Fracture, Fatigue and Failure of Materials
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Lecture 38
Strain Controlled Fatigue (Contd.)

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NPTEL ONLINE CERTIFICATION COURSES

Fracture, Fatigue and Failure of Materials

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Module 02: Fatigue

Lecture 38 : Strain-controlled Fatigue

Hello, everyone. We are here at the 38th lecture of this course, Fracture, Fatigue and Failure of Materials, and in this lecture also, we are going to discuss some more on the Strain Controlled Fatigue.

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Concepts Covered

Transition Life

Rainflow Counting

The particular topics that will be covered in this lecture are the following. We will be talking about the transition life. When the life changes from the high cycle mode to the low cycle fatigue mode, and we will be also talking about a very interesting method of life estimation known as the Rainflow counting.

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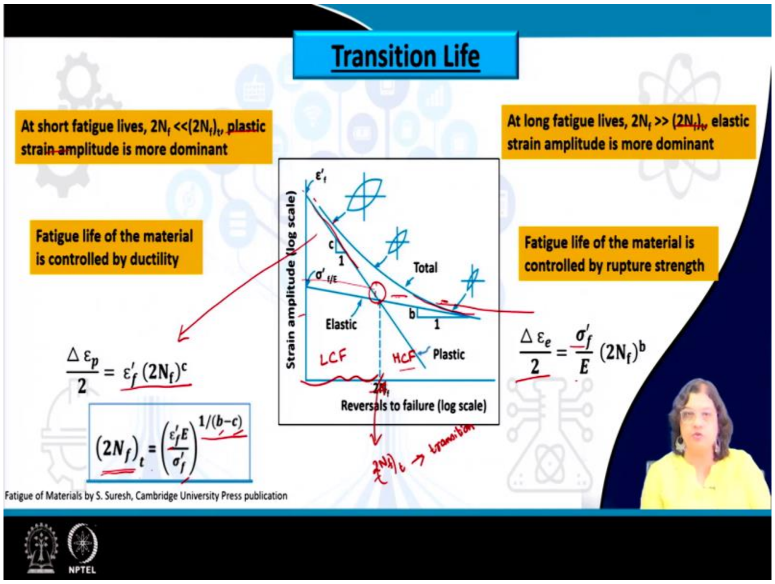

Transition Life

At short fatigue lives, $2N_f << (2N_f)_t$, plastic strain amplitude is more dominant

At long fatigue lives, $2N_f >> (2N_f)_t$, elastic strain amplitude is more dominant

Fatigue life of the material is controlled by ductility

Fatigue life of the material is controlled by rupture strength

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon_f' (2N_f)^c$$

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma_f'}{E} (2N_f)^b$$

$$(2N_f)_t = \left(\frac{\varepsilon_f' E}{\sigma_f'}\right)^{1/(b-c)}$$

Fatigue of Materials by S. Suresh, Cambridge University Press publication

So let us start with the transition life, in the last lecture we have seen that there are two relations covering mostly the elastic strain amplitude and the plastic strain amplitude. And the experimental strain versus life actually follows the combination of both these relations. So we see a curve something like this when the initial part or the, the one when we are applying high strain amplitude, like right here. This mostly follows the Coffin-Manson plot here.

And the relation is given by something like this, $\Delta\varepsilon_p/2$, which is nothing but the plastic strain amplitude, and this is equivalent to ε'_f $(2N_f)^c$, where ε'_f is the fatigue ductility coefficient, and c is the fatigue ductility exponent. On the other hand, when we are talking about application of lower values of strain amplitude, certainly the life of the specimen and the component will increase significantly.

And this is what we are seeing that at this part the experimental curve is typically following the elastic relation, which is given by Basquin's relation which is the elastic strain amplitude, that is equivalent to $\sigma'_{f/}/E$ and $(2N_f)^b$. So the σ'_f is nothing but the fatigue strength coefficient and b is the fatigue strength exponent, whereas e is the elastic modulus.

And the experimental curve actually has a transition from this mode at the very high strain amplitude when it is following the Coffin-Manson relation to the one when lower strain amplitude is been used, and it is following the Basquin's relation. And this transition is happening at this point, the elastic and the plastic curve actually intersects at this point. So that is termed as $2N_t$. So this is the load reversal at which the transition is occurring, that is known as the transition life, and typically signified as $(2N_f)_t$.

Now, we are having two different parts in the life cycle. So first of all we are having this part here which signifies the short fatigue life, which means that the $2N_f$, the load reversal number is lesser than or much, much lesser than this transition life of $(2N_f)_t$. And if such is the case, then plastic strain amplitude is more dominant, it is basically controlled by the plastic strain amplitude, and ductility is the most significant parameter, most significant properties here, to control the behavior, the fatigue behavior of the material.

On the other hand, when we are talking about fatigue life, which is much greater than this transition life of $(2N_f)_t$, at this point, the elastic strain amplitude is more dominant, which means the fatigue life is controlled by the rupture strength of the material. So that is how we are moving from the low cycle fatigue for a higher strain amplitude but lower number of cycles for failure to high cycle fatigue in which case, we are applying lower values of strain amplitude but we are receiving we are obtaining higher life cycle for the component.

So this transition is occurring at this point, which is termed as $(2N_f)_t$. So t here, stands for transition, and as we mentioned that because there are two load reversals in a typical cycle, that is why there is this factor 2, is also here. Now, this transition life which is the intersection of the elastic and the plastic strain amplitude relations, that is the Basquin's and the Coffin-Manson relation, this actually is dependent on all these factors, all these parameters that we can determine from those relations.

