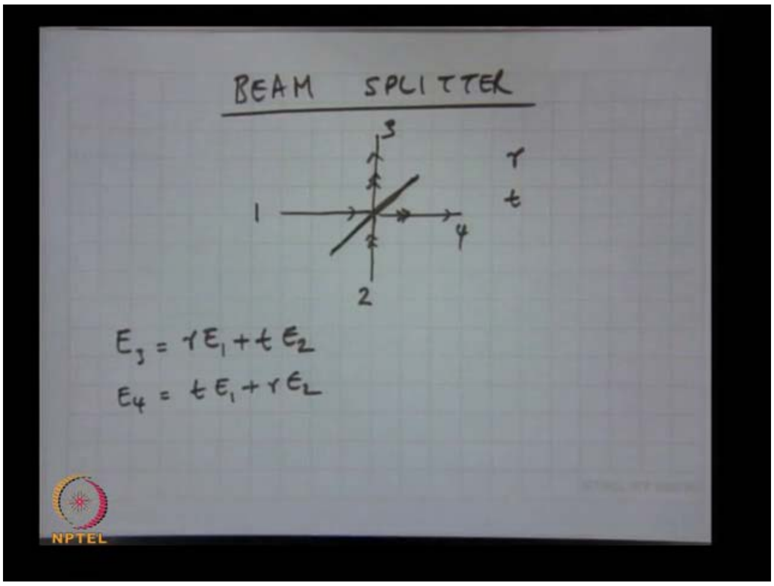

Now, to simplify the mathematics I am going to assume a beam splitter which is completely symmetric. That means, the reflected, the amplitude reflection of this wave is the same as the amplitude reflection of this wave on the other side. So, this wave comes reflects this wave when it comes when I am drawing double arrows here, its partly deflected and partly transmitted. So, the single arrow gives you a single arrow reflected light a single arrow transmitted light, a double arrow incident light gives me a double arrow reflected light and a double arrow transmitted light. And by symmetry, I am assuming that if this reflection coefficient is small r this reflection coefficient is also small r. If this transmission coefficient is small t, this transmission coefficient is also small t. I can actually generalize to asymmetric situations, but we will restrict this.

So, let me call the amplitude reflection coefficient as small r and the amplitude transmission coefficient as small t. So, let me call this the port one, port two, port three and port four. There are two input ports and there are two output ports. This is a very general device actually. A lot of devices have 2 inputs and 2 outputs. Please note even if you send like only in one port, the other port is still there. You may not be considering the second port, but there is a second port and I will show you that in classical, you can forget about the second port, in quantum, you cannot forget the second port because vacuum is incident from second port all the time. Even if you do not send a light beam from here, there are modes all modes are occupied all the time with vacuum at least. So, you cannot forget the fact that there is vacuum in this input. Incidentally and that matters a lot in quantum mechanics, because if you neglect that you have problem in computational relations.

Now, so first let me do a classical analysis of this beam splitter and gets some general relationships and then we will quantize that through a particular procedure and I will tell you the procedure of getting the quantized relations between the annulation operators of these fields 3 and 4 with respect to the fields 1 and 2. So, if I have incident waves, let me call this E1 E2 are incident E3 and E4 are the output ports. So, what is E3? E3 will be r times E1 plus E times E2 and E4 will be t times E1 plus r times E2. If the r and t s were not the same I have to write t prime and r prime for the second 1. And please note r and t are in general complex. it is an amplitude reflection coefficients and amplitude transmission coefficients not energy reflection coefficients. So, these are complex

Now, I need to satisfy energy conservation. So, the sum of the energies in 3 and 4 must be equal to the sum of the energies in 1 and 2. The energy, certain energy is incident from here, certain energy is incident from here,that must be either in 3 or 4, I am neglecting any absorption or any other scattering or anything else. So, let me try to calculate. So, the energy is proportional intensity which is proportional to the mod modulo square of the electric field. So, let me calculate what is mod E3 square plus, mod E4 square.

