Fracture, Fatigue and Failure of Materials
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Lecture 42
Fatigue Crack Propagation

Hello everyone, we are at the 42nd lecture of this course Fracture Fatigue and Failure of Materials. And in this lecture also we will be extending our knowledge on the concept of fatigue.

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The particular topic that will be covered in this lecture is Fatigue Crack Propagation.

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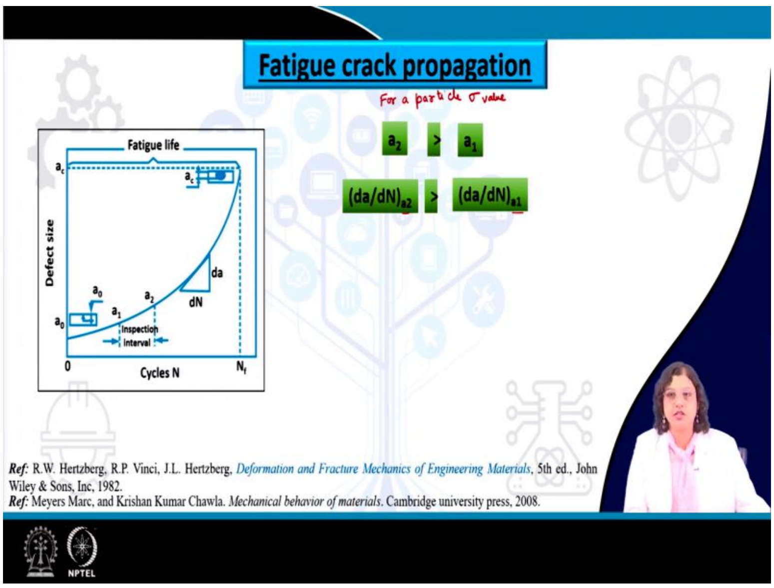

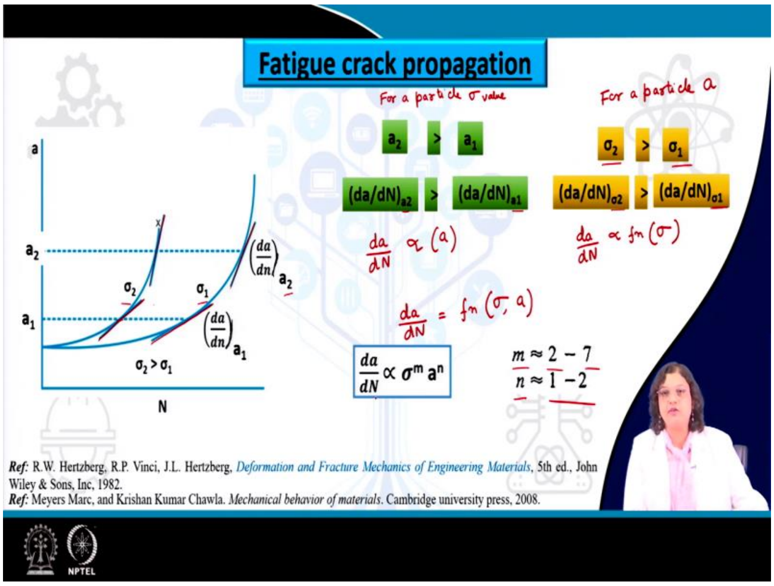

So, we have seen in the last lecture, where we have simply introduced the concept of fatigue crack growth is that, when we plot the defect size versus the number of cycles, we see that there is a trend in the curve and at every point as the crack length is increasing the corresponding da / dN or the crack growth rate, the way by which the crack is increasing per number of cycle that is also increasing as the crack length is increasing.

So, that means, that if in this case a_2 is greater than a_1 as we can see the point a_2 is here and point a_1 is there. So, a_2 being larger than that of a_1 actually the crack growth rate for the case of a_2 is greater than that for the case of a_1. Another thing to specify here is that unlike any other kind of rate, where we are comparing the properties with respect to time, in this case for the crack growth rate, actually the comparison is with respect to the number of cycles.

So, that means that any kind of rate that we are talking in case of fatigue is per number of cycle how much the crack is advancing per number of cycle. So, for the case of a_1 being less than a_2, we see that the crack extension rate is also quite less compared to that for the case of a_2. So, not only that, now, this is for a particular stress value.

