Fracture, Fatigue and Failure of Materials
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Lecture 10
Plane Stress and Plane Strain Fracture Toughness (Continued)

Hello everyone. We are now, at the 10th lecture of this course Fracture, Fatigue and Failure of Materials.

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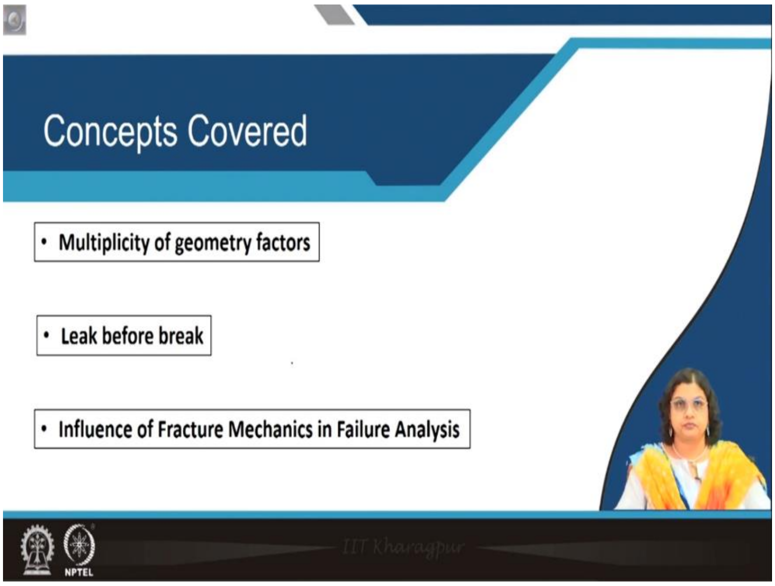

And in this lecture, we are going to discuss about the multiplicity of geometry factors, particularly, for the determination of fracture toughness. And we will see that how based on the crack size, shape, position, how the value of K changes or in other words the fracture toughness value changes. We will also see a specialized condition known as leak before break, particularly, used in case of nuclear reactors.

And how the fracture mechanics the concepts are helping in developing this kind of condition. And we will see that how fracture mechanics is used in case of failure analysis in some real incidents, how can we use these concepts to generate the idea about the fracture toughness or from this leak before break condition, how we can design various components.

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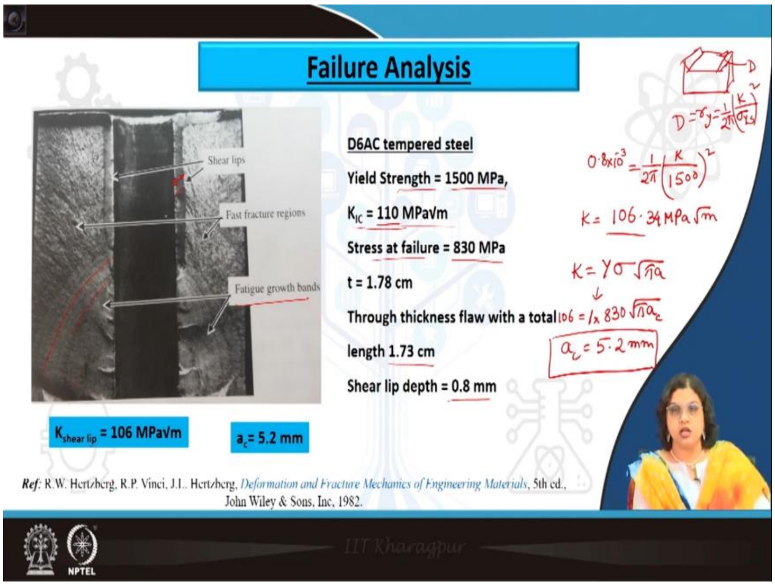

So, before we move on, as we have seen in the last lecture that whenever there is a fracture from the fracture surface itself from the fractograph, it is possible to determine the fracture toughness at the point of fracture and this is done typically considering the shear lip width. So, if this is the shear lip width, which typically occurs because of the plane stress condition, we can determine the shear lip width.

And this is nothing but equivalent to the plastic zone size under plane stress condition. And that is given by D=r_y is given by $\frac{1}{2\pi}(\frac{K}{\sigma_{ys}})^2$. In a similar fashion, we will see how this is employed in the real failure analysis, how this information will be helpful. So, this one here shows the fracture of a lab component and what we can see here is development of some shear lip at this point which is very clearly visible.

