Fracture, Fatigue and Failure of Materials
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Lecture 13
Plane Stress Fracture Toughness

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Hello everyone, we are here for the thirteenth lecture of this course on Fracture, Fatigue and Failure of Materials. And today we are going to discuss a very important concept of fracture mechanics and fracture toughness, which is the plane stress fracture toughness.

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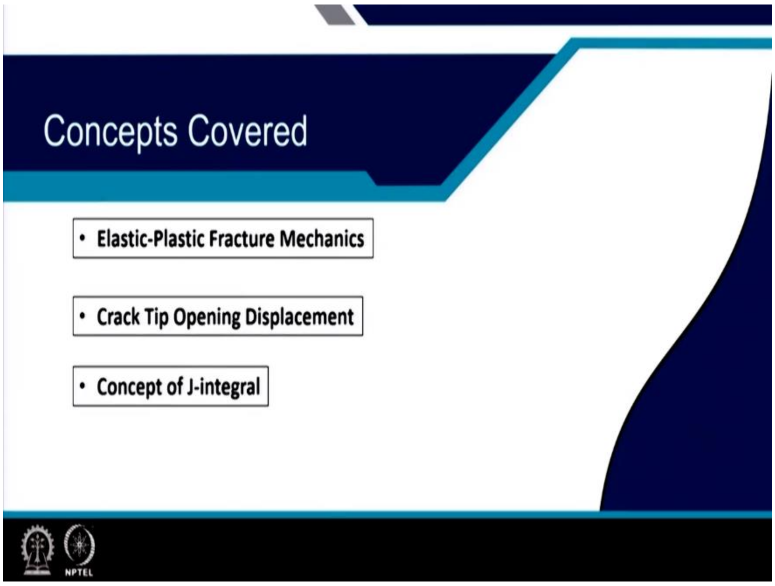

So, these are the concepts which will be covered in this course, in this lecture particularly, we will be talking about the elastic-plastic fracture mechanics. So, far we have seen about the linear elastic fracture mechanics we have discussed about that and from there we are moving towards the elastic-plastic fracture mechanics, which is particularly valid for materials which undergo some amount of plastic deformation prior to failure.

So, to find out the fracture mechanics behavior of such material under such deformation category, we will evaluate it through the crack tip opening displacement and the concepts will be introduced and discussed in this lecture. And this will be followed by another very important concept, which is the J integral.

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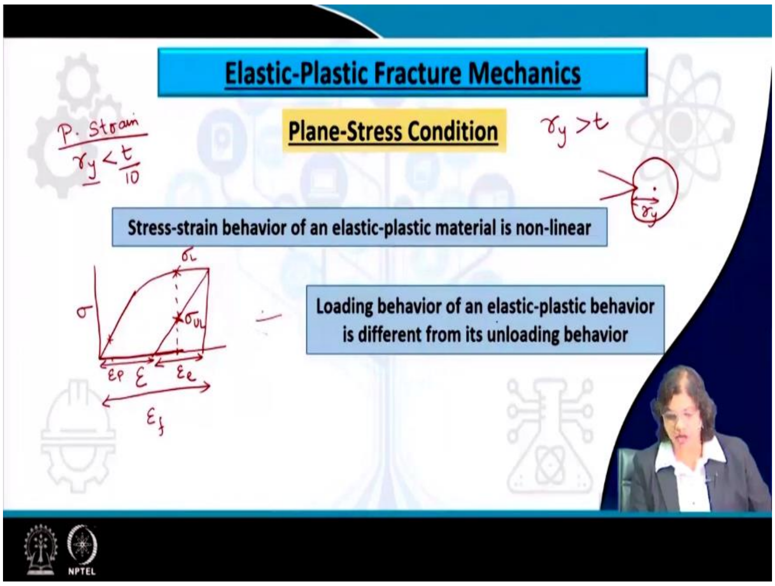

So, let us begin with the first topic itself, which says the elastic plastic fracture mechanics and this is particularly valid for the plane stress condition. So, so, far we have seen that how a material behave or fail in a brittle fashion, when it undergoes failure or fracture in the plane strain condition. And we have seen that Griffith criterion is being followed for most of the cases, so, that we can estimate the fracture toughness or fracture strength according to that relation.