And in case of fatigue, we do not talk about one single stress value that is of interest rather the stress range or the maximum and the minimum stresses or the mean stress amplitude or any kind of this stresses are important. So, this is for a particular σ value. On the other hand, if we are imposing different values of σ, then also we are noticing a change in the crack growth rate.

For example, now, we have the 2 conditions for one, we again have this A versus number of cycles as the y and x axis and we can see that the crack growth rate follows a particular trend for the case of σ_1 and that is different for the case of σ_2. Now, σ_2 is actually higher than that of σ_1, which means that for any particular crack length.

Now, if we are talking about keeping the crack length as a constant parameter and let us see how the influence of stress modifies the crack growth rate. So, now we see that for this particular crack length of a_1, we are getting here, this slope, which is equivalent to da / dN for σ_2 and the one here is da / dN for σ_1.

And obviously, what we can very clearly see here is that for the same crack length σ_1 is giving us a lesser slope compared to the one that is given by σ_2, the σ_2, the slope is steeper right and that means, that higher the stress faster will be the crack growth rate. So, this is what is being shown here that if σ_2 is greater than σ_1, then once again we are having da / dN for the case of σ_2 is higher than that for the case of σ_1.

Now, this is for a particular crack length value. And obviously, if we are increasing the crack length now, if you are talking about a_2 condition, then of course, the crack growth rate for the case of a_2, for σ_1 is actually higher than that of a_1. And again, if you are comparing this between σ_1 and σ_2, the crack growth rate for a_2 for the case of σ_2 is higher than that for a_2 for the case of σ_1.

So, essentially what we are seeing here is that da / dN is a function of σ (da / dN or da / dN is directly proportional to the stress level. On the other hand, what we have seen in the first case is that the da / dN is directly proportional to the crack length as well. So, overall we can safely assume that da / dN is actually a function of both sigma and a.

Typically, this can be represented by a relation like this that the da / dN $\alpha\ \sigma^m\ a^n$, this m and n are the material parameters and the values of m lies between 2 to 7 for that of n lies between 1 and 2. Now, this is very much simplification of the factor that how da / dN is related to the stress level as well as the crack length.

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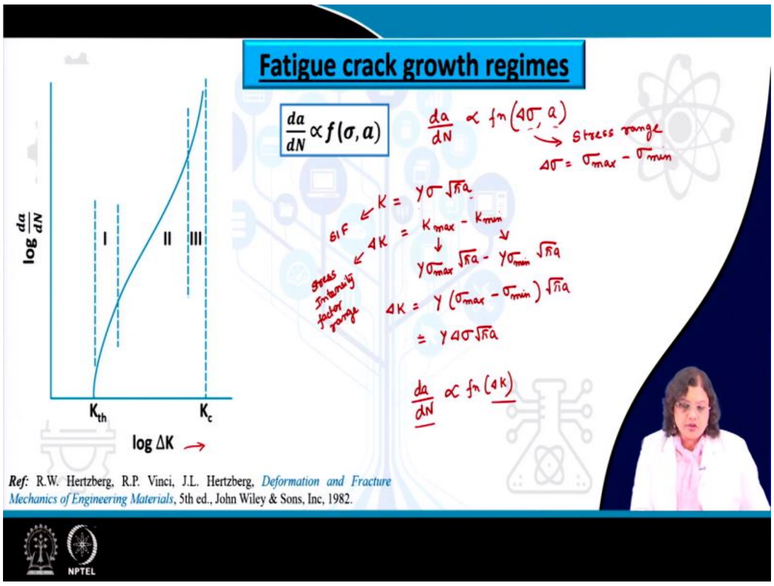

What we can better understand from here is that, since the da / dN is a function of σ and a and we also have known so, far that it is not a particular stress level rather the stress range or any other kind of stress such as mean stress, stress amplitude, those are of importance. So, let us say if you are talking about a cyclic loading, let us assume the maximum and the minimum cycles there, then obviously, the parameters that of interest is Δσ.

So, we can also rewrite this as da / dN is a function of Δσ and a. Now, if we want to simplify this the relation between sigma and a is also correlated through another very important parameter that we have seen for the case of fracture mechanics, that is the stress intensity factor K, let me just write it down here once for the reference that Δσ is the stress range and that means that Δσ is equivalent to the maximum stress minus the minimum stress.

