Mechanical behavior of materials

Dr. Niraj Mohan Chawake

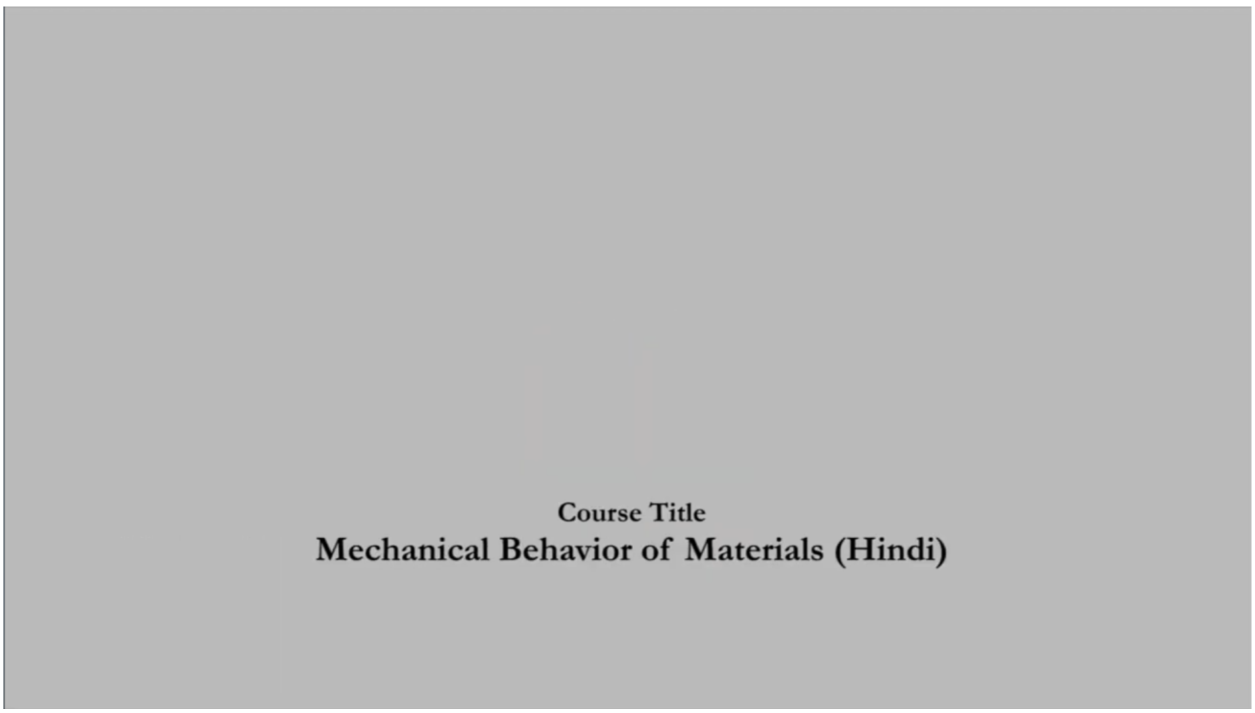
Department of Materials Science and Engineering

