Fracture, Fatigue and Failure of Materials
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Lecture 35
Strain Controlled Fatigue (Contd.)

Hi there, so we are at the 35th lecture of this course Fracture, Fatigue and Failure of Materials and in this lecture we will be introducing another concept on fatigue which is the strain controlled fatigue.

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So the concepts that will be covered in this lecture are the following. We will be talking about the cyclic strain controlled fatigue and stress strain hysteresis that are generated from there and what we can understand from there, basically the hardening or the softening of the behavior of material based on the cyclic loading that is what we can understand from the cyclic strain control fatigue and that will be also discussed in this lecture and finally we will have a look on the cyclic stress strain curves.

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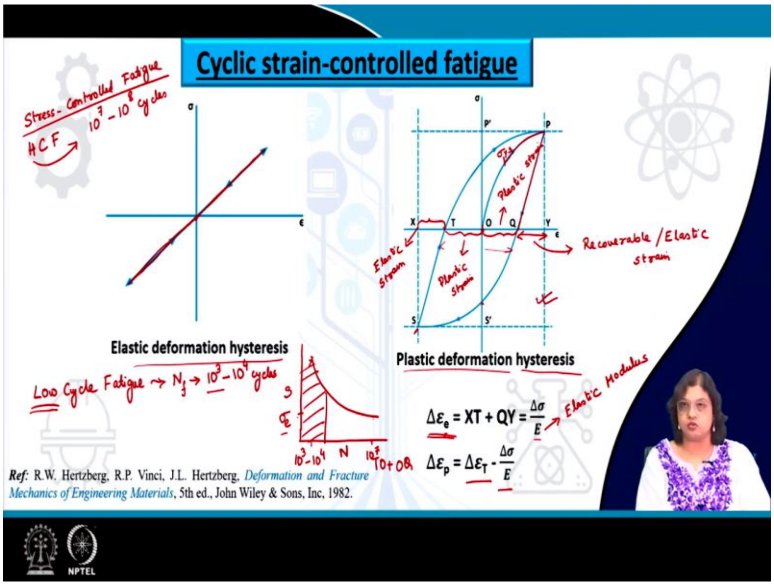

So when we are talking about cyclic loading or repeated loading so far while considering the stress control fatigue we have seen that the fatigue strength of a material can be achieved even at a stress amplitude value which could be lower than the yield strength of the material so that manner specimen is supposed to undergo only elastic deformation and there should not be any failure or any plastic deformation at all.

So if such is the case we know that if there is a stress versus strain plot for the case of elastic loading we typically get a stress straight curve which will be coming back to its original position if we want to unload or if we are doing this on the compression side that is on the fourth quadrant there also we will see that if you are compressing it up to certain extent up to the within the elastic limit and then if we are unloading coming back to the zero load it should also come back to the origin without any significant changes in the values or in the nature of deformation and as such.

So this is typically the elastic deformation hysteresis which is nothing but a straight line but things will change, if we are applying stress above than the yield strength of the material, so this is what we typically employ for the case of cyclic strain control fatigue when we say strain control fatigue we typically mean that we are applying the stress level exceeding the yield strength of the material so that means that there will be some amount of permanent deformation in the material.

And the exact strain that the specimen is encountering that is being determined and based on that we control the strain parameter and the response of the material in turn is being determined and analyzed. So this one here shows a typical stress strain hysteresis of specimen when it is undergoing plastic deformation, so let us say we are starting from the origin from the point O and at some point here the material is yielding so this could be the yield strength of the material and we are applying stress beyond this value so that means that it is having some amount of permanent deformation and this could be apparent if we are unloading it.

So if we are unloading it basically unloads following the elastic slope and that means that some part can be recovered so this is actually recoverable strain, now recoverable strain is nothing but the elastic strain of a material so that part Q to Y that is the recoverable strain on the other hand O to Q this is the non recoverable strain or this we can term as the plastic strain.

