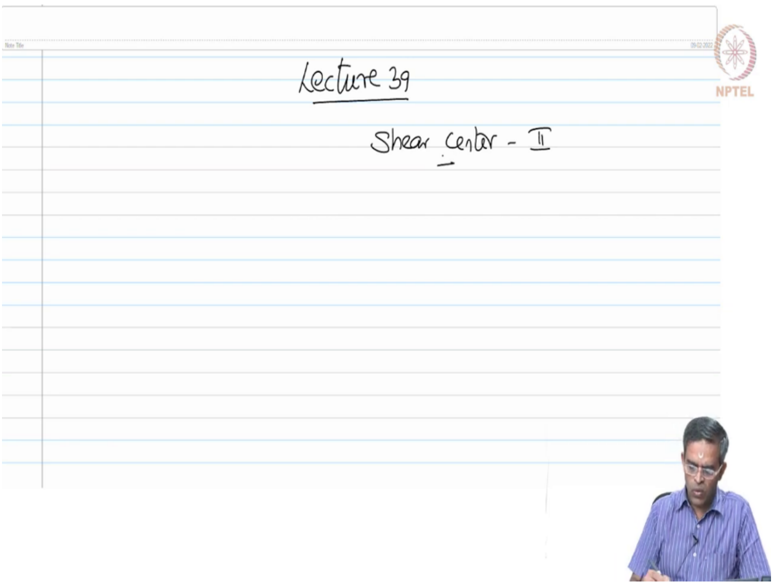
Advanced Design of Steel Structures
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 39
Shear center - 2

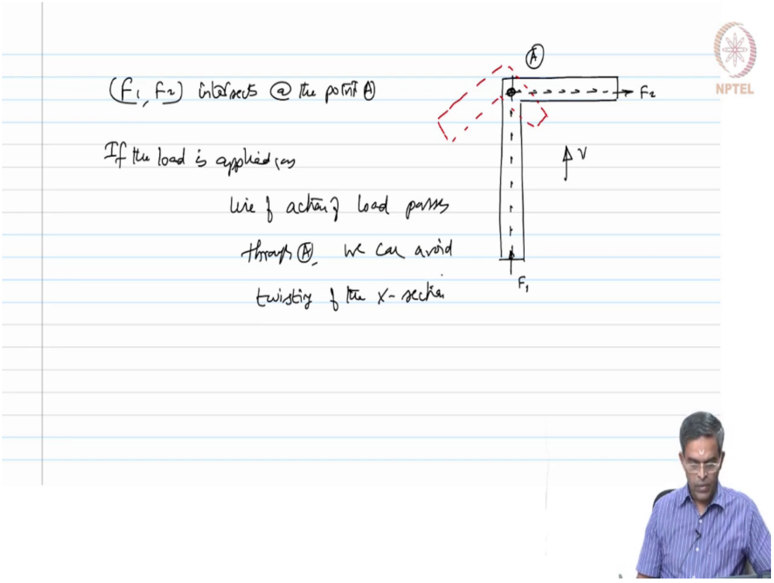
(Refer Slide Time: 00:23)



Friends, welcome to the lecture-39 on Advanced Steel Design course. We are discussing about the Shear center. So, we will call this lecture as Shear center - 2 because in the last lecture we discussed about the conditions under which twisting of cross-section should happen and we stated clearly that if the load is applied or pass through the shear center you can avoid twisting of the cross section.

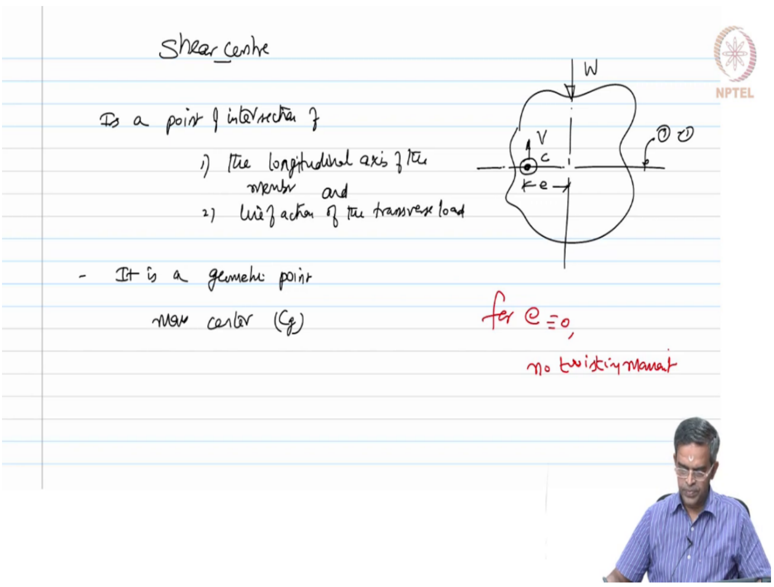
Therefore, friend shear center is a geometric point in the cross-section which is actually the intersection of the loading plane with the bending plane, this is what we said.

