**Fracture, Fatigue and Failure of Materials**
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**Lecture 40**
**Notch effect**

Hello everyone, we are at the 40th lecture of this course Fracture Fatigue and Failure of Materials. And in this lecture also we will be continuing the module 2 that is fatigue of materials particularly we will be looking at the notch effects on fatigue.

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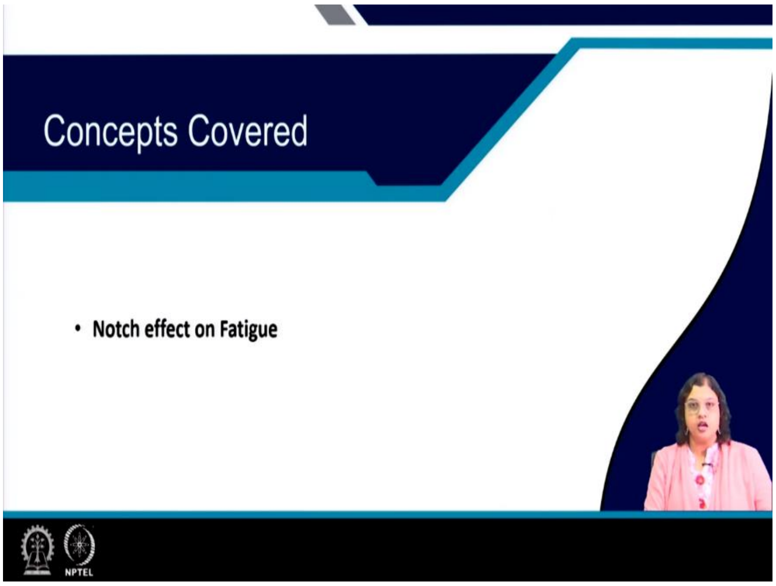


So, the concepts that will be covered here is typically the notch effect on fatigue. In the last lecture, we have seen that even for the case of high cycle or low cycle fatigue, when we do the characterization do the testing using smooth specimen, which means unnotched specimens till there also are possibilities that there could be some scratches or machining marks or dents that can act as the stress concentration sites.

We have also seen that how different methods different processing can be done to increase the strength of the surface or to remove the scratches or defects to the maximum extent possible which can increase the fatigue performance of materials. But still, we do know that there could be notches and often it is necessary to characterize the fatigue performance in presence of such notches.

So, we will go to in the depth of that topic later. But for now, let us assume that if there is a notch present in the fatigue specimen that has been tested for fatigue characterization, how would it react or influence the fatigue performance of the materials?

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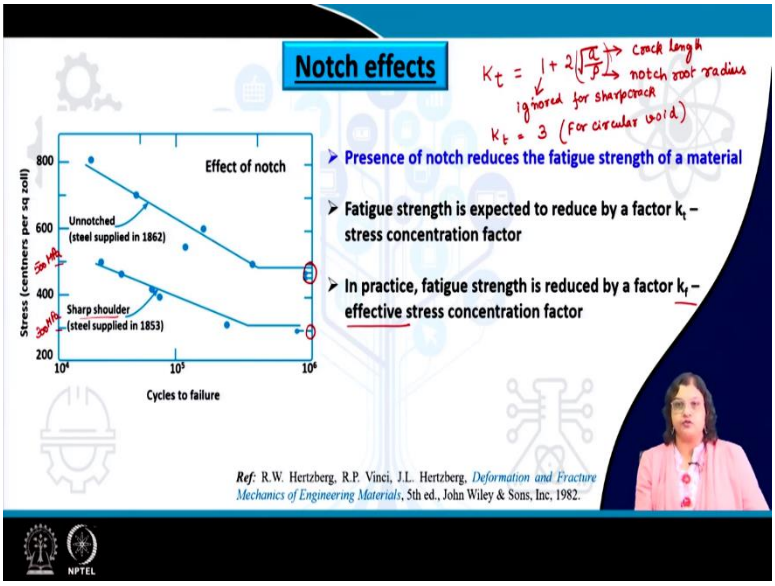


We have seen this earlier also while we were discussing about the SN curve that is the stress amplitude versus number of cycles to failure curve, we have seen that in presence of notch that is for a north specimen this SN curve is quite below the typical SN curve of an unnotched specimen.

So, we have seen exactly the same curve which shows that the upper one here is for the unnotched specimen and we can see that if we are considering certain number of cycles as the endurance limit or the fatigue strength calculation for example $10^6$ in this case, so for the case of unnotched specimen or smooth hourglass shaped specimen, we can see that that may have a fatigue strength something like around 500 MPa.

On the other hand, if there is a notch, there like in this case there is a sharp shoulder and for that the fatigue strength for this $10^6$ life has come down to almost 300 MPa. So, 200 MPa reduction in the fatigue strength is actually a very big deal and we should be careful about that. So, notches actually affect the fatigue behavior quite drastically fine.

So, let us see how the presence of notch is actually influencing the fatigue behavior. Actually, whenever there is a notch or for that matter notch is nothing but a source of stress concentration.

So, this could be not necessarily crack but this could be also some sharp corners or some holes or some kind of discontinuities in the overall smoothness of the specimen or component.