And there are some other mechanism that are also active, as you can see here, the stresses for the fatigue crack growth bands, but for now, we would see that for this particular fracture, considering the shear lip, how this can be helpful in generating the fracture toughness values. So, when this kind of failure occurs, we always check with the documents that are present to consider the various properties of the materials that are already established and that has been provided by the supplier.

For example, in this case, the yield strength of the material is quite high. It is a high strength steel of 1500 MPa. Fracture toughness of this material is expected to be 110 $MPa\sqrt{m}$. And stress values at the point of failure is 830 MPa. Thickness is around 1.78 centimeter. And there is a through the thickness flaw which has a total length of 1.73 centimeter. And the shear lip depth is 0.8 millimeter.

So, based on this we can figure out that whether the fracture toughness value at the point of fracture matches with the given provided fracture toughness or the expected fracture toughness values or not. If not, then there could be some other reason for failure that has lead to the local enhancement in the fracture toughness values or if it matches then there could be some other reason not the typical materials property that is of any concern.

So, in this case, let us use this shear lip depth value of 0.8 millimeter to see if we are finding out the K value. So, yield strength of the material is already known as 1500 MPa or 1.5 GPa. So, that leads to K value of $1500^2 x\ 2\pi\ x\ 0.8\ x\ 10^{-3}$ and square root of that is giving us the value of K which is 106.34 $MPa\sqrt{m}$.

$$D = r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

$$0.8\ x\ 10^{-3} = \frac{1}{2\pi}\left(\frac{K}{1500}\right)^2$$

$$K = 106.34\ \ MPa\sqrt{m}$$

$$K = Y\sigma\sqrt{\pi a}$$

$$106.34 = 1x830\sqrt{\pi a_c}$$

$$a_c = 5.2\ mm$$

So, that is what we are seeing here and interestingly what we see is this 106 $MPa\sqrt{m}$ matches pretty well with the provided fracture toughness values, which confirms that it has satisfied this criteria. So, there could be some other reason which we can figure out based on the failure analysis. But we can at least say that there is no other defects or something that has lead to change in the behavior of the material.

The expected value is being met by the fracture toughness values that we can determine right from the fracture surface itself. Now, it has been said that there is a through the thickness flaw, actually, this is considered as flaw in this case, but it actually is the design parameter which looks some kind of change in the geometry because of the design consideration and that may have acted as a flaw. So, that total length is 1.73 centimeter.

We can also figure out that what is the critical flaw length that would have initiated the fracture that would have led the fracture. So, for that we can use the typical relation as K is $Y\sigma\sqrt{\pi a}$. Now, in this case Y is 1 because it is a through the thickness flaw and σ is given as 830 MPa.

What we need to figure out is the critical value of a. And K we have already estimated as 106. So, with this we can find out the value of a or ac the critical value. So, that comes around 5.2 millimeter. So, if there was a crack of just 5.2 millimeter that would have been sufficient to lead to this fracture and instead we are having here a much larger or much longer flaw that have led to this failure.

So, obviously, it has something to do with this flaw itself. And this the values that has been determined, K from this shear lip width is 106 $MPa\sqrt{m}$ which matches pretty well. And the critical value of flaw size that led to fracture is 5.2 millimeter. So, the total length would be 10 millimeter that would have been sufficient for fracture to occur.

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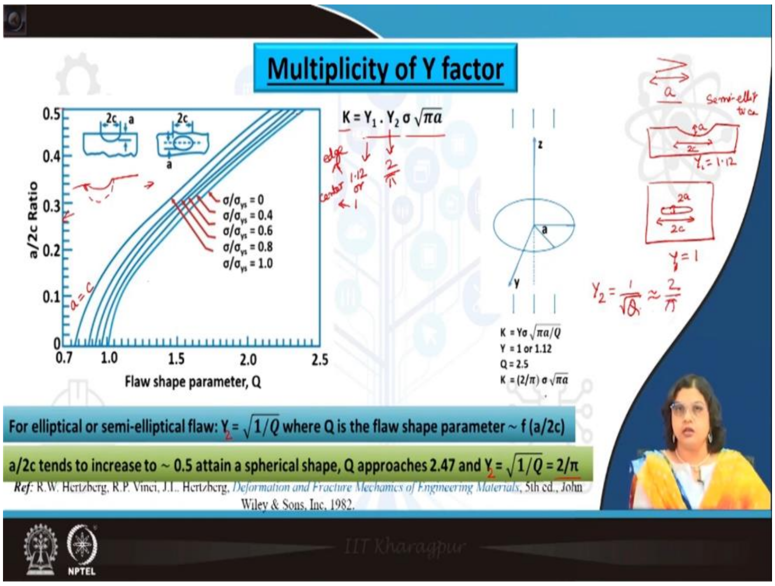

Now, along with that we also need to consider the shape of the flaw. So, so far we have seen that there are notches which are either machined or change in the parameters of the the geometry which leads to a sharp crack. And for that we considered particularly the crack length or half crack length being a.