So, what we were discussing is that, this Δσ or sigma and a these are correlated through the relation of K where we have seen that K is the stress intensity factor and K is given by the relation $K = Y \sigma \sqrt{\pi a}$ where Y is the geometrical parameter we have seen that the values of Y can be different in case the crack or the defect is present at the centre versus at the ages of the component and we can have different values of Y, sigma is the applied stress and a is the crack length.

So, if we are talking about the cyclic loading again the parameter that is of interest is ΔK which is the stress intensity factor range, stress intensity factor range. And that is actually given by $\Delta K = K_{max} - K_{min}$ and this K_{max} is nothing but when we are applying the maximum stress whatever is the value of the corresponding stress intensity factor is known as the K_{max} as well as similarly K_{min} is given by $K_{min} = Y\ \sigma_{min}\ \sqrt{\pi a}$. So, essentially that means, that ΔK is nothing but $Y\ (\sigma_{max} - \sigma_{min})\ \sqrt{\pi a}$ or that is simply $Y\ \Delta\sigma\ \sqrt{\pi a}$. So, if such is the case, so, we can safely say that da / dN is a function of ΔK. So, if such is the case, then we can actually plot this da / dN versus ΔK to figure out what is the actual relation between the crack growth rate and the stress intensity factor range.

So, this is what is shown here in the y axis we have the da / dN and in the x axis there is the ΔK or the stress intensity factor range and both of the y and the x axis are in the logarithmic scale.

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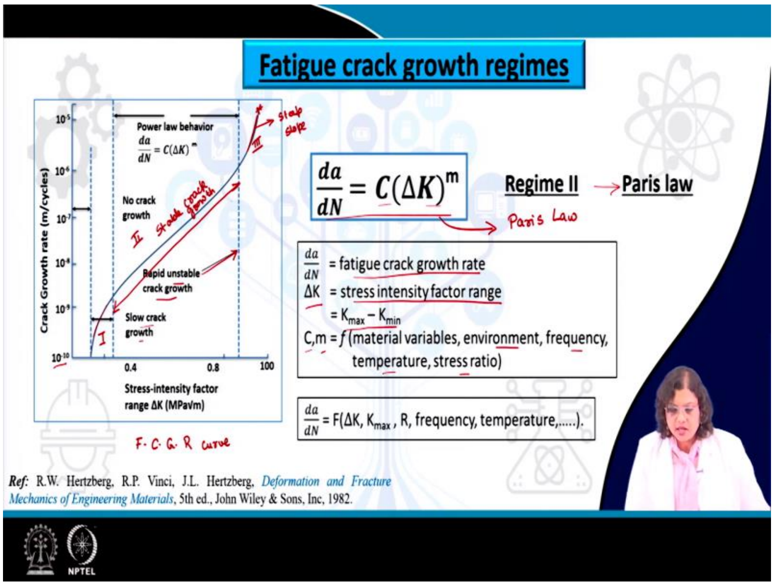

So, if we look into this curve carefully, we basically see that it has three different regimes. Particularly the first regime here is where it is having initially very slow crack growth, the crack actually will start from growing here. So, we can see that there is a finite value for the crack growth rate instead of 01 this is not starting from the origin but it is starting from a very low value.

So, this signifies that although there is a notch or a crack or a defect already existing in the crack, what is the stress intensity factor range that is necessary to start the growth of that crack, but once it does, it actually have quite a steeper slope. On the other hand, this middle portion here or the regime 2 that is the most prolong one and that covers almost the entire span of the ΔK range and this regime is the regime 2, and then we have the third regime, where once again the crack growth rate is very, very high.

So, very steep slope that we can see. And this essentially signifies the point at which fracture occurs at this particular point when K_C or the critical value of stress intensity factor is reached particularly by the application of the maximum stress at this point fracture occurs. So, here we see the unstable growth of the crack.

So, this is for the regime 3 we can see this unstable growth of the crack leading to failure for regime 1 there is very slow crack growth and for regime 2 actually there is a stable crack growth. So, very stable crack growth and that maintains a particular relation with ΔK. So, the typical relation for the regime 2 is given by this here.