And that means that there will be some non-uniformity in the stress level and this non uniformity is coming into the picture based on the stress concentration factor. So, if there is a notch present typically the fatigue strength is expected to reduce by a factor of the stress concentration factor, which means that let us say the fatigue strength of a material is 210 MPa and there is a circular hole in there.

So, let us recap the concept of stress concentration factor we have seen that stress concentration factor or $k_t$ is actually related to the geometry of the notch or the crack or whatever discontinuity is there. So, this is actually given by this relation $1 + 2\sqrt{\frac{a}{\rho}}$, where a is the crack length or half crack length and rho is the notch root radius.

Now typically for the case of very sharp crack we ignore this parameter 1 here because this is not influencing it in a big way the factor $2\sqrt{\frac{a}{\rho}}$ itself is a very huge number that increases the applied stress to a large extent. So, we ignore this one factor. So, this is ignored for sharp crack but when we are talking about a circular void then this factor here is equivalent to 1, $\sqrt{\frac{a}{\rho}}$ and that means that $k_t$ for a circular void is 3.

So, in that case we should not ignore the 1 term and the total $k_t$ value is coming to 3. So, in case we know that the endurance limit or the fatigue strength of component a smooth component is let us say 210 MPa in that case if it is having a circular void in it which is having a stress concentration factor of 3 just because of the presence of this circular void now it will have the endurance limit of the order of around 210/3 which is nothing but 70 MPa.

So, there is such a reduction in the endurance limit just with the presence of a circular void. And you can imagine it very well that in case of a sharp crack this $k_t$ the number or the value of the stress concentration factor is even large which means that the fatigue strength will come down to quite a higher extent. Certainly, we need to find more about this and to our understanding what we have seen is that in practice fatigue strength is actually reduced by a factor $k_f$ which is the effective stress concentration factor.

So, not exactly by the stress concentration factor, but by a factor which is very similar to $k_t$ which is known as the effective stress concentration factor and we will see that how this effective stress concentration factor is related to $k_t$ and how this can be determined. So, it is very important to know, to understand that if there is a notch or a crack we need to determine this stress concentration factor or more practically the effective stress concentration factor to determine the fatigue strength of a notched component.

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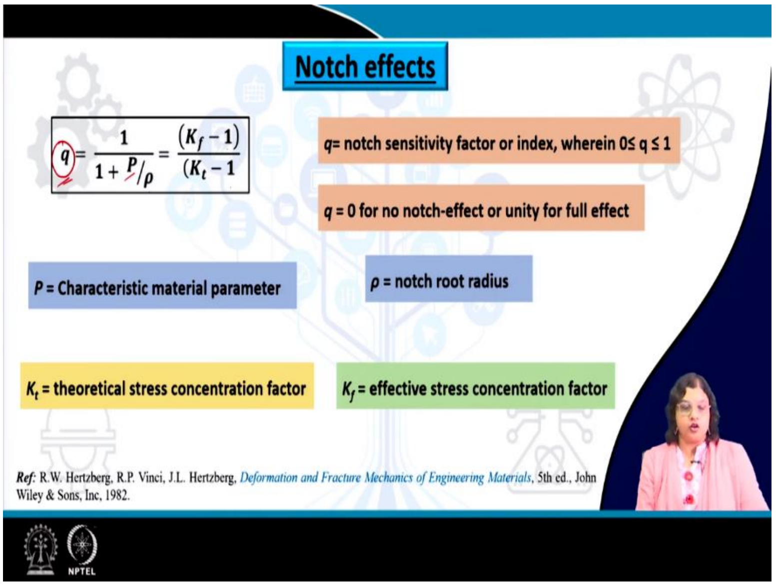


So, typically this is related to another parameter here which is known as notch sensitivity factor. How much the notch is affecting the fatigue behavior of the material that is reflected through this parameter q. Typically the value of q lies within 0 and 1. When it is 0 that means that there is no notch effect or there could be no notch at all. And the value of q could be 1 or unity in case there is a complete notch effect is active.

Now depending on the material the geometry of the notch, the position of the notch, the loading direction with respect to the notch, how severely the presence of notch can affect the fatigue behavior of the material, this is related to all those factors. And we need to figure out this notch sensitivity factor to understand that how dangerous the notch could be. This notch sensitivity factor in turn is related to the relation $\frac{1}{1+P/\rho}$.

Now ρ is the notch root radius, we have already discussed that, p on the other hand is a characteristic constant or it is a material parameter. It may vary for the different materials. In a more simplified form notch sensitivity factor is typically expressed as the ratio of $\frac{k_f-1}{k_t-1}$, where

$k_f$ is nothing but the effective notch concentration factor and $k_t$ is the notch sorry $k_t$ is the theoretical stress concentration factor as such.

So, once again we have seen the importance of $k_f$ and even if we need to figure out that how much the notch will be effective in reducing or for that matter influencing the fatigue behavior of the material, we need to understand that what is the value of $k_f$ or the effective notch concentration factor.

