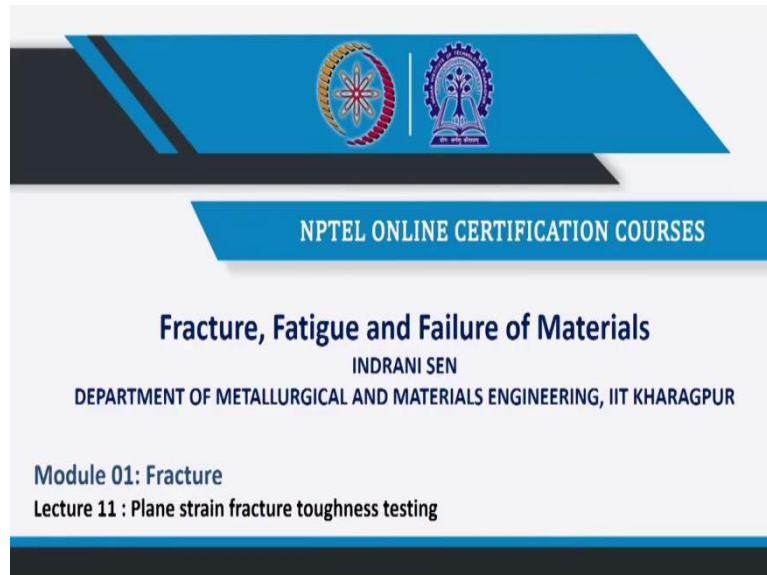


Fracture, Fatigue and Failure of Materials
Professor Indrani Sen
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Lecture 11
Plane Strain Fracture Toughness Testing

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Hello everyone and welcome back to the eleventh lecture of this series, Fracture Fatigue and Failure of Materials.

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And in today's class we are going to discuss about a very important concept, which has already been discussed and we will be discussing about how we can practically determine the fracture

toughness of a material under plane strain condition. So, the experimental details will be discussed in this class.

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Plane strain fracture toughness testing

Importance of plane strain fracture toughness

1. Lower bound value.
2. Constant value, doesn't decrease with thickness
3. Mode I is the most detrimental loading condition, so K_{IC} is determined

Important to validate plane strain condition:

- To check if lower value of K_{IC} is possible for the material?

Ideal plane strain condition: $t, a \geq 2.5 (K_{IC}/Y_S)^2$

Handwritten notes on the slide include:
 $t > 10 r_f$
 $r_f < t/10$
 $t \sim 50 r_f$
 $r_f = \frac{1}{\sigma_f} \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$
 $t \sim 50 \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$
 $t \sim 2.67 \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$

Ref: R.W. Hertzberg, R.P. Vinci, J.L. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 5th ed. John Wiley & Sons, Inc, 1982.

So, we have seen in the previous few lectures that plane strain fracture toughness is the one which is considered as the materials property, and this is particularly because of two important factors, one is that the fracture toughness values under plane strain condition is the lowest limit for any material that is the lower bound value, which does not change even if we vary the thickness of the component. So, that means that it gives us a constant value and we can quote this value as a materials property.

And also, we typically apply mode 1 condition while experimentally determining the fracture toughness and for considering this as a materials property because this is the most detrimental one, mode 1 is the most detrimental loading condition, it is a crack opening mode when the crack is directly perpendicular with respect to the loading direction and hence K_{IC} or the fracture toughness under plane strain condition is determined.

So, let's see what are the typical parameters or factors that are of concern when we want to determine the fracture toughness of any material in plane strain condition. First of all, when we say plane strain or plane stress, when we want to differentiate between these two, we know that this is particularly dependent on the thickness of the component if the thickness is more or if it is a thicker specimen, we tend to get the plane strain values and if it is a thin sheet or plate, then the plane stress condition becomes active and we get the fracture toughness values which

are not a lower bound value or it is not a constant one. So, that means that we will require a thicker specimen for plane strain fracture toughness.

We have also discussed that this thickness or higher thickness, lower thickness, these are all qualitative terms. And what it actually depends on is the plastic zone size, if the plastic zone size is almost equivalent to the thickness or even more than the thickness of the specimen or component or structure, that is a plane stress condition. On the other hand, for the case of plane strain, we know that the plastic zone size is actually less than one tenth of the thickness.