In some cases, we did consider ρ, particularly, for the calculation of stress concentration factor, but most of the cases, especially, for stress intensity factor, we particularly rely on ‘a’ as the primary crack or the notch or the defect dimension. Now, there could be also possibilities that this defect would not be this much of sharp; rather, it can be an elliptical defect.

For example, let us say a component which looks like this. So, instead of a surface crack in this case we have a elliptical or a semi elliptical void. So, this is a semi elliptical which is having a total length for the major axis as c and the minor axis, this is again the half length and that is equivalent to a.

Or in other words, we can have a component in which there is a through the thickness elliptical crack, in which case, once again the major axis is 2c and the minor axis is 2a. So, in that case along with the value of Y being 1.12 or Y being 1 considering that $\frac{2a}{W}$ is sufficiently less, we also have to consider another parameter which is another type of Y.

So, we also have to consider Y_2. This parameter is related to the shape of the crack. And in this case being elliptical, so, this is related to $\frac{1}{\sqrt{Q}}$, where Q is the flaw shape parameter. We can see here these are practical values that has been determined for such different configurations of crack and the different positions of crack and what we are seeing is this is $\frac{a}{2c}$ ratio is increasing with the flaw shape parameter.

Now, when we have a crack of something like this shape, it has a natural tendency to form a semicircular or a circular void. So, let me draw it here itself. So, if we have a configuration like this, let us say this is the thing and we are applying stress in this direction, in this direction. So, it will try to form a circular shape.

That means a=c, which obviously means the major axis and the minor axis being same. So, this leads to a spherical or a circular shape. And for that matter, when we have this kind of circular shape, this is for the energy balance; anything tends to attain the spherical or the circular shape. So, for that we are getting the value of Q as 2.47.

You can see this has been experimentally validated also for this ratio of $\frac{a}{2c}$ to 0.5, we are getting a value of something like close to 2.5, something like 2.47 to be more specific. So, that makes $\frac{1}{\sqrt{Q}}$ equivalent to $\frac{2}{\pi}$. So, this is what we are seeing this Y parameter or this we often term as Y_2 in this case. So, this Y_2 parameter is $\frac{1}{\sqrt{Q}}$ and that comes to $\frac{2}{\pi}$ because it tends to grow to a circular one. So, as $\frac{a}{2c}$ tends to 0.5, Q tends to 2.47 and y tends to or Y_2 tends to $\frac{2}{\pi}$.

So, essentially, that means that when we have cracks or defects of such shape and such position, we can actually have K which is determined by multiplying this Y_1 and Y_2 along with this typical relations of $\sigma\sqrt{\pi a}$. So, this Y_1 is based on its position. So, this could be 1.12 or 1.

So, this is for edge crack, this is for center crack, we have already seen that and σ is the applied stress πa are all that we already know. And Y_2 is coming because of the shape of this defect or notch and this is for the case of semi elliptical one. This is always $\frac{2}{\pi}$, because it tends to be a circular one. This is the same thing that has been shown here. So, based on this, there are different kind of calculations which are typically done based on this defect size and shape and the position, so that we can predict the K value in a more accurate way.

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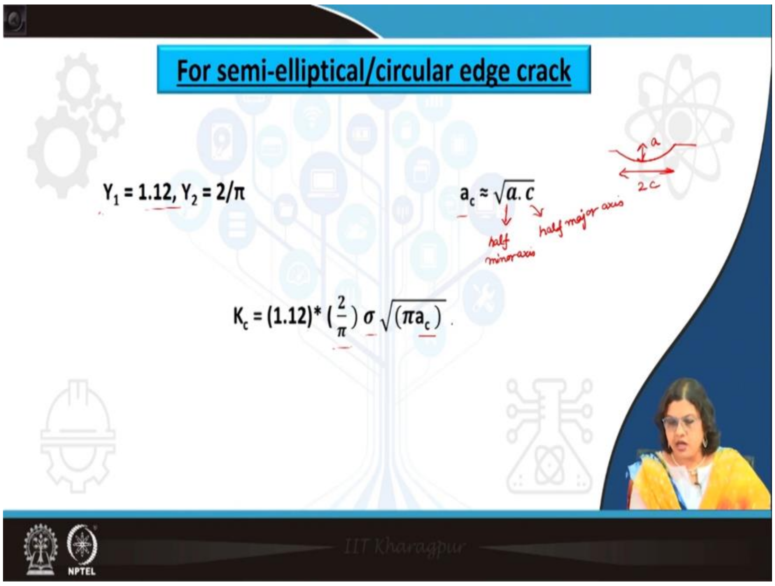