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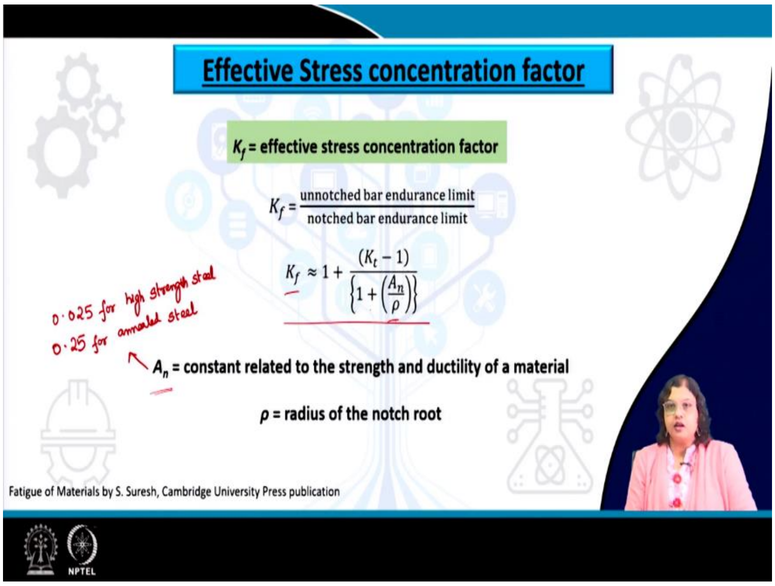


So, the $k_f$ is actually given by the ratio of the unnotched bar endurance limit or fatigue strength divided by the notched bar endurance limit. So, that dictates the value of $k_f$ in a more generalized form we can express $k_f$ by this relation here. So, $k_f$ is given by $1 + \frac{k_t - 1}{1 + (\frac{A_n}{\rho})}$. Once again, we are introducing another parameter here which is $A_n$ that is a constant related to the strength and ductility of a material.

So, again this is somehow related to the materials characteristics. The value of $A_n$ could vary from material to material and not only for different materials even for the same material if we are changing the microstructure, if we are changing the mechanical properties by doing some heat treatment that may also affect the value of the $A_n$. For example, for the case of steel this value of $A_n$ could be of the order of 0.025 for high strength steel.

On the other hand, the value of $A_n$ could be just 0.25. So, one order of magnitude different for annealed steel. So, just by changing the heat treatment condition, if you are varying the mechanical properties that is supposed to influence the effectivity of the notch also. And that is quite understandable considering the fact that it is the microstructure or the mechanical properties such as the yield strength of the material.

That dictates whether a material is able to undergo plastic deformation or not, or there could be a fracture in a brittle mode or ductile mode and that typically dictates all the fracture behavior, we have seen this in details in the fracture mechanics module of this course. So, if we know this $A_n$ parameter here, we should be able to find out the $k_f$ also considering that we also know the geometry of the notch. We need to know the ρ value the, a value crack length value and then we can figure out the effective stress concentration factor as well.

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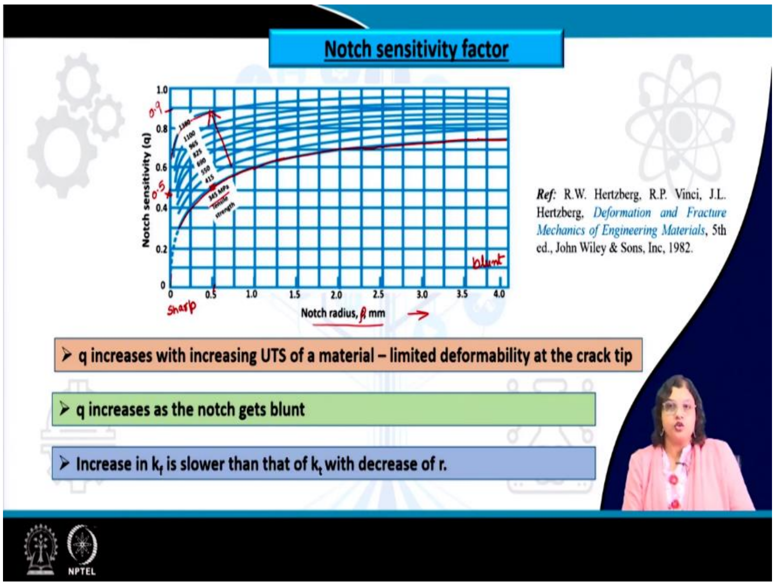


So, here is a graph showing the experimental results, for varying the notch sensitivity with the notch root radius. This is typically explaining the q versus ρ curve and what we are seeing here is that for each particular strength value if we are talking about the tensile strength of the material each graph actually signifies the tensile strength remaining constant. So, in this case the first one is 345 MPa.

So, we can see that this is the material with the tensile strength of 345 MPa. And if we are varying the notch root radius ρ like making it more and more blunt, notch radius is 0 here like very very sharp and then gradually it is increasing along the x axis which means the crack is getting blunter and blunter.

On the other hand, the y axis shows that how the notch sensitivity factor changes. And in all cases, we have seen a continuously increasing notch sensitivity factor, not only that if we are doing this performing this kind of experiments for materials having different values of ultimate tensile strength. Again, we have seen that there is an increasing trend in case the UTS is increasing.

For example, for a particular notch root radius of let us say 0.5 what we can see is that the first graph here is for the tensile strength of 345 MPa and for that the notch sensitivity factor is around 0.5 something like that. So, around 0.5 but if we are talking about a component or a material which is having this ultimate tensile strength value of 1380 for that the even for the same notch root radius the notch sensitivity factor turns to around 0.9.