Now, this gets difficult if we are talking about a material which undergoes some amount of plastic deformation and notable amount of plastic deformation in front of the crack tip while we are applying stress, we have also seen that this plastic zone size is related to the thickness of the material or vice versa thickness of the material if it is more than the plastic zone size or even the plastic zone size is comparable to the thickness or little bit more than the thickness then only we can term that as plane stress condition.

On the other hand, in case of plane strain one we have seen that the plastic zone is very very small actually lesser than one tenth of the thickness and that is the reason that we could ignore that amount of plastic zone, it is almost negligible. On the other hand, in plane stress condition, plastic zone size is either almost equivalent or even could be more than the thickness. So, obviously, this cannot be ignored anymore. And we have to think of some other way of determining the fracture strength and fracture toughness.

Now, this may not sound like a problem as of now that, if we have the plastic zone ahead of a cracked it, something like this, and we know the relation to estimate the plastic zone size, then what is the problem, we can determine that and we can calculate the effective length, crack length. And we can do all the calculations there. But things are not so simple. If we are talking about the stress strain behavior. Let us see how it looks like. So, this is the stress on the y-axis and strain on the x-axis had not been an elastic failure, we would have observed just a linear curve, which follows the Hooke's law and we know that all the stress and the strain values are correlated.

So, if we are talking about a particular value of strain here, that should always give me a stress value of something like this, whether I am loading it or whether we are unloading the specimen, we are still getting the same value of stress, but this is being violated if we are exceeding the building point of the material and it undergoes some amount of plastic deformation. So, we can see that from this point onward. There is a permanent deformation which is happening because of the dislocation movement or because of some defect interaction.

We all know about how to estimate this yield strain there is a method called 0.2 percent offset by which this yield strength value is being measured. But after this point, if we are trying to unload it, it is not going to follow back the same path rather it will come something like this. So, there will be some part of this deformation. Let us say this is the total elongation to failure. And out of that this much is the strain that is being recovered. So, that is basically the elastic strain. And here is the one which is not being recovered. So, this is the plastic strain.

Now, we again know this thing, we also know about how to calculate the elastic strain and the plastic strain, so, the problem is still not obvious, but if we are trying to find out the stress and the corresponding strain value, or vice versa, let us say we have a particular strain. And we want to figure out the stress at that point, that is the whole purpose, we need to find a correlation between the stress and the strain. So, if we want to do that, let us say we are talking about a strain of this value, and this is the dashed line that I am joining to find out the stress value.

The obvious question there would be which stress we are talking about is it the loading stress or is it the unloading stress because if you are talking about the loading stress, this is the value let us say σ_L, if at the same amount of strain, we can also get a value which is this one σ unloading, for example. So, obviously, whatever relation we might have figured between the strain and this loading stress is not valid for the strain versus this unloading stress. So, that leads to all the complex cases and that leads to think about some other mechanism by which the fracture behavior of such material which shows similar kind of behavior can be explained.

So, this is particularly suitable for the material which has an elastic-plastic and nonlinear curve, this elastic-plastic fracture mechanics is used in such cases. And this is once again as we have explained that the loading behavior and the unloading behavior are different for the material undergoing elastic plastic behavior. And we have to figure out a different ways for that.