(Refer Slide Time: 01:19)



Furthermore, if I have an angle section as shown in the screen now we already know that the internal force distribution will be as F_2 and F_1. If you apply a force V and F_1 and F_2 intersect at the point A. So, one can say that F_1, F_2 intersects at the point A, if the load is applied or the line of action of load passes through the point A, then one can avoid twisting of the cross-section.

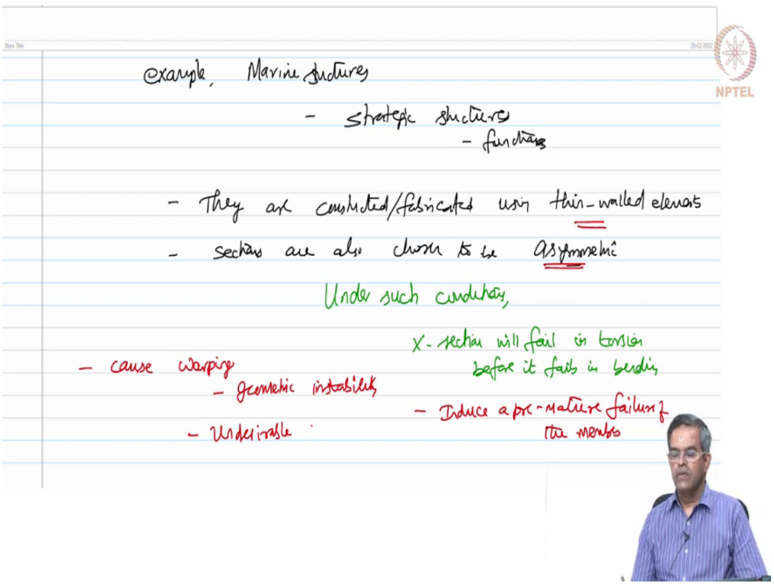
(Refer Slide Time: 03:16)



We already said that shear center is the intersection of the bending plane and the loading plane. We can also define shear centre in a more geometric sense. Let us say I have a cross-section of some shape this is my point of action w and this is the shear center offset e from the Cg and we have applied a load V. Therefore friends, shear center is a point of intersection of the longitudinal axis of the member that is this axis and the line of action of the transverse load which is point C.

So, therefore friends, shear center is a geometric point whereas, mass center is identified as C g. If the load does not pass through the shear center, then the section will be subjected to twisting moment. So, we can say straight away here for e equals 0; no twisting moment.

(Refer Slide Time: 05:38)

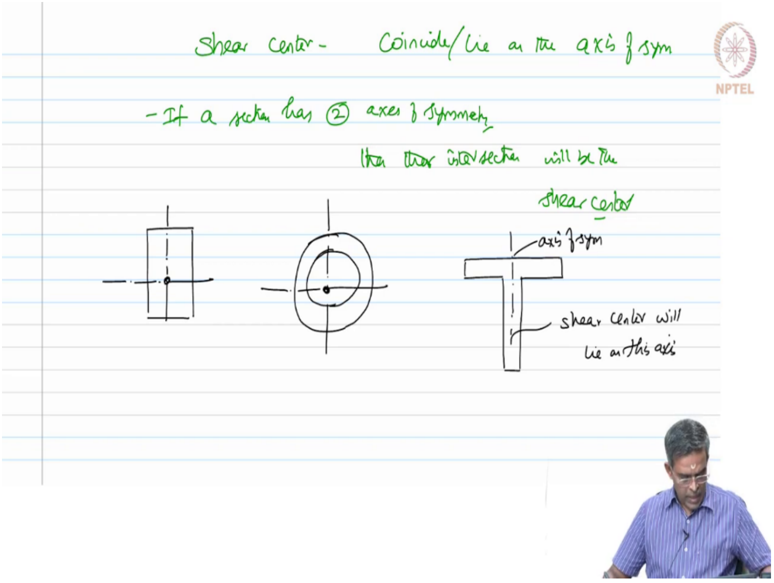


Now, let us quickly see where are shear center very important in what kind of structures. Let us take for example, marine structures. We already know marine structures are strategic structures constructed for specific functions. Usually, they are constructed or I should say fabricated because we use essentially steel as a material using thin-walled elements and sections are also chosen to be asymmetric.

Therefore, under such conditions the cross-section will fail in torsion because it will be weak in torsion before it fails in bending. So, this twisting will induce a premature failure of the members. So, that is a very important status. So, therefore, shear center or location of shear center are important in such kind of structures where they are thin-walled elements and the sections are as symmetric in nature.

Furthermore, this will also cause warping which will induce geometric instability and which is one of the most undesirable characteristics in design.