So, that means that the notch will be very very effective in case we are talking about a high strength material. More the strength of the material is, more effective the notch will be to reducing the fatigue performance or to affect the fatigue behavior of the material. So, we should be more careful if you are talking about the high strength material.

Typically, high strength material means that there is a lack of ductility, we have seen this that strength and ductility, strength and toughness, these are more or less inversely related. Except for one condition, we have also discussed about the grain refinement thing, let us not talk about that. In general strength and toughness are inversely related and this is also affecting the fracture behavior as well as the fatigue behavior.

So, fatigue is finally leading us to fracture only. So, higher the strength of the material, the ductile deformation will be restricted and that will make the notch more effective to break the bond to lead to final fracture or to initiate the crack, for the case of fatigue or cyclic loading. Now this is one major findings that has been noted here that q increases with increasing ultimate tensile strength of the material because of the limited deformability at the crack tip.

Now this is quite understandable the next point that we already discussed is that not only that even for a particular value of tensile strength we have seen that as the notch is getting blunter. So, this is the blunt side of the notch and this is the sharp one. So, as the notch is getting blunt, we have seen that the noise sensitivity factor also increases.

Now this is a little bit difficult to appreciate at the very first instance considering that we have always understood or realized or imagined that sharper the notch more detrimental it is there is no doubt in that. This is the first time we are seeing that actually a blunt notch could be more detrimental because that could be more effective in reducing the fatigue behavior of the material. A sharp notch will be preferred because it will be less active or less effective in influencing the fatigue performance of the materials.

Of course, we need to understand this in more depth to appreciate what actually is happening. In fact, this q the or the north sensitivity factor that we are talking about this is related to not only $k_t$ or the stress concentration factor but the effective stress concentration factor and it is time to look more in more depth what exactly is $k_f$ how is it controlling the deformation behavior or more precisely the notch sensitivity factor in general.

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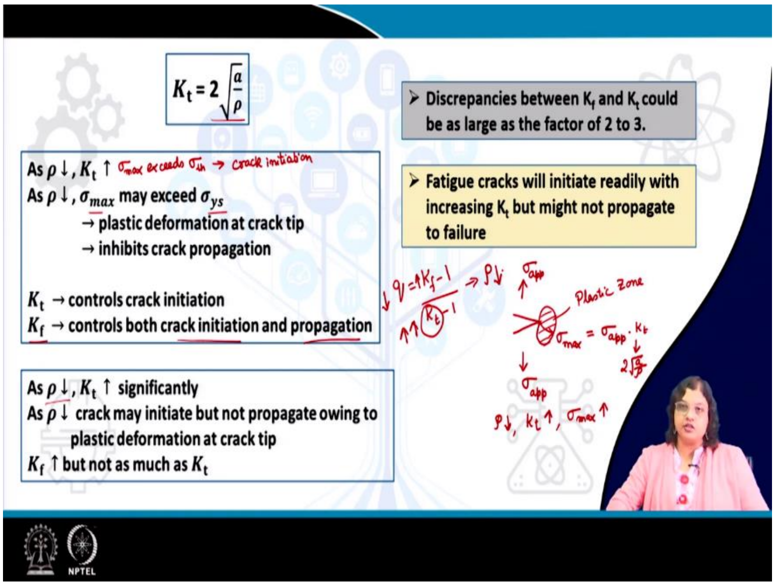


So, what we have seen or what we know so far is that $k_t$ is typically given by $2\sqrt{\frac{a}{\rho}}$ for a sharp crack. So, let us see if there is a sharp crack like this on which we are applying stress σ or $\sigma_a$ for example then at the tip of the notch we are getting $\sigma_{max}$. Because of the stress concentration and $\sigma_{max}$ is nothing but $\sigma_a$ multiplied by the stress concentration factor and this is nothing but $2\sqrt{\frac{a}{\rho}}$.

Now as ρ decreases of course $k_t$ will increase because it is inversely related to ρ. So, $k_t$ and ρ are inversely related. So, as we are decreasing ρ there will be an enhancement in $k_t$ no doubt about that. So, that means that as we are decreasing ρ there will be an enhancement in $\sigma_{max}$ also. So, typically there is no doubt in that and as long as the $\sigma_{max}$ exceeds.

So, in this case in case the $\sigma_{max}$ exceeds the theoretical cohesive strength of the material it is sufficient to break the bond and breaking the bond means it is actually leading the crack initiation. Fatigue is all about the initiation of the crack that is what we have seen so far, typically initially when there is no dominant notch or crack present in there the first thing that is necessary for fatigue failure to materialize is the crack initiation.

And that can happen if the $\sigma_{max}$ exceeds the theoretical value locally at that point, it may exceed this theoretical value and that can lead to the breaking of the bond near to the that defect or near to that stress concentration site and that can lead to crack initiation. So, there is no doubt in that.

Now for the case of $k_f$ if you want to estimate the $k_f$ it is not only related to breaking the bond but rather what happens particularly for the case of ductile materials is that, whenever $\sigma_{max}$ exceeds the yield strength of the material then there is the plastic deformation that commences and that means that this plastic deformation also, we have seen this that it can it has the capability to make the crack tip blunter.

So, officially we are not making the corrective blunt but virtually it is getting blunt because of the presence of this plasticity. And that essentially inhibits the crack propagation once the crack tip gets blunt it means more and more applied stress will be required to overcome this barrier of plastic deformation, so that the crack can progress. So, since now this $k_f$ is actually controlling both the crack initiation and propagation.