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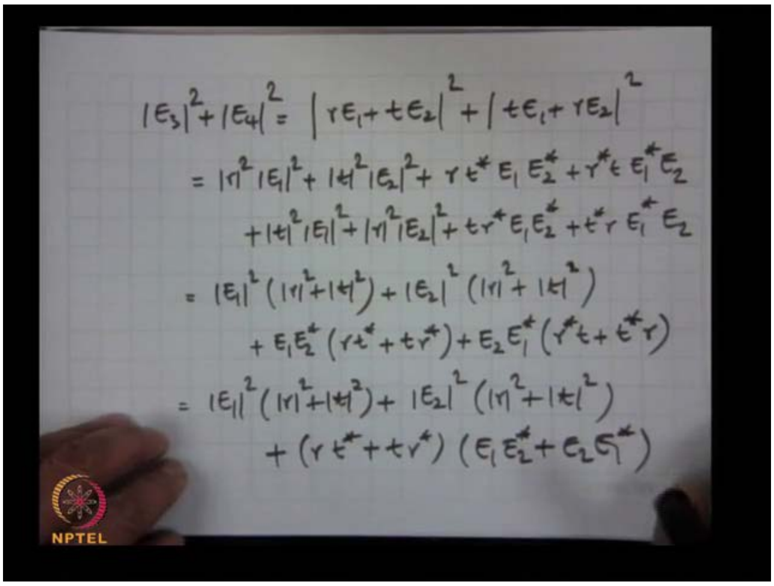

So, mod E3 square plus mod E4 square is equal to modulus of r E1 plus t E2 whole square plus modulus of t E1 plus r 2 whole square. This is equal to modulus r square modulus E1 square and please note, E1 and E2 could also be complex. They are phases plus mod t square mod E2 square plus r t star E1 E2 star plus r star t E1 star E2 plus mod t square mod E1 square plus mod r square mod E2 square plus t r star E1 E2 star plus e star r E1 star E2. So, let me collect the E1 square term. So, this is mod E1 square into mod r square plus mod t square, this and this term plus this and this gives me mod E2 square into mod r square plus mod t square plus E1 E2 star into r t star E1 E2 star E1 E2 star plus t r star plus E2 E1 star into r star t plus t star r. So, this is same as this is r t star plus t r star r t star plus t r star. So, this is actually mod E1 square mod r square plus mod t square plus mod E2 square mod r square plus mod t square plus r t star plus t r star into E1 E2 star plus E2 E1star.

Now, what is the condition mod E3 square plus mod E4 square must be equal to mod E1 square plus mod E2 square irrespective of the values of E1 and E2. I can choose E1 is equal to 0, I can choose E2 is equal to 0, I can choose E1 is equal to E2, any combination I can choose and still all the time I must have mod E3 square plus mod E4 square as equal to mod E1 square plus mod E2 square. So, what are the conditions I get, this must be equal to 1 and this must be 0. Other wise I cannot satisfy this equation for all values of any combination you need to. So, I get just simply by energy conservation. I get Mod r square plus mod t square is equal to 1 and r t star plus r star t is equal to 0. So, let me write r is equal to mod r exponential I phi r and t is equal to mod t exponential I phi t. So, this is mod r into mod t into exponential I phi r minus phi t plus exponential minus I phi r minus phi t r t star plus r star t.

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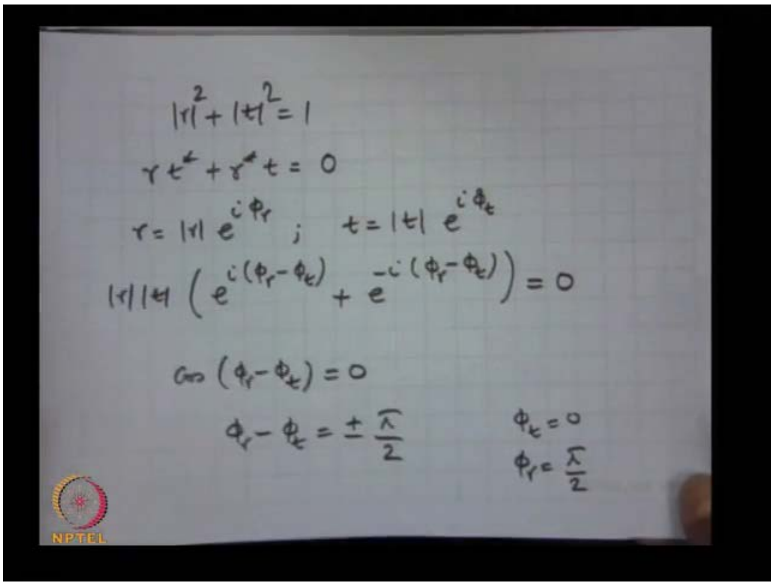

