Fracture, Fatigue and Failure of Materials
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Lecture 43
Fatigue Crack Propagation (Contd.)

Hello everyone, we are at the 43rd lecture of this course Fracture Fatigue and Failure of Materials. And in this lecture also we will be talking some more about fatigue and particularly, we will be focusing on the fatigue crack propagation in notched component. So, let us see, what we have in store for this lecture.

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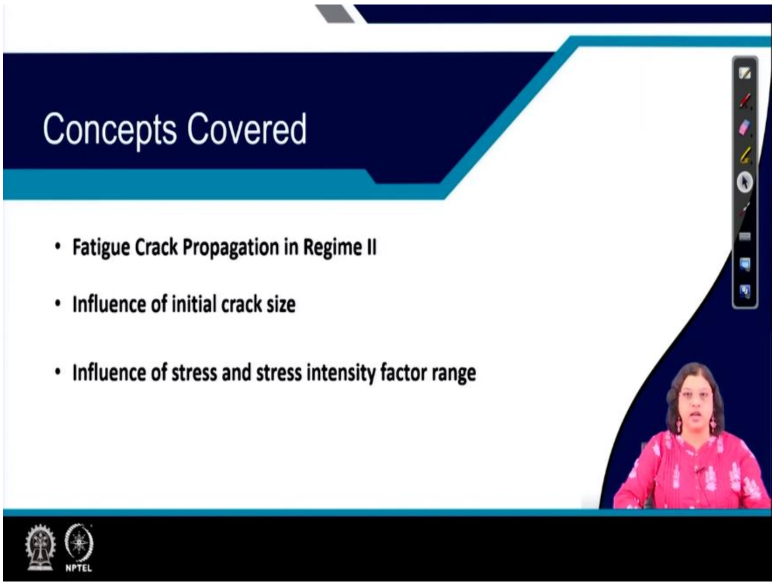

We will be talking about particularly the fatigue crack propagation in regime-II. In the last lecture, we have seen that based on this da/dN versus ΔK curve, there are typically three different regimes or stages through which the fatigue crack propagation materializes, and we will be talking about the middle region or the regime-II which accounts for the major part of the fatigue crack growth rate curve.

Particularly, we will be looking at the influence of the initial crack size and also the influence of the stress, the stress level that are applied and the stress intensity factor range and how they are modifying the total fatigue life or the crack growth rate etc will be discussed.

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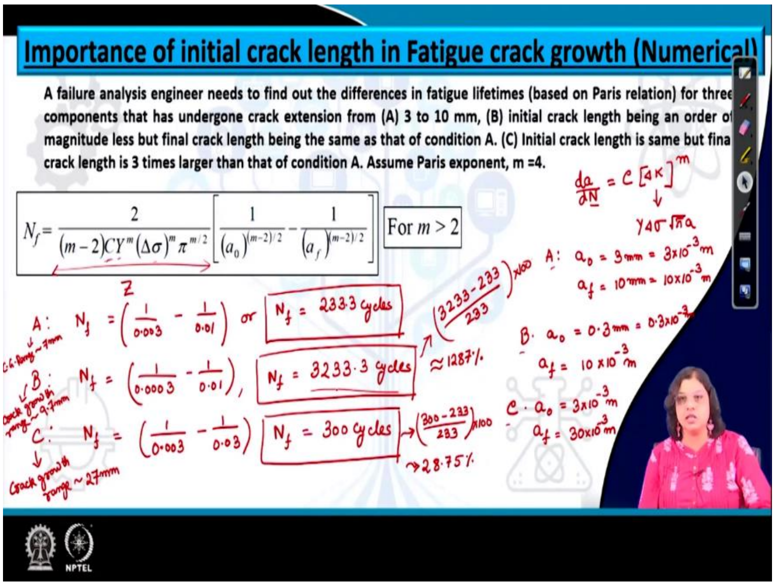

Let us start today's lecture with a numerical at the very first hand. So, what it says is failure analysis engineer needs to find out the differences in fatigue lifetime based on the Paris relation for three components that has undergone crack extension for the three conditions, for the first one, the crack extension is occurring from 3 millimetre to 10 millimetre.