And if we are coming back to the zero loading scenario at the point of Q and we try to impose the compressive load then from Q to S' it is still having some amount of deformation and then it is moving forward up to the point S so that the yielding is happening here and then it is moving up to the point S where there is only permanent deformation and if we are unloading again back to the zero load as per the elastic hysteresis it should have come to the origin but in this case once again we are seeing the part which are being recovered.

So this XT is the elastic strain and there are some part T and O, so this part is basically the plastic strain, now elastic strain is pretty straight forward, so elastic strain range this is how it is being denoted, the delta signifies the range that we have already seen and the small subscript e, e of the small letter this signifies the elastic strain, so the range is coming around XT plus QY and this can be represented as the stress range divided by the elastic modulus.

So this is as per the Hooke's law, we know that the elastic part follows the Hooke's law so that means the total elastic strain range is given by the total stress range divided by the elastic modulus of the material so that is quite straight forward, let me also write it down here as the elastic modulus.

The plastic part on the other hand is TO plus OQ so this plastic strain range is given by TO plus OQ or we can simply say as TQ so that can be determined based on the total strain range minus the elastic strain range that is what is the plastic strain range and we often determine this based on the the elastic strain range because that is quite easier to estimate following the Hooke's law and considering the total strain range we can determine the plastic strain range as well.

So that is the typical loading scenario when we are talking about the strain control fatigue and what is very important to understand here is that in comparison to the previous lecture so far on fatigue when we were talking about the stress control fatigue we have shown how the SN curves are generated and the fatigue strength can be determined.

So if we are talking about S as the stress amplitude and N as the number of cycles we get a curve something like this and based on the number of cycles as 10^7 or this could be 10^6 we can determine the fatigue strength or the endurance limit, let us name this as σ_e, so that means that and we have also seen that this fatigue strength is less than the yield strength of the material,so as I mentioned that it is within the elastic strain range.

But that does not mean that at every point of this curve this SN curve we applied only the elastic strain we have seen that we start the test at some values which is typically 50 to 60 percent of the ultimate tensile strength of the material certainly exceed the yield strength of the material and for that we receive, we obtain very very lesser number of cycles so from the SN curve we are mostly interested for this endurance limit and the number of cycles at any particular stress amplitude.

Now when we are talking about the plastic strain controlled fatigue or the typically the strain control fatigue we are talking about the part when there is some amount of plastic deformation which is certainly this part of the curve, so here the strain range that is of interest is 10^3 to 10^4.

So let me write this down clearly this is what is known as low cycle fatigue when the number of cycles to failure so N_f is typically within the range of 10^3 to 10^4 cycles, so that is what is low, the part low here means lower number of cycles around 10^3 to 10^4 we have seen for the case of stress controlled fatigue. We mostly look for the SN curve or we mostly determine the high cycle fatigue part and this high stands for 10^7 to 10^8 or sometimes 10^6 number of cycles also, so that is the major difference.

Now if we are telling that this is low cycle fatigue certainly it fails at a lower number of cycles now 10^3 only so that means that we must have applied higher stress amplitude that is why it has failed for lower number of cycles and that means that higher stress amplitude can be well explained if we are talking about in terms of strain in the sense that it is having plastic strain or not.

In case we are not applying plastic strain it can achieve life as high as 10^7, if we are applying permanent or plastic strain it can survive only for lower number of cycles of 10^3 or 10^4, so that makes strain control fatigue very very important if we are talking about the low cycle fatigue and then we always have such kind of hysteresis for stress and strain which is of interest.

Now things are not so straight forward here what we need to understand is how the stress is generated in a material if we are controlling the strain it is not always obvious that the corresponding stress values at any particular strain will follow the same behavior as it would have been for the case of the elastic loading so the linear kind of behavior.