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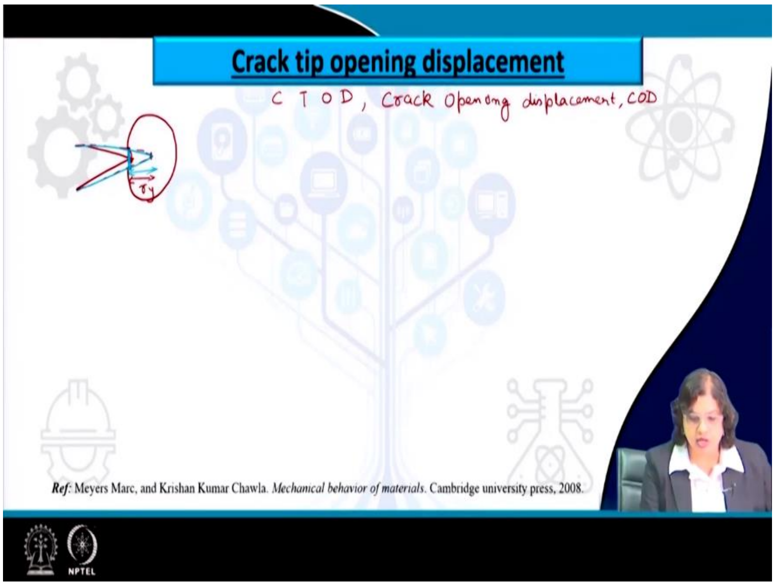

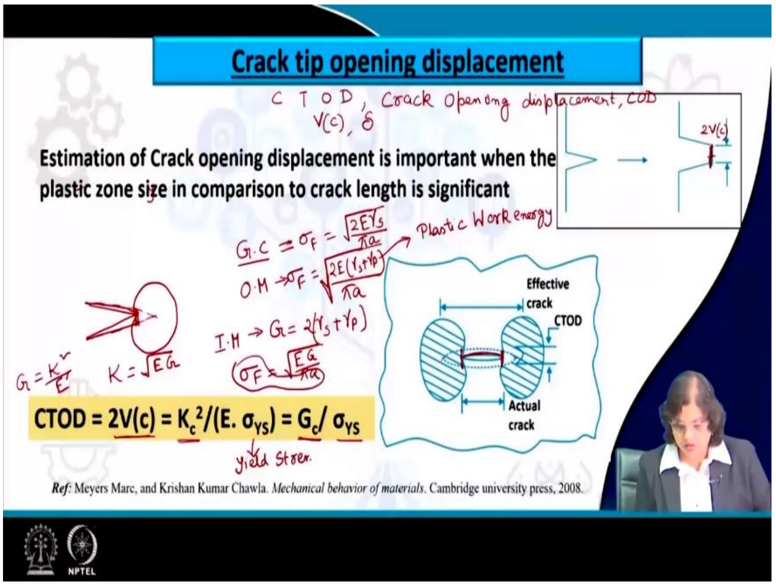

So, one of the typical way by which this can be analyzed to some extent is the crack tip opening displacement also known as the CTOD. Sometimes, also this is considered as or this is termed as crack opening displacement or COD. So, what does this mean crack tip opening displacement, so, essentially it means that something is happening to the tip, the tip gets opened up.

So, this is particularly again relevant when there is some amount of plastic deformation in the material ahead of the crack tip. So, that is the whole point of plane stress deformation. So, we have a cracked and there is a plastic zone ahead of the crack tip. So, that leads to some r_y and we know that just that also changes the effective crack length. Now, the crack which was supposed to be very sharp at this point like, which was the tip of the crack now, it has actually been extended till this point, which means, the tip is now here and the crack actually form something like this.

So, this again is not a typical growth of the crack, rather what we can see here is the crack tip, which used to be sharp here, now, get blunt at this part. So, let me change the color of this pain to show you what we mean by this tip. So, this is the effective crack. So, the crack tip and this effective this growth of the crack is not a real one this is a virtual growth in the sense that the crack tip is getting blunt. So, what was sharp here now is getting a radius something like this magnitude.