And for the second condition, initial crack length being an order of magnitude less, but final crack length is still the same as that of A and the third condition is when the initial crack length is same as that of A, but the final crack length is three times more than that of A. So, basically what we want to find out here is the number of cycles for three different sets of crack length both the initial and the final lens.

And for the second regime, we have already seen that, that how da/dN or the crack growth rate is typically related to the stress intensity factor range ΔK through the Paris equation, we have seen this that for the Paris relation da/dN or the credit growth rate is equivalent to C which is a constant ΔK stress intensity factor range to the power of m where m is the Paris exponent and for this particular case the value of m is provided as 4.

So, let me give you another hint what we have seen in the last lecture is how to determine the n value from here and there is a simplified approach already there for different values of m for m equals to 2 or m greater than 2 for this case, of course, m is greater than 2 and we can use a relation something like this.

But once again you do not need to memorize this relation, we can simply put the values of the different parameters here like C and ΔK and ΔK can be expanded to $Y\Delta\sigma\sqrt{\pi a}$, and we can figure out this integration condition and we can get the same values, for solving this I just keep this handy because in this case, what we have here are the three conditions.

So, let me just write out the conditions first. In the first case, for condition A what we have is initial crack length is equals to 3 millimetres. So, that is $3 \times 10^{-3}\ m$ and final crack length is 10 millimetre, again which is $10 \times 10^{-3}\ m$ or 10^{-2} m as a whole.

Then we have condition B for which the initial crack length is actually an order of magnitude less so, that means that it is 0.3 millimetre, which is equivalent to $0.3 \times 10^{-3}\ m$ and a_f is still the same. So, it is $10^{-3} \times 10\ m$ and then we have conditioned C, where the initial crack length is same, so, a_0 is same as that of a so, that means, it is 3 millimetre and final crack length in this case is 30 millimetre 3 times larger. So, that means $30 \times 10^{-3}\ m$.

Now, if we look at the span for the crack growth, so, for the first case it is just 3 to 10 which is 7 millimetre the crack has to grow for the case of B it is 0.3 to 10 so, essentially 9.7 millimetre that crack has to grow, for the third case, this pan is the maximum which says 3 to 30 millimetres so, that means 27 millimetre the crack needs to go.

So, obviously at the very first instance we may think that for the condition C since the crack length that is required here is 27 millimetre that is the total length that needs to be grown obviously, that may require a higher number of cycles. So, let us know solve this and find out the answers. Now, for this particular condition since we do not have the factor C here, we can overall consider this as some constant let us say Z considering that these values are same like C or Y or Δσ or π all these values are same for all the three conditions.

So, we can simply consider this particular number or you can use any particular value for your reference to get an idea of this. So, that means for condition A what we have here is apart from Z the number of cycles to failure will be given by something like this $\frac{1}{0.003} - \frac{1}{0.01}$. I am doing this in the shortest way, so that we can do this finish this in time, but I encourage you to please follow all the procedures and you can do this by putting some either some values of C and Y or you can use a constant value such as Z.

So, this comes around N_f actually turns up to 233.3 or 233 cycles. So, this is for condition A on the other hand, if we are talking about condition B then now, we have the initial crack length

10 times smaller so, that means $\frac{1}{0.0003} - \frac{1}{0.01}$ so, this on the other hand turns to 3233.3. Again, this 0.3 cycles can be ignored.

And for the condition C for which case we have seen that the crack growth is for the maximum span here we can see that since the initial crack was of the same length, so, it should be $\frac{1}{0.003}$. But in this case, this is the final crack length is or $\frac{1}{0.03}$ and this however, turns out to only 300 cycles.

So, now this is quite surprising, if you think that for the case of C although crack growth range here is 27 millimetre, but we need only 300 number of cycles on the other hand for the case of B which actually required the maximum number of cycles. And in this case, however, initial crack length is 0.3 and the final crack length is 10.

So, that means that for this particular case crack growth range is actually 9.7 millimetre, so much lesser than that of a condition C, but still we require more than an order of magnitude higher number of cycles for this case. So, obviously there is some parameter or some particular feature that is of particular concern for example, if we consider that A is the beginning where we are having this crack growth range is only 7 millimetre, so basically the list of all the three.