So, that is the prime criteria to check whether a component designing is perfect to analyze the plane strain fracture toughness condition. So, this is the first checking we need to do based on this criteria that $r_y < t/10$, one tenth of t , that means that thickness of the component should be greater than 10 times the plastic zone size.

Now, for actual laboratory scale experiments, we want to be on the safe side and instead of this factor as 10, we consider this as 50. So, we make the thickness of the component as large as 50 times the plastic zone size which means that in no way this will be under plane stress condition this will definitely be in very much plane strain condition and whatever fracture toughness values that we are determining is supposed to be constant and will not vary any further even if we increase the value of t .

So, just to be on the safe side we consider this factor as 50 times. So, 5 times more than the expected one. So, to do that, we again have to specify now plastic zones are something that we cannot see right at the beginning or it is very difficult to measure for that matter. So, we have to determine the thickness even before running the experiment just for designing the sample or specimen itself. And this can be done on the basis of the relation that we have seen that plastic zone size is related to the fracture toughness as well as the yield strength of the material. We

know about this relation as $r_y = \frac{1}{6\pi} \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$

Now, this typically could be K by the stress intensity factor divided by the yield strength of the material, but at the point of fracture this can be considered as a fracture toughness. So, based on that if we want to make the thickness more than 50 times or at least 50 times r_y , then we need to have the thickness equivalent to 50 times this relation.

Now, at the first hand, this looks a little bit surprising that how to get this because we have not yet determined the K_{IC} value and our target is to determine this K_{IC} factor itself. So, even before

that we need to have some basic understanding about the fracture toughness, the expected value of fracture toughness of any material.

Now, when I say that, we do not need to know the exact value, rather we want to know the expected or the order of magnitude. For example, if the fracture toughness of a material is something like 50 MPa root meter or it is 500 MPa root meter that is of concern, it does not matter if we are writing this equation on the basis of K_{IC} as 50 or 55 or 60. But, if we are making this as 500 then there will be some error in the calculation or the estimation. We anyway are going to recheck this back at the end of this experiment.

So, that we know that we can validate the plane strain condition being active. So, this comes around if we solve this, the $t \approx 2.65 \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$, and to make this a round number, we instead of using 2.65, it is used as 2.5 times K_{IC} by yield strength square of a material and both the thickness as well as the crack length are expected to be more than this.

So, we can make this 2.65 also to be specific or we can make this 2.5. Because, we always are on the safe side as we are using this 50 times r_y . So, based on the literature value for the yield strength of the material and the expected fracture toughness of the material, we determined some value of t and a that we need to use for designing the specimen at the very first hand.

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Numerical on validity of plane strain fracture toughness

Compact tension specimens with crack length and thickness of 4 cm and 1 cm are prepared from an as-cast (A) and heat-treated (B) steel alloys. The plane strain fracture toughness values for alloy A and B are 25 and 35 MPa√m respectively, whereas the yield strength values are 560 MPa and 460 MPa respectively. Would the specimen dimensions be suitable for valid plane-strain fracture toughness conditions?

A (As-cast)



$a = 4 \times 10^{-2} \text{ m}$
 $t = 1 \times 10^{-2} \text{ m}$
 $K_{IC} = 25 \text{ MPa}\sqrt{\text{m}}$
 $\sigma_{ys} = 560 \text{ MPa}$

$t, a \geq 2.5 \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$
 $\approx 4.98 \text{ mm}$
 Valid Plane Strain Condition achieved

B (Heat-Treated)

$a = 4 \text{ cm}$
 $t = 1 \text{ cm}$
 $K_{IC} = 35 \text{ MPa}\sqrt{\text{m}}$
 $\sigma_{ys} = 460 \text{ MPa}$

$2.5 \left(\frac{35}{460} \right)^2 \approx 14.5 \text{ mm}$
 $40 \text{ mm} \uparrow$
 $a > 14.5 \text{ mm}$
 $10 \text{ mm} \leftarrow t < 14.5 \text{ mm}$
 Plane strain condition is not achieved

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So, let's see based on a numerical that how we can practically do that in case of material, which might have different values of fracture toughness or yield strength of the material. So, here is the compact tension specimens with crack length and thickness of 4 centimeter and 1 centimeter

are prepared from the as-cast and heat treated steel alloy. So, there are two conditions of the same material, but one is in the as-cast condition and one is in the heat treated condition.