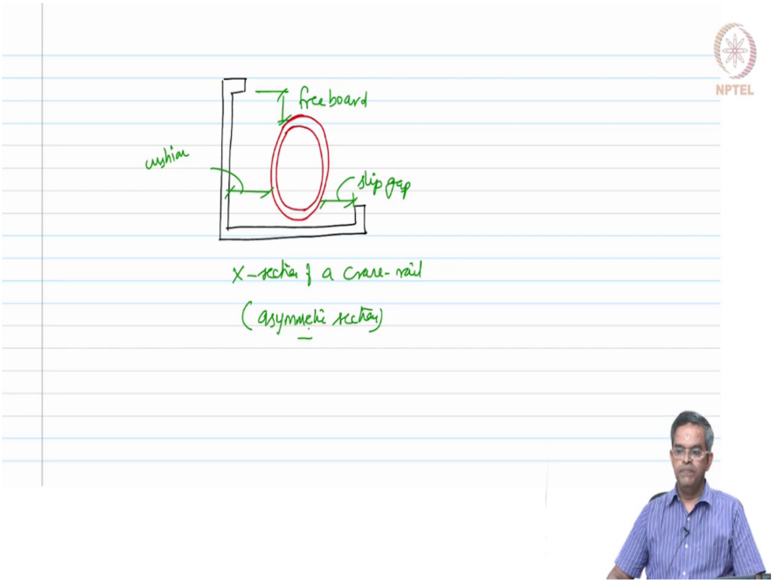
(Refer Slide Time: 08:34)



Friends, shear center location is made comfortable by a simple logic. Shear center will coincide or will lie on the axis of symmetry. If a section has two axis of symmetry, then their intersection will be the shear center. So, sometimes in most of the sections shear center can be located by simple observation. We can show some examples. Let us say rectangular section where there are two acts of symmetry.

So, this becomes my shear center, here tubular section which is commonly used in marine structures where there are at least 2x of symmetry that becomes the shear center. If I have a T-section which has one axis of symmetry, shear center will lie on this axis.

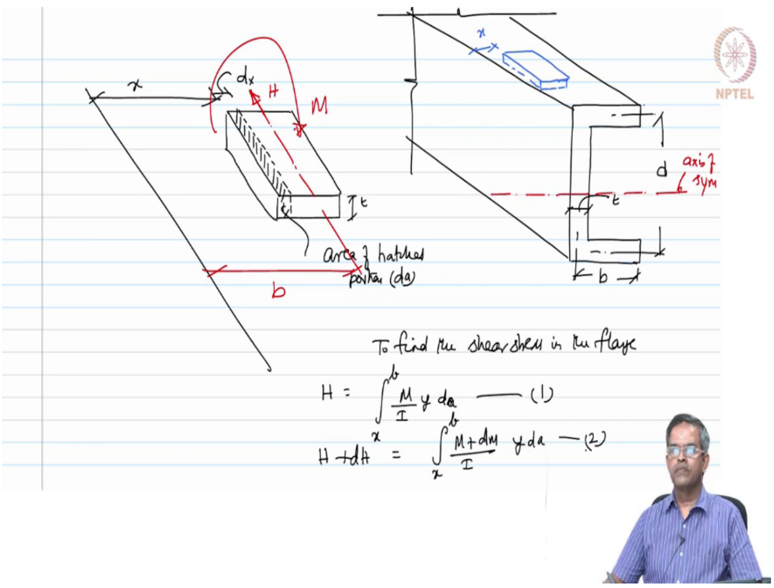
(Refer Slide Time: 10:41)



Now, the question comes where are asymmetric sections popular? You must have seen in crane rail. Crane rail has a section like this. This becomes my crane rail. This will have a cushion effect here and this is what we call as a slip gap, now this is the free board. So, if the rail jumps of the crane rail bends still the wheel should not touch the top. This is a typical cross-section of a crane rail. This is a classical example of an asymmetric section.

So, in heavy industrial structures friends, you will see that asymmetric sections are very commonly used as load carrying elements which are also crucial elements in the whole setup.

(Refer Slide Time: 12:20)



Having understood this, let us try to derive the basic equation for finding the shear stress. Let us say we have a flange of a channel section. Now, depth d and breadth b and thickness t we know that this section has an axis of symmetry. Shear center has to lie on this axis. Let us imagine this along its length at a distance x, let us cut the strip let us say at a distance x let us cut a strip.

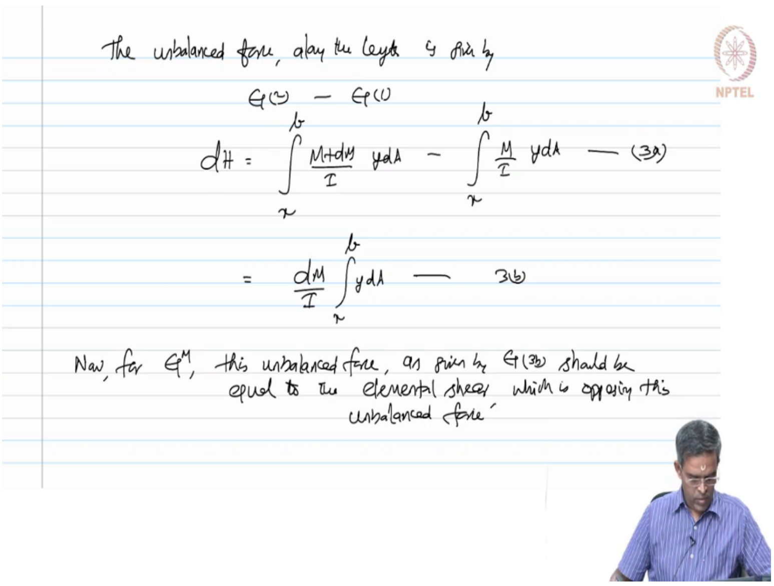
$$H = \int_{x}^{b} \frac{M}{I} y \, da$$