So this $(2N_f)_t$, is actually given by this relation which is the ratio of the fatigue ductility coefficient to the fatigue strength coefficient, multiplied by the elastic modulus. And this, to the power of 1 divided by b minus c, where we have already known that b is the fatigue strength exponent and c is the fatigue ductility exponent. So if we have all this information about the fatigue strength coefficient, fatigue strength exponent, that is σ'_f and b and then the fatigue ductility coefficient and fatigue ductility exponent, we should be able to find out what is the value of this transition life.

And based on that, we can determine what is the limit up to which we can consider the LCF, and from, up to what extent, or from whatever is the lower limit for the high cycle fatigue, can be also determined based on this transition life. So there is a way to quantify this number of $(2N_f)_t$.

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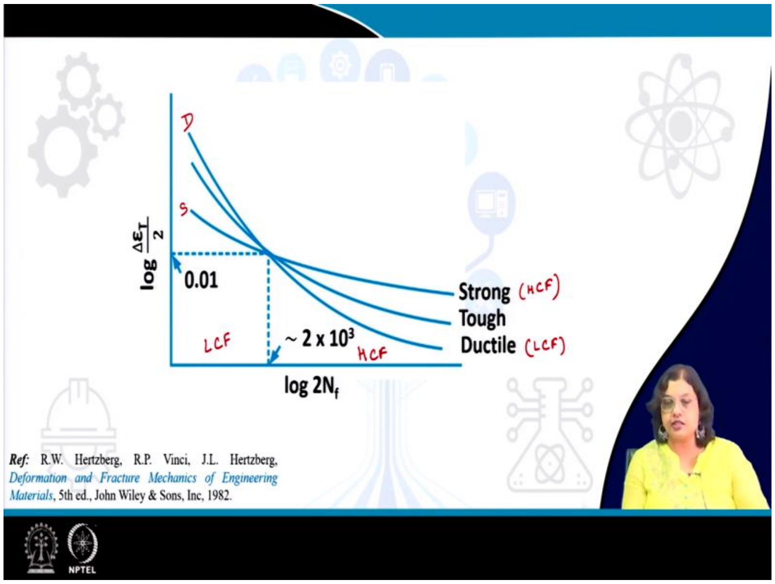

Ref: R.W. Hertzberg, R.P. Vinci, J.L. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 5th ed., John Wiley & Sons, Inc, 1982.

So this is the typical strain amplitude versus number of cycles to failure for three different kind of material, one which is strong, one which is tough and one which is ductile. Now, if we are talking about the low cycle fatigue part, for which we are applying higher strain amplitude, for that case actually, the ductile material, the one which is having the capability to deform to the maximum extent, that kind of material will give us the highest fatigue strength.

And whereas, the strong one in that case, will not be having so much high fatigue strength. So if we are talking about the low cycle fatigue behavior, we know that there will be significant dislocation activities in the material, so the one which is having that kind of capability, the one which is ductile, which is having the dislocation motions already active or which has the ability for the dislocations to move with the application of stress or strain, that kind of material, so the ductile materials will be preferred because it will lead us to higher fatigue strength.

On the other hand, if we are talking about the high cycle fatigue behavior of a material, then the strong one will be favored. Now, the strong one will be the one in which it is very difficult for the crack, for the fatigue crack to initiate, and because it needs to, the bond needs to be broken, if we want to initiate the crack. That is why a strong one will always have a higher fatigue strength, and hence a strong material will be favored for high cycle fatigue, and a ductile material will be favored for low cycle fatigue. So when we are thinking of designing a specimen, we should also keep this in mind.

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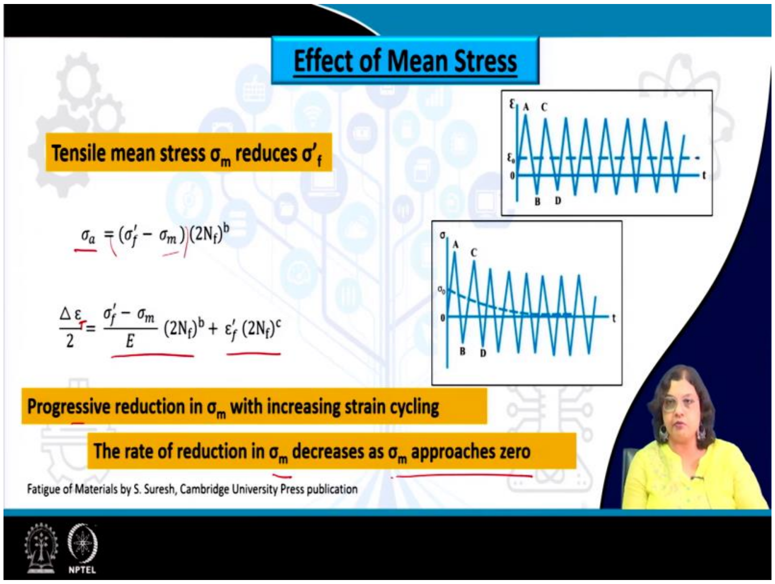

Effect of Mean Stress

Tensile mean stress σ_m reduces σ'_f

$$\sigma_a = (\sigma'_f - \sigma_m)(2N_f)^b$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

Progressive reduction in σ_m with increasing strain cycling

The rate of reduction in σ_m decreases as σ_m approaches zero

Fatigue of Materials by S. Suresh, Cambridge University Press publication

So now that we understand how the fatigue life changes, and how the plastic and the elastic strain amplitude controls the fatigue life. Let us now move on to another interesting fact which means the effect of mean stress. We have already seen in the case of high cycle fatigue that if there are mean stress present in a material, then obviously there is some influence, if we have higher mean stresses versus lower mean stresses versus zero mean stresses, there is certainly some influences on the fatigue strength and overall fatigue performance of the material.