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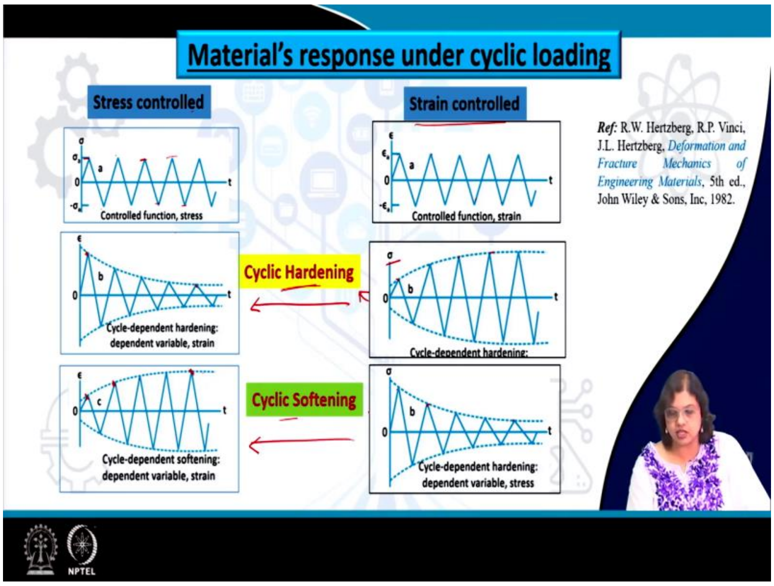

So lets see here how the materials response is under cyclic loading when we are applying higher amount of stress or permanent deformation in the material, so in case of the stress controlled one so far we have seen about the stress controlled so let us talk about this first, so the stress controlled one means that we are applying stress of certain magnitude, so magnitude which is known to us we are asking the machine to apply a stress value of lets say +100 MPa and -100 MPa and then again +100 MPa, -100 mpa and we keep on repeating this for n number of cycles.

Now there could be two ways in which the strain can be represented, strain can have a different values at every cycles so it can either have a lower value at every cycle so it keeps on reducing with cycles, had it been elastic loading the value of strain should have been the same and the stress and strain can be simply correlated with the Hooke's law but this is not the case here and based on the materials particularly we are talking about the metallic systems depending on the material's properties, different other parameters we can see either a reduction in the strain at every cycle.

So please remember that we are still applying the same magnitude of load in this case both positive and negative load of the same magnitude every cycle, so ideally we should have received or we should have obtained the same value of strain at every cycle whatever it is but we are not seeing that, rather the strain is reducing at every cycle in this case so this is a typical example of cyclic hardening.

If we have done a simple monotonic test or the tensile test we have would have seen only this value but since we are doing this test under repeated loading we see that after N number of cycles the strain value has come down from this level to that level, so this is because the material is getting hardened because of the cyclic loading, there could be the another case in which the strain is increasing at every level, so this is the first cycle when we are seeing a lower value of stress and the next cycle a little bit higher value and so on it keeps on increasing and that is why we call this as cyclic softening.

So the strain is increasing at every cycle although in both these cases we are applying the stress of the same magnitude for each cycle. Material can either show a cyclic hardening behavior or a cyclic softening behavior or a mixed mode kind of behavior often, so let us see how the behavior would happen if we have used the strain control mode as we typically do this for the low cycle fatigue behavior.

So in the strain control mode again we are applying same values of strains positive and negative at each cycle, so let us say we are talking about a strain value of +1 % and -1 % and then again +1 %and – 1 % and so on, and we keep on cycling that with the same value of the strain amplitude, however the stress, the corresponding stress response could be different and in this case we are seeing in the first case that the stress output is continuously increasing at every cycle.

So for this is considered as cyclic hardening as the stress value is increasing. So this is same as what we have seen for the case of strain, the stress controlled response also when we were talking about the strain being reducing at every cycle, similarly there could be another case in which the stress response is decreasing at every cycle and this is known as cyclic softening as we have seen similar kind of behavior in which the strain response is increasing had we done the stress control mode.