So $da / dN = C (\Delta K)^m$ where da / dN is of course, the fatigue crack growth rate of growth of a or the crack length per number of cycles and ΔK as explained is nothing but the stress intensity factor range which is $K_{max} - K_{min}$, on the other hand C and m these are the constants are these other related to the material variables particularly the environment, frequency, temperature, stress ratio R value.

So, all this actually influence the C and the m values and that dictates how the crack growth rate should be. So, that essentially also dictates that if we are changing the environment of the cyclic loading or the frequency of loading we are doing it fast or slow or changing the temperature, stress ratio whether we are applying higher minimum stress or lower minimum stress that that is a very very significant here and all that controls the crack growth rate here.

For this regime 2, this regime 2 is actually the most important part of this da / dN versus ΔK curve also known as the fatigue crack growth rate curve, if FCGR curve or fatigue crack growth rate curve or fatigue crack propagation curve, which essentially means, the variation of the crack growth rate with the stress intensity factor range. This regime typically is known as the Paris regime because this relation is known as the Paris law. And since this Paris law is being followed for regime 2, regime 2 is also known as the Paris regime.

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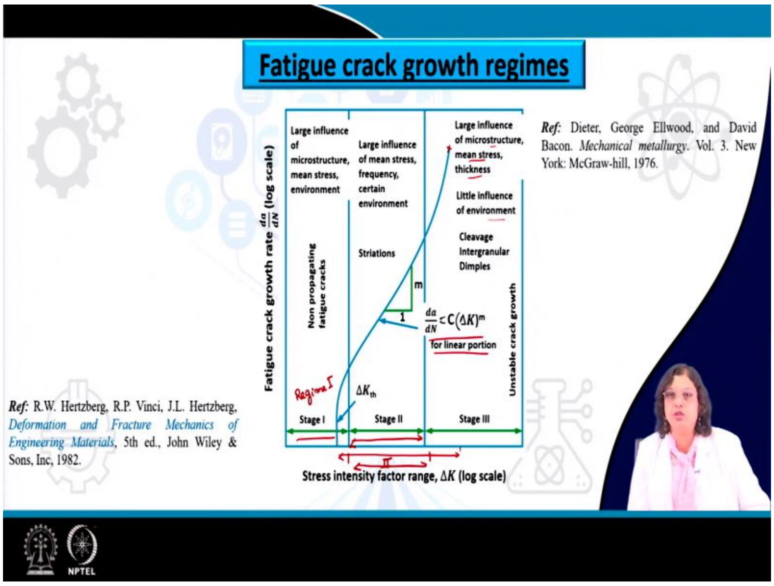

So, let us also see that, how all these different regimes are actually different from one another and what are the particular factors that control the behaviour or the crack growth rate for each of this regime, particularly for the stage 1 or regime 1 where there is essentially no propagating fatigue crack.

So, even if there are defects that may not be propagating, and here the large influence of microstructure is actually seeing other than the effects to mean stress as well as environment microstructure is the one that controls the behaviour here. So, we can modify the microstructure to tailor the properties particularly for stage 1 or regime 1.

For the case of stage 2 actually, here there is not much influence of the microstructure rather the experimental parameters or the external factors such as the environment or the loading conditions such as the mean stress frequencies, etc are the ones that control the behaviour or the crack growth rate particularly for this regime 2.

On the other hand, if you are talking about the regime 3, but the crack grows in an unstable fashion, here again, once again, we can see the influence of microstructure along with mean stress and obviously, thickness. And when we are talking about thickness, I hope you are now able to appreciate the importance of thickness particularly for the case of fracture mechanics.

So, that concept is gets valid once again when we are talking about regime 3 because finally, at the point of fracture, the key value out of this ΔK the K_{max} value has to reach the K_{1C} condition and whenever we are talking about K_{1C}, we also know the importance of plane strain and plane stress. And for plane strain, plane stress again the thickness becomes very important. So, that is how thickness is one of the major factor for controlling the stage 3 fatigue crack growth rate in an unstable fashion.

However, there are little influence of environment, because the crack is at the verge of failure at this point. So, not much influence of environment is seen for the case of regime 3. Now, if we are talking about this entire span of ΔK within which this regime 1, 2 and three are valid, we see that this major part major shear is being covered with regime 2.