And we have seen how Goodman relation is used to correlate the mean stress with the stress amplitude and fatigue strength and ultimate tensile strength of a material. So here also, mean stress is certainly very important, and particularly, if we are having a positive mean stress, as we can see here, that there is a positive mean stress, this dashed line here signifies the mean stress or the average of the maximum and the minimum stresses, which is quite in the positive direction, above zero.

And this can be possible if we are having some amount of residual stresses in the material, particularly tensile mean stresses, if that is present there, that actually reduces the σ'_f. σ'_f is the fatigue strength exponent that we have seen in the previous relations.

And in such case, if there is a positive mean stress or tensile mean stress, then this σ'_f is being reduced by this factor of the mean stress, that is already existing in the system. And the overall Basquin's relation will be modified like this.

So σ_a now will be equivalent to σ'_f minus σ_m. So this factor multiplied by $(2N_f)^b$. So obviously that that means that the total strain amplitude $\varepsilon_t/2$ that will be given by this modified Basquin's relation plus the Coffin-Manson relation. So that leads to the overall estimation of life quite different, and we have to take care of that. Now because of this presence of this mean stress, actually and the residual stresses in turn, this with cyclic loading, this mean stress actually keeps on decreasing.

So there is a progressive reduction. It keeps on decreasing at every cycle unless it reaches more or less near to the zero. And the extent by which this reduction is happening, this rate of reduction in σ_m, that also decreases as the σ_m approaches zero. So that is how the system neutralizes itself, just with repeated loading, the σ_m whatever positive tensile mean stress is available, that is being reduced continuously with every cycle unless, until and unless the σ_m equals to zero condition is being reached.

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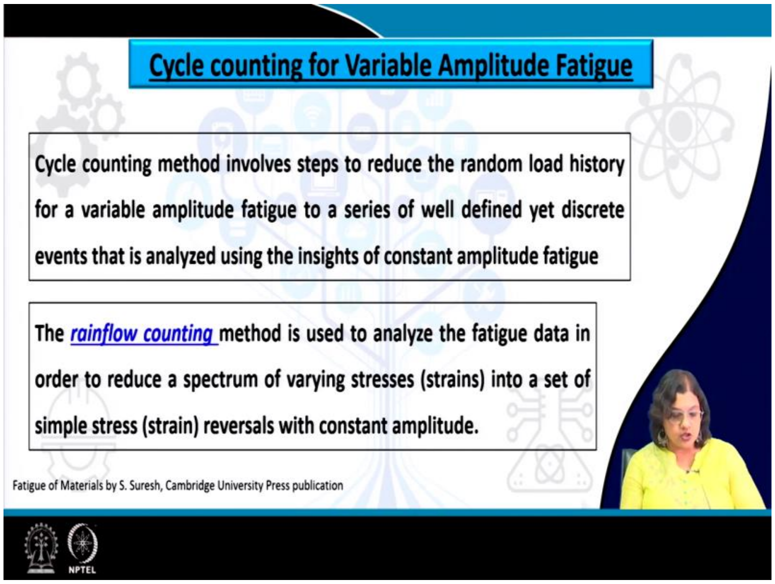

Cycle counting for Variable Amplitude Fatigue

Cycle counting method involves steps to reduce the random load history for a variable amplitude fatigue to a series of well defined yet discrete events that is analyzed using the insights of constant amplitude fatigue

The *rainflow counting* method is used to analyze the fatigue data in order to reduce a spectrum of varying stresses (strains) into a set of simple stress (strain) reversals with constant amplitude.

Fatigue of Materials by S. Suresh, Cambridge University Press publication

So with this, let us move on to the next topic which says about the cycle counting. So we know that there are repetition of loading, there are cyclic loading, and it is very, very essential to count the number of cycles. Now, this had not been a problem so far because we were talking about a particular kind of loading, mostly the strain amplitude or even previously, when we have the stress amplitude these are kept constant.

In case of variable amplitude loading, we have already seen for the case of high cycle fatigue that how Miner’s law is applicable, and we have also done the numerical to see that how the life of the bridges or any other such things can be determined based on this Miner's law.

Now, similarly for the case of the strain controlled fatigue, sometimes the correlation between strain and stress is a little different, particularly because they do not follow the linear relationship between strain and stress anymore when we are talking about the plastic deformation.

And we have now made this clear that when we are talking about strain controlled fatigue there is always some amount of plastic deformation. So certainly the relation between stress and strain is complex, and when we are applying variable amplitude loading or variable strain amplitudes, that can lead to a completely different scenario if we are talking about the stress output.

So we need to find a way to count the number of cycles in such complex cases. And there are different kind of cycle counting method, which actually involves the steps to reduce the random load history if there is a different kind of variable, amplitude loading are being used so that random load history is being reduced to a series of well defined, yet discrete events, and those kind of well defined events can actually be analyzed on the basis of the constant amplitude fatigue.

And one such cycle counting method which is very, very popular and very interesting also is the Rainflow counting method, and that is used to analyze the fatigue data in order to reduce a spectrum of varying stresses and strains to a set of simple stress and strain reversals with constant amplitude.

So we have, we know that there are variable amplitude loading and we just need to divide it into such a way that there, this, we can see that there are some blocks of constant amplitude that makes it easier to analyze the data. So let us see how this is actually done in practice with a particular example.