So, crack growth range in this case is 7 millimetre obviously, it took lesser number of cycles also 233. But, if we are decreasing particularly the initial crack length to by one order of magnitude, we see that there is a significant enhancement in the number of cycles actually, if we try to figure out the percentage change here for the case of B with respect to that of A. So, that will be something like this $\frac{3233-233}{233}$.

So, that into 100 will turn to around 1287 percent like that is a huge number just if we are changing the initial crack length and not only changing we are reducing the initial crack length on the other hand, for the case of C when here also the crack range is quite higher and we have increased the final crack length instead of the initial crack length.

Now, we are considering the final crack length but, in this case, the number of cycles enhancement is not that significant and this should be $\frac{300-233}{233} \times 100$ so, this percent enhancement is something like around 28.75 %.

So, not so, significant obviously, that means the major number of cycles for this case for the condition B out of these 3233 number of cycles 233 will be used up for growing the crack out of the 3233 number of cycles, 233 will be used up for growing the crack from 3 millimetre to 10 millimetre, but the rest 3000 cycles will be used up just for growing the crack from 0.3 to 3 millimetre.

So, major part or the most share of this number of cycles would be used up just for having the crack of certain particular length. So, lesser is the initial crack length longer and longer number of cycles the component can survive. So, that gives us another lesson that if we cannot avoid the presence of the defect at least we can try to make this as smaller as possible.

Because smaller defects will require much number of cycles just to grow to a particular length and afterwards more is the length of the craft lesser and lesser number of cycles would be required we have already seen that the crack growth rate is directly proportional to the crack length.

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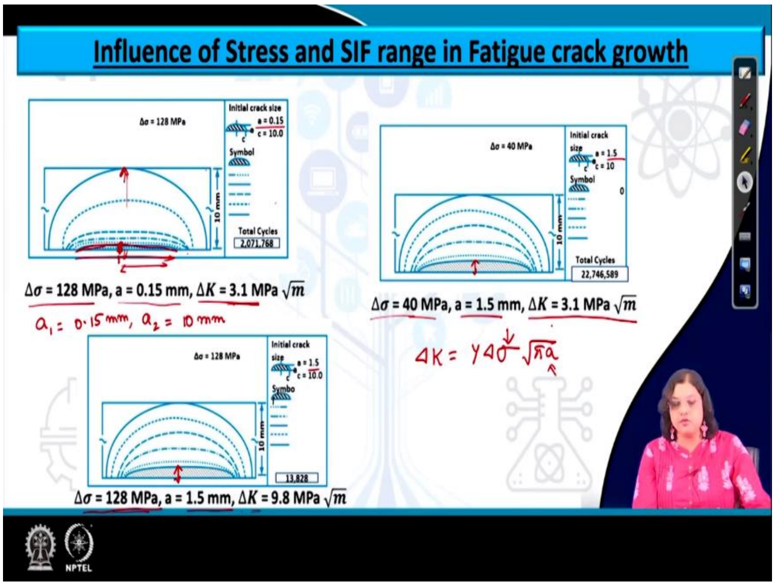

Now, this is one part and the next thing that we want to see here is the influence of stress as well as the stress intensity factor range in fatigue crack growth and there is a problem which already explains this in the sense that there was a crack initial crack something like this, which has this a what we can see here is that for the case of having a surface defect here, the depth of the thickness of the crack is 0.15 millimetre.

And the overall thickness of this component is 10 millimetres. So, that means that the crack length grow up to 10 millimetres, so that it can achieve this leak before break kind of condition and that is the critical value the crack and grow in such case if we are applying a $\Delta\sigma$ or the stress range as 128 MPa we can see that it requires around 2×10^6 number a number of cycles for the crack to grow from 0.15 millimetre up to 10 millimetre.

So, a_1 in this case is 0.15 millimetre and a_2 is 10 millimetre, on the other hand the same situation, but if we have now the crack length or the crack depth being actually one order of magnitude higher. So, now we have the a as 1.5 millimetre and we are still applying the same value of $\Delta\sigma$ of 128 MPa what we can see here is that the number of cycles that is required for the crack to grow from 1.5 millimetre to 10 millimetre is only 13,000 cycles 13,828 to be precise.

But of course, much lesser than this one this condition here when the crack initial crack length was very very small or one order of magnitude smaller, we have already seen this earlier also. But, now, let us bring another twist to it in the sense that we have the third condition when the crack length initial crack length is same as that of the second condition.