So, this is what is shown here. So, Y_1 as 1.12 for the case of edge crack and Y_2 is $\frac{2}{\pi}$. So, this is typically for semi elliptical or semicircular edge crack. Now, if we have a elliptical one and in which case the c and the a are distinctly different numbers and it is not semi circular but semi elliptical, in that case, we often needs to find out the a_c value.

Now, a_c here is not the critical one but this is a combined one, considering both the major and the minor axis. And this is given by square root of the product of the half major axis and half-minor axis. So, this is half major axis because unlike a sharp crack or unlike a circular one. Now, we are considering a elliptical one. So, we have to consider their overall radius which is given by a product of root square of half-major and half-minor axis. So, this is how the relation can be then formulated for an edge crack, where Y_1 is 1.12, Y_2 is $\frac{2}{\pi}$, σ is the applied stress and a_c is $\sqrt{a}.c$.

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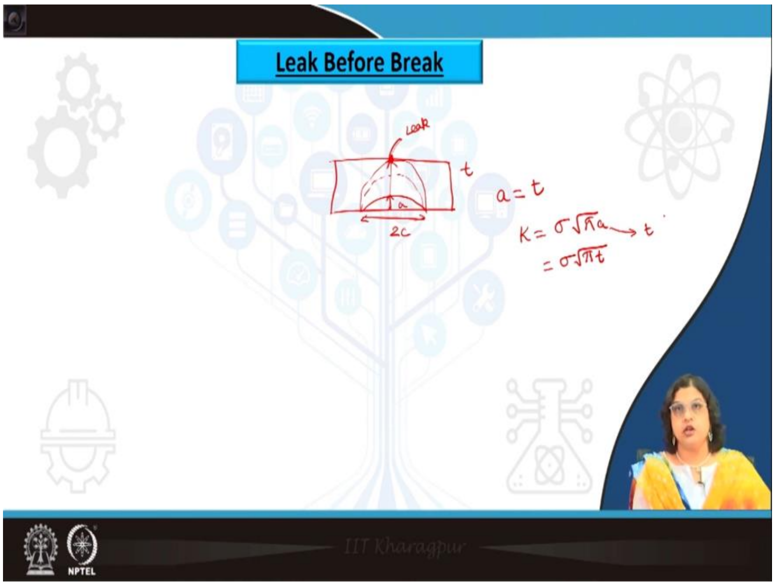

So, this leads us to a very interesting mechanism of leak before break. Now, what happens in case of particularly applicable for nuclear industries? In case of pressure vessels, when we have a cylindrical vessels in which the pressurized fluids are being kept. So, if there is any kind of cracks that generates on the inner wall of those kind of cylinders that can lead to either a catastrophic failure or a leak before break condition. Of course, the leak before break one is preferred to avoid the catastrophic failure.

Let me explain this in a more clear way. So, what happens is that. So, let us consider a cylindrical pressure vessel and in which there is a semi elliptical flaw on the inner surface of the pressure vessel and let us say that this is the thickness. So, what happens is that there is a hoop stress that is being applied and as a result because of this hoop stress, this crack is expected to increase its dimension. So, this is 2c once again the major axis and the minor axis is a.

Now, what happens is that as we discussed that this always has a tendency to grow to a semicircular one. Just to minimize the energy balance. So, in that case if it grows such that this a or the length of the minor axis which is now equivalent to the half-length of the major axis, when it becomes equals to the thickness of the cylinder then there will be leakage exactly at the point of contact between this crack faces as well as the cylinder wall.

So, this leads to a leakage and whatever pressurized fluid is stored inside the cylinder that can come out from this and there will not be any catastrophic failure. So, that although is a failure even but that helps in avoiding the catastrophic failure. So, that is why leak before break condition is favored.