If there are some students who are from the metallurgy background will know that materials, if they undergo some kind of heat treatment or thermal mechanical treatment, there is some change in the microstructure and that will lead to change in the mechanical properties as well. So, as-cast is the one which has been directly cast and then if there is some kind of heat treatment that is what is considered as B.

So, as you can see the plain strain fracture toughness values of alloy A and B are 25 and 35 $\text{MPa}\sqrt{\text{m}}$ respectively. So, let me write it down here also simultaneously for condition A which is the as-cast one, we have crack length as 4×10^{-2} meter, 4 centimeter and since it is a compact tension specimen we now know that compact tension looks something like this.

So, that means the a here is half crack length. So, it is not $2a$ it is just a that is provided here and it has a thickness of 1 centimeter for both the cases. And so, let me write it down for B also. So, B is the heat treated one. In this case also we have the same values of 4 centimeter and t is 1 centimeter. What we are seeing here is that the fracture toughness values has been changed for the case of A fracture toughness values the expected or the literature values that has been found is 25 $\text{MPa}\sqrt{\text{m}}$. And in this case, because of this heat treatment, we can see that fracture toughness actually increases to 35 $\text{MPa}\sqrt{\text{m}}$.

And interestingly what we are seeing here is that the yield strength values are 560 MPa and 460 MPa respectively for A and B alloys, which means the one which is having higher fracture toughness is supposed to have lower yield strength. So, B is having 460 MPa as well as A which is having lower fracture toughness is having higher yield strength of 560 MPa. So, this also shows us we will go through the details of such behavior later, but as of now, this numerical itself gives us an indication that fracture toughness and yield strength of a material are actually inversely related. Higher the fracture toughness lesser will be the yield strength of the material.

So, for both the cases if we want to design this compact tension specimen, we need to check for this condition so that the $t, a \geq 2.5 \left(\frac{K_{IC}}{\sigma_{YS}} \right)^2$ and this should come to K_{IC} is 25 / 560. So, that leads to around 4.98 millimeter.

Now, component size already have 4 centimeter as the crack length and one centimeter as the thickness. So, we are seeing that both t is 1 centimeter or let me write this as 10 millimeter and

a which is 4 centimeter, so which is 40 millimeter, both are actually greater than this number. So, that means that we can do perform a valid plane strain fracture toughness testing from this condition for A. So, valid plane strain condition is achieved in this case.

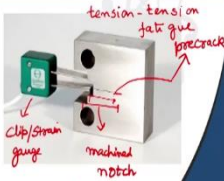
Now, let us see what happens to B. In this case we are seeing that this factor 2.5 and what we have here is $35 / 460$. So, this turns to 14.5 millimeter. And what we are seeing here the thickness which is 1 centimeter is not validating this criteria, although a is greater than 14.5 millimeter, we have a of 40 millimeter, so, that is validating this criteria, but t is not.

So, t which is only 10 millimeter is actually less than 14.5 millimeter and unless both this a and t satisfies this condition, then the plane strain condition will not be achieved. So, in this case plane strain condition is not achieved. So, that means that if we are heat treating this and we are enhancing the fracture toughness of the material, we would need a thicker specimen to perform a valid plane strain fracture toughness testing.

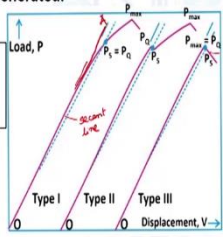
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Plane strain fracture toughness testing for metallic/ductile material

- A test specimen C(T) or a 3-pt bend bar sample is fatigue loaded to extend the machined notch.
- Clip gauge is placed at the mouth of the notch to monitor crack displacement when load is applied.
- Load-Displacement curve is generated.



Through the origin of the load-displacement curve a secant line with a slope of 5% is drawn.



P_3 - load at the intersection of secant line with the load-displacement curve

Ref: R.W. Hertzberg, R.P. Vinci, J.L. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 5th ed., John Wiley & Sons, Inc, 1982.

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Now, let's see the details of how the fracture toughness testing is being performed in lab under plane strain condition. So, initially CT specimen or a compact tension specimen or a 3-point bend specimen or sometimes even a center cracked tension specimen are used for fracture toughness testing and all these specimens are having a machined notch.

Now, you can see this is just a picture for reference and you can see that in this CT specimen there is a particular notch which is being machined up to this level and there is a clip gauge. So, this is nothing but a clip gauge also known as strain gauge to measure the crack growth. So, this vertical growth of this crack is being monitored through this clip gauge or strain gauge.