That is 1.5 millimetre and the final crack length that it is allowed to grow is still 10 millimetre, but in this case, instead of applying the same stress range of 128 MPa we are applying the same stress intensity factor level as that of the first condition. So, in this case, for the condition one, we were applying $\Delta\sigma$ as 128 MPa.

And for this particular crack length of 0.15 millimetre that leads to a stress intensity factor range of only 3.1 $MPa\sqrt{m}$. So, for the condition 3, we have now the crack length initial length being one order of magnitude higher, but in this case the stress intensity factor range is being maintained as that of condition 1 and now, what we are seeing is that, this actually survived for the longest number of cycles and this is 2×10^7 number of cycles actually.

So, quite a higher number of cycles. And the reason for that is if we are applying the stress intensity factor range of 3.1 MPa that is same as this value, but since the crack length in this case is much higher that turns the $\Delta\sigma$ to a much lesser value, because we have known this that ΔK is directly proportional to both $\Delta\sigma$ as well as a or the crack length.

So, in this case, since crack length is increasing that means, Δσ is reducing and if we are applying lesser value of Δσ certainly the crack can requires much larger number of cycles to survive. So, this can be also solved with a numerical to get a more detailed understanding.

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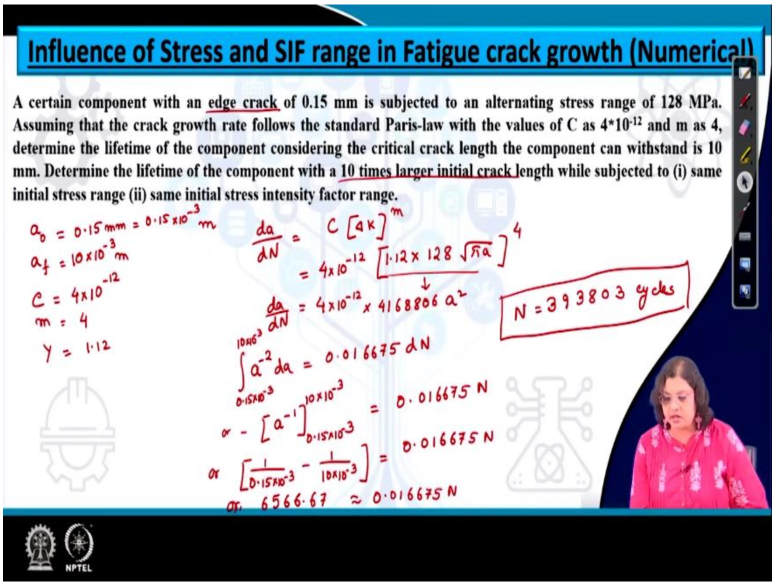

So, what it says here is that a certain component has an edge crack of 0.15 millimetre and that is subjected to an alternating stress range of 120 MPa the same value of stress strain. And assuming that the crack growth rate follows a standard Paris law, the values of C and m are provided here and what we need to find out is the lifetime of the component considering the critical crack length that it can grow is still 10 millimetre.

So, once again almost the same thing here, but, what we need to find out here is a_0 is 0.15 millimetre and af is 10 millimetres so, $10 \times 10^{-3}\ m$ and this will be $0.15 \times 10^{-3}\ m$. The values of C and m are given here unlike the previous numerical, so, this will help you understand how to solve this problem without even remembering the expanded form of the formula.

So, why in this case will be however 1.12 since it has been mentioned that this is an age crack. So, what we can see here is based on the Paris law, da/dN is $C[\Delta K]^m$. So, if we are plugging in the values here delta K will be 1.12 for the edge crack and then stress range which is 128 and then $(\sqrt{\pi a})^4$.

So, if we do that so, this factor actually turns to 4168806 you can do the calculations and a^2. So basically, we can then say that this is $a^{-2}da$ equals to this we can do the multiplication and this turns to 0.016675 dN. So, if we are integrating this from 0.15 millimetre to 10 millimetre, this will turn to a^{-1} if you are simply solving the integration, I am still doing it step by step for you to understand this better.