So, in case of leak before break condition, actually, the typical relation as K equals to $\sigma\sqrt{\pi a}$, the one that we have developed from the Griffith criterion itself, this is considered as $\sigma\sqrt{\pi t}$, because at the league before break condition, this a is equivalent to the thickness of the cylinder.

So, to make the leak before break condition applicable, we have to see, we have to check that whether this a equals to t can be obtained and still it should not exceed the typical fracture toughness value of a material. When we use a material for any kind of design, we first of all need to know the fracture toughness values of that. And this fracture toughness value means the plane strain fracture toughness value which is supposed to be a constant one.

Now, in practice we need to also figure out that what would be, under certain conditions, what would be the fracture toughness values and whether that exceeds or whether that is less than the predicted value determined for different conditions. So, in this case for this leak before break condition to be active, we need to determine the K value and see that whether that exceeds the expected fracture toughness values, in which case there will be catastrophic failure, or it is below the expected plane strain fracture toughness value. So, that the leak before break condition can be applicable.

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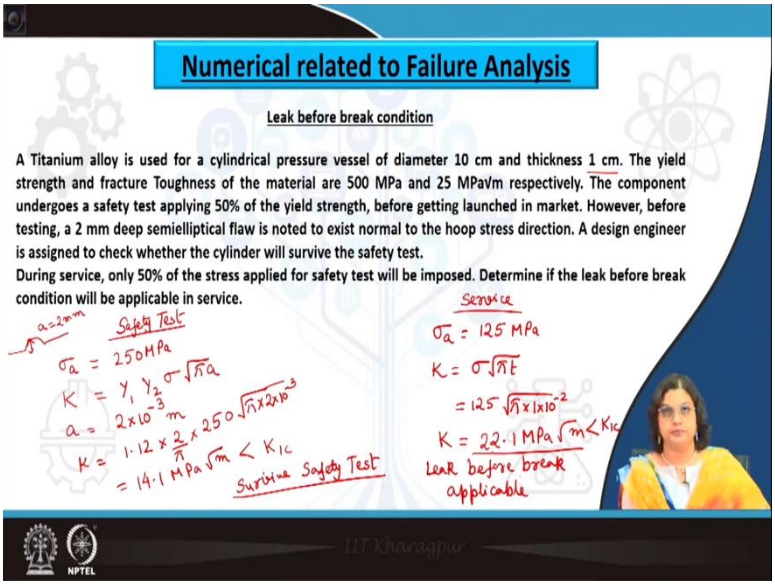

So, let us do a numerical to make this clear. What it says is a titanium alloy which is used for a cylindrical pressure vessel, which is having a diameter of 10 centimeter and thickness of 1 centimeter. The yield strength and fracture toughness of this material are 500 MPa and 25 $MPa\sqrt{m}$, respectively.

And the component undergoes a safety test applying 50 percent of the yield strength before getting launched in market. So, this is very essential before something is being launched it is always tested and tested at a condition which is more severe than the actual service condition. But before the safety test it is found that there is a 2-millimeter deep semi elliptical flow, which exists normal to the hoop stress direction, and a design engineer is assigned to check whether the cylinder will survive the safety test.

There is another part of this problem, which says that during service only 50 percent of the stress is applied. The stress that is applied for safety test is being imposed. So, that means for the case of service, it is actually half of whatever has been applied for the safety test. And we need to determine the leak before condition whether that will be applicable in service or not.

So, let us first solve the safety test criteria. So, what it says is that the applied strength, stress is 50 percent of σ_{ys}, so, that means σ_{ys} is 500 MPa that leaves to σ_a as 250 MPa. And we need to find out the K value as $Y_1Y_2 \sigma\sqrt{\pi a}$. a in this case is given as 2 millimeter deep. So, that is half the length of the minor axis.

So, this is given as a = 2 millimeter. So, 2 x 10^{-3} m. And that leads to Y_1 as 1.12 because we are having a surface crack and then Y_2 is $\frac{2}{\pi}$ because it is an elliptical one. And then we have σ which is 250 MPa and π x 2 x 10^{-3}.

So, if we solve this quickly, we see the value as the following. So, this is coming as something like 14.1 $MPa\sqrt{m}$. So, that means that the stress intensity factor value that is being generated for the safety test is much lesser than the K_{IC} value, which is 25 $MPa\sqrt{m}$.

So, obviously, this means that it will survive the safety test. So, that is for sure that it is not going to break down at the safety test, although, there is a flaw which is 2 millimeter deep and semi elliptical. Now, let us check the condition of leak before break for the service. So, in case of service it is seen that the stress applied for service is actually 50 percent of that used for safety test. So, that means it is 125 MPa and if we want to see the leak before break condition, we typically need to find out the K for this case. So, it says $\sigma\sqrt{\pi t}$.