So this is important to understand that what we are controlling and what response we are getting, in case we are controlling the stress the stress value is not supposed to change the strain values will change and then depending on whether the strain is decreasing we can call this a cyclic hardening or if the strain is increasing we can call this a cyclic softening.

Similarly, for the case of strain controlled one since we are applying the same level of strain, here the important parameters that we want to look at is the stress and if the stress values are increasing at every cycle that is typically the cyclic hardening behavior and if the stress values are decreasing at every cycle that is nothing but a cyclic softening behavior.

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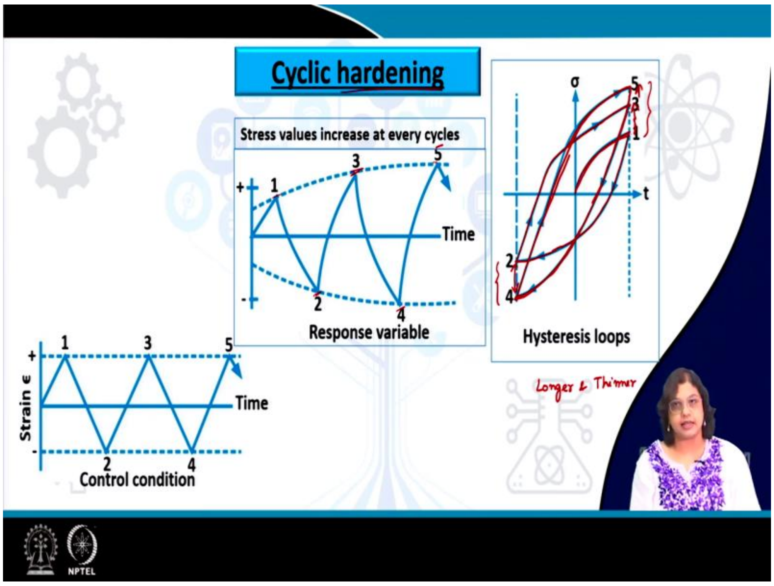

Now, let us see how this is reflected in the hysteresis loop also. So as I mentioned for the strain controlled one this is the stress output in this case for the case of cyclic hardening, the stress is increasing at every cycle so we are talking about three cycles here 1, 3 and 5. So 1, 2, 3, 4, 5 and we can see that the stress values in both the positive and the negative directions are higher from the the previous values and that is nothing but a cyclic hardening.

So the hysteresis loop will look something like this, it starts from the 0 it moves to 1 and so this is the first cycle 1 and then the second cycle so that means also in the compressive side this is the first cycle here but if we are repeating the test the next one that is cycle 3 is having a value higher than that of 1 by this much magnitude, similarly the 1 the step, the compression cycle will have this magnitude higher than that of 2 and so on, for the case of fifth cycle also we are seeing that there is some extent in the enhancement of the stress values for the level 5.

So if we are comparing this from 1 to 5 so only for this five number of cycles we can see some discrepancies in the values of stress so from the starting one to the final one, overall if we look into the stress strain curve carefully we will see that although we initially started with this curve here the cycle 1 and 2 we end up having a thinner and longer hysteresis loop.

So if we are simply joining 4 to 5 we can see that the hysteresis loop is getting longer and thinner. So this is just an approximation if we are looking at the hysteresis loop and seeing that it is getting slick and at the same time it is elongating in the total dimension that can be considered as cyclic hardening.