And if we want to figure out the correct growth rate actually that can be done if we are considering this Paris relation here because here is this linear relation that is being maintained mostly at the regime 2 and in no other sections or no other stages for example, regime 1 and 3. So, that is the reason that we pay more attention or we try to correlate this relation to get or to predict the number of cycles that a specimen or component can survive or the time duration that is required for the growth of the crack.

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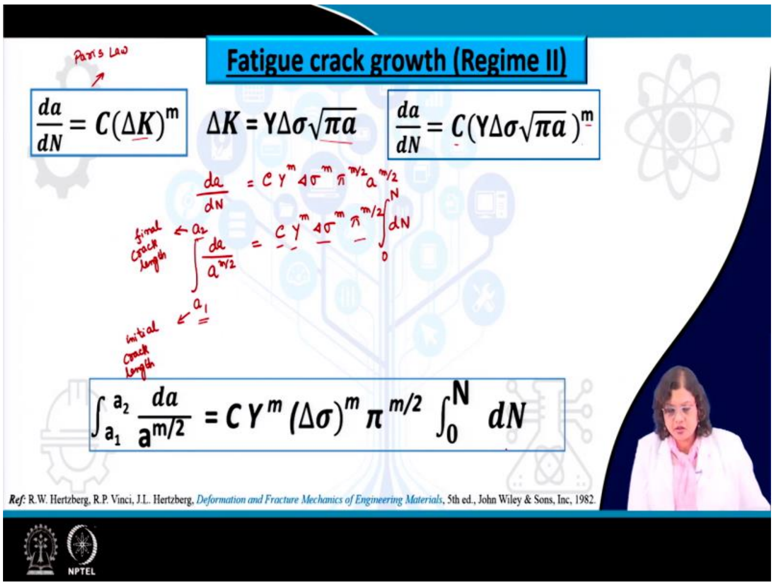

So, let us talk in more details what this relation signifies and how we can use this for practical significance. As we have seen that da / dN is nothing but C $(\Delta K)^m$. So, that is the Paris law that is valid for the case of regime 2 or the typical stable crack growth rate and in the case that it is unstable for the case of regime 3, it is very difficult to find out any relation any particular valid relation between da / dN and ΔK for regime 3.

It can be done only when it is a stable portion. Now, instead of ΔK, we can simply use this relation as $Y \Delta\sigma \sqrt{\pi a}$ I have just shown. So, that means that da / dN can be rewritten as da / dN equals to $C (Y \Delta\sigma \sqrt{\pi a})^m$. If such is the case, then we can also elaborate the relationship a little bit further to see how this can be used for solving this relation.

And to understand the number of cycles that will be required for growing the crack. For example, let us say from a certain length, initial length to certain final length. So, let us see how this can be done we can simply expand this relation something like this. So, that leaves us that $da / dN = C Y^m \Delta\sigma^m \pi^{m/2} a^{m/2}$.

In other words, we can also write this as

$$da / a^{m/2} = C Y^m \Delta\sigma^m \pi^{m/2} dN$$

Now, C is a particular constant value Y also we know based on the position of the crack and attend different values, but that is a constant one for a certain situation $\Delta\sigma$ is a loading condition we need to know that the stress range and once we know that that is also being maintained constant for a particular loading condition pi once again has a constant value.

So, if we can plug in all these values we are left out with dN. So, that means that if we try to figure out the growth rate for the crack, we can simply integrate this between certain crack lengths for example, a_1 to a_2 where a_1 is the initial crack length and a_2 can be the final crack length. If that is the case, then actually dN can should also be integrated between 0 to n.

Now, 0 means when we are just starting the process, so, at that point there is a crack of length a_1 please note that we are talking about the notched fatigue here. So, that means that crack or defeat or notch is already existing we have also emphasized about the short crack and long crack and how a defect can be termed as crack.

So, we are already considering the factor that crack is already existing here and the initial length of the crack is a_1 when there was 0 cycle and then we keep on applying the cyclic loading, and we want to figure out that after how many number of cycles a_2 can be achieved.

Now, this a_2 can be the final crack length leading to failure or it can be a certain crack length that we are interested to know. So, if such is the case, we can simply expand this relation and solve this integration and we can then find out the number of cycles that will be required for the crack to grow from a_1 to a_2.