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Mod r and mod t are not 0 need not be 0. So, this tells me cosine of, though this means phi r minus phi t must be equal to, this is symmetric situation. If you do not have a symmetric situation, you get a different combination of phases of r r prime t t prime etcetera, you will have another equation there. But here I get and this is in general, I have not assumed anything. This is actually a general 4 port device. 2 input port, 2 output ports the phases of the reflected in the transmitted components must be different by phi by 2, if it is a symmetric device.

So, for example, if my beam splitter was a 3 d 50 percent beam splitter, which means it reflects 50 percent of the light and transmits 50 percent of the light, so, what I can do is I choose for example, phi t is equal to 0 and phi r is equal to pi by 2 so as to satisfy this condition. So, t is real and r is imaginary, phi r is pi by 2. So, if I have a 50 percent beam splitter, I have r is equal to what is the magnitude of r, 1 by root 2 and there is an exponential I phi r so, this is I by root 2 and t is equal to, this is for a 50 percent beam splitter.

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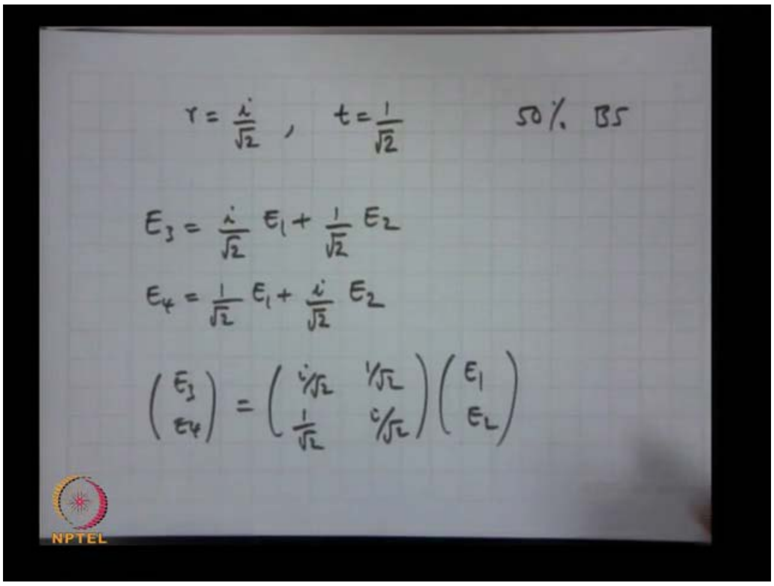

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The phase difference is independent of whether you are choosing 50 percent beam splitter or 80 percent beam splitter, it is always pi by 2 in this symmetric situation. So, if I go back and write the earlier equations, I find E3 is equal to I by root 2 E2 plus 1 by root 2 E2 and E4 is equal to 1 by root 2 E1 plus I by root 2 E2.

Now, when we do phi t equal to 0 pi I equal to pi by 2 for all you're given, if you have the matching generated to it? Yeah, but symmetric what I am saying is, I have assumed r is equal to r prime is equal to t t prime, if you do not have symmetry for example, if I have glass air interface and interface is not symmetric situation. So, the reflection coefficient from here and transmission coefficient complex, I am talking about complex reflection and transmission coefficient could be different. So, I have to work it out and it is very simply. It is just the same analysis except that I need to ensure that when I write this equation, I do not write I write r and t and t prime and r prime here, that is all, and I do the same analysis and you can find out the relationship between the phases of the various coefficients.