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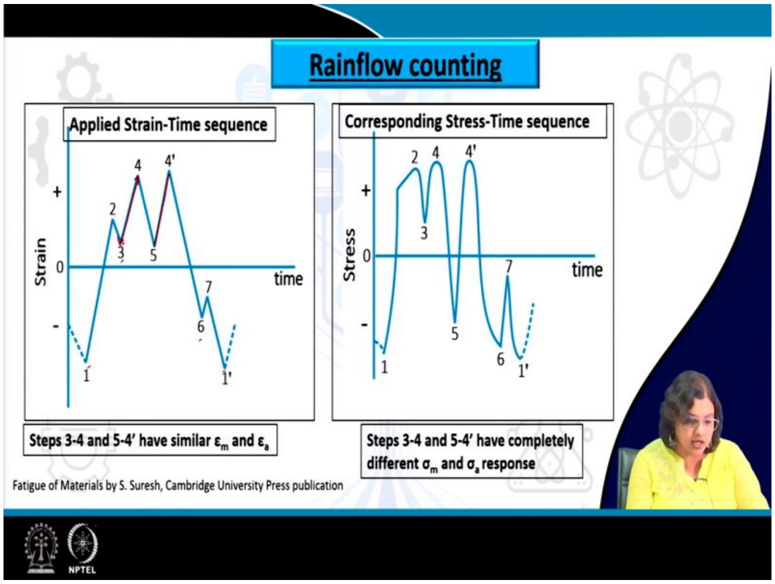

Rainflow counting

Applied Strain-Time sequence

Corresponding Stress-Time sequence

Steps 3-4 and 5-4′ have similar ε_m and ε_a

Steps 3-4 and 5-4′ have completely different σ_m and σ_a response

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So this is a strain time sequence that is being shown here, and what we can see is that there is a compressive strain initially, one step 1, and then there is some positive strain up to step 2 and then again some unloading up to step 3, but it is still the positive value, going on to 4, and then coming down to 5, but still positive value, and then again to 4′.

Now, the magnitude of 4 and 4′ are the same, that is why it is named as prime, and then again some unloading and compressive loading is there, so we have the step 6 when there is a compressive strain available. 7 is having somewhat higher strain than that of 6 but it is still in the compression zone. And then we have this 1′ which is again have the same magnitude as that of 1, but, in this case, that is why this is named as 1′ and so on.

Now, if we look into the corresponding stress response of this, so stress time response, this is not so straight forward. We see that there is, certainly this kind of parameters are mostly maintained that 1 is a negative 1 and because we are talking about compressive strain, so that will certainly lead to a compressive stress response also but not that it is following exactly the same way in which the strain-time sequence has been maintained or controlled.

Particularly, if we are looking this in a quantitative manner, you can see that step 3 to 4, so this span of strain is same as 5 to 4′. And they have similar values of mean and the amplitude of strain. On the other hand if we are looking for this to particular step, you can see that 3 and 4 and 5 to 4′ are absolutely different. So the mean stress as well as the stress amplitude are completely different for these two cases.

So this obviously, gives us an indication that whatever method we are used for strain-time, the number of cycles in this case may not be directly applicable to the stress-time response because they are not following this linear relationship. So how do we do that?

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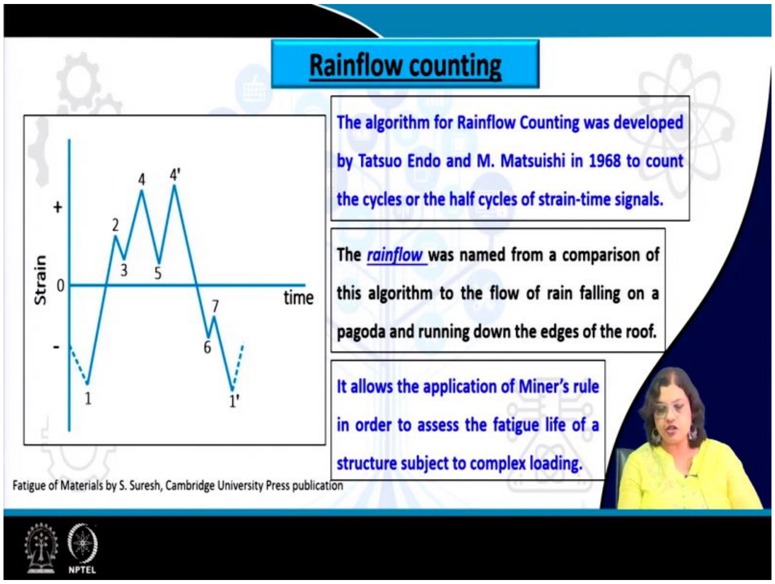

Rainflow counting

The algorithm for Rainflow Counting was developed by Tatsuo Endo and M. Matsuishi in 1968 to count the cycles or the half cycles of strain-time signals.

The *rainflow* was named from a comparison of this algorithm to the flow of rain falling on a pagoda and running down the edges of the roof.

It allows the application of Miner's rule in order to assess the fatigue life of a structure subject to complex loading.

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Now, we apply the rainflow counting by using a method which is very, very interesting. So let me discuss this first of all a little bit history, that this algorithm for Rainflow counting was developed by two famous scientists Endo and Matsuishi in 1968, so long back, to count the cycles or the half cycles of strain-time signals.

And how this is being done that is based on the principle of rain falling on the pagoda roof. So pagoda is the, the typical temple like patterns or the temple like structures that we see, and the rains running from this roof of the pagoda is from which this concept has been taken. So how it is done, it also implements the Miner's rule to assess the fatigue life of a structure subjected to complex loading scenario.