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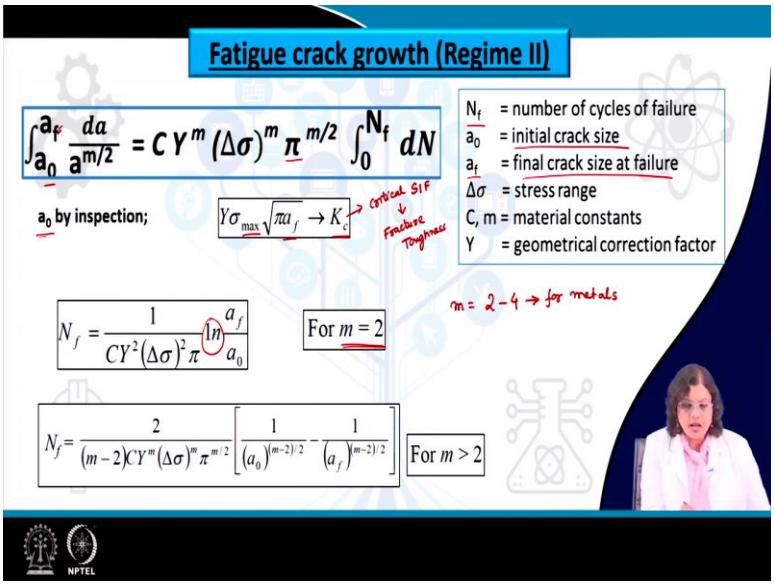

Let us expand this some more time to see how this is relevant when we are talking about using this for any practical application. Now, N_f in this case, please note that the things that have changed here from the previous slide is that instead of a_1 and a_2 now, we are talking about a_0 and a_f particularly a_0 is the initial crack size similarly to a_1 and the other hand a_f is the final crack size at the point of failure.

So, this is the critical crack size in cases when we want to figure out that how many number of cycles the component can survive prior to failure, we can determine this based on the final crack length. And this initial crack length can be understood based on the inspection, for the case of fatigue we often need to inspect this periodically to see if there is any development of crack.

Even in a completely unnotched component or specimen because of the repeated loading some internal defect may lead on to crack and we need to figure this out continuously monitoring this as you might understand that this is very very relevant if you are talking about an aircraft structure or a bridge structure, we often need to do a thorough inspection to find out if there has been any crack that has been developed.

So, whatever that crack size is through inspection that is considered as a_0. On the other hand, a_f is the critical crack size that leads to failure and that can be obtained again by using the concepts of fracture mechanics and that is based on the K_C or the critical stress intensity factor. So, that is nothing but the fracture toughness of a material and under plane strain condition this can be termed K_{1C} which is a constant value not supposed to change any more.

On the other hand, for the case of plane stress condition, we know that this is applicable for a thin component in which case the plastic zone size is quite signified with respect to the thickness and in such cases the fracture toughness values do vary with changing the thickness anyway, so, this is the critical stress intensity factor or simply fracture toughness.

And so, if we are talking about fracture that can happen only at the condition of maximum stress right at the point of minimum stress there should not be any fracture at all because it will not achieve the highest value of K of course, that can be achieved in a cyclic loading condition only at the point of maximum stress. So, maximum stress is of importance here and based on that we can find out what is the value of a_f which is nothing but the a_c.

Another thing to emphasize here is that since we are talking about the concept of crack growth, it considering that there is an already existing crack, we should also note that there is no point in loading it under compression because compression compressive loading is actually helps in closing the crack. So, when we are doing the experiment, at least in the lab scale, we typically apply tension and tension kind of loading.

So, we are applying higher values of tension and σ_{max} and lower values of tensile loading for σ_{min} or we also do apply the 0 as the minimum stress and some positive value of stress and the maximum side, fine. So, based on doing this fracture mechanics base estimation, we can figure out the a_f value and if we can do that, we can simply plug on these values to find out the relation.

Now, this solving has been done for the case of m equals to 2 if we have m equals to 2 actually the things get quite simplified as typically we have this π which is m if we consider m as 2 then this will be simply just π on the other hand a in such case will be simply a. So, da / a and that keeps the ln factor coming into the picture here for the case of m equals to 2.