So, that leads us if we are considering the minus sign and nullifying this so, this will come something like this $\frac{1}{0.15\times10^{-3}} - \frac{1}{10\times10^{-3}}$ and this is equivalent to this number multiplied by N if we simply solve this, this turns around 6566.67 simply solving this will lead to N = 393803 cycles.

So, that is the total number of cycles that are required for the crack to grow from 0.15 to 10 millimetre. Now, that was the simplest case, which has already been explained with the schematic in the previous slide. Now, what we need to find out is for the 2 conditions when for the first case we are applying the same initial stress range and for the second case we are applying the same initial stress intensity factor range.

So, for the first case, it is almost similar kinds of calculations only difference is that now, we have actually one order of magnitude the initial crack is 10 times larger now.

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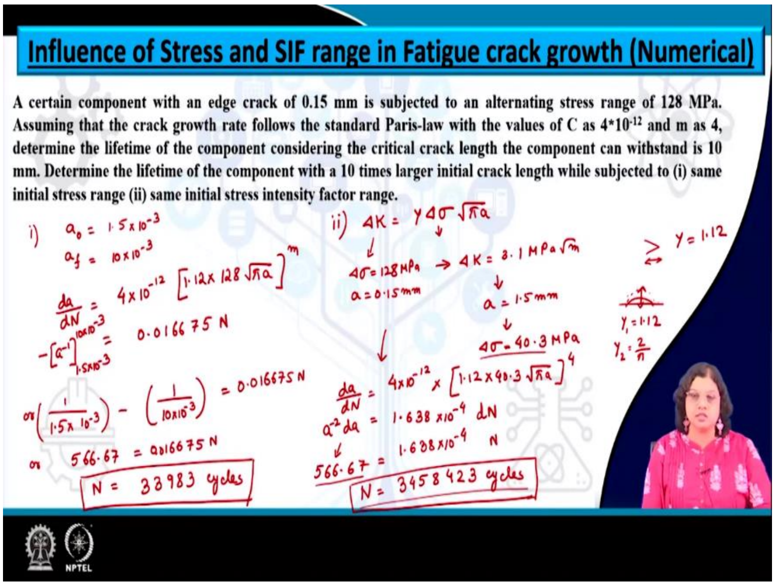

So, for the condition 1 what we have is a_0 is 1.5×10^{-3} and a_f is still the same of course, there is no further scope for the crack to grow. So, again if we can plug in the values, so, C is 4×10^{-12} and this 1.12×128 because we are applying same values of the stress range and if we solve this turns to the same thing.

So, we do not need to do the calculations. Since we have already done this and in this case however, this one will be from 1.5×10^{-3} to 10 into 10×10^{-3}. So, that leads to $\frac{1}{1.5\times10^{-3}} - \frac{1}{10\times10^{-3}}$, so, that will make a whole lot of a difference and this turns to only 566.67, and that makes our n equals to 33983 cycle.

So, this is much lesser than the previous one. So, now we have only around 3.3×10^{4} number of cycles while in the previous case we have 3.9×10^{5} cycles, so, which was quite higher value. Now, let Us see what happens if we are applying the same initial stress intensity factor range.

So, like it was shown in the example 1 here also we can see that the ΔK will be obtained from this relation here. And what we need to figure out is the Δσ here, so, basically for the condition or the initial condition what we have the ΔK value considering a stress range of 120 MPa and crack length of 0.15 millimetres so, that actually leads to ΔK value of 3.1 $MPa\sqrt{m}$.

And if we are using the same value of K for this case or the stress intensity factor range for this case, now, what we are having is a equals to 1.5 millimetre and that leads to our Δσ equals to 14.3 MPa. So, again the same relation if we are putting these values as da/dN equals to $4 \times 10^{-12} \times 1.12 \times 40.3$ in this case and $(\sqrt{\pi a})^{4}$ if we solve this, this comes around 1.638×10^{-4} and we are doing the integration also simultaneously.

So, this will be again the same thing. Now, here actually this the left side will be same as that for the condition 1. So basically, we are having the same values 566.67 and that is equivalent to 1.638×10^{-4} dN and if we solve this, so, what we are seeing here is this is getting the same values as the previous condition 1 and that lead to the total value of N as something like 3458423 cycles.