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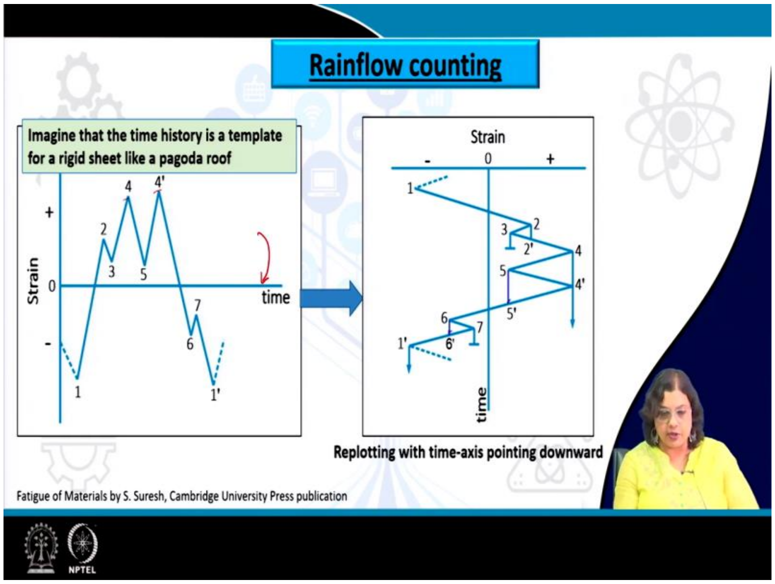

Rainflow counting

Imagine that the time history is a template for a rigid sheet like a pagoda roof

Replotting with time-axis pointing downward

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So initially when we have the strain time history it is considered like, like each of this peak, we can consider as the roof of the pagoda structure. You can very well imagine that like this is a pagoda structure, like that, in this temples, and then these are the roof, and if there are rain, that is supposed to fall down.

To understand it in a more lucid way, it is actually turned 90° such that the time is now facing downward, so we have simply turned or rotated it by 90°. And now we have exactly the same level, same steps as shown here 1, 2, 3, 4, 5 etcetera, whatever is there, but we have some additional steps to that which I am going to explain soon.

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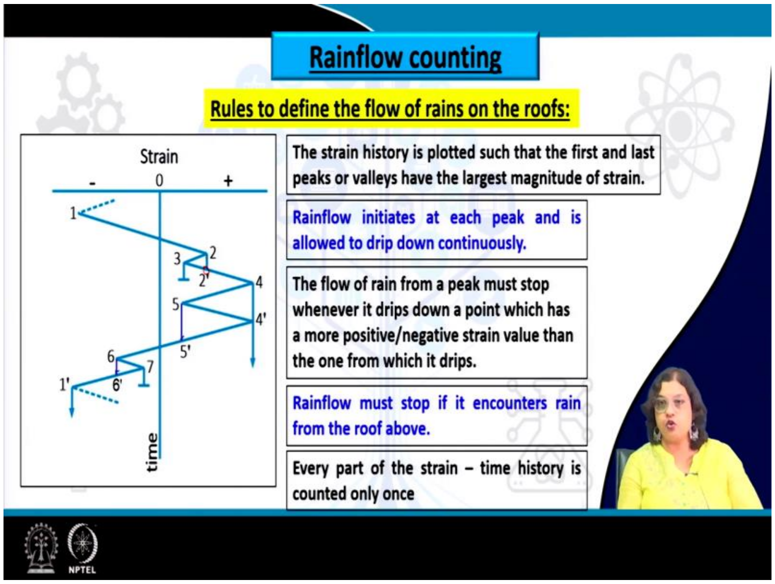

Rainflow counting

Rules to define the flow of rains on the roofs:

The strain history is plotted such that the first and last peaks or valleys have the largest magnitude of strain.

Rainflow initiates at each peak and is allowed to drip down continuously.

The flow of rain from a peak must stop whenever it drips down a point which has a more positive/negative strain value than the one from which it drips.

Rainflow must stop if it encounters rain from the roof above.

Every part of the strain – time history is counted only once

So before that, let us set the rules first, to define the flow of rains on the roof. So when we do that, when we do such kind of plotting the strain versus time, and then turning it to 90°, we have to make sure that the strain history is plotted such that the first and the last peak or valleys, whatever it is, the high point or the lower point, that should have the largest magnitude of strain. Otherwise, the rain will not flow at all or will not fall at all.

So if that is the case, then Rainflow initiates at each peak and it is allowed to drip down continuously. The flow of rain from a peak must stop whenever it drips down a point which has a more positive or negative strain value than the one from which it drips. So by dripping actually, let us say this 2 to 2′, that is a kind of dripping that can happen. And Rainflow must stop if it encounters rain from the roof above.

For example, if it is coming from 2 to 2′, and it there is also from 3 to 4, so there is an encounter at this point, then it has to stop. And every part of the strain-time history should be counted only once, so there should not be any repetition of this thing. It may still sound confusing to you, but if, in the next slide if, we are discussing each of these steps and see that how these are progressing, this will be very, very interesting. I hope you will find this interesting.