So, again this cannot be ignored and that means that all the stress concentration factor which is dependent on the crack tip radius of curvature, that is getting manipulated again. So, we cannot ignore that anymore and we have to think of a way to estimate that. So, this is known as the crack tip opening displacement the crack tip is getting opened up to this amount here, it says that the estimation of crack opening displacement is important when the plastic zone size in comparison to the crack length is significant. So, that leads to whatever was a sharp crack initially gets a blunt tip here, so, this is getting extended. So, this is the total tip blunt lifts that has been obtained and we need to measure this as the crack tip opening displacement.

So, this is typically measured by a symbol often known as V or V(C), sometimes this is also determined with a term with a symbol δ. Now, this is actually 2V one for the upper part one for the lower part that is how we determine this and the C stands for the the crack or the critical value at which fracture occurs. So, this is once again a way of demonstrating the crack tip opening displacement and this is happening for both the cases. So, both the tip is getting opened up by this much amount as you can see here the crack tip which was sharp initially is getting opened up by this much amount.

So, this certainly is not ignorable anymore and we need to find out a way to measure that. And once again for you to understand that this entire thing this dashed one is not how we are seeing like the crack is physically not growing up to this part, this is only the plastic deformation which is making the crack tip blunt.

So, crack is essentially in this part and then it is getting blunt like this, so, we can see the crack something like this and if we want to measure this amount this to Vc term here like this entire crack, tip bluntness or crack tip opening displacement that is related to the G, strain energy release rate and at the point of fracture, this is related to the critical value of Gc divided by the real strength of the material or since, we know that G is related to K once again as per the Irwin's modification, we have seen that Griffith criterion is giving us this relation. So, $\sigma_F = \sqrt{\frac{2E\gamma_S}{\pi a}}$ and we have seen that how Orowan has modified this included the plastic work energy and how Irwin has modified it.

So, Irwin has introduced the concept of G. So, as per Orowan's modification, this becomes $\sigma_F = 2E(\gamma_s + \gamma_p)$ where γ_p is the plastic deformation divided by πa whole under root and so, this is the plastic work energy, I am writing this once again as a recap of what we have learned so far. Because we often will need to use it again and again going back and forth to the different concept that has been introduced and Irwin has introduced the term G which is equivalent to this $\gamma_s + \gamma_p$. So, that makes us the Griffith criterion is being modified as $\sigma_F = \sqrt{\frac{EG}{\pi a}} \sigma_F = \sqrt{\frac{2EG}{\pi a}}$

Now, this term again being equivalent to K is given by $\sqrt{EG}$ and that makes as $G = \frac{K^2}{E}$. Now, since we are converting this from the plane strain to the plane stress, we are also introducing or this has already been discussed that instead of E we are using E prime, where this E prime will be related to the Poisson's ratio of this material as it is undergoing a volume change.

So, we will look into this in the next slide, but for now, this is how we are getting this crack tip opening displacement is related to not only G but also inversely related to the yield strength of the material since, we are talking about the plastic deformation in this and G on the other hand is related to K as per this relation as $G = \frac{K^2}{E}$. So, this is what we are getting, eventually that $2V_c = \frac{K^2}{E.\sigma_{ys}}$ or the yield strength of the material.

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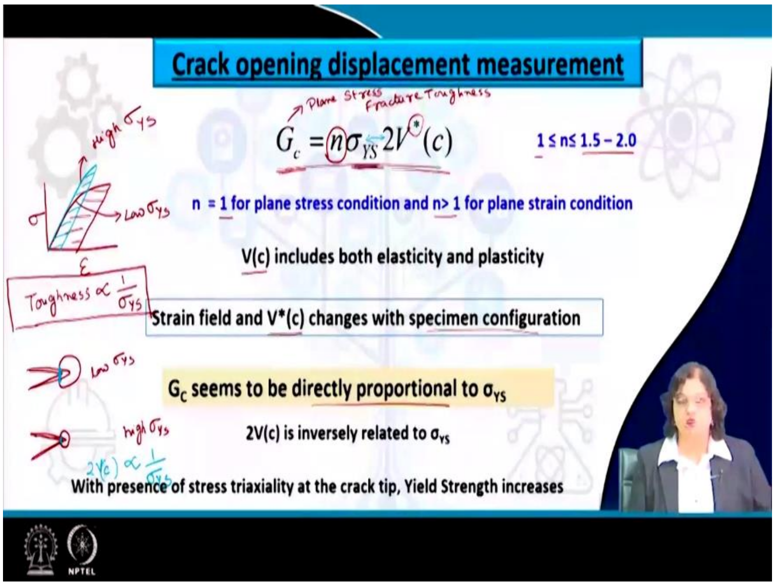