So, σ is 125 and t is 1 centimeter. So, that makes K value of around 22.1 $MPa\sqrt{m}$. So, we see that leak before break condition will be applicable because this value that is generated is less than the K_{IC}. So, it will fracture at the point of contact at the leakage point that will lead to the leakage. But the margin is very very low because K_{IC} is only 25 MPa and we are getting a value of 22 MPa, the margin is really close. And that should also be considered while using this in service.

$$\sigma_a = 250\ MPa$$
$$K = Y_1 Y_2 \sigma\sqrt{\pi a}$$
$$K = 1.12\ x\ \frac{2}{\pi} x\ 250 x\ \sqrt{\pi\ x\ 2\ x\ 10^{-3}}$$
$$K = 14.1\ MPa\sqrt{m} < K_{IC}$$

$$\sigma_a = 125\ MPa$$
$$K = \sigma\sqrt{\pi t}$$
$$K = 125\sqrt{\pi\ x\ 1\ x\ 10^{-2}}$$
$$K = 22.1\ MPa\sqrt{m} < K_{IC}$$

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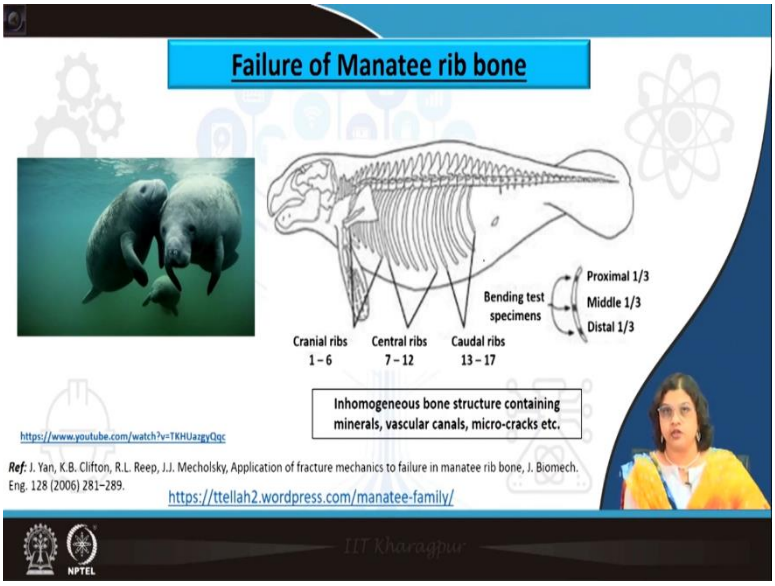

So, let us move on to another real life example, where the concept of fracture mechanics is very very helpful. So, these animals here are known as manatee also known as sea elephant, sometimes as seahorse. These are found in the sea of Florida and these are giant animals, very very big something like of the size of around 200 or 240, 250 centimeter.

But these are being constantly killed by the watercrafts, water vehicles that are used on that water ways, for example, the steamers, the small ships or the crews, they simply hit this animal and their bones get fractured, particularly, their rib bones get fractured and they die. So, there has been a lot of initiative to find out that even the path of the watercrafts or their speed limits the kind of stress that they can generate to figure out a way to save these animals.

So, let us see how we, while talking about fracture mechanics, how we can use our knowledge to be of little help. So, what we are seeing here is the internal structure of the manatee bone structure and what we are seeing here is the the rib base. So, there are around 17 to 19 pairs of rib bones in the different parts and people have done some testing to figure out that how impact with the different water crafts that is happening with this ribs, how can that lead to fracture and what is the typical fracture toughness of their bones.