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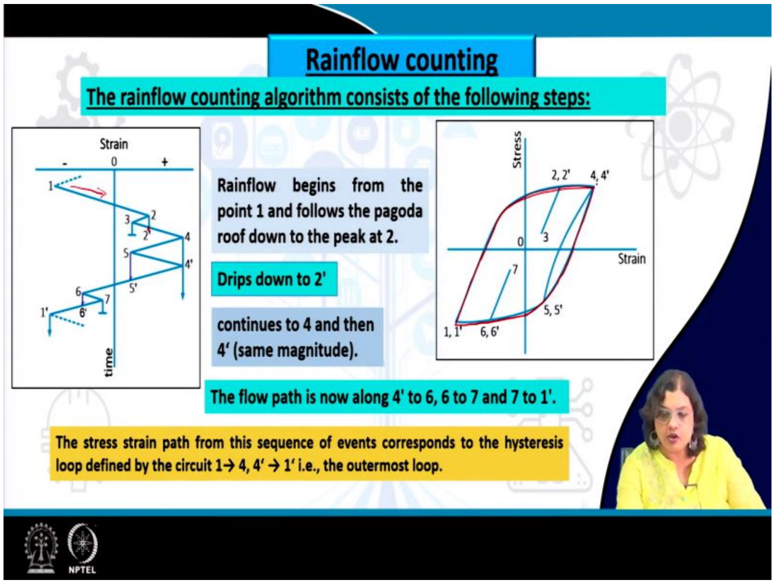

Rainflow counting

The rainflow counting algorithm consists of the following steps:

Rainflow begins from the point 1 and follows the pagoda roof down to the peak at 2.

Drips down to 2'

continues to 4 and then 4' (same magnitude).

The flow path is now along 4' to 6, 6 to 7 and 7 to 1'.

The stress strain path from this sequence of events corresponds to the hysteresis loop defined by the circuit 1→ 4, 4' → 1' i.e., the outermost loop.

So let us start with the point 1 here, the step 1. Rainflow begins from the point 1 and follows the pagoda roof to the peak 2. So you see when, if you are imagining some rain here, or rain drops, that will certainly flow along this direction because that is how the nature of the slope is, which will make it easier for slide down from 1 to 2.

And if it does so, at the point of 2, it will automatically drip down to 2′, that is why again we get another point at 2′, which was initially not there at the initial strain versus time history. So if that is the case, the next thing is that it continues up to 4 and from 4, it again drips down to 4′.

The flow path is now along 4′ to 6. So it is passing through this 5′ also, to 6 and then 6 to 7 and 7 to 1′. And in between, wherever there is this possibility of dripping, they are also dripping to 6 to 6′ and 5 to 5′, and so on. So the stress strain path from this sequence of events corresponds to the hysteresis loop.

So basically we are, for each of this path, actually we are defining the hysteresis loop. Strain versus time, that is what was leading us to the corresponding stress response, and if we are having the stress and the strain, we know for strain control fatigue, that is what is leading us to the hysteresis, stress-strain hysteresis.

And if we have the stress-strain hysteresis, we can have the rest of the relations, all the relations for the elastic strain amplitude, the plastic strain amplitude can be applied to it and we can determine the, the life cycle based on the Basquin's or the Coffin-Manson relation.

Now, for this overall path, how it goes from 1 to 2 and then 2 to 2′ to 4 to 4′ and then 6, 7, 1′, all this path can actually lead to this hysteresis loop defined by the circuit like this. 1 to 4 or 4′ to 1′. So basically, this outer loop of this hysteresis. So let me draw this in red for your understanding.

So this is what we are typically obtaining if we are considering this Rainflow mechanism. You can tally it yourself that how this is moving from 1 to 2 and then 2 to 2′. Basically, this is the same point, and then to 4 and 4′ and then to 5 and 5′ and 6 and 6′ and 1 and 1′.

So although it looks quite simple, but it was not appearing so simple if you are looking for the strain-time. By using this Rainflow counting method, you can at least have this outline of this hysteresis loop that can be obtained.

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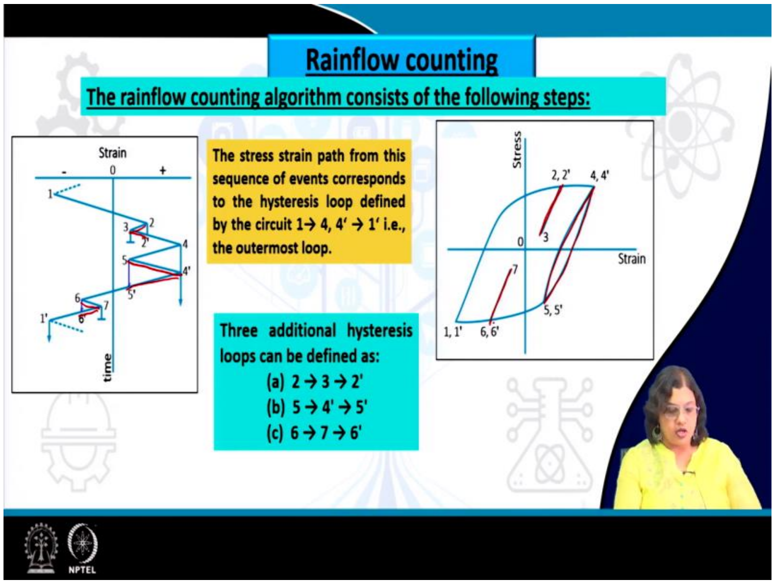

Rainflow counting

The rainflow counting algorithm consists of the following steps:

The stress strain path from this sequence of events corresponds to the hysteresis loop defined by the circuit 1→ 4, 4′ → 1′ i.e., the outermost loop.

Three additional hysteresis loops can be defined as:

(a) 2 → 3 → 2′

(b) 5 → 4′ → 5′

(c) 6 → 7 → 6′

Now we are not done here, this is the just the outer loop that we have obtained, and there are three additional hysteresis loops that can be defined as 2 to 3 to 2′. So basically this section here, 2 to 3 to 2′. So that is signified by this thin hysteresis here. So 2 to 3 to 2′. Coming down to 3 and moving back to 2′, so that forms a hysteresis.