$$H + dH = \int_{x}^{b} \frac{M+dM}{I} y \, da$$

So, I am drawing that strip in enlarged view here. Let us say this is my plane of reference thickness be t. Let it be cut at a distance x and a strip is being cut of thickness dx. An area of this hatched portion is da, and let us say we have an axis which is measured as b distance from there and along this axis we have H and we have M acting in the cross-section.

Now, I want to find the shear stress in the flange. Now, H will be given by M by I into y of dx sorry da x to b equation number 1. H plus dH will be M plus dM by I into y da which will also run from x to b equation, number 2 equations from first principles.

(Refer Slide Time: 16:51)



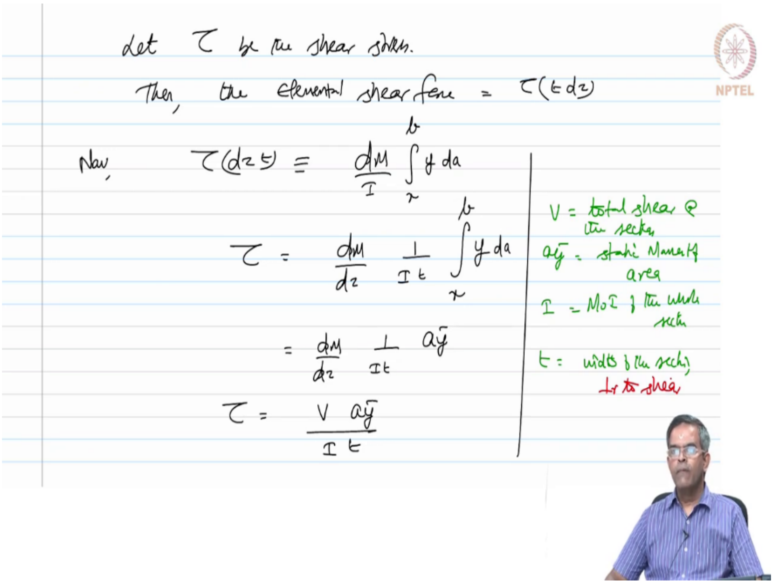
$$dH = \int_{x}^{b} \frac{M+dM}{I} y \, dA - \int_{x}^{b} \frac{M}{I} y \, dA$$

$$dH = \frac{dM}{I}\int_{x}^{b} y\, dA$$

Now, I want to find the unbalanced force along the length is given by equation 2 minus equation 1 which will be dH which will be x to b M plus d M by I y dA minus integral x to b M by I into y dA. We calls equation number 3. Now, I can say it is dM by I into x b y dA equation number 3 b we call this 3 a.

Now, for equilibrium, this unbalance force as given by equation 3 b should be equal to the elemental shear which is opposing this force.

(Refer Slide Time: 18:51)



$$\tau(dz\, t) = \frac{dM}{I}\int_{x}^{b} y\, da$$

$$\tau = \frac{dM}{dZ}\frac{1}{It}\int_{x}^{b} y\, da$$

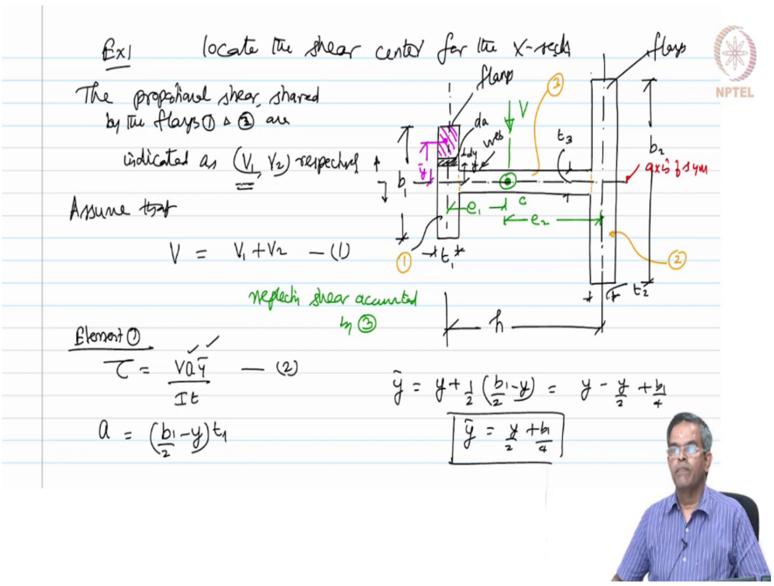
$$\tau = \frac{dM}{dZ}\frac{1}{It}\, a\overline{y}$$

$$\tau = \frac{V a\overline{y}}{It}$$

Let tau be the shear stress. Then, the elemental shear force will be tau times of t dz. So, now for equilibrium this should be equated dM by I integral x to b y da. So, now, I can say the shear stress is going to be dm by dz 1 by I t integral x to b y da which can be dm by dz 1 by It a y bar because y da integral is a y bar whereas, tau is now going to be V a y bar by I t, where V is a total shear at the section.

ay bar is a static moment of area; I is the moment of inertia of the whole section and t is the width of the section perpendicular to shear. Having said this, let us try to do a problem to locate the shear center for a section.