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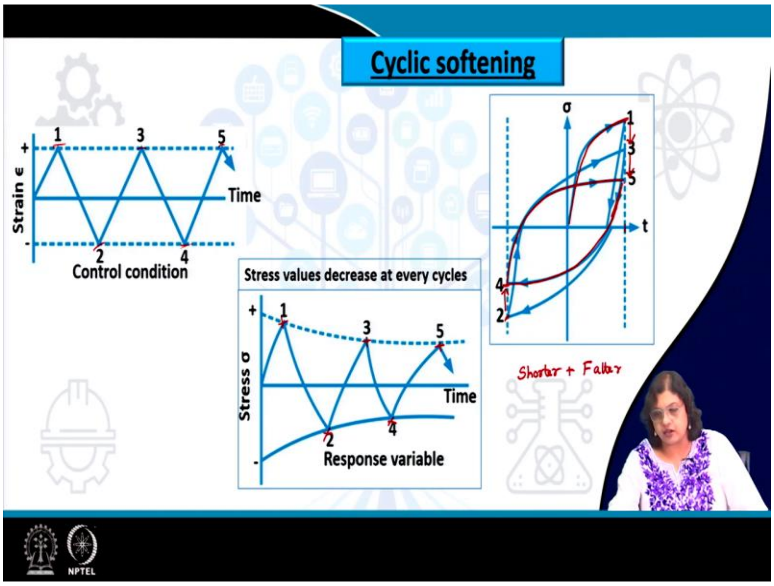

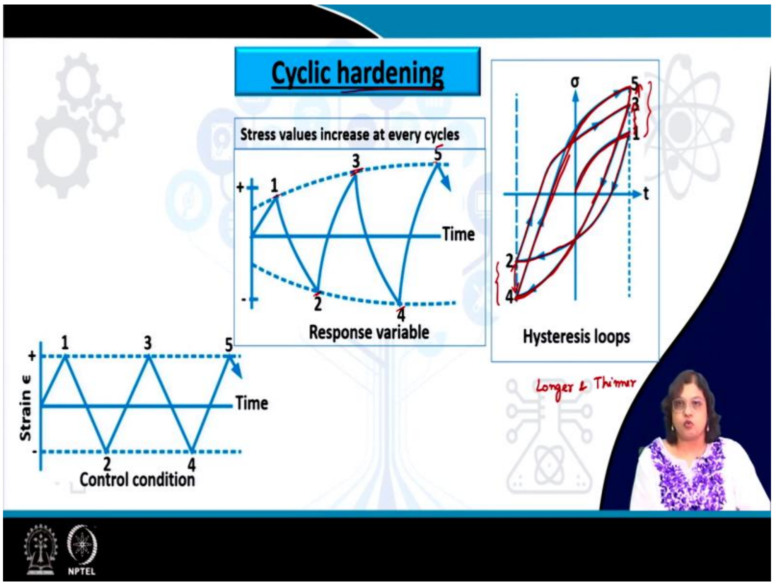

On the other hand, if we are talking about the softening part under cyclic loading this is how it looks like, we are controlling the strain here once again so at every cycle 1, 2, 3, 4, 5 we are applying the same magnitude of strains, positive one along 1, 3 and 5 and negative one along 2 and 4 but the stress response will be something like this when the stress values are decreasing at every level.

So we are having some stress value in the positive part at 1 and at 3 as well as at 5 and we can very well see that the stress level for 3 is less than that of 1, 5 is less than that of 3 and so on, similarly for the compression side also we can see that the stress level for cycle 4 is less than the stress level for cycle 2 and in the hysteresis loop this is also being reflected so we started here with this line here with the first cycle and we can see that if we are doing it under compression it has achieved this maximum value of 2 but in the next cycle, for cycle 3 we have seen that there is some reduction in the stress level.

Similarly, for the cycle 4 we can see that the stress level for 4 is less than that of 2 and so on the stress level for 5 is even lesser than that of 3, so once again if we are talking about among these five cycles if we are talking about the final cycle here so let us then just join these stresses here so 4 to 5 and what we can see here in this case the hysteresis is getting shorter and fatter as if it has been swollen, so that signifies the cyclic softening that strain is being increased and if the strain is being increased to maintain the volume constant the stress actually being decreased.