So, in fact, this is a in a matrix form E3 E4 is equal to I by root 2 1 by root 2 1 by root 2 and I by root 2. So, these are classical relations, you know quantum mechanics in this these are completely classical equations. Now, use the following procedure to understand the property of the interferometer in the quantum terms. So, these electric fields now become operators and I will for example, I have a relationship between the annulation operators of the mode coming out at 3 and the modes 1 and 2, similarly the annulation operators for 4 1 and 2. So, by quantization I would say, that the annulation operator a 3 corresponding to the field coming out in the port 3 is actually I by root 2 a 1 plus 1 by root 2 a 2 and a 4 is 1 by root 2 a 1 plus I by root 2 a 2. A quantum mechanical operation of the beam splitter is written in terms of the annulation operators of the fields coming out in 3 and 4 ports, their relationship with the annulation operators on the fields coming in 1 and 2.

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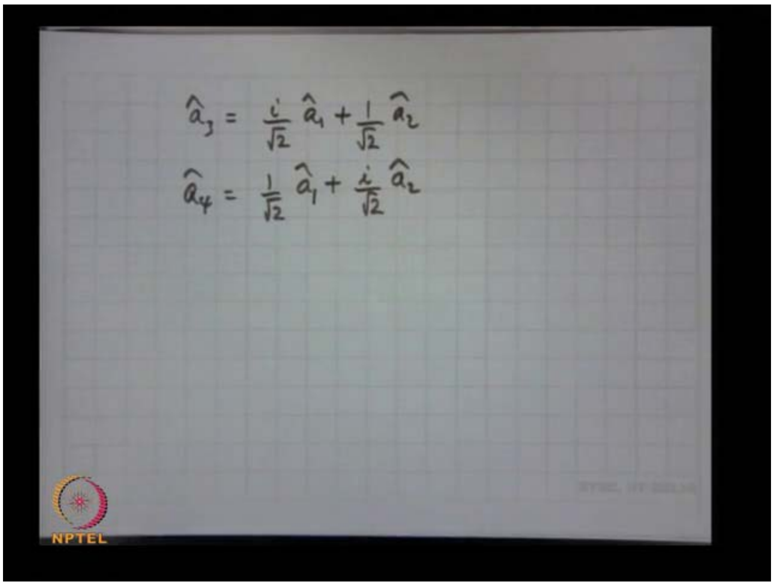

Sir, why is it the same? I am just using, the sort of proposing here, that I will quantize this by this operation. And I will show you that a 3 and a 4 satisfy the commutation relations that are required for a 3 and a 4. a 3 must satisfy a 3 a 3 dagger must be 1 and a 4 a 4 dagger must be 1. That is also satisfied from this equation. So, actually what I am trying to do is, I am just replacing, which I expected to be all right, replacing the classical electric field operator by quantum electric field operators, classical electrical fields by quantum electric field operators. Electric fields are replaced by operators.

Some negation condition might be satisfied by some other combination for a 3 and a 4. No, once I, this is the quantum equivalent of this beam splitter, which has this classical relationships. Of with quantum expression, which expression this expression I have got classical. This is classical I replace the electric fields by the corresponding electric field operators.

I can see that but. I do in, but why isn't it exactly that. Actually I must in a in a completely correct picture I must represent the beam splitter by a unitary operator and work out its effect on the input wave function or the input ket state that is coming in, that is much more complex, a complex analysis has to be done, but this gives me the same result. I am not showing you, I am not proving that this is right, but I am actually expanding these classical equations into corresponding quantum equations and this quantum equations will predict the results of all measurements that I do with is beam splitter for example. This is where I have actually gone from a classical picture of beam splitter to quantum picture of beam splitter.