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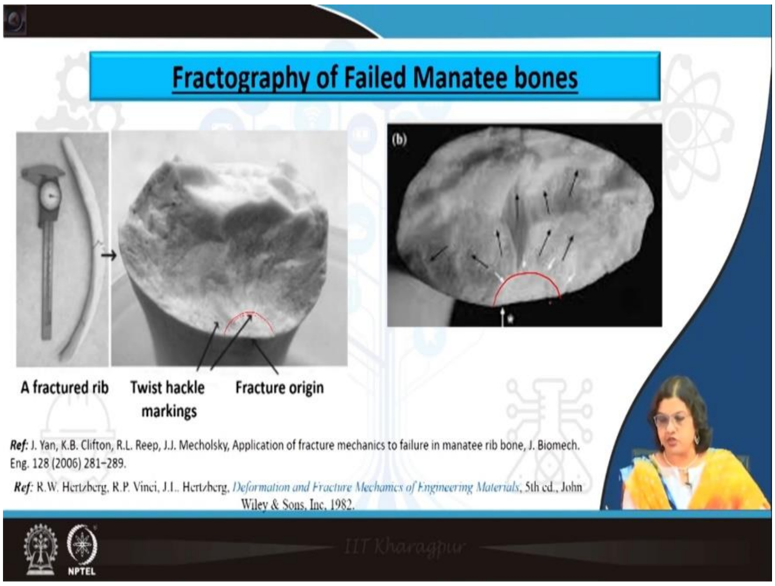

So, these are some of the fractographies from the fractured rib bones and what from there itself we can figure out their fracture toughness values. Because, otherwise, their bones are actually a little a compact one compared to human bones and they also have some pores in their bones and that leads to the failure that acts as the crack or the defects that leads to the fracture.

So, after the fracture has happened, we need to figure out from the fracture surface itself that what has been the impact, how much load or how much stress or what would have been the stress intensity factor or fracture toughness that might have led to the fracture. So, this is once again a post mortem study for the failure analysis part.

So, from these images here, you can see that the crack initiation or the fracture initiation site is somewhere near the surface. Here it is very clearly if we can see that there might have been some kind of defect of the semicircular or semi elliptical origin that might have led to the fracture.

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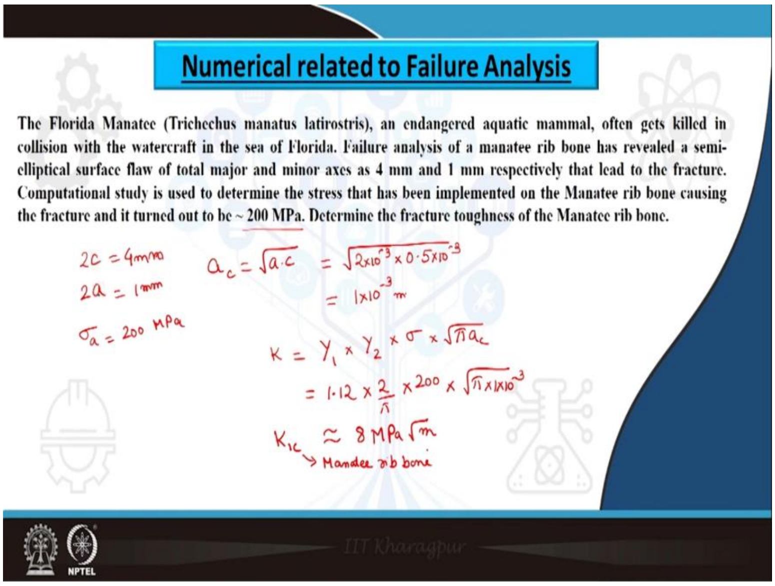

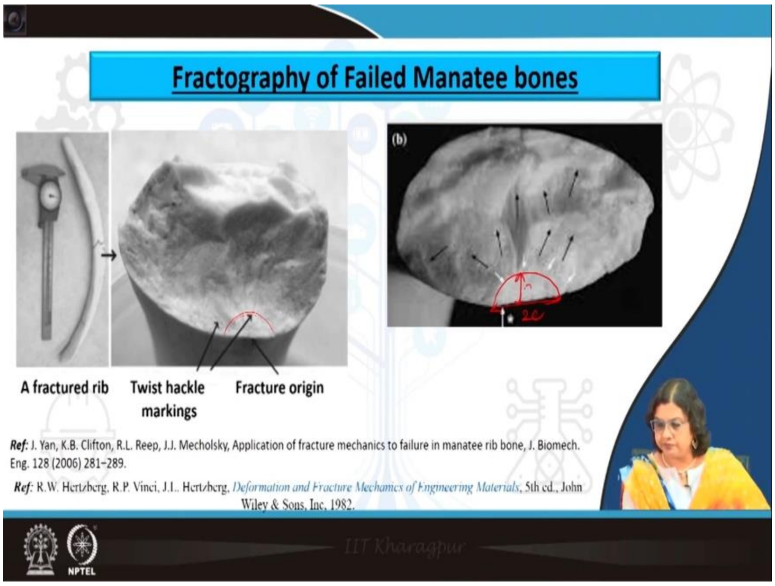

And to solve that here is one more example or a numerical through which we will see that how much could be the fracture toughness, estimated fracture toughness of this manatee bone. So, it says this Florida manatee which is having a scientific name like this an endangered aquatic mammal often gets killed in collision with the watercraft in the sea of Florida.