So, in case there is any possibilities of this corrective blunting that does not extend or does not increase the $k_f$ up to that extent at way as at which $k_t$ is happening. So, overall if we are reducing the value of rho both $k_t$ and $k_f$ is supposed to increase because somehow both are inversely related to $\rho$, there is no doubt. So, $k_t$ will increase significantly because this is related to only crack initiation.

On the other hand, $k_f$ may not increase to that extent as $k_t$, $k_f$ will increase but not very highly as that of $k_t$, why? Because this crack may initiate, locally crack may initiate because at near the tip of the crack where the stress concentration value is very very high at that point it may break the bond and that can lead to crack initiation.

But even if it initiates, if there is significant plastic deformation that is sufficient to stop the crack and that means the crack won't propagate. And that will actually hinder the growth or the increase of $k_f$ to a very large extent as that of $k_t$. And that makes the effective enhancement in q reducing. Because q is still related to this factor here $\frac{k_f - 1}{k_t - 1}$.

So, in case $k_t$ is increasing to a very very high number, whereas $k_f$ is increasing not to that high extent then obviously q will gradually come down and that is what we are seeing as ρ is decreasing. So, that makes q and ρ directly proportional to each other. This discrepancies between the effective and the theoretical stress concentration factor could be as high as around 2 to 3.

So, overall what we have seen here is that fatigue crack may initiate readily because of the increasing $k_t$ with reducing ρ value or increasing creative sharpness. But that may not propagate always to failure and that leads to not so high enhancement in the effective stress concentration factor and essentially that leads to a reduction in the notch effect.

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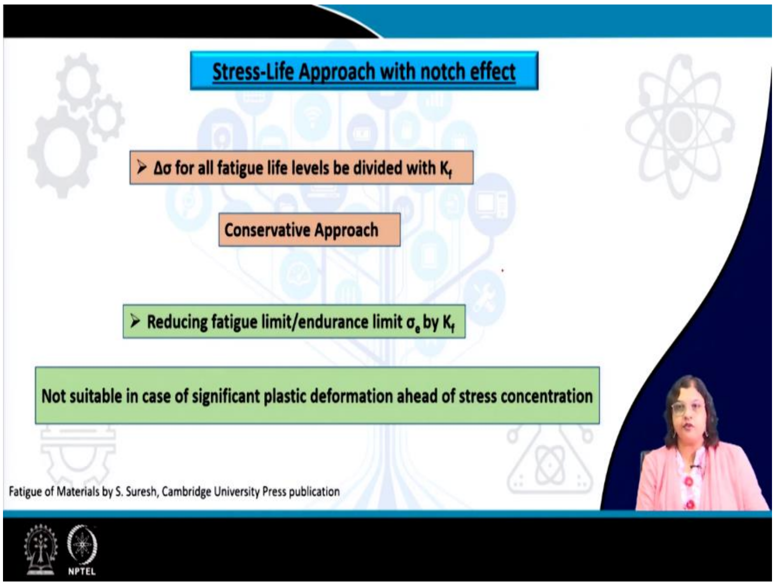


Now not only that, if there is this notch effect already active, we should also think about how the stress life can be determined the fatigue strength or the fatigue life can be determined in presence of a notch already existing. So, either there are two ways to do that either we can divide the entire Δσ or the stress range that has been applied by $k_f$ for the effective stress concentration factor but this approach will be too conservative.

It is not always applicable that all the stress ranges even the higher stress ranges versus the lower stress ranges for a fatigue cycle that will be equally reduced by the $k_f$. On the other hand, it is also possible to reduce just the endurance limit or the fatigue strength by $k_f$. So, that is another approach by which we can modify the fatigue strength or the fatigue life of a component. But this way is also not suitable in case there is a significant, there is a possibility of significant plastic deformation ahead of the stress concentration.

So, that can also lead to some discrepancies in the value and we should be careful of choosing what way we are considering the notch effect to be active.

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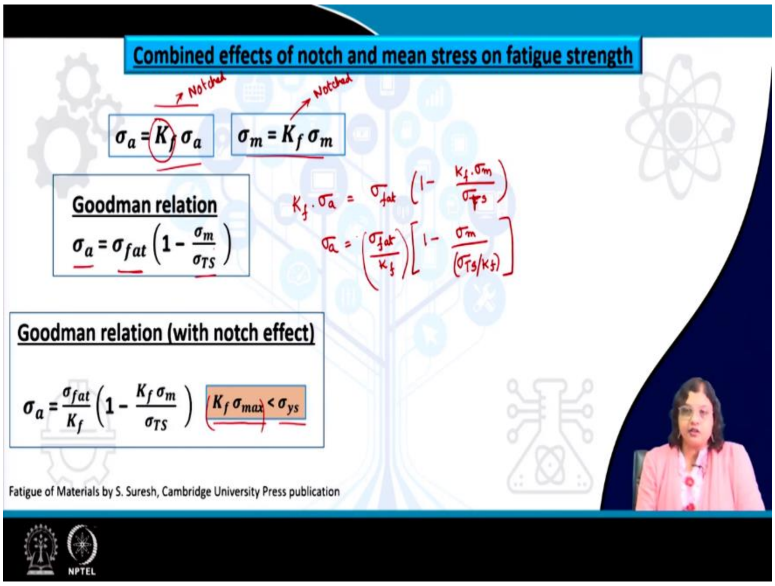


Now, this is interesting to note also that in presence of the notch, now we have seen earlier that if we are varying the mean stress of the mid of the cyclic loading then the fatigue performance is varying completely. We have seen this on the basis of the Goodman diagram and Soderberg relation as well as this Gerber relation etcetera. So, let us see how does it affect in in case of a notch present in a so called smooth bar now.