Sir, but if we in that expression for e 3 and e 4, that we write classically, if we replace the quantum operators, the operators were e 3 and e 4? No, assume that this is like postulate, I have actually postulated here in this in this here, but normally, I would have to understand the quantum mechanical operation beam splitter by taking it as a operator operating on input ket state etcetera etcetera. That's much more complex, but this is a procedure which I employ and I find everything is consistent. For example, I will tell you, suppose I had not taken the second port into account and I had said that let me look at the following problem, I have a wave incident on beam splitter, E1 is incident, generates E2 and E3, E3 and E4, I would have got E 3 is equal to r E1. So, I would have replaced this by. Now, if I calculate the commutation relation a 3 a 3 dagger is equal to r a 1 r star a 1 dagger, which is equal to mod r square a 1 a 1 dagger, it is a mod r square., that is not 1, it is not a correct relationship. So, this why I said I cannot neglect this port.

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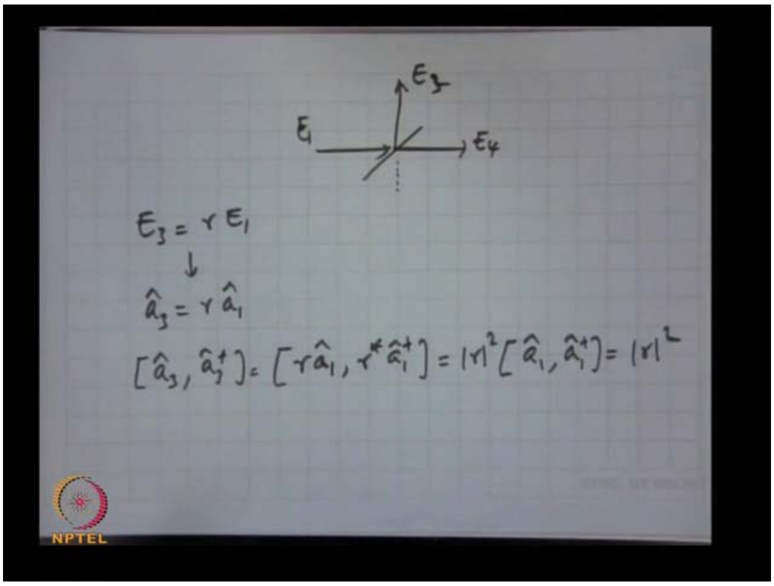

But it is, see, I think just writing the unity replace the x for the electric field operator poised at x we still have a dagger into b r dagger I will have different k for all the 4 inputs. So, it is not intended for drag correctly, you know, replace the electric field by the operator, exact operator that we derived? Yes . Then it is not introduced still to write in the previous expression.

No, what I am trying to say is, I am giving you a procedure, let me assume, that I give you a procedure, to get a quantum equations of the beam splitter from the classical equations. This is done in many other areas where I get the classical equations and I replace the electric the variables by corresponding quantum mechanical operators and I get a relationship and of course, the final thing is whether this is correct or not will be predicted by whether will this equation predict correct experimental observations. There must be surly more complicated procedures of getting these equations from by analyzing the beam splitter as a quantum mechanical operator etcetera etcetera. But I think it is much more complicated and finally, it seems the result is this is the right result.

So, let me assume that this is the corresponding operator equations representing, connecting the operators of the transmitted and reflected fields to the operator in the input fields 1 and 2 and the input 1 and 2. In fact, I will do a similar thing when I come to parametric down conversion. Remember in parametric down conversion we had, did a obtained an equation relating the electric field at the output with the electric fields at the input of the signal and idler. I can go into the quantum mechanical picture by replacing those electric fields by annulation of creational operator.

And I will show you that is the equation I get there is the equation which is feasible. Although this is not a very rigorous procedure, I am not following a rigorous procedure of deriving these equations, but because of these classical equations, I am sort of applying a procedure I should say, which is, which seems to be correct, that I replace these electrical operators by the corresponding annulation of electric fields by the corresponding annulation operators and get a relationship between the annulation operators in ports 3 and 4 with respect to ports 1 and 2.