And 5 to 4′ to 5′. So here 5 to 4′ and to 5′. So that is being shown here, so 5 to 4′ and that to 5′. So that is a separate hysteresis loop, and then once again we have this one here which is 6 to 7 to 6′. So we can see here again another thin hysteresis, 6 to 7 to 6′.

So this is not a straight line. Since we are moving back and forth, that does consist of a hysteresis that we need to take care of. So eventually we are having then the four hysteresis loops here. One is the outer one, so 1, 1′, 2, 2′, 4, 4′, 5, 5′, 6, 6′, and then we have this additional 2, 3, 2′ 4, or rather 5, 4′, 5 prime, and then 6, 7, 6′.

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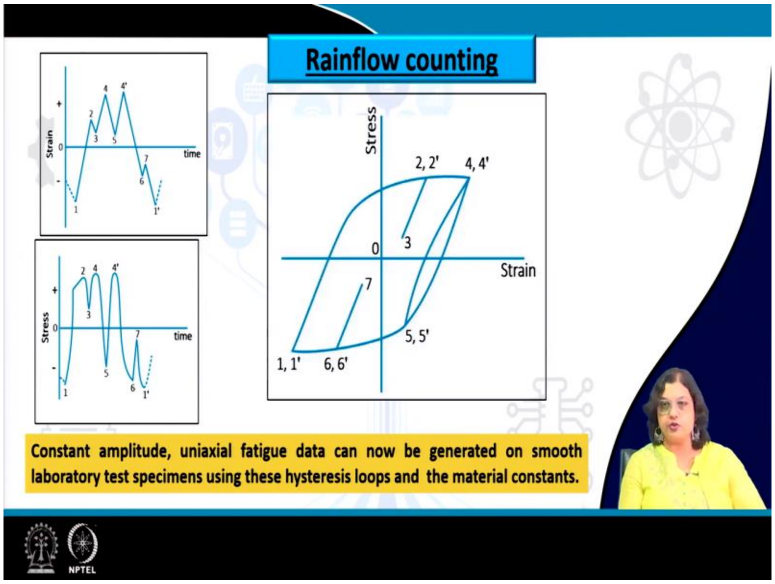

Rainflow counting

Constant amplitude, uniaxial fatigue data can now be generated on smooth laboratory test specimens using these hysteresis loops and the material constants.

So this kind of four different hysteresis loops can be obtained and we can then apply the, the constant amplitude condition for each of the hysteresis loop for the uniaxial fatigue results, can now be generated on smooth laboratory test specimens using these four different hysteresis loop condition.

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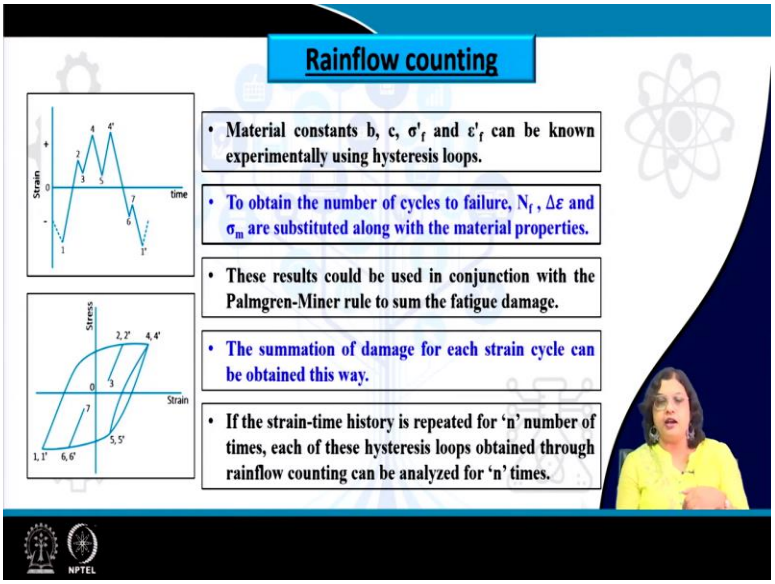

Rainflow counting

- Material constants b, c, σ'_f and ε'_f can be known experimentally using hysteresis loops.
- To obtain the number of cycles to failure, N_f, $\Delta\varepsilon$ and σ_m are substituted along with the material properties.
- These results could be used in conjunction with the Palmgren-Miner rule to sum the fatigue damage.
- The summation of damage for each strain cycle can be obtained this way.
- If the strain-time history is repeated for 'n' number of times, each of these hysteresis loops obtained through rainflow counting can be analyzed for 'n' times.

And now, the most interesting part is that we can, for each of these hysteresis loop basically, we can determine the b, c. So those are the fatigue strength exponent, fatigue ductility exponent, fatigue strength coefficient, fatigue ductility coefficient, so all those parameters can be determined for each of these loops.

And to obtain the number of cycles to failure, N_f, we can also find out the, the strain range or the strain amplitude to total strain amplitude and all the other parameters from the based on the materials properties. And then, we can also employ the Miner's rule here to sum the fatigue damage.

The summation of damage for each strain cycle can be obtained in this way, and we can repeat it for let us say, if the entire strain-time history is being repeated for n number of times, so that means that each of these hysteresis loops will be repeated for n number of times. So in this case we have seen that there are four different hysteresis loop.

For each of this hysteresis loop we can find out the all the parameters and then each of the hysteresis loop should run for n number of times to get the final life cycle determination. So that would be possible if we are employing this Rainflow counting method.