And finally, the load displacement curve is generated, but prior to that there is this specimen is initially being fatigued loaded. So, that means, that this is being repeatedly loaded particularly under tension-tension fatigue condition. So, tension-tension, which means that we are applying a tension value of higher magnitude and a tension value of lower magnitude.

For example, let us say we are applying a stress level of 100 MPa and 50 MPa repeatedly for a certain number of cycles. So, this is known as tension-tension or simply fatigue pre cracking. So, this pre crack means that whatever this notch is a machined notch. So, this part here is a machined notch and typically, for the case of brittle material machining a notch itself is a problem, but still we can machine a very sharp notch. Problem is when we are talking about ductile material, machining of notch will not give us a very sharp crack tip.

And we see we have seen that plane strain fracture toughness is nothing but fracture toughness under brittle mode of failure we have seen that plane strain fracture toughness leads us to very small plastic zone which means that there is not enough dislocation motion happening in front of the crack tip and that leads us a flat fracture surface which is indication of brittle fracture.

So, brittle fracture is what we have also seen for the case of Griffith criterion going back to the where we have started as I mentioned that most of the cases all the fracture mechanics leads to or it starts from the Griffith criterion itself. So, basically we are trying to make the Griffith criterion valid in such condition and for that we have to make our specimen thick enough and considering that Griffith criterion to be valid, it needs a very very sharp crack, we have also determined that we how sharp a crack tip needs to be.

So, if we cannot do that by machining, typically, for the case of ductile materials, what we do is we simply fatigue pre crack it. So, this pre crack means, whatever the machined notch, so, machined notch act as a defect that is the point from which the crack is supposed to grow. And if we are pre cracking it by applying some very low values of fatigue loading, then this is going to be extended by a certain value. So, this is the this is the pre crack part, this red part here.

And, the total crack length is considered as the machine notch plus the pre crack length. Based on this now, when the pre cracking is completed, and how much the pre cracking length should be, how much should be the length of the notch machined notch what should be the width of the specimen, everything has been described as per the ASTM standard American Society of Testing Materials. So, all these things are being prescribed.

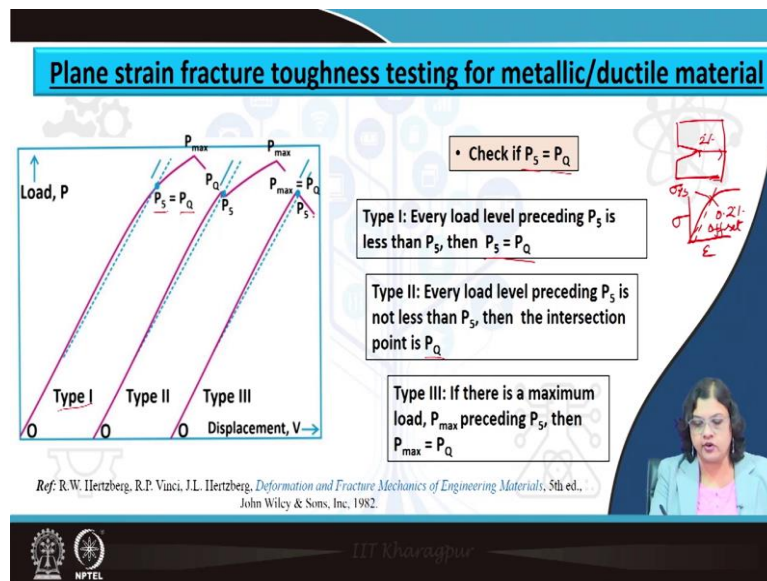
So, after this is done the fatigue pre cracking is done, then we simply apply the load till it fails apply tensile load that makes it mode 1 till it fails and in turn we are generating the load displacement curve something like this. So, these are the three examples that has been shown here, but anyway any of this curve will look like one of this mode here.

So, this is the origin and it simply increases the load increases as the displacement also increases. So, we see something trend like this. Now, this is what similar kind of behavior we also see for the case of let us say tensile testing also although there or some other activities and plastic deformation, etc., are apparent in this case we see mostly the load increases very steeply and the displacement also varies and more or less a linear relation is being maintained up to certain extent.