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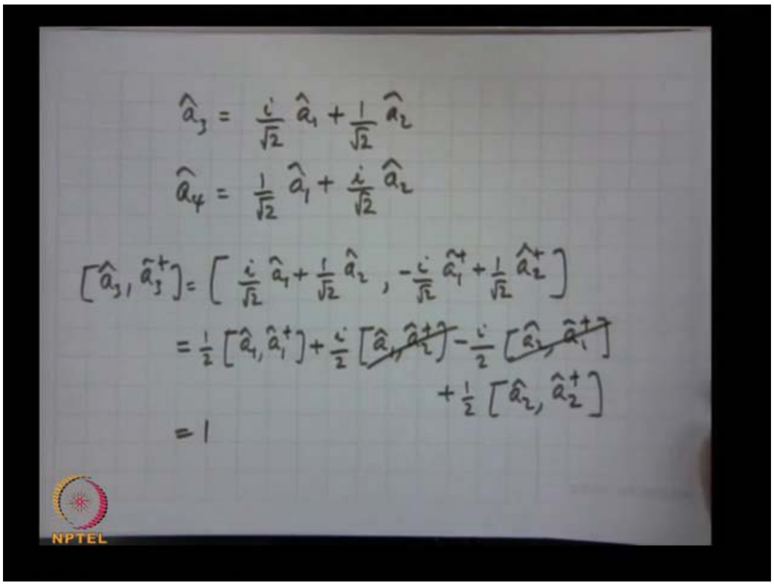

Now, you can check for example, here a 3 a 3 dagger is I by root 1 a 1 plus 1 by root 2 a 2 minus I by root 2 a 1 dagger plus 1 by root 2 a 2 dagger. So, this is equal to half a 1 a 1 dagger plus I by 2 a 1 a 2 dagger minus I by 2 a 2 a 1 dagger plus half a 2 a 2 dagger. Now, please note the inputs 1 and 2 are completely independent of each other. So, the annulation operator a 1 and a 2 commute. This is 0, this is 0, this is equal to 1, this is equal to 1 and I get a 3 a 3 dagger is equal to 1.

Similarly, you can show a 4 a 4 dagger is 1. If I neglected the second port at the input, as I showed you, I am not able to satisfy the commutation relation at the output annulation operator and that is not a correct transformation. So, let me look at one example of this operator equations and that is an example where I assume a single photon is incident on a beam splitter from port 1. So, this is a 1 photon state and here it is vacuum. So, I want to find out what comes out of this beam splitter.

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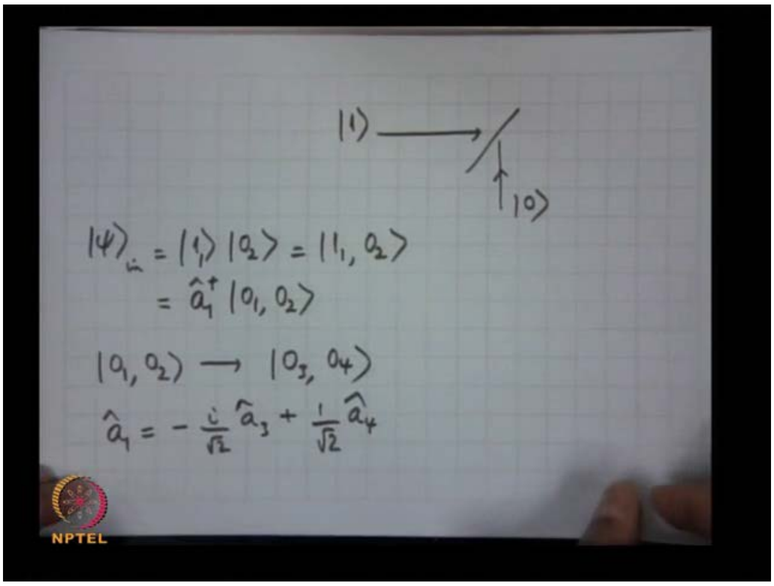