(Refer Slide Time: 21:17)



So, we will say example one locate the shear center for the given cross-section. Let us draw the cross-section. Let us mark the geometric dimensions of this. Let us mark the centerline of this, call this dimension as h, this thickness t_1 and this thickness t_2 and this breadth as b_2 and this breadth as b_1.

We also know that this line is automatically the axis of symmetry. Let us assume that the shear center is located here, where the external load V is acting to avoid twisting of this cross-section. We call this distance of offset as e_1 and e_2. So, our job is to find either e_1 or e_2 because they are connected by relationship e_1 plus e_2 will be equal to h. Let us take this thickness of the section as t.

So, this is my 1st element, this is my 2nd element and this is my 3rd element where the intersection is cut here. Let us say the proportional shear shared by the flanges 1 and 2 are indicators. By the way we know that they are flanges and this is called as a web or indicated as V_1 and V_2 respectively. So, we assume that the total shear V is counterbalanced by these two shear.

So, we are neglecting shear accounted by the element 3. So, now, let us cut a section at a distance y of thickness dy and area shaded is da and this is the part above the section under consideration. This part has a C g let us say that is y bar from here from the axis of symmetry or from the line where the shear center is being located.

Now, tau is V A y bar by It. This known equation we just now derived, let us put it a small a. So, the small a for the component 1, this is for element 1, component a is going to be b_1 by 2 minus y that is the hatched portion in neon color b_1 by 2 minus y into t_1. Let us say y bar is y plus b_1 by 2 minus y half of that which will be y minus y by 2 plus b 1 by 4. So, I can now say y bar is y by 2 plus b_1 by 4 because we need y bar here we need a here.

$$\tau = \frac{Va\overline{y}}{Ib} = \frac{Wa\overline{y}}{It_1}$$

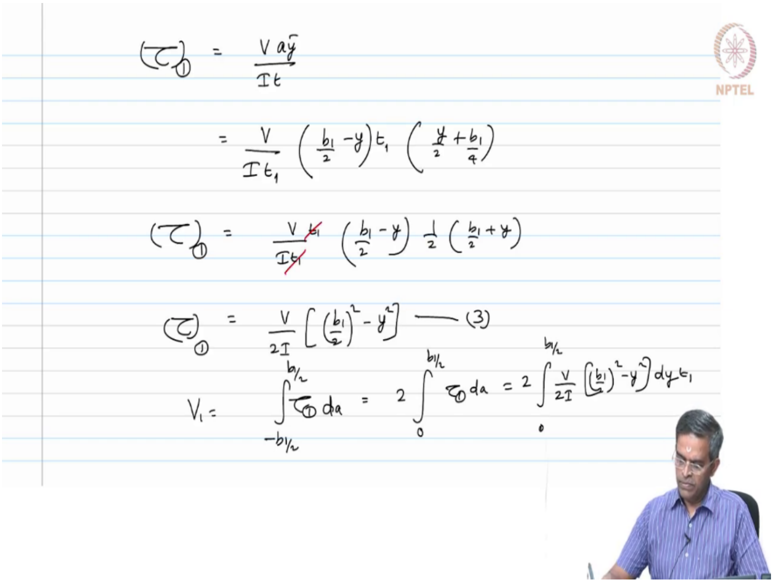
$$A = \left(\frac{b_1}{2} - y\right)t_1$$

$$\overline{y} = y + \frac{1}{2}\left(\frac{b_1}{2} - y\right) = \frac{b_1}{4} + \frac{y}{2}$$

$$\tau = \frac{W}{It_1}\left[\frac{b_1}{2} - y\right]t_1\left[\frac{y}{2} + \frac{b_1}{4}\right]$$

$$\tau = \frac{W}{2I}\left(\frac{b_1^2}{4} - y^2\right)$$

(Refer Slide Time: 28:31)



Now, I can substitute tau of number 1 is V ay bar by I t which is V by I t_1; ay bar is just now we computed which is b_1 by 2 minus y of t_1 and y bar is y by 2 plus b_1 by 4. Let us simplify this tau of number 1 will be V by I t_1 of t_1 b_1 by 2 minus y half b_1 by 2 plus y, can I write like this?