But at every point we also have to be careful that you see this dashed line, these dashed lines are a particular one this is because the strain at any point is being maintained constant and the response we are seeing only in terms of stress, so strains are practically not increasing or decreasing or changing for that matter because we are controlling the strain but the stress is so in case of softening instead of enhancement in the strain what we are seeing this is reflected as reduction in the stress level, so shorter and fatter hysteresis loop can signify about cyclic softening.

But we should not be stuck with this term shorter and fatter for cyclic softening or longer and thinner for cyclic hardening because these are also relative, shorter with respect to the initial one, so with respect to 1 and 2 it is getting shorter and fatter for 4 and 5. Similarly, for the case of cyclic hardening with respect to the first cycle we are saying that the 4 and 5 hysteresis this looks like thinner and taller so this depends on its qualitative enhancement with respect to the first cycle.

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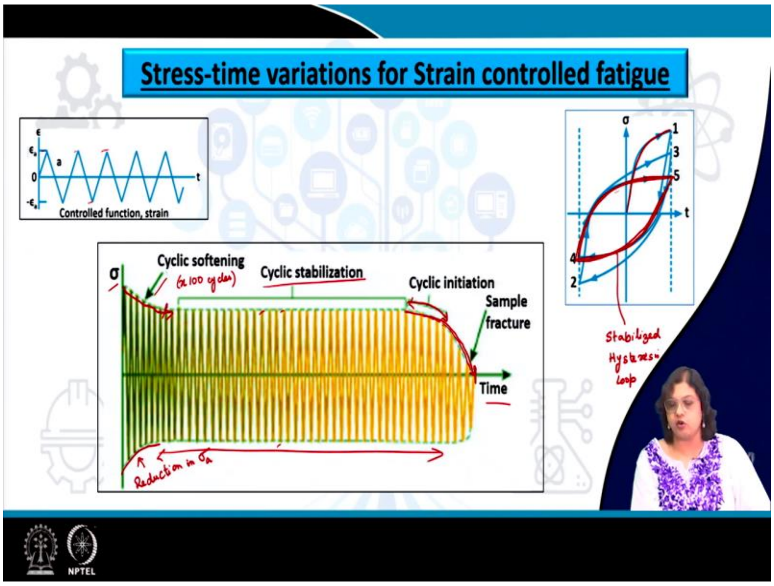

This will be apparent if we are seeing this in a broader picture and we also should note that this kind of hardening or softening behavior is not going to continue till the fracture rather what happens is that, that if we are for example we are controlling the strain versus time here we are applying the same values of strain at every cycle and in this case this one particular example shown where we are seeing some cyclic softening, softening because we are talking about the stress versus time response now, and we can see that there is a reduction in the stress output level.

And because of this we can see this reduction both in the tension and the compression side, so because of this reduction in the stress level, this reduction in stress or stress amplitude is seen for both the tension and the compression part that is why it is termed as cyclic softening but this happens only up to certain number of cycles typically some 100 number of cycles, so let us name this as x 100 cycles.

And after that it attains a stabilized condition when there is no change in the stress output that is what is expected, if we are applying certain value of strain whether this is in elastic condition or plastic condition it should have some particular value of stress amplitude, so after some initial cycles of this discrepancy of hardening or softening under cyclic loading we are seeing the major part under cyclic stabilization when the corresponding stress output will have the same value at every cycle so this is called as the cyclic stabilization and this is what is important if we are talking about the life cycle estimation.

Because if you are looking into this carefully you will see that major part like this is the point at which failure occurs and major fraction of this is being actually covered by the stabilized cycle part, now near towards the end if we are doing this for many number of cycles there is a damage accumulation which is happening at every cycle and since this is a plastic deformation one the damage accumulation is actually happening at a faster rate.

So, that means that this is being signified here where you can see that there is a reduction in the stress level, continuous reduction in the stress level until it fractures, so this is the part when the damage accumulation has been severe and that leads to the onset of the unstable fracture and that is this like the stage 3 that we have seen in the previous video.