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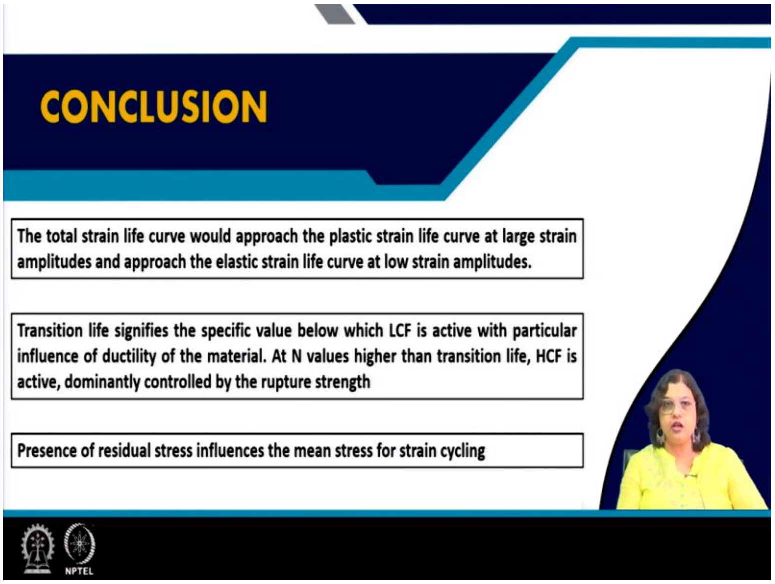

CONCLUSION

The total strain life curve would approach the plastic strain life curve at large strain amplitudes and approach the elastic strain life curve at low strain amplitudes.

Transition life signifies the specific value below which LCF is active with particular influence of ductility of the material. At N values higher than transition life, HCF is active, dominantly controlled by the rupture strength

Presence of residual stress influences the mean stress for strain cycling

So based on this, let us conclude this lecture with the following. The total strain life curve would approach the plastic strain life curve at large strain amplitude, and we can obtain the elastic strain life curve in case we are applying low strain amplitude. Basically, transition life signifies a specific value of $2N_f$.

So if we are having the value of N_f being lesser than this transition life, then LCF is active, and we have known that low cycle fatigue means when the life cycle is around 10^3 to 10^4 number of cycles, and in this case the ductility of the material is a prime importance, and that signifies the fatigue or that controls the fatigue performance of the material.

On the other hand, if the number of cycles to failure exceeds the transition life, then we can say that that is equivalent to the high cycle fatigue behavior of the material, and that is something like of the order of 10^6 to 10^8, particularly 10^7 and in such cases the fatigue performance is particularly controlled by the rupture strength of the material.

So, so far we have seen that low cycle fatigue and high cycle fatigue, first of all these two terms, low and high are relative term, and then we also have defined based on the number of cycles that if low cycle is 10^3 to 10^4, high cycle is 10^6 to 10^7, but again we were not sure that how this are being based on, what is the basis for considering such number of cycles for low or high.

And now, we understand that it is based on this transition, and this transition from the elastic strain amplitude being the dominant one or the plastic strain amplitude being the dominant one is actually controlling the low cycle of the high cycle fatigue behavior. And we have also seen that how a strong material would be favoured for high cycle fatigue condition.

But if we want to employ a material for low cycle fatigue condition, then we should better use a ductile material because that will have higher fatigue strength. There is an effect of mean stress on the overall fatigue performance of the material, that has been also seen from this lecture.

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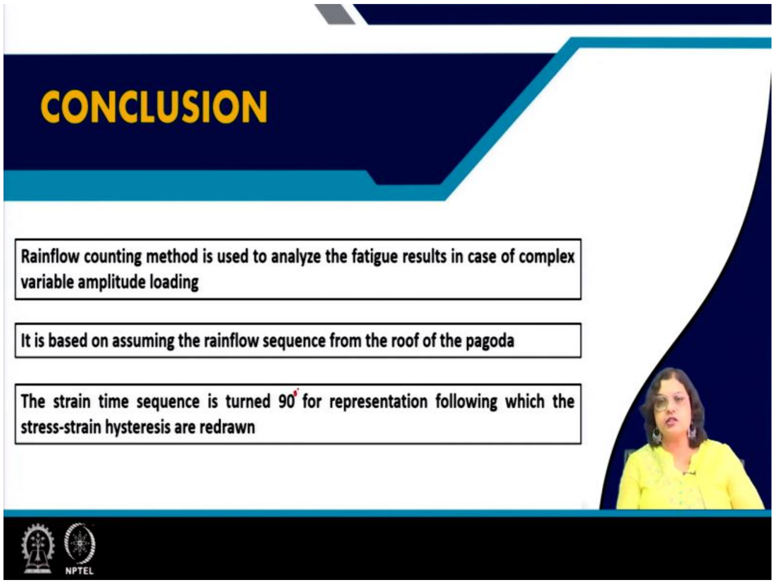

CONCLUSION

Rainflow counting method is used to analyze the fatigue results in case of complex variable amplitude loading

It is based on assuming the rainflow sequence from the roof of the pagoda

The strain time sequence is turned 90° for representation following which the stress-strain hysteresis are redrawn

And we have also discussed about a very interesting method of Rainflow counting by which the fatigue results in case of a complex variable amplitude loading is being simplified to cases where we can employ the constant amplitude loading conditions. We have seen how a strain-time history, which is not exactly corresponding to a similar kind of stress-time history, that can lead to formation of four separate hysteresis loops.

And for each of these hysteresis loops, we can employ all the relations that we are aware of and we can determine the fatigue performance of the material. Now Rainflow counting, the name comes from and the basic assumptions is based on the rain, the sequence of rain that flows on the roof of the pagoda. And the strain time sequence is turned usually to 90° for representation and that, let us understand how the stress strain hysteresis are redrawn.

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So following are the references that are used for this lecture. Thank you very much.