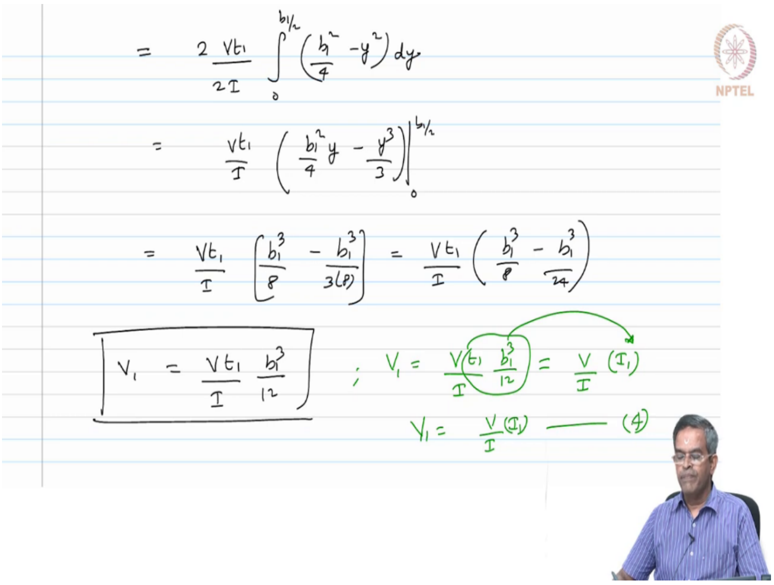
Because I get a product here a minus b and a plus b so, which can be V by 2I as t_1 goes away 2 I of b_1 by 2 b_1 by 2 a square minus b square equation 3. This is my tau of the first number. So, therefore, I can write V_1 is integration of this of the whole area varying from minus b_1 by 2 to plus b_1 by 2, see this figure minus b_1 by 2 to plus b_1 by 2 that is from this section here from this section here.

So, that is the validity of the shear force V_1 which is shared by the flange 1. Let us substitute and find out this value which will be 2 times of 0 to b_1 by 2 tau of 1 d a because of symmetry. So, which will be 2 times of integration 0 to b_1 by 2 tau 1 we already have.

So, V by 2I b_1 by 2 the whole square minus y square of dy of t_1, That is the strip area which is integrated from minus to plus limits of this, That is da.

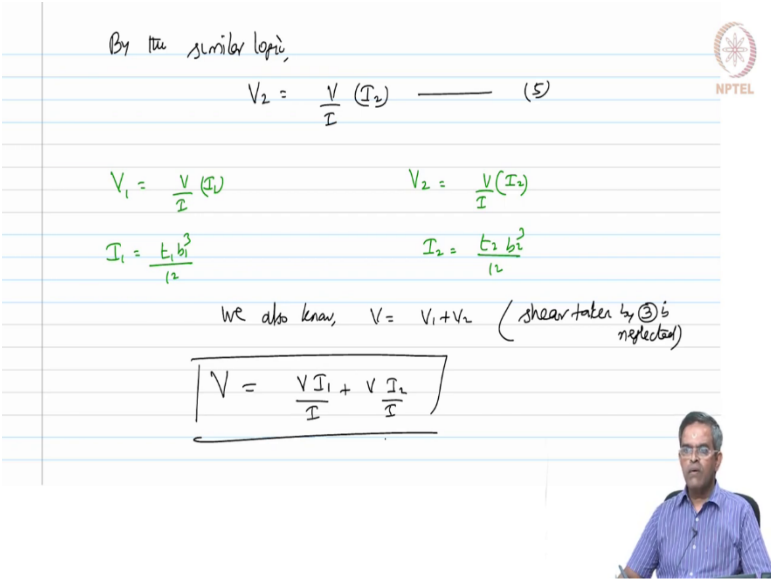
$$
\begin{aligned}
V_1 &= \int \tau \, da \\
&= \frac{W}{2I} \int_{-b_1/2}^{b_1/2} \left(\frac{b_1^2}{4} - y^2 \right) t_1 \, dy \\
&= \frac{W}{2I} \int_{-b_1/2}^{b_1/2} \left(\frac{b_1^2}{4} t_1 - y^2 t_1 \right) dy \\
&= \frac{W}{I} t_1 \left(\frac{b_1^3}{12} \right) \\
V_1 &= \frac{W}{I} I_1
\end{aligned}
$$

(Refer Slide Time: 31:40)



So, that is what I am having here. So, I replace this as I_1. So, I can now say V_1 is V by I of I_1 equation 4.