So, eventually what we are seeing here is that if we are keeping the stress intensity factor range same, yet we are applying lesser value of the applied stress range and that can lead to significant enhancement in the number of cycles that is required for the crack to grow even for higher or lower span. So, that means that apart from the crack length or particularly the initial crack length it is also very important to maintain lower value of stress range.

So, that to achieve a component can survive for larger number of cycles or for longer duration. So, those are the things that we need to take care of while we are considering improved fatigue performance or particularly the notched fatigue crack growth of a component. So, you may also wonder that why we are getting different values considering the previous slide that we have shown with the schematic versus this one.

So, please note that in this case, we are considering an edge crack, which is supposed to be a sharp one and we are mostly considering the crack length which is of importance here. If you remember the schematic that we have shown that has an elliptical crack, which is supposed to grow in a semi-circular or semi spherical condition.

And for that, we have already discussed this in the fracture mechanics concept that for such cases in the case of the semi elliptical crack actually we need to consider both the Y_1 and Y_2 condition where Y_1 is related to the position of the crack like whether it is an edge crack or centre crack. And in this case of course, Y_1 should be 1.12.

Whereas, on the other hand Y_2 is related to the shape of the elliptical semi elliptical crack and we need to consider this $\frac{2}{\pi}$ factor and actually this may also vary depending on the exact shape of the crack and with respect to that for the component. On the other hand, to simplify things for this particular numerical problem, we have simply considered an edge crack which is sharp in nature.

And of course, in this case, we just have this Y value, which is equivalent to 1.12. But, overall what we are seeing here and what has been already shown in the schematic is that, for any case, if we are applying lesser value for the stress range, we are achieving higher number of cycles for failure although the stress intensity factor range is still being kept constant.

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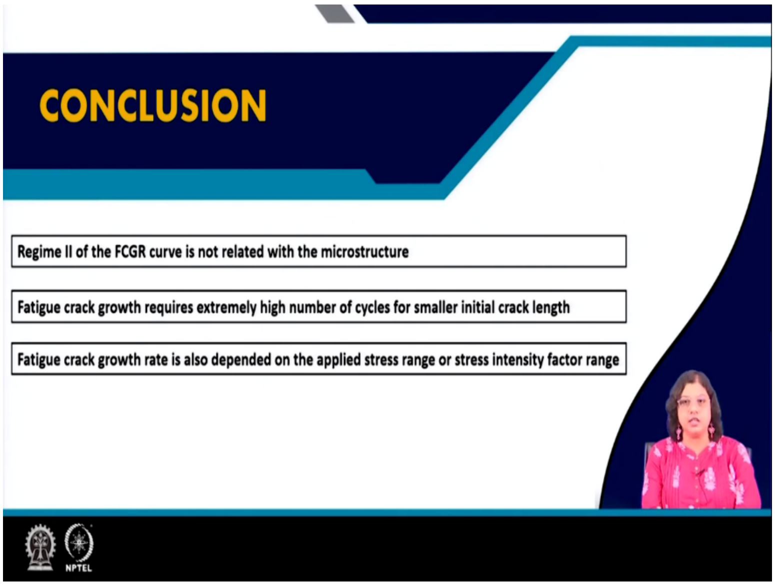

So, in conclusion, what we can see here is that, this regime-II of the fatigue crack growth rate curve or the da/dN versus ΔK curve is typically not related much to the microstructure rather it is more or less dependent on the experimental conditions or the actual service conditions.

And for that, the most important part is that of course, the crack length have a major control on the fatigue crack growth rate as well as the total number of cycles that a component can survive, but out of the crack length, it is the initial crack length that is of significance here and if the initial crack length is quite low, then we have seen that it requires higher and higher number of cycles to attend up any particular crack size.

For that case, we actually need to maintain the initial cracking to be as small as possible, so, that we can know that the component can survive for longer duration. On the other hand, fatigue crack growth rate is also related to the applied stress range or the stress intensity factor range. In fact, we have seen through these numerical problems that even if we keep the stress intensity factor range constant.

And we are still applying higher crack length or the initial crack length being higher, there also if we are applying lower value of the stress range to maintain same values of the stress intensity factor range that also can lead to significant enhancement in the number of cycles for failure. So, these are the things that needs to be considered for an actual application and to have improved fatigue characteristics.

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Following are the references that are being used for this lecture. Thank you very much.