So, eventually, this is the exact relation would be also related to the term n, where n is a constant, but the value of n varies from either 1 to more than 1. And n is typically 1 for the plane stress condition, that is why we use the term $G_c = 2V_c \, x \, \sigma_{ys}$, the star here signifies the critical point at which fracture occurs. So, often there are different kinds of nomenclatures that are being used. So, the star is also used to represent that.

And, in case of plane strain condition, the value of n is greater than 1 and this could be this value could be varying between 1.5 to 2, this Vc actually the crack tip opening term that includes both the elasticity and the plasticity. Of course, because of this plasticity only the zone is forming and the cracked tip is getting blunt. So, obviously, it includes not only the elastic behavior, but also the plastic deformation behavior as well.

And this term also changes as we change the specimen configuration like the thickness of the specimen and this is once again as per the fact that we know that plane stress fracture toughness is always changing with the specimen dimension particularly the thickness we have seen that how with thickness as we are increasing the thickness fracture toughness is going down until it reaches the plane strain condition when the fracture toughness gets a constant a lower bound as well as the constant value.

So, for the plane stress condition, however, fracture toughness is always dependent on specimen configuration or specimen dimension and this is true for the case of crack opening displacement also. But, another significant observation from this relation is that, we are seeing here that the crack tip opening displacement, this relation we have seen that the strain energy release rate or the critical value of that which signifies the plane stress fracture toughness, this is directly proportional to the yield strength of the material.

So, this part here signifies the plane stress fracture toughness. Even when we are talking about just the toughness like the tensile toughness, a tensile toughness is the one that can be obtained from the area under the stress-strain curve write something like this let us say we have a stress versus strain like this and we can draw this.

So, the area under the stress strain curve for plastic deformation that is giving us the toughness value. Now, this is for certain yield strength value if we are increasing the yield strength of the material then it again let me change the color of the pen. So, then it gets something like this, its total elongation to failure also decreases or the ductility is also decreasing.

So, eventually we are seeing that this this one here is the low yield strength material and this one is the high yield strength material and what we are seeing even for the tensile toughness is that toughness is inversely related to the yield strength, more the yield strength less will be the toughness, this is true for also fracture toughness, we will look into more details of this in the subsequent lectures, but for now, I draw this analogy with the tensile toughness because this might be familiar to all of you. And we should appreciate this fact that toughness is typically in most of the cases toughness is inversely related to the yield strength of the material.

Also, there are ways by which we can increase both the toughness and the yield strength, which again will be subsequently discussed, but for now, this is what we always see for most of the cases that toughness and yield strength are inversely related.

However, as per this relation here we are seeing that toughness and yield strength are getting directly proportional, which is strange, which is like violating whatever we have learned so, far, but it is not this is a very well established relation. So, obviously, there are something else which is happening which we might have been overlooked. So, the point is that these two are not exactly directly related rather there is a relation between yield strength as well as the crack tip opening displacement also.

Now, more is the yield strength of the material more difficult it will be to have a plastic zone or higher is the yield strength lesser will be the plastic zone this is obvious. So, that means that the let me again draw this here so, let us say we have a crack like this and there is a plastic zone forming like this, so, that we have a crack tip opening. Now, this is for low yield strength material.