But the quantify the fracture toughness value or the fractures strength at the load values are at the point of fracture what we need to do is to analyze it in a more detailed way. So, for that what we do is from the origin, so, this pink line here if we extend this so, this will lead to a tangent, so, OA will be a tangent to the curve to the load displacement curve and to that we actually draw a second line that is having an angle 5 degree less than this tangent line.

So, this dash line here is the sec 5 degree line shown for all the cases and where even these two intersect, so, the pink line which is the load displacement curve and the blue one dashed one which is the second line wherever this intersects that point or is known as the P_5 and again there are different ways by which we can find out the P_Q . P_Q is actually a trial value of load that is considered as the load required for fracture.

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So, let us look into this in more details. So, all the calculations will be done based on this P_Q and P_5 is used just to experimentally determined this Q term. If you are getting confused, let me also tell you that what is the basis for drawing a line which is having a slope less than 5° to the tangent line, it is considered as the crack grows by 2% so, 2% crack extension that leads to fracture. We have already seen that in case of unstable fracture and plane strain fracture toughness is nothing but dealing about unstable fracture or brittle fracture what happens is that as soon as the crack starts growing.

So, this is the fatigue crack and then as soon as the crack starts growing, it leads to catastrophic failure moves to to the edge and that leads to final failure. So, we consider the growth of the crack by only 2% and that 2% is determined based on this second line here which is having a slope of 5° . So, this analogy between these can be drawn as we use the yield strength 0.2% offset method.

So, if we have the stress versus strain curve something like this what we do is to determine exactly the specific location specific value of stress at which the plastic deformation commences for that what we do is we simply draw this 0.2% offset strength value and wherever this meet that gives us the yield strength of the material. So, with the similar analogy here also we use this dash line to find out to 2% crack extension.

Now, again there could be three different ways by which the curves might have generated. So, we need to determine whether P_5 that we are getting is equivalent to P_Q or not. So, there are three conditions type one, either we get a curve as simple as this and in this case all the loads

preceding P_5 is actually lesser than the value of P_5 . So, in this case P_5 is just P_Q , we can use this value of load as the fracture load.

Alternately, in case of load two we see that there are not all the values are less than a P_5 rather we can have some maximum values also here and for that if we have some thing like this we can consider the intersection point as the P_Q one.

Or in the third case where you can see that there is a maximum point and then it reduces which means that fracture has already commences at this point of P_{max} . So, for that case we consider P_{max} is equivalent to P_Q , so, that is how we can determine P_Q .

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Relation between K and PCompact Tension [CT] specimen

• Convert P_Q to K_Q

$$K_Q = \frac{P}{BW^{3/2}} f(a/W)$$

$$f(a/W) = \frac{(2+a/W)}{(1-a/W)^{3/2}} \times [0.886 + 4.64a/W - 13.32(a/W)^2 + 14.72(a/W)^3 - 5.6(a/W)^4]$$

Ref: <https://commons.wikimedia.org/wiki/File:Single-Edge-NotchBending.svg>
 Ref: S.K. Kudari, K.G. Kodaacha, 3D stress intensity factor and T-stresses (T11 and T33) formulations for a compact tension specimen, *Fat. Integritia Strutt.* 11 (2017) 216-225

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3-pt bend Single Edge Notched Beam (SENB) specimen

$$K = \frac{P S}{BW^{3/2}} f(a/W)$$

$$f(a/W) = \frac{3(a/W)^{3/2}}{2(1+2a/W)(1-a/W)^{3/2}} [99-(a/W)(1-a/W)(2.15-3.92(a/W)+2.7(a/W)^2)]$$

Center Cracked Tension (CCT) specimen

$$K = \frac{P\sqrt{\pi a}}{BW} f(a/W)$$

$$f(a/W) = \sqrt{\sec \frac{\pi a}{W}}$$

Starvin, M. S., K. C. Ganesh, and R. Pandiyarajan. "Correlation of fracture parameters during onset of crack in middle tension specimen." *Journal of Computational Design and Engineering* 4.3 (2017): 169-177.

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Once we determine P_Q our next job would then be to convert P to K . Our target is to find the value of K or the stress intensity factor and critical value of stress intensity factor for fracture. So, far we have determined P and we are also able to find out the critical value of load, critical load at which fracture has occurred.

So, next target would be to convert the P_Q to the K_Q term or the stress intensity factor. So, this is done based on this kind of relation. So, this is for a compact tension specimen, we see that if the relation between K and P is quite straightforward it is related to the dimension of the component or the specimen, it is related to the thickness. So, once again let me write it down here B equals to thickness.