Now, typically for the case of metal the value of m is lies between 2 to 4 in case we have higher value of m more than 2, then the relation will be something like this, this has been simplified you do not need to remember this relation rather you need to simply solve this integration yourself in case we want to use this for any practical application. So, based on this relation, we should be able to figure out that what are the number of cycles that are required for a crack to grow from a_0 up to a_f or a_1 or a_2 that is of significance.

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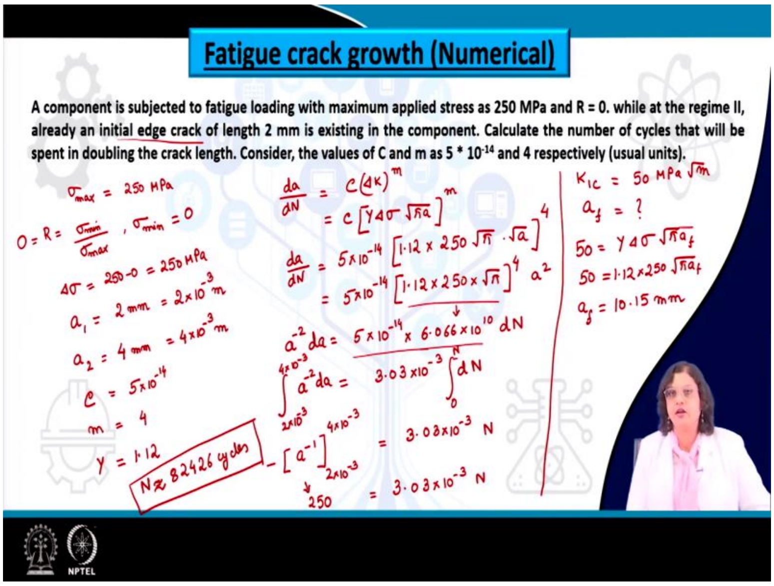

So, let us solve a numerical here to understand these concepts better, what is stated here is that a component is subjected to fatigue loading with a maximum applied stress of 250 MPa and R equals to 0. So, let me write this also parallelly which says that σ_{max} equals to 250 MPa and R equals to 0, so R equals to actually σ_{min} / σ_{max}.

So, if that is equivalent to 0 that actually gives us the fact that σ_{min} equals to 0. So, in that case $\Delta\sigma$ or the sigma range equals to once again 250 - 0 which is nothing but 250 MPa. The next line says that while at regime 2 that means that the Paris region already an initial age crack of length 2 millimetre is existing.

So, that means, a_1 in this case is given as 2 mm. So, since in case of fracture and fatigue we prefer to use the unit of length as meter. So, let us convert this to 2×10^{-3} m. Calculate the number of cycles that will be spent in doubling the crack length.

So, what is of interest here is a_2 which is double the initial crack size that means 4 mm right and that is just simply 4×10^{-3} m. The values of C and m are also given here the value of C is 5×10^{-14} and m is given as 4. So, let us see that how we can solve this and without even remembering the relation that has been shown in the previous slide.

So, let us simply write the Paris relation here, which says that da / dN = C $(\Delta K)^m$. So, let us expand this relation as C $(Y\ \Delta\sigma\ \sqrt{\pi a})^m$. Now, we also need the value of Y here and what it mentions is the presence of an initial edge crack and we have seen this from the fracture mechanics concept that Y for an edge crack is 1.12.

So, that gives us that da / dN = 5×10^{-14} $[1.12 \times 250 \times \sqrt{\pi a}]^4$ so, let me simplify this even further. So, what it says is 5×10^{-14} and here we can write $1.12 \times 250 \times (\sqrt{\pi})^4 \times a^2$. So, if we are simply using the calculator to solve this, this comes around the value of 6.066×10^{10}.

And the value of C should be multiplied to this and what we are doing here is that we are simply doing the integration or preparing it for integration. So, we are rewriting this as a^{-2} because a square is coming in this side da = 5×10^{-14} into this value here which both of these are now constant.

And the multiplication of this turns around 3.03×10^{-3} dN and we need to integrate this from 2×10^{-3} m to 7×10^{-3} m whereas, dN will be integrated from 0 to N, we need to figure out the value of N from here.