In comparison, if we are talking about material with the same crack length as the previous one and if we are talking about the high yield strength, then the plastic zone size will be even less and that will make the effective crack length as well as this crack opening displacement very, very small. So, in this case for the high yield strength it is only this much the blue part whereas, in case of low yield strength, it is quite high we can see that.

So, eventually what we are seeing is that this 2Vc term is inversely proportional to the yield strength. Higher is the yield strength, lower will be the plastic zone size and lower will be the crack opening displacement. So, in case we are increasing the yield strength of the material we end up having lower value of this 2Vc and that will eventually have lower value of this Gc also.

So, eventually we are getting an inverse relation between fracture toughness and yield strength still valid, because these two are inter related with an inverse relation. And with the presence of stress triaxiality at the crack tip, yield strength actually increases and that further complicates the situation. So, this is what we understand about the crack tip opening displacement and how we can estimate that and to find out the effective plane stress fracture toughness as per this method.

In the last lecture, we have also seen that based on the G curve as well as the resistance to fracture curve for the R curve that is also one of the way to find out the plane stress fracture toughness and this one like the crack opening displacement measurement technique is another one to estimate the fracture toughness under plane stress condition.

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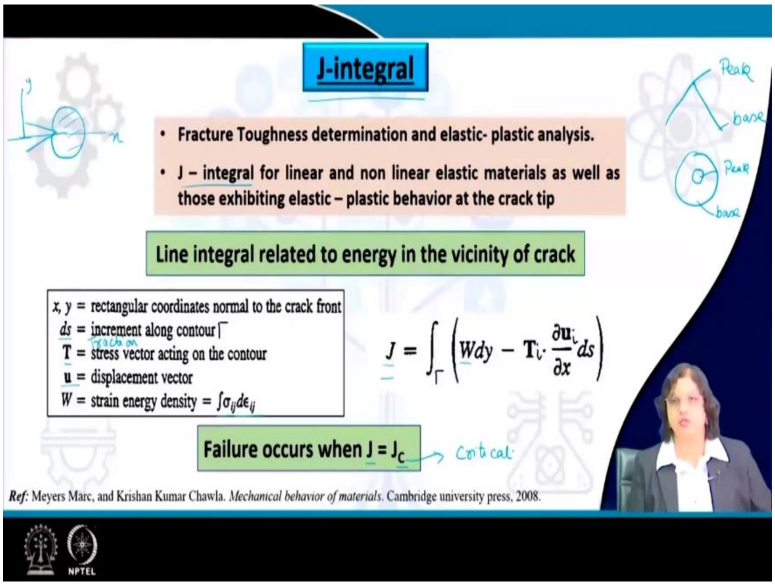

But so, far the mostly used method for plane stress fracture toughness or for that matter, if we are talking about the elastic-plastic fracture mechanics is related to the J integral. So, this is a very widely used method by which the fracture toughness of an elastic plastic material is determined. So, fracture toughness determination for the elastic-plastic analysis is particularly what is taken care of by this J integral method. And this is valid for linear and nonlinear elastic material as well as those exhibiting elastic-plastic behavior at the practice.

So, what it essentially does this J integral although it sounds a little odd like the crack tip opening displacement, we can just from the terminology itself we can understand the meaning J integrals in that sense sounds totally mathematical. So, what it actually this is a mathematical concept indeed, and what it actually signifies is the energy in a certain contour.

Now, when there is a plastic deformation happening, there is a volume change that is happening in the material in the region ahead of the crack tip and we want to figure out the energy criteria at this region in this zone for plastic deformation to occur. So, to do that, what we try to figure out is the energy change in the contour ahead of the crack tip. So, that gives us a possibility to determine this in a 2D plane.

For example, if we have a hill something like this, if we look into the top view of that, what we will see is like this is the peak and this is the base of a hill if you are looking into the top view, what we are seeing is this as the peak and this as the base. So, this is the contour that is being developed also in that case, if we have a crack and there is something some kind of deformation that is being happening. So, this is the contour and if we are projecting this in the 2D plane, we should be able to find out the energy situation here in this contour that is our area of interest.