So, now it is not any more smooth and if we do know that there is a presence of notch in it. So, what happens there is because of this notch and we have seen that it is the effective notch concentration factor that controls all the behavior of the material in front of the crack tip. So, now $\sigma_a$ which is supposed to be the stress amplitude, now is being multiplied by this term here as $k_f$ $\sigma_a$ for the notched component, so this is for a notched component.

On the other hand, the mean stress is given by $k_f$ $\sigma_m$. So, again for the notch component we have to multiply the $\sigma_m$ or the mean stress with this effective stress concentration factor. And if we do so, obviously the effect of mean stress on the fatigue performance of the material will be also influenced and we should be able to figure that out. Let us see, how?

We have seen that how the Goodman relation explains the relation between stress amplitude, fatigue strength, mean stress, tensile strength of the material etcetera. So, combining both the monotonic and the cyclic behavior of the material. So, this is given by this relation as $\sigma_a = \sigma_{fat}(1 - \frac{\sigma_m}{\sigma_{TS}})$.

So, now that we know that sigma a for a notched component is given by $k_f$ $\sigma_a$ and on the other hand mean stress with this relation, we can simply plug this values here and we will see that instead of $\sigma_a$ we can write $k_f.\sigma_a = \sigma_{fat}(1 - \frac{k_f.\sigma_m}{\sigma_{TS}})$.

So, this is, we can also rewrite this to make it familiar or make it similar with the Goodman relation as something like this, $\sigma_a$ equals to $\frac{\sigma_{fat}}{k_f}(1 - \frac{\sigma_m}{\sigma_{TS}/k_f})$. So, basically, we have a change here $\sigma_{fat}$ is now represented as $\frac{\sigma_{fat}}{k_f}$and $\sigma_{TS}$ is now represented as $\sigma_{TS}/k_f$.

In more simple way, we can write it in this way as $\sigma_a = \frac{\sigma_{fat}}{k_f}(1 - \frac{\sigma_m.k_f}{\sigma_{TS}})$. Now in all this condition for the notch effect to be valid we need to maintain one relation which is that the $k_f$ $\sigma_{max}$. So, that means at the highest value of the applied stress multiplied by the stress concentration and the effective stress concentration factor, this value should be less than the yield strength of the material.

So, we are not considering the plastic deformation here. Because if we have the plastic deformation it is very difficult to numerically assess the behavior. We have seen this how plane stress condition will be active and then all these relations will not be valid anymore. But typically, if this $k_f$ $\sigma_{max}$ is less than the yield strength of the material.

So, that means plastic deformation has not commenced and it has all the possibility to initiate the crack. At that condition because of the presence of the notch with the variation in the mean stress, we can determine how the stress amplitude mean stress and fatigue strength will be related.

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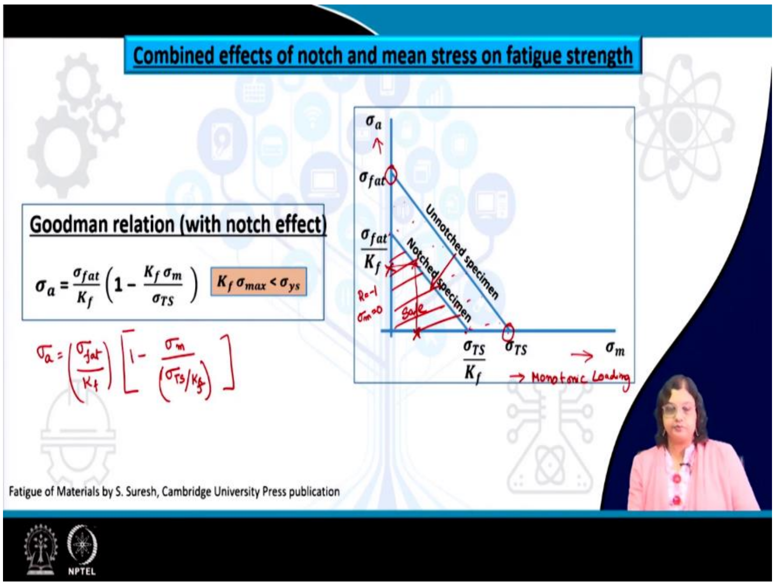


And this is actually determined or this is once again plotted based on the $\sigma_a$ versus $\sigma_m$ curve we have explained this earlier also that sigma m is the mean stress and at this point the x axis which means that $\sigma_a$ is 0. So, basically this means the monotonic loading, so similar to tensile testing. And for that case fracture occurs at the point of ultimate tensile strength of the material.