So, basically what we have here is that a^{-1} and we can simply do the integration here and that is equivalent to 3.03×10^{-3} N. Now, this value here will be nothing but 250 = 3.03×10^{-3} N. So, that leads us to the value of N something like 82426 cycles.

So, that is the number of cycles that is required for the crack to grow from a length of 2 mm to a length of 4 mm, we do not need to be very very precise here as I mentioned that we do not need to really mentioned number of cycles after the decimal it does not make sense. And if we want to approximate the value, we should better take the conservative approach for example here we can simply say that 82,000 cyclists will be required to grow the crack from 2 to 4 mm rather than very precisely as 82,426.

And in no case we should actually approximate this to a higher value, that we can never say this as 82,427 or 500 cycles something like that rather we can be on the lower side to have an conservative approach. Now, along with that, just a small example is what I would also like you to show is that in case we know that the fracture toughness of the material, or the K_{1C} of the material, let us say is 50 MPa√m.

So, we can also find out that what is the critical size of the crack up to which it can grow and then it will have a catastrophic failure. So, that means that we can typically find the a_f value here. So, a_f value should be what is of interest so, we have said or we have seen that how many number of cycles will be required for the crack to grow from 2 to 4 mm but what is actually the size of the crack up to which it can grow before it fails.

So, again we can use the relation for K and which is given by Y Δσ $\sqrt{\pi a_f}$. So, Y as usual this is 1.12 here for the case of edge crack, Δσ we know is given us 250 and if we are using this relation. So, if we are simply using this relation we turn out to af equals to 10.15 or simply 10.1 mm. So, that means, the component has a provision that the crack can grow from 2 mm to 10 mm prior to it fractures.

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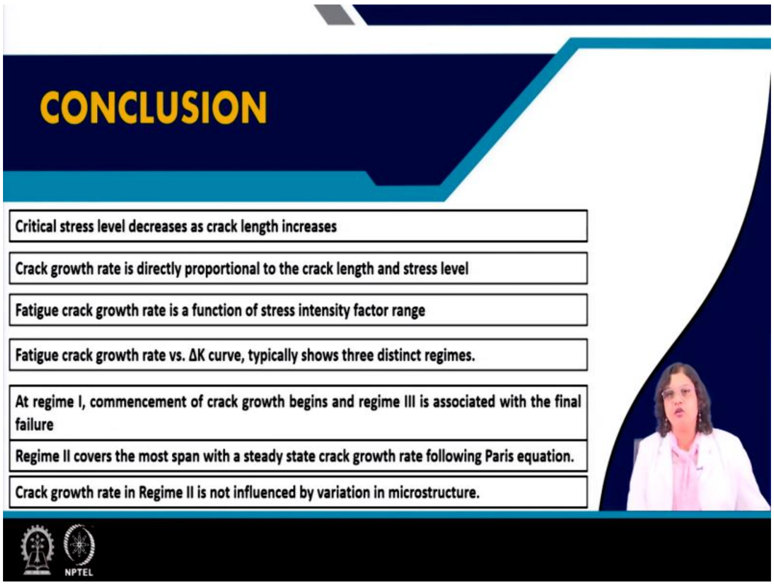

So, let us conclude this lecture with the following take home thoughts that critical stress level decreases as the crack length increases, the component can survive for up to lower stress value if it has a higher crack length in it. And crack growth rate is directly proportional to both the crack length as well as the stress level.

And this can be properly explained by the concept of stress intensity factor range and basically fatigue crack growth is a function or directly proportional to the stress intensity factor range. Now, the fatigue crack growth rate versus delta K curve typically known as da / dN versus ΔK curve shows three distinct regimes.

The first one or regime 1 is for the commencement of the crack growth even if there is a defect that needs to grow or start growing that happens at the region 1 and region 3 is associated with the final failure. However, regime 2 covers the most span of this ΔK over which the crack growth occurs in a very stable fashion following the Paris relation which is $da / dN = C (\Delta K)^m$.

And if we try to figure out the number of cycles that is required for the crack to grow in a stable fashion from one length to the other length, we can do that based on this Paris relation. Crack growth rate in regime 2 is typically not influenced by the variation in microstructure but rather particularly influenced by any changes in the environment or the looting situations.

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So, following the references that are used for this lecture, thank you very much.