So another thing to which is interesting here to note at this curve is that, this kind of reduction in the stress level is particularly seen on the tension side and not much on the compression side because we have already known this fact that under compression the crack is supposed to close even if the crack has already initiated, the fatigue crack which the dominant one has initiated that which can lead to fracture this is particularly relevant if we are talking about the tensile cycles.

So this is reflected only in the tensile part of the curve and then it fractures but in either case for the both the tension of the compression one we can see that the major part is being followed by the cyclic stabilized part and this is what is being reflected in the stress strain hysteresis which will talk about this in more details soon.

But one more thing I would like to clarify here is that although we started from this one here and because of the initial softening let us say this 4, 2, 5 is the one that we are seeing at this level here, just prior to stabilization after that what happens is that within a few hundred cycles once it gets stabilized then the points will be just repeating or the graph will be just repeating itself so this red hysteresis will be the one which will be the stabilized. So that there is no change in the stress strain curve, so this is the stabilized hysteresis loop.

And this is very important and just by looking at the graph we can say that this is the stabilized hysteresis loop because you can see that since there will be the repetition of the data points here this graph will be more clear and broader actually, broader in the sense the line thickness of the point thickness will be more because there are more number of data points there.

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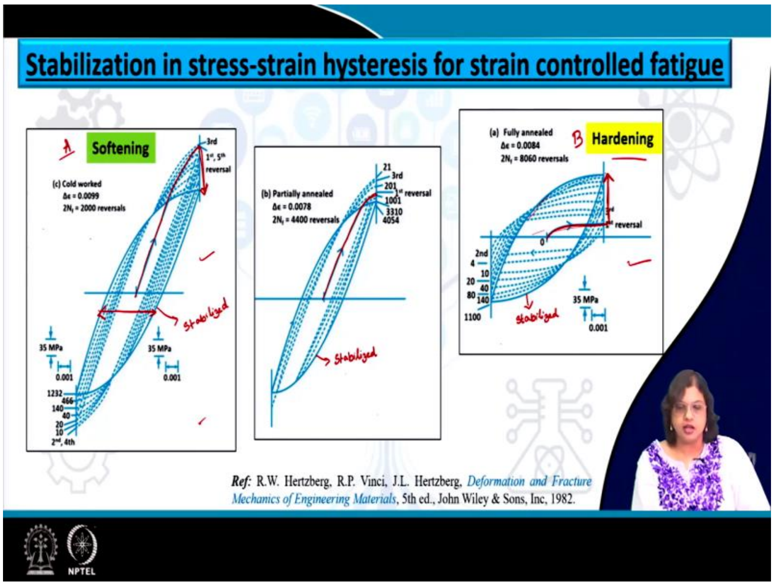

So let us see how this practically looks like from some example of a material, so in this case what we are seeing here is the the first cycle which is starting from 0 and then here is the first cycle, so let us see somewhere here and just by looking in this curve we can see that the rest of the things are all dashed and this one here, the solid line blue is the stabilized hysteresis loop, this will be always apparent and can be very clearly visible if you are talking about a stress strain hysteresis because there will be repetition of data point at this hysteresis loop for many many number of cycles.

And what we can see here is that this stabilized one has the maximum stress much lesser than the first one, so certainly this indicates that this is a softening behavior, on the other hand this is another one which shows that the first one is this one here as well as the stabilized one as you can see very clearly the solid one as my pointer is following so this is the stabilized hysteresis loop.

So let me also name this as stabilize hysteresis loop and we can very well see here that there is an enhancement in the stress level from the first cycle to the stabilized loop so obviously this includes or this is as per the cyclic hardening. Now previously I have also mentioned that if the hysteresis loop is getting fatter and shorter that indicates softening and if the stresses loop is getting longer and thinner that indicates hardening, what we are seeing here is not exactly following that.

If we are looking at this graph at the very first instance we will, it appears that this one is the longer and thinner hysteresis loop, so accordingly it should have been hardening but what we are seeing here is softening we can very well see that there is a reduction in the stress level, on the other hand what we are seeing here it appears like this is a fatter and shorter one but what we are seeing here is that the stress level is increasing from the first cycle so definitely that means it is a cyclic hardening one, so why is this anomaly?

Actually we also need to be careful about the fact that we are not comparing this hysteresis with this one, no, that is not our target, we are comparing the same hysteresis loop itself so if we are talking about the first cycle hysteresis loop, first to second to that if we just closed it versus the final stabilized hysteresis loop we can still see that this one is getting a fatter and shorter one compared to the initial cycle.

These are two completely different tests on two different materials so we should not compare A and B, let us name this as A and this as B, so in the A condition itself if we are comparing between the first cycle to the stabilized cycle then we will see that there is a shorter and fatter hysteresis loop which signifies softening, on the other hand if we are comparing the first cycle, so first hysteresis loop including the first and the second cycle and so on we will see that it definitely achieves a longer and thinner hysteresis loop in this case and that signifies hardening.

There could be also situations for certain material when the first cycle, so first cycle in this case is somewhere here and the stabilized one you can see the solid lines here so the stabilized one and the first cycle hysteresis loop are more or less tallying with each other so that means that it is a kind of neutral so there is no softening or hardening that is typically seen in such condition.

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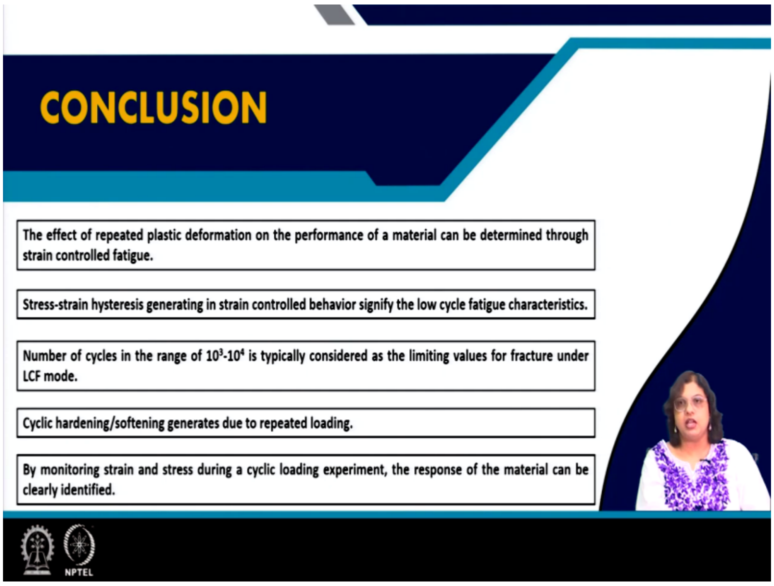

So in conclusion what we have discussed in this lecture is the following, the effect of repeated plastic deformation on the performance of a material can be determined through strain control fatigue and basically what we are generating here from the strain control fatigue is the stress strain hysteresis, so instead of a straight line for the elastic loading now we have a hysteresis curve and that signify the low cycle fatigue characteristics.

Now what is low cycle fatigue, low cycle fatigue is when we are talking about the number of cycles to failure which is varying within the range of 10^3 or 10^4 that is the limiting number of cycles for fracture and we have also seen based on the hysteresis loop that there could be cyclic hardening or cyclic softening depending on the response in the stress amplitude level, although we are controlling the same values of strain amplitude for each cycles, the stress amplitude could be either and there could be an either enhancement or a reduction in the stress amplitude level.

The former in case there is an enhancement in the stress amplitude level that could be named as cyclic hardening if there is a reduction in the stress amplitude level that could be cyclic softening. And by monitoring the strain and the stress during a cyclic loading experiment basically we can identify the response of the material and the behavior of the material in more details.

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So these are the references which are been used for this lecture. Thank you very much.