On the other hand, y axis which is the stress amplitude and where mean stress is 0. So, mean stress could be 0 only when in case of equal magnitude of maximum and the minimum stress but in opposite direction, when R equals to -1. So, under such condition failure will occur at the point of fatigue strength or endurance limit. So, but with the presence of notch what we have seen is that this $\sigma_{TS}$ is now being reduced to this point here $\sigma_{TS}/k_f$.

On the other hand, the fatigue strength is the sigma fatigue by $k_f$ that is what we have seen earlier also that this relation can be written similar like this. So, this is one side the y axis as a limiting value and on the other hand the x axis limiting value will be given by this and what we are seeing here is that this is now the safe space. So, if we are applying the mean value of something like this then this is the limiting value for the stress amplitude that can be applied.

So, we have to be careful and we should know the safe and the unsafe space obviously the safe space has been now reduced by quite some extent compared to the notched versus unnotched specimen and so much of reduction in the safe space has obtained because of this notch in the specimen.

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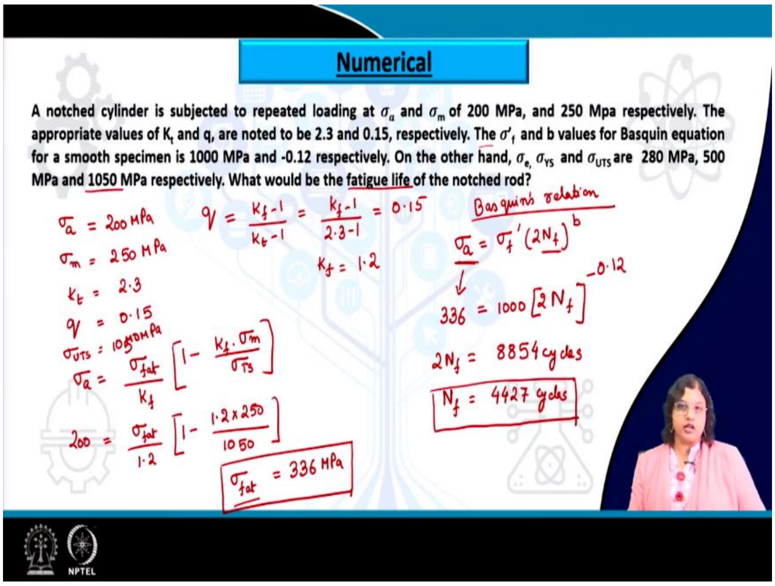


So, this could be better understood, how this understanding can be related to the fatigue strength as well as the fatigue life of the specimen that can be appreciated based on this numerical. So, what it says here is that there is a cylinder which has a notch in it and that is subjected to repeated loading all the parameters of loadings are given here as $\sigma_a$.

And that means a $\sigma_a$ is given as 200 MPa and the $\sigma_m$ is given as 250 MPa, if such is the case because of the presence of the notch the notch which is having a notch concentration factor. So, this is the theoretical notch concentration factor is 2.3 and notch sensitivity factor is given as 0.15.

And the other values are also given here for the Basquin's relation, we will come to that later on. First of all, let us see this if we are trying to apply the modified goodman relation how can we figure out what should be the limiting fatigue strength of the material. So, let us use this relation that we have seen that $\sigma_a = \frac{\sigma_{fat}}{k_f}(1 - \frac{\sigma_m . k_f}{\sigma_{TS}})$.

So, if we try to figure out the sigma fatigue part here, then we can put the other values as 200 $\frac{\sigma_{fat}}{k_f}$. So, we are stuck here we need to find out the value of $k_f$ at the very first instance. Now $k_f$ is related to q as $q = \frac{k_f - 1}{k_t - 1}$. Now $k_t$ is given here already. So, if we plug this value on that gives us q is 0.15. So, that makes $k_f$ the value of $k_f$ turns around 1.2.

Now if we plug this value here instead of $k_f$, now we write 1.2. So, it comes around 1.2 into mean stress is 250 MPa and ultimate tensile strength is given as 1050 MPa. So, you can solve this and what we can obtain from here is that, the value of $\sigma_{fat}$ is given by 336 MPa. So, that is the limiting value of fatigue strength of the material having a notch in it.

You can also do parallelly that in case there is a no notch present there that means that $k_t$ or $k_f$ is not active anymore or we can simply use the actual Goodman relation. What would have been the value of this limiting fatigue strength? So, this you can do as an exercise and you can understand the difference in the value of fatigue strength just with the presence of notch and without the presence of notch.

Now we are not yet done. What next we need to find out, is there life. So, what is given here the next part of the problem is that the sigma f prime remember the fatigue strength coefficient and b which is the fatigue strength exponent, this values for the Basquin's relation is given as 1000 MPa and minus 0.12 respectively.

Now Baskin's relation is actually something like this. So, this $\sigma_a = \sigma_f'(2N_f)^b$. Now one thing I need to clarify here, is that the $\sigma_a$ at this part is the stress amplitude at which fatigue occurs or failure occurs. So, this is nothing but the fatigue strength itself.

This is not just any stress amplitude of a loading cycle but the limiting value of the stress amplitude at which fracture occurs, that is why we consider this $N_f$, f here signifies the failure. We are applying $N_f$ number of cycles at the point of failure what is the value of $\sigma_a$. So, this $\sigma_a$ is sigma fatigue actually.