So, far we have used t as thickness, but for some cases B is also referred to as thickness. So, you should not get confused these are the terminology that are being used particularly in the case of the standards and this relation between K and P is related to the thickness of the specimen and the width of the specimen as well as a function of this a by W . This looks very much complicated, we do not need to memorize this, although we just need to find out that this is somehow giving us the stress value and also considering the dimensions of the crack length and the W .

So giving us these parameters in MPa root meter or that includes both the stress and the crack length part. And all this relations can be found actually from any of the handbooks of ASTM or American Society of Testing Materials or also in any of the mechanical behavior books book by Hertzberg or Meyers, Chawla all these books also add the appendix typically this kind of relations are being provided. So, this is for the most commonly used specimen as a compact tension specimen.

Also, the relations I have just simply shown you the relations for a three point bend single edge notched beam typically known as a SENB specimen here also K is related to the load level. So, this should be our P_Q value whatever P_Q we have determined we can use this, S is the span length and span length we know is four times the W of the specimen see this is again as per the standard the specimen dimensions are and we see that the span length, the distance between the two loading points are 4 times the width of the specimen and it is also related to the thickness and width of the specimen and the function of a by W again the function is being shown here as well as for the case of center cracked tension specimen.

So, you see this is looks like a flat dog bone shaped tensile specimen but there is a crack at the middle. So, that is why this is known as center cracked tension specimen and here also K and P are related something like this, we also have the a value considering in this center crack tension specimen.

So, once we know that what kind of specimen we are using and which kind of relations are being active for that we should be able to convert the obtained P_Q to the K_Q one. And if we are able to do that our next term would be to find it that whether this K_Q will be a valid one.

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So, for that we need to check if the K_Q is equivalent to K_{IC} . So, how we are doing that, so, far all K_Q is being determined and we need to validate that this is a valid plain strain condition. To do that, we again have to go back to the initial relation as t and a , this t can be also referred to as B , just to make sure that both B and t represent thickness of the specimen. So, this t and a , is nothing but half crack length. I am repeating these terminologies again and again just for you to be familiar with these terms and should not get confused. So, $t, a > 2.5 \left(\frac{K_Q}{\sigma_{YS}} \right)^2$. So far we have designed this we have use this relation for designing the specimen and we have used the value of K_{IC} instead of K_Q to determine the values of thickness and the crack length.

And now, since we have determined K_Q we have to put it back to this relation to check whether it is still validating. So, if we are getting some K_Q value and we are putting in this relation, and then we are seeing that whether the t and a the initial the starting crack length as well as the thickness of the specimen if that is maintaining this relation, then we will know that this

condition is being valid, plane strain condition is being valid and whatever K_Q that we have determined that is same as K_{IC} . So, now, we can safely term this as plane strain fracture toughness which is supposed to be a materials property. So, then we are confident to report this and quote this value.

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CONCLUSION

For executing valid plane strain fracture Toughness testing, thickness and half crack length of the specimen should be equivalent to 50 times the plastic zone size. $t, a > 2.5 \left(\frac{K_{IC}}{\sigma_{YS}} \right)^2$

Plane strain fracture toughness is estimated based on 2% crack extension

C(T), CCT or SENB specimens are mostly used for laboratory scale fracture toughness testing, the former being particularly used .

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So, that leads us to end of this lecture, which says that for executing a valid plane strain fracture toughness testing thickness and half crack length of the specimen should be equivalent to 50 times the plastic zone size and this is what we are getting that this $t, a > 2.5 \left(\frac{K_{IC}}{\sigma_{YS}} \right)^2$. So, this 50 times of the plastic zone size actually is coming as 2.65 and we are simply rounding it up to the value of 2.5.

Plane strain fracture toughness is estimated based on 2% crack extension. So, this is also very important to understand that we are not taking up any arbitrary value since this is an unstable growth of the crack and it goes very very fast. So, we have to put a value and that is equivalent to 2% crack extension which is obtained by using the second line having 5%, slope of 5% less than that of the tangent line and CT or compact tension, center cracked tension or SENB Single Edge Notch Beam specimens are mostly used for laboratory scale fracture toughness testing, however, for most of the cases CT specimen is particularly preferred.

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So, following other references that we have used for this lecture.

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THANK YOU!

Thank you very much.