(Refer Slide Time: 34:18)



$$V_1 = \frac{V}{I}(I_1)$$

$$I_1 = \frac{t_1 b_1^3}{12}$$

$$V_2 = \frac{V}{I}(I_2)$$

$$I_2 = \frac{t_2 b_2^3}{12}$$

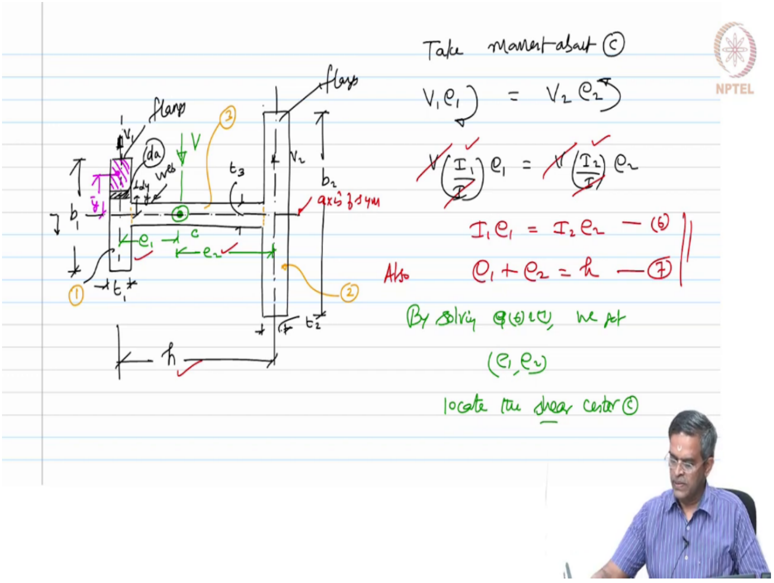
$$V = V_1 + V_2$$

$$V = \frac{VI_1}{I} + \frac{VI_2}{I}$$

By the same logic I can apply this to piece number 2 or flange number 2 I will get by the similar logic V_2 will be V by I of I_2 equation 5. Now, we have two equations V_1 is V by I_1 V_2 is V by I_2, where I_1 is t_1 b_1 cube by 12 and I_2 is t_2 b_2 cube by 12. We also know total shear is sum of these two, shear taken by member 3 is neglected we already stated that. So, the total force shear will be V I_1 by I plus V I_2 by I.

So, that is the control equation we have with us. Now, let us take moment about the point C. See this figure, I will copy this figure.

(Refer Slide Time: 36:12)



$$V_1 e_1 = V_2 e_2$$

$$V\left(\frac{I_1}{I}\right)e_1 = V\left(\frac{I_2}{I}\right)e_2$$

$$I_1 e_1 = I_2 e_2$$

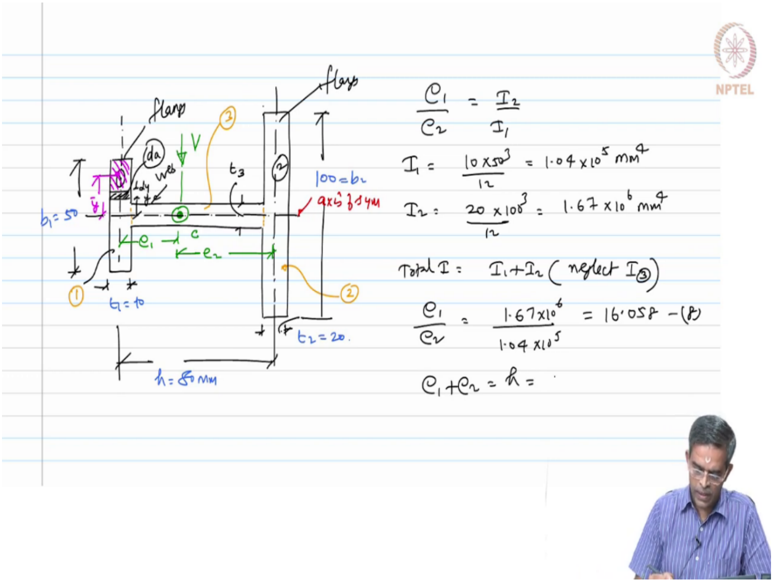
$$e_1 + e_2 = h$$

Now, let us say we will take moment about shear center C. So, V_1 into e_1 which is clockwise this is V_1 this is V_2 should be equal V_2 into e_2 which is anti-clockwise. So, what is V_1? Please see from this equation which is V I_1 by I of e_1 and V_2 see from this equation I_2 by I, V I_2 by I of e_2.

So, from this relationship we get a concept I_1 into e_1 is I_2 into e_2 we get one relationship from here. We call equation number 6 here. Also, we know that from the geometry e_1 plus e_2 is h, see here the solving equation 6 and 7 because we know I_1 and I_2 we already have them here see I_1 is known, I_2 is known. So, I already have I_1 known, I_2 known.

So, there are two unknowns e_1 and e_2, I have two equations. So, by solving equations 6 and 7 we get e_1 and e_2 and I can locate the shear center. That is a very easy approach from first principles. We found out the shear center very easily. Yes friends, it is a very easy illustration for applying the fundamental logic to obtain the shear stresses. Let us apply on to a problem and see this quickly. So, we will say the same dimension, I will just copy this figure again.