The input psi 11, which is actually, and this is actually a 1 dagger, because 11 is a 1 dagger 01. Now, as far as the states are concerned, I know that if there is vacuum incident in 1 and vacuum incident in 2, I will get vacuum coming out of 3 and vacuum coming out of 4. So, this beam splitter transforms this state to an output which is both vacuum and the beam splitter transforms a 1 in terms of a 3 and a 4. So, I have to invert this equation and calculate a 1 in terms of a 3 and a 4. So, if I multiply this, I can show that a 1 is equal to, I get a 1, I multiply this by I invert this equation. So, let me give you the inverted equation here. Minus I by root 2 a 3 plus 1 by root 2 a 4. Estimated we find out a 2 from here in terms of a 3 and a 4. So, a 1 depends on a 3 and a 4 and so, if this is the input state 0102 goes to 0304 a 1 dagger gets replaced by 3 and 4 operators and the output state becomes minus I by root 2 a 3. So, it is a dagger. So, plus I by root 2 a 3 dagger plus one by root 2 a 4 dagger operating on 0304. So, I replaced a 1 dagger by the corresponding operators in terms of a 3 and a 4 and I have replaced the 0102 state by 0304.

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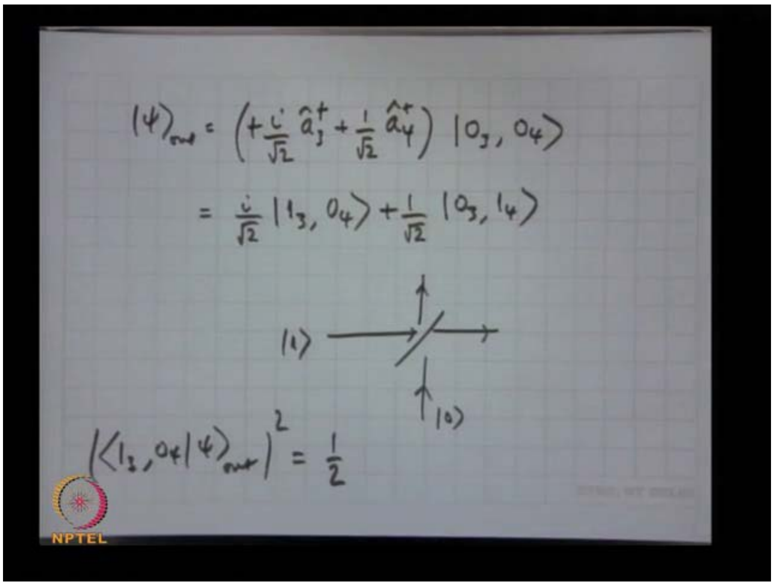

So, what is this? This is I by root 2 a 3 dagger on this. 1304 plus 1 by root 2 a 4 dagger on this gives me a 3 dagger operates on 03 it gives me 13, A 4 dagger operates on 04 to give me 04. What is the state? This is a superposition state which we had discussed earlier what it implies is the output state is a superposition of the photon being present in port 3. So, if I have a single photon coming from here, you generate a superposition state of 3 and 4, where the photon could be either here or here, actually in both, which probability is half, because of this 1 by root 2 amplitudes here. The probability of detecting 11304 is, which is 1 by 2 and similarly the probability of detecting 0314 is also half.

So, the single photon now goes in to a superposition state of being in this part of the beam splitter and in this part of the beam splitter. It will continue to remain in the superposition state unless it is disturbed and please note that I can go on for a billion miles and still be in a superposition state of both arms and if you were to detect, put a detector on one of the arms, you will suddenly detect it or not detect it.

Now, look at a wave packet, see this is a single photon state, but single photons can be in a wave packet. So, as if problem is, it is as if the photon is in a superposition state in a both arms, but suddenly collapses. This is a problem in quantum mechanics. There is a collapse of the wave function. A measurement collapses in the wave function and that collapse has no space variable in this picture. So, it is not that the beam splitter is either reflecting or transmitting the photon. It is reflecting and transmitting. It is in both arms. In fact, what I will do in next class is to build a mach zehnder interferometer from here and what you will find is that there is an inter phase effect.

The single photon enter this beam splitter the mach zehnder interferometer and interferes with itself, the probability amplitudes actually are interfering. It will not be the part or the particle is going here, part of the particle going here. The probability amplitudes of the both the arms are interfering to produce a probability of being detected at the output in either of the arms. I think we will we will stop here. Any questions?

Thank you.