So, this is what you should understand that the limiting value of $\sigma_a$ at which failure occurs is nothing but the fatigue strength of the material. So, if we are putting this value here as 336 and the value of $\sigma_f'$ is already given as 1000, what we need to figure out is the fatigue life that means $N_f$. So, we do not know the $N_f$ yet but we can find that out.

So, if we plug the other values in this relation you will see that $2N_f$ comes around 8854 point something cycles and to be on the safe side I would like to take this as 8854 cycles. So, that means $N_f$ is half of that which is 4427 cycles. So, overall, what we have seen is due to the presence of a notch in a component the life cycle is actually this one here.

So, once again you can find the values of fatigue strength for the unnotched specimen and then you can plug those values and see that what number of cycles the specimen could have survived in case there was no notch there. And how does a notch of certain $k_t$ value can change the overall fatigue performance of the material. So, this is a very good way to understand and appreciate that.

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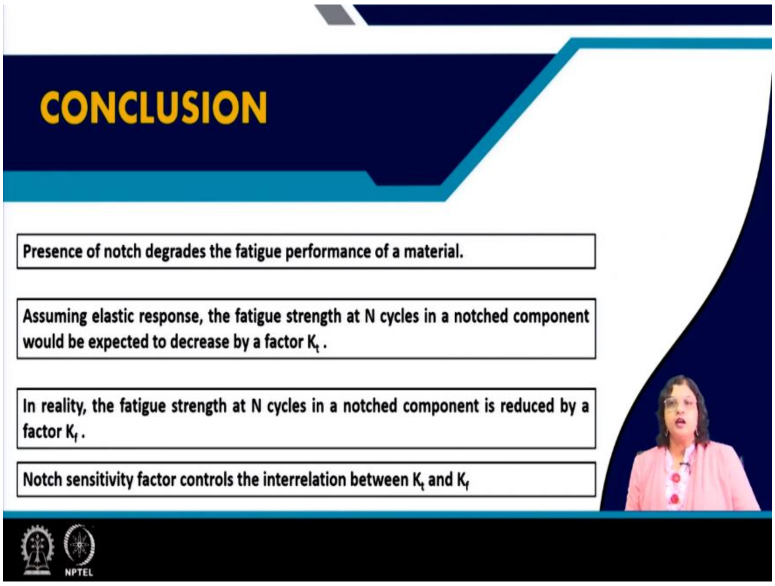


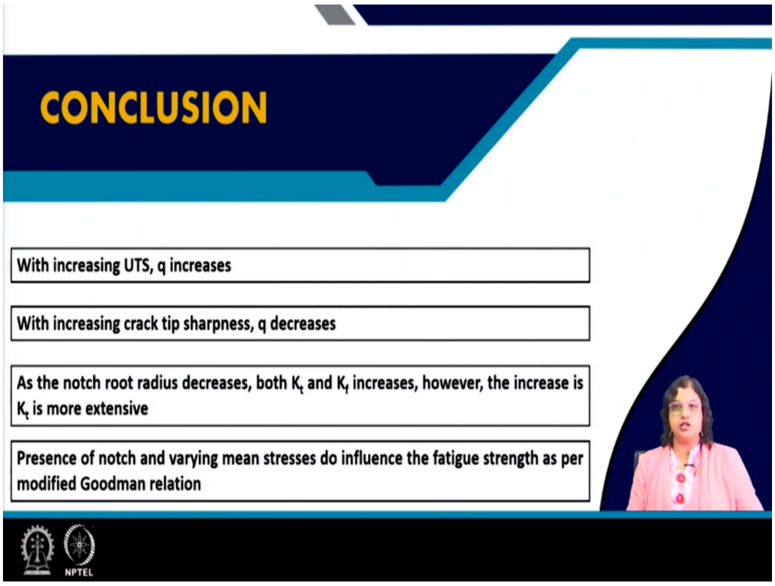


So, let us conclude this lecture with the following points that what we have seen here, is presence of notch certainly degrades the fatigue performance of a material, and considering the elastic response, the fatigue strength at n cycles in a notch component would decrease by a factor $k_t$. But this is not happening in practice rather the fatigue strength is reduced by a factor $k_f$ or the effective stress concentration factor.

And notch sensitivity factor controls the interrelation between this theoretical stress concentration factor and the effective stress concentration factor. And interestingly what we have seen is that q or the notch sensitivity factor increases or is directly proportional to the ultimate tensile strength of the material. Now, this is expected based on the limited ductility as the strength increases.

But more interestingly, what we have seen is that q is directly proportional to the notch tip bluntness. So, more the blunter the notch tip is more effective it will be in degrading the fatigue performance of the material. And this can be explained based on the interrelation between $k_t$ and $k_f$ to crack initiation and crack propagation.

We have seen that as the notch root radius decreases, which means the notch is getting more and more sharp $k_t$ increases and that is primarily related to the crack initiation but $k_f$ on the other hand which is related to not only crack initiation but crack propagation that is not increasing up to that extent as the $k_t$ one.

And overall, we see that q value is coming down because of that as the notch root radius decreases. We have also seen that the presence of this notch as well as varying mean stress do influence the fatigue strength, as well as the life based on the modified Goodman relation and then we can implement that to the Basquin's relation as well to see how the life of the material can change just with the presence of a notch.

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So, following are the references that has been used for this lecture. Thank you very much.