And if we can figure that out, we can find out that how much energy is being spent how much energy is required for the crack to propagate to lead to failure. So, that is what is obtained from the J integral. So, it is a line integral related to the energy in the vicinity of the crack. And this the energy at any point of this contour is given by a relation something like this, which says that J equals to this integral of this entire term $(Wdy - T_i \frac{\partial u_i}{\partial x} ds)$. So, these are all the terminologies that has been used here x and y are the coordinates.

So, if we have a crack like this, this is the coordinate system that we have used. So, this is y this is the x and W actually signifies the strain energy density this is again related to the integral of the stress and strain when the change in strain value Ti or T is actually the stress vector or also the traction vector at any point on the contour.

First of all, we considered a contour of Γ this is the total contour like anything any space like this having a typical contour morphology we can say that that is the within this contour at any point the traction vector should be represented by T and ds represents the increment along the contour the overall increment in the contour and on the other hand, u signifies the displacement vector. So, these are all the terminologies or all the parameters that are used to determine the energy value at any point in this contour.

And we will see in the next lecture, how this can be expanded and how we can find out the energy of the entire contour through this method of J integral for determining the plane stress fracture toughness. And at the point of failure, if we are able to figure this J value for each of this point and at the point of failure, this J should achieve a critical value. So, that that can lead to a total catastrophic failure and that is what is of interest for us as the plane stress fracture toughness.

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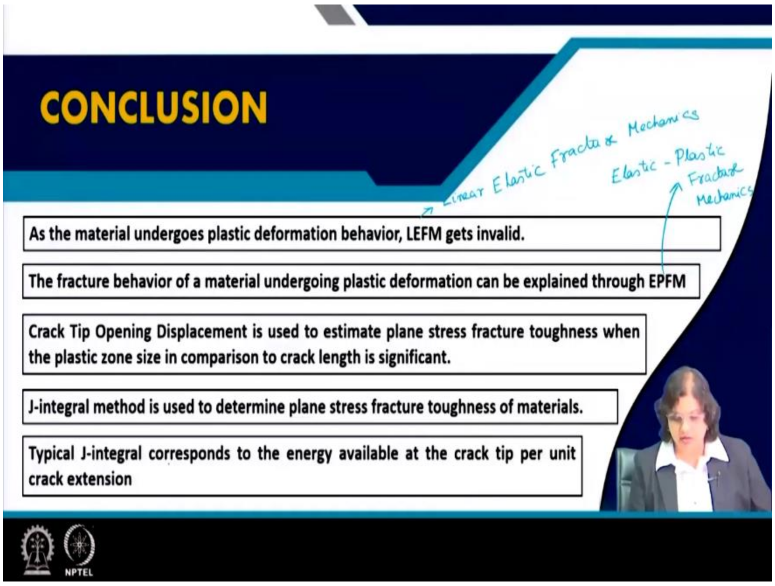

So, this leads to the conclusion for this lecture. As the material undergoes plastic deformation behavior, then the LEFM or the linear elastic fracture mechanics gets invalid. So, once again let me just write down the terminologies linear elastic fracture mechanics gets invalid and the fracture behavior of material undergoing plastic deformation can only be explained if we are considering that plastic deformation. As we have seen that during the loading and unloading cases for a particular strain value, the stress could be different and that can be only understood on the basis of elastic-plastic fracture mechanics.

And one of the ways to determine that is through crack tip opening displacement which is used to estimate the plane stress fracture toughness, when the plastic zone size in comparison to the crack length is significant and we cannot ignore that anymore typically when the plastic zone size is either equivalent to or more than the thickness of the specimen.

And, another or the most widely used method is the J integral method to determine this plane stress fracture toughness method and it is related to the energy available at the crack tip per unit crack extension.

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So, following other references particularly the books from Hertzberg and Meyers and Chawla and by Professor Kumar, these are typically being used for this lecture. Thank you very much.