And failure analysis of one of this manatee rib bone revealed a semi elliptical surface flaw of total major and minor axis of 4 millimeter and 1 millimeter, respectively. So, such flaw size we can very well determine from that semicircular or semi elliptical size that we have seen here, we can determine the major axis and the minor axis very well.

And computational study is used to determine the stress that has been implemented on the manatee rib bone causing the fracture and this stress that has led to the fracture turn to 200 MPa. What we need to find out is the fracture toughness of the magnetic rib bone. So, what we have here is. So, that means that a_c which is given by $\sqrt{a}.c$ will be $\sqrt{2\ x\ 10^{-3} x\ 0.5x\ 10^{-3}}$. So, that comes to very simple. So, that comes to 1 millimeter.

What else we have is the applied stress which is 200 MPa. And what we need to find out once again is K which is related to $Y_1 Y_2 \sigma\sqrt{\pi a_c}$. So, that comes to, this is once again a surface flow. So, $1.12x\frac{2}{\pi}x200x\sqrt{\pi\ x\ 10^{-3}}$. So, that turns out to something like this. So, it turns around 7.99 or let us approximate this as 8 $MPa\sqrt{m}$.

$$a_c = \sqrt{a.c} = \sqrt{2\ x\ 10^{-3} x\ 0.5\ x\ 10^{-3}} = 1\ x\ 10^{-3}\ m$$

$$K = Y_1 Y_2\ x\ \sigma\ x\ \sqrt{\pi a_c}$$

$$K = 1.12\ x\ \frac{2}{\pi}\ x\ 200\ \sqrt{\pi\ x\ 10^{-3}} = 8\ MPa\sqrt{m}$$

Fracture toughness values of manatee rib bone turns out to be 8 $MPa\sqrt{m}$. So, first of all we see that this value is quite less compared to the metallic parts that we were discussing. So, far and all the other values that we have considered, the bones typically in this case the manatee bones has much lesser values of fracture toughness. But not only that when we do this real calculations and often found that these are again dependent on the size of the animal, the size of the bones, the impact, etcetera lot of other factors that also need to be taken care of.

But the point here was to discuss about this little off bit topic is to see that how fracture toughness can be used, the concepts of fracture toughness can be used in the different aspects, something which is typically different from the structural problems that we deal with the industries.

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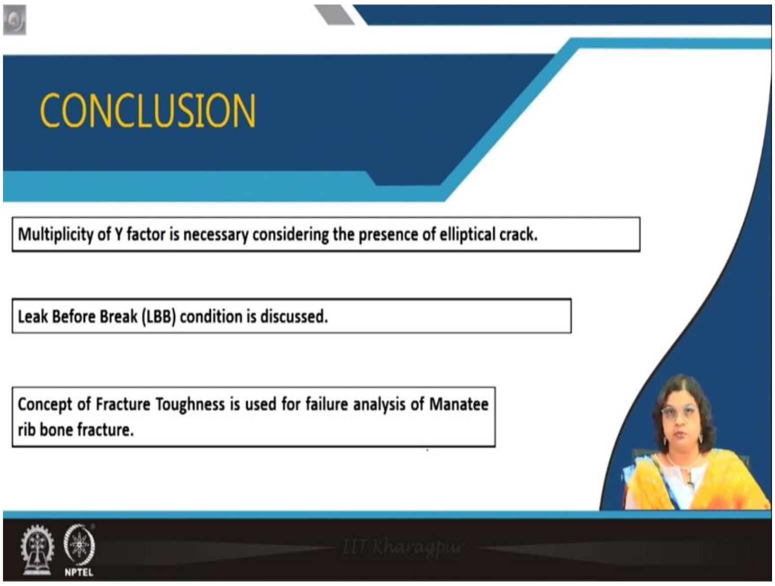

So, let us come to the conclusion for this lecture. What we discussed here is the effect of this multiplicity Y of Y factor due to the presence of elliptical or semi elliptical crack. And we have also considered or discussed about the leak before break condition and how that can be generated, how to take care of this factor while designing a component for such critical application. And the concept of fracture toughness is used for the failure analysis of manatee rib bone fracture.

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So, following are the references that has been used for this lecture. Thank you very much.