(Refer Slide Time: 39:09)



$$\frac{e_1}{e_2} = \frac{I_2}{I_1}$$

$$I_1 = \frac{10\times50^3}{12} = 1.04\times10^5 mm^4$$

$$I_2 = \frac{20\times100^3}{12} = 1.67 \times 10^6 mm^4$$

$$I = I_1 + I_2$$

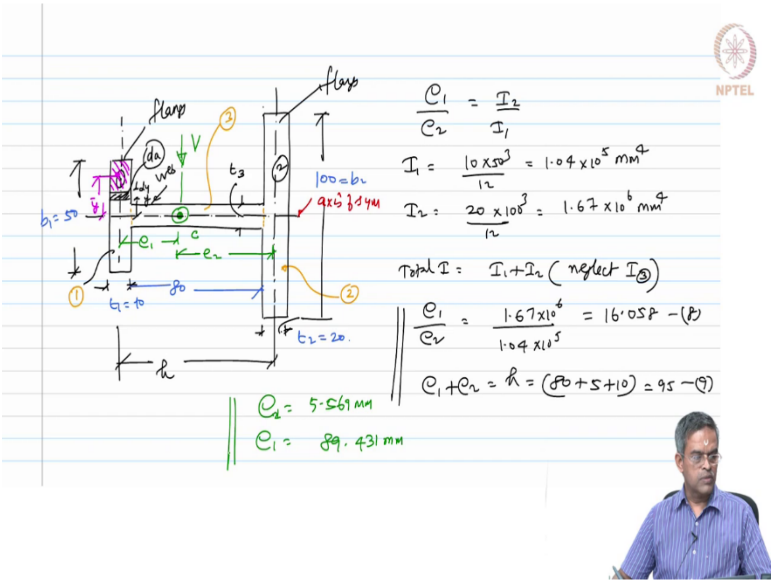
$$\frac{e_1}{e_2} = \frac{1.67\times10^6}{1.04\times10^5} = 16.058$$

Let me give these dimensions as b_2 is 100. I write in a different color maybe 100 which is b_2; b_1 is 50; t_1 is 10; t_2 is 20 and h is 80 all dimension millimeters. So, we have an equation e_1 by e_2 is I_2 by I_1 can you have this equation here. e_1 by e_2 is I_2 by I_1. So, let us compute I 1 for this problem which is going to be 10 into 50 cube by 12 which is 1.04 10 power 5.

Let us compute I_2, this is the second member, this is the first member 20 into 100 cube by 12 which is 1.67 10 power 6. We say the total moment of inertia I is I_1 plus I_2. Let us neglect the

I from member 3 this is member 3. So, let us say e_1 by e_2 is 1.67 10 power 6 by 1.04 10 power 5 which is 16.058, equation let us say 8.

(Refer Slide Time: 41:45)



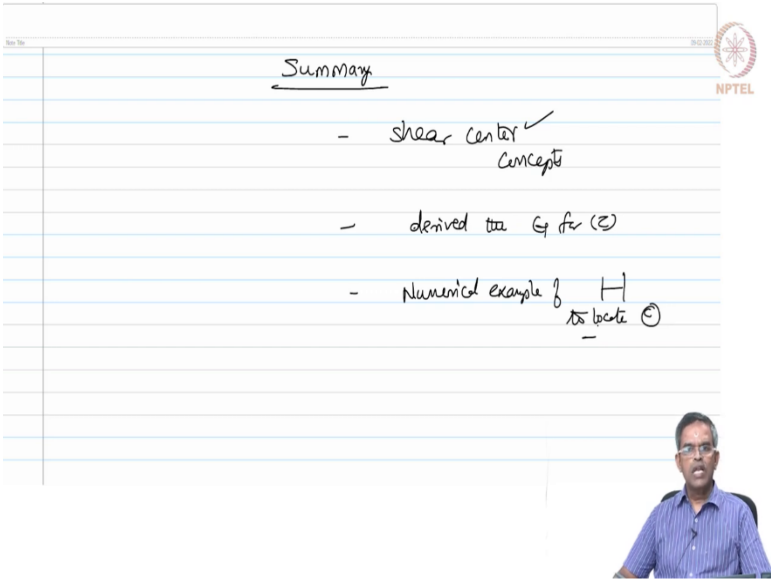
$$e_1 + e_2 = h = (80 + 5 + 10) = 95$$

$$e_2 = 5.569\ mm$$

$$e_1 = 89.431\ mm$$

e_1 plus e_2 is h which is, we will say this is 80 in this figure this is anyway h, but we have this dimension. This dimension the clear dimension is 80. So, h will be now equal to 80 plus 5 plus 10 which is 95 equation 9. So, by solving these two equations I get e 1 as 5.569 millimeters and sorry, e_2 as and e_1 as 89.431 millimeters. This is a simple illustration what we had to solve this problem.

(Refer Slide Time: 42:33)



Friends, in this lecture we learned more about the concepts of shear center, we also did a numerical example. We derived the equation for shear stress, then we did a numerical example of an I section to locate the shear center. We did this using first principles and the derivations are very clear therefore, there is no question or any confusions.

So, friends we will do couple of more examples in the coming lectures to illustrate about the calculation of shear center more in detail. We will also take up shear center for unsymmetrical sections. We will try to elaborate this more on coming lectures.

Thank you very much and have a good, bye.