Fracture, Fatigue and Failure of Materials
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Lecture 16
J-integral and J_{IC}

Hello everyone and once again, welcome back to the sixteenth lecture of this course, fracture, fatigue, and failure of materials. And in this lecture also, we will be talking a little bit more about J integral and J_{IC}, in fact.

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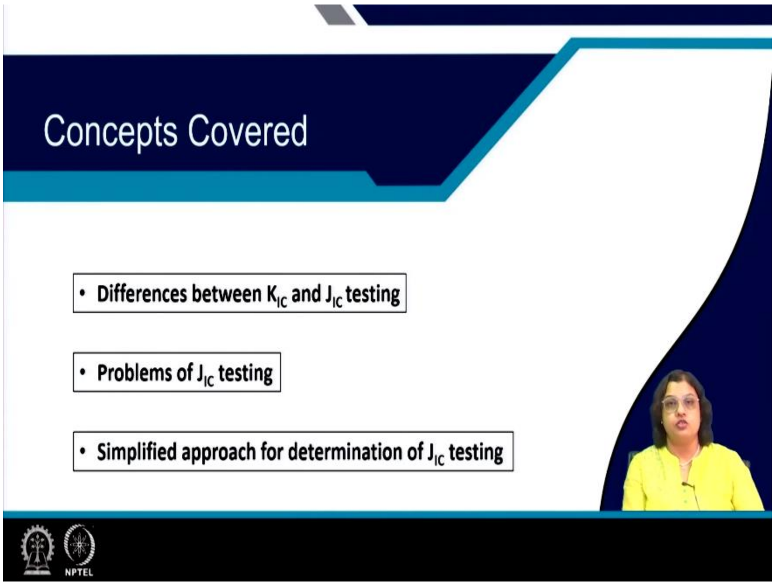

So, the basic concepts that will be covered in this lecture are the following. We will be talking about, J_{IC} testing one more time. And then we will like to find out the differences between the K_{IC} and the J_{IC} testing modes. And we will also like to indicate the problems that are associated with the J_{IC} testing and our simplified approach for the determination of J_{IC}. So, this will be covered in this lecture covering the overall span of plane-stress fracture toughness testing.

So, before we begin to the differences between, the similarities and the analogy between K_{IC} and J_{IC}, let us continue from where we left. In the last lecture, we were talking about J- integral and J_{IC}, which is the summation of the elastic and the plastic component.

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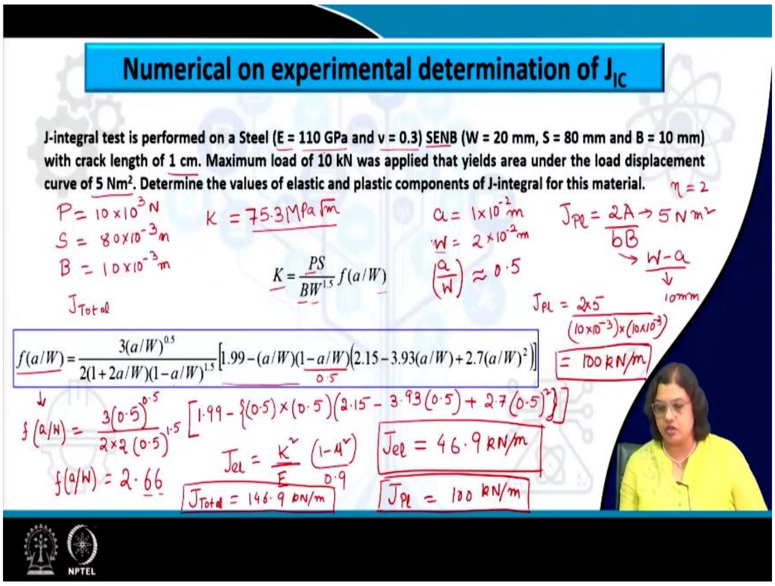

So, let us just solve simple numerical to make this idea a little bit more clear. So, here is an example for a steel component, an SENB a single edge notched component, which has the elastic modulus and elastic modulus of 110 GPa and the Poisson's ratio is 0.3. And the other parameters for the SENB specimen are as follows. The width is given as 20 millimetre, span length, which is the distance between the loading point as 80 millimetre.

And the thickness is 10 millimetre. It also has a crack length of 1 centimetre or 10 millimetre. Maximum load of 10 kN was applied and that yields area under the load-displacement curve of 5 N/m^2 and we need to find out the elastic and the plastic components of J integral and the overall J_{IC} value in general. So, the relation, all the relevant relation which are obtained from the standards of the handbooks are already provided, what we need to perform here is modification of the applied load to the stress intensity factor.

And this is related to load and the span length, as well as the specimen configuration, including the thickness and the width of the specimen, as well as a function of a/W where the details of the a/W relation has also been provided, and we do not need to memorize this. Of course, we need to understand and appreciate that how, if we are changing this a/W thing how the overall K value is changing. So, at the very first hand, let us determine the ratio of a/W.

So, what we are seeing here is that crack length is given as 1 centimetre. So, which means $1 \times 10^{-2} m$. And W on the other hand is 20 millimetre or 2 centimetre. We can write this for convenience as $2 \times 10^{-2} m$. So, which makes life quite easy in the fact that a/W is given by

0.5. So, if that is, so let us determine the, f(a/W) first because this at the first hand looks scary, but this is just a simple mathematics, very high school level.

So, f (a/W) is, I am just putting these values together and you can do the calculations. So, instead of a/W now we are putting as 0.5, and that makes this as 2×0.5 as 1. So, I am writing this as 2 instead. And 1 – a/W is 0.5. So, that makes it $0.5^{1.5}$. And this one here goes something like this 0.5 into this also comes to 0.5. And this one here has $2.15 - 3.93 \times 0.5 + 2.7 \times 0.5^2$.

So, if we are doing this calculation, which is pretty simple in that sense. So, what we are getting is the f (a/W) is 2.66. You can do the calculation very, very precisely, but it does not really matter because when we are talking about the fracture toughness, any variation in the values by digits, beyond the decimal, more than 2 does not really make much offer change.

So, now, if we need to find out the K value, we also need to put the other parameters. So, let us see what we already know here is P which is given as 10 kilo Newton. So, which means $10 \times 10^3 N$ and span length is $80 \times 10^{-3} m$. And B is already, B is 10 millimetres. So, $10 \times 10^{-3} m$ and W already we know that this is 20 millimetre.

So, once again, if we put all this values here, we end up getting the value of K as 75.3 $MPa\sqrt{m}$. And, so that gives us a chance to find out the elastic component of J as we know that J elastic is given by $\frac{K^2}{E}(1 - \mu^2)$. So, μ here is 0.3, which makes this as 0.9. And if we are plugging the values of K now, and, so K is like 75.3 $MPa\sqrt{m}$, and E is, given us 110 GPa.

We need to consider these units as MPa and convert GPa to MPa. And if we plug these values, we are getting J elastic 46.9 kN/m^2. So, we often need to convert from Pascal to Nm^2 or whatever is the relation that is most convenient and we have to use this. So, this relation is pretty straightforward. And I want you to do this along with me so that you can have a feel for this and also should know about the values which are typically seen for different kind of materials.

So, this is an example of metallic system. You can go ahead with some of the solve problems or the problems given in the exercise of the various textbooks that have been followed. And then you will have an idea about the J elastic and the J plastic component or overall fracture toughness values of materials, which is very, very important. Particularly, when we are talking about the failure analysis and we want to see that how this fracture toughness can be correlated with the failure mechanisms, so on and so forth.

So, this is what we are getting for the elastic part. For the plastic part on the other hand, let me make it in a box and the plastic part on the other hand is given by, so this is SENB. So, that means eta equals to 2. So, that makes it J plastic is 2A by the ligament length and, thickness the product of that. So, area is also given area of this low displacement curve as 5 Nm^2 and all the other parameters of SENB specimen are given.

So, from there we can also find out the B value, which is $(W - a)$, and that leads to, so W is 20 millimetre and a is 1 centimetre or 10 millimetre. So, that makes our small B as also 10 millimetre, as well as the thickness is also 10 millimetre. So, if we plug these values, then it is quite straightforward and we can, get the values of, J plastic as, so, there is not enough space, but let me still write it down here, 2 x 5.

So, this is and so, here it is $10 \times 10^{-3}m$ and $10 \times 10^{-3}m$. So, which makes it 100 kN/m. So, this is the value of the plastic component of J and overall what we are getting, this is being done in a little bit haphazard way, but you can figure out how we are doing this based on this lecture and what we are seeing is that, let me write it down here for clarity.

So, J plastic is equivalent to 100 kN/m and overall J then would be the summation of, these 2 terms here, or let me write it down here. So, J total will be given as 146.9. So, this is the value that we are getting from here.

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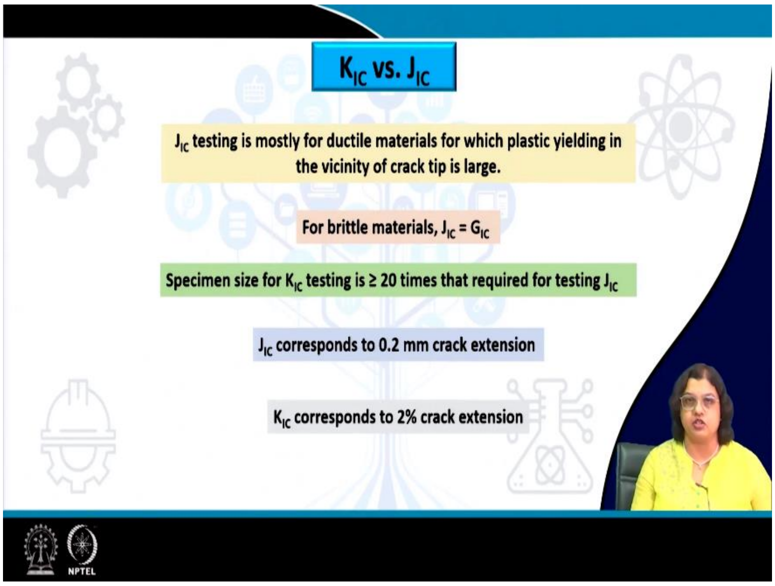

Now, moving on to the next part, let us see that how K_{IC} is different from that of J_{IC}. First of all, we have seen that, J_{IC} testing is mostly used when we are talking about ductile material and when there is a significant amount of plasticity and plastic yielding in at the vicinity or in the at the tip of the crack, which is very, very large, and we cannot ignore it.

In fact, the plastic zone size is almost equivalent, or even more than greater than the thickness of the specimen. We see that K_{IC} of the linear elastic fracture mechanics is not valid anymore. And we have to consider the plastic deformation and the elastic plastic fracture mechanics becomes applicable. However, for brittle material, if the material is brittle and we anyway do not need to go for J_{IC} at all.

So, J_{IC} for brittle material is same as a J_{IC} that we have seen earlier. So, we do not need the plastic component and include that in the calculations and the relations. So, for that it is quite easy and straightforward. The most significant difference is that we have seen that how plane-stress fracture toughness is applicable for thinner specimen and if the specimen is getting thicker and thicker the fracture toughness values decreases until it reaches a constant and the lowest value which is nothing but the plane-strain condition fracture toughness and the plane-strain condition.

So, there is certainly a difference in the specimen size that is required for plane-strain or the plane-stress fracture toughness testing, although it can be SENB or it can be CT specimen, but the thickness of the specimen can be different of the overall, if the thickness is different, all the other parameters of an SENB or a CT specimen are related to the thickness, the width then the, even the machine notch length, everything is related, the small width or the ligament length.

So, if, for that matter, actually the specimen size for K_{IC} testing is, it is seen that this is 20 times or even more than, that required for J_{IC} testing. So, that means that we will require quite bigger specimen if we are talking about K_{IC} testing. And, another important fact is that what we have seen in the last lecture is that this J_{IC} value, the critical value at the onset of fracture is determined based on 0.2 millimetre of crack extension.

So, if the crack, extends beyond that, so that is the particular point, which is of interest and we determine the critical value based on that point. On the other hand, K_{IC} correspond to 2 percent of crack extension. So, we do an offset method. We use the second line to determine just 2 percent of the crack extension. And that is the onset of the unstable fracture of the brittle fracture.

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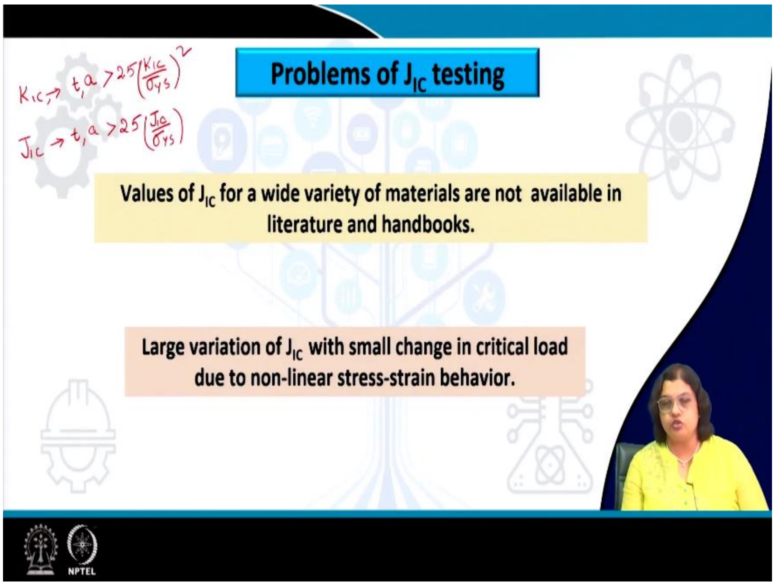

So, these are the major differences between K_{IC} and J_{IC} testing and, coming to the problems or the issues associated with J_{IC} testing. Actually, we have seen, even for plane-strain or plane-stress fracture toughness that we need to understand, or we need to have some idea or expectation about the fractured toughness values, right? For any case, if we need to figure out the K_{IC} or the J_{IC} values, we need to know the K_Q or the J_Q values.

And this is dependent on the initial, calculation that we need to do to, understand or determine this specimen dimension. In case of K_{IC} testing, we have seen that the thickness, t or B, and a crack length, this should be greater than 2.5 K_{IC} by the yield strength square. So, we need to have some idea about the K_{IC}, this value we can get from the literature very well for almost all different kind of material.

So, this is not an issue when we are talking about K_{IC} testing, but when it comes to G_{IC} testing, once again, here, also, we have seen that the thickness, as well as the ligament length, they should be greater than 25 times the J_Q or J_{IC} value by the yield strength. So, in this case, also, we need to know that what is the J_{IC} value if we are doing this for aluminium or titanium we need to have some idea about that material.

However, most of the cases, J_{IC} values for that particular thickness is very difficult to obtain from the literatures. K_{IC} is a standard value. It is a constant value, so we can get this quoted for almost all materials, but J_{IC} depends on the specimen size. So, if, we are doing a test here, whereas someone else at the other corner of the world is doing the test on the same material and the same specimen configuration, but maybe with slightly different, be the thickness.

And that can lead to difference in the J_{IC} value. So, we cannot use that data for our purpose. So, that makes it quite difficult to use or to use the concept or to determine it experimentally because, there is a literatures are not, available for all different kind of material for all different kind of thicknesses as well. And, this gets particularly critical because J_{IC} values change to a lot extent if there is, any change in the specimen dimension, or even with the change in the critical load due to this non-linear stress-strain and behaviour and that makes the assessment of J_{IC} even more difficult.

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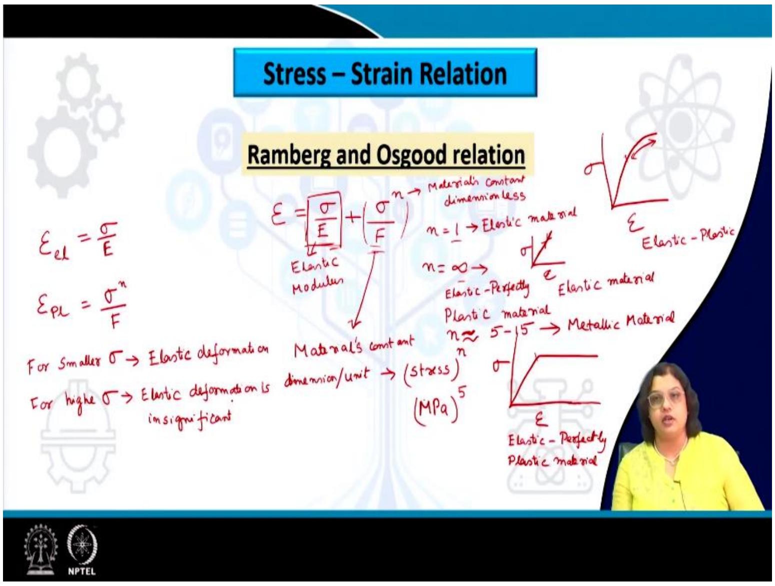

Now, to deal with that, there are other ways to look at this problem, which has been found and practiced by some of the groups, some of the scientists. So, this is based on a relation known as the Ramberg and Osgood relation this signifies, or this determines a stress-strain relation for an elastic-plastic material. So, if there is a stress strain curve like this, this Ramberg and Osgood relation could be valid.

So, what it says, what, this relation actually stands on the value of strain, which is given by $\frac{\sigma}{E}$, E is once again the elastic modulus and sigma is the stress value, epsilon is the strain value and this plus $\frac{\sigma^n}{F}$ so n is an exponent or so it is like the strain hardening exponent or so. So, it is a materials constant. And, typically it is dimensionless, which means that it has no dimension.

The values of n are, something like n equals to 1 for elastic material. So, this is, let me clarify this. This is the elastic-plastic behaviour. We see some part, the initial part as the elastic, and then we have the plastic part here. The value of n is 1 for the elastic material. So, that means, that in this case the stress train curve will be something like this. So, it is following a straight line, both when we are loading as well as when we are unloading it.

So, that is the elastic curve. So, that is elastic material. On the other hand, the value of n equals to infinity, and this is for elastic, perfectly plastic material. So, in this case, the stress strain curve looks like this. So, this is the stress axis on the Y and strain axis on the X. And in this case, elastic perfectly plastic looks like something like this is yields, and then it has a perfectly straight horizontal relation with the strain.

So, that means that there is no enhancement in the stress values any further. So, this is known as elastic-perfectly some common example of elastic, perfectly plastic, material, metallic material is for example, titanium. And, so coming to that, E is the elastic modulus as already mentioned. And F on the other hand is, material's constant, let me, so that is also a material's constant typically F has a very large value and not only that it has a dimension, so it is not dimensional.

It has a dimension of, or, say unit, which is the unit of stress times the unit of n or, n stress times, stress to the power of n. So, that means if we are using the, stress, the unit of stress as MPa, and the value of n for example, is let us say 5. So, the unit will be $(MPa)^5$. So, we have seen that the values of n also varies from 1 to infinity, 1 for elastic material and infinity for elastic, perfectly plastic material.

For typical metallic material, the value of n let me write it down here itself so n varies from 5 to 15 for metallic material. Now, this is typically the Ramberg and Osgood relation but what we are seeing from here is that it has two distinct part. The first part here, is nothing but that elastic strain. The elastic strain is given by σ/E, which is very familiar to all of us. We know about the Hook's law and how stress and strain are related, particularly elastic stress and strain are related to the elastic modules.

And this is exactly what we are seeing. So, that is, nothing of concern to us. What is important also to notice here is the plastic part, which is given by $\frac{\sigma^n}{F}$. So, that signifies the plastic deformation. Now as I mentioned, that the value of f is typically very, very high. So, that makes, this relation of this plastic part value at n a very less value.

If we are talking about, lower stress values, such that the deformation then is mostly being dominated by the elastic part. And the second part here attains a very lesser value is σ is small. So, for smaller σ values, we are seeing mostly elastic deformation, this is something that we can also appreciate and understand, but when we are increasing the σ value, then the second part increases to a high extent because it is also having to the power of n.

So, that makes the second part very, very large and for higher σ value actually the elastic part becomes insignificant. The plastic part dominants. So, elastic deformation is insignificant and the plastic deformation dominates.

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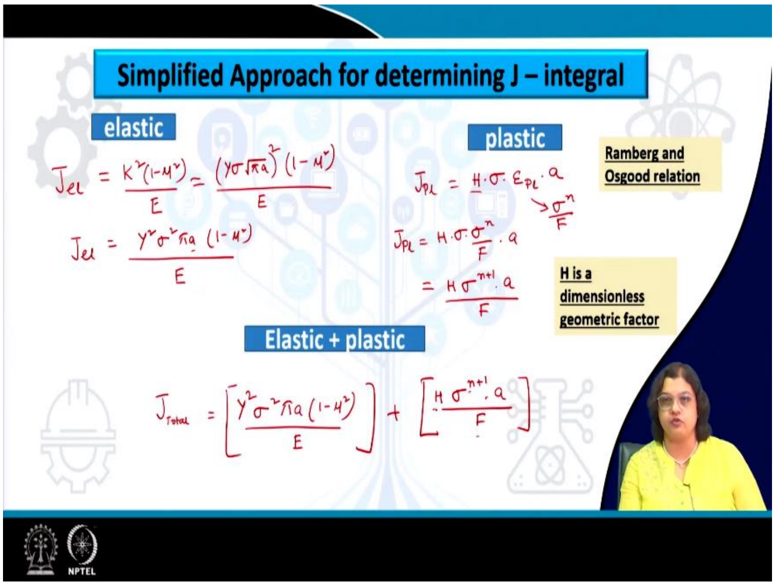

Now, if such as the case, actually, we, came to this concept because, this is often used as a simplified approach for determining the J integral other J_{IC} values. Why? Because we have already seen the problem with J_{IC} testing is that not many values are available in the standards or the handbooks. So, let us see how we can do that. So, J elastic, once again is not so critical because we know that this is given by $\frac{K^2}{E}(1-\mu^2)$..

And if we are using the value of K as $Y\sigma\sqrt{\pi a}$, so the very old relation that we have used so far between K and σ, so that, and $1-\mu^2$ multiplied by the elastic module. So, this is quite straightforward. So, J elastic is given by $Y^2\sigma^2\pi a(1-\mu^2)/E$ and the plastic part on the other hand is given by a relation, which is $H \times \sigma \times \varepsilon_p$, which is the plastic strain and a, which is the crack length, a is crack length here as well.

And we have introduced the, term, another term H here. Now H is material's constant, which is, geometric factor essentially, and that is one, again, a dimensionless one. And in case this material follows the Ramberg, this is particularly suitable for the material, which follows the Ramberg and Osgood relation for that case. We have seen that E or epsilon plastic part is given by $\frac{\sigma^n}{F}$.

So, we can rearrange this relation and write that J plastic equals to H sigma and instead of epsilon plastic, we can write $\frac{\sigma^n a}{F}$. So, that is $\frac{H\sigma^{n+1}a}{F}$. So, overall, we can see that J or J_{IC} or J total can be obtained, something like this we can simply sum this up and solve this. So, this is the elastic part, as well as we can find the plastic part also.

So, we have used the 3 constants here, n, H and F. And if we have these values, then we can determine the J quite simply, we do not need this area under the curve, or do not need to do this test. So, for most of the cases, these values are not so difficult to find rather than the J values itself. So, that makes it quite easy and straightforward and simplified approach to find out a J-integral.

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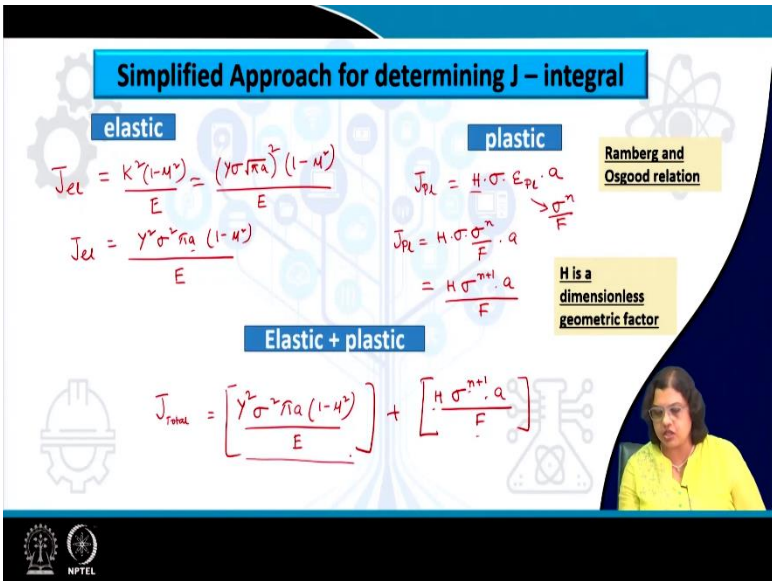

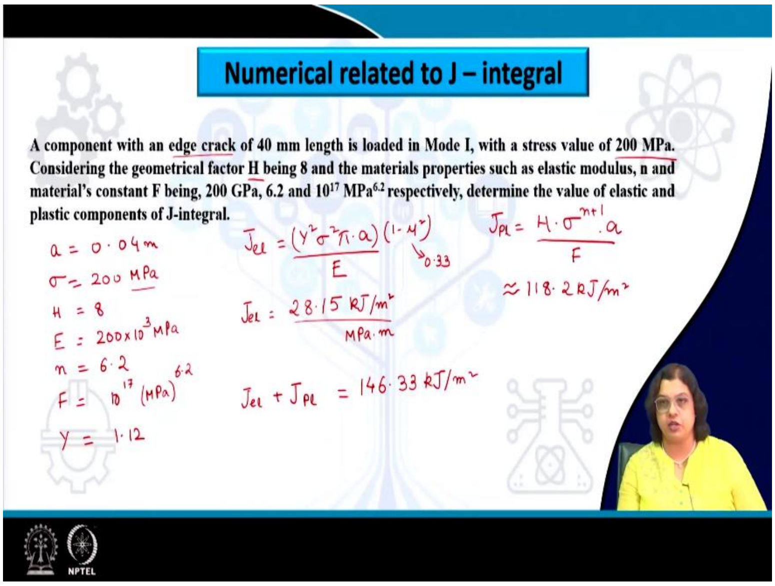

So, here is a numerical related to that, which says that there is a component with an edge crack of 40 millimetre length. So, that makes the, a value, as let us say, 0.04 meter. And this is loaded in mode I with the stress value of 200 MPa. So, σ equals to 200 MPa, and the geometric factor H is considered as 8. And the material property such as the elastic modules in. So, elastic modules is 200 GPa.

So, for clarity, let me write this as 200 x 10^3 MPa. n is given as 6.2 and F as I mentioned, that F has a very high value. So, it is 10 to the power 17 and interestingly, it has a unit also $MPa^{6.2}$. So, such as the case, we can determine the J elastic and the J plastic values from this relation here. And J elastic is once again, as we have seen from the last slide itself.

So, we can plug this relation here. And, since this is, edge crack that is mentioned here, so we should also consider the value of Y as 1.12 only for centre crack, we have Y equals to 1 for edge or surface crack. We have Y equals to 1.12. So, we can plug all this relation here to $Y^2\sigma^2\pi a$, and this multiplied by $1-\mu^2$. So, μ we can consider this as 0.33, and this divided by E so if we are doing this, we can see that the value of J elastic is coming something like, 28.15 kJ/m^2.

Actually, if we plug all the values here in terms of MPa and m then what we are getting this unit in MPa meter. And, we have to use this relation as, the relation between Pascal and joule as, Pascal is J/m^3. And then if we use that, we can figure this, unit as well. And coming to the J plastic term, let me write it down here. So, J plastic is H into sigma to the power of n plus 1.

So, in this case, n + 1 will be 7.2 into a, and so divided by F so, if we do that once again, we can get the value of J plastic as 118.2 kJ/m^2. So, once again, we have to convert the MPa meter to J/m^2 , and we can attain this value. So, overall J elastic plus J plastic will be. Let me do this quickly. So, this is so, that comes around 146.33 kJ/m^2. So, that is how we can J-integral or the J_{IC} values, quite in a simplified way. And with this we are at the end of this plane-stress factor toughness testing.

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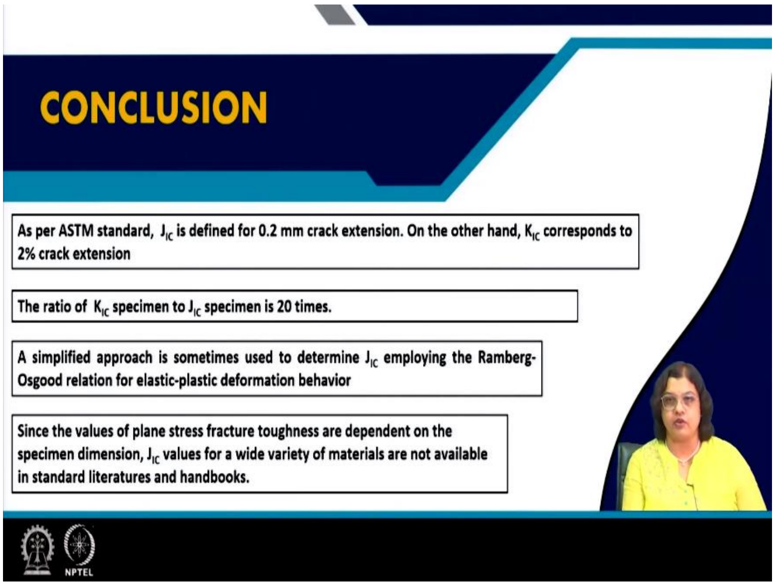

Let us conclude this lecture with the following points, which are the take away thoughts from this lecture which says that the J_{IC} is defined for 0.2 millimetre crack extension that we have seen, and the ratio of the K_{IC} specimen to J_{IC} specimen is 20, which makes a J_{IC} specimen quite large.

And a simplified approach has also been shown here to determine J_{IC} and employing the Ramberg and Osgood relation. And since the values of plane-stress toughness are dependent on the specimen dimension, J_{IC} values of a wide variety of materials are not available in standard literatures and handbooks. So, that is a drawback of J_{IC} testing. Of course, if we have a ductile material, which we know that undergoes a significant plastic deformation ahead of the crack tip, we cannot, go ahead with the K_{IC} testing.

Then we do need to do the J_{IC} testing, and actually in practice, we often need to perform the J_{IC} testing in the actual condition for the actual specimen of the component dimension, and configuration to find out the exact values. So, if necessary, we do need to use this. So, that makes a J_{IC} determination also, or J-integral for that matter, the concept of this very, very useful. And we should have a pretty good understanding about the fact.

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So, these are some of the references used in this lecture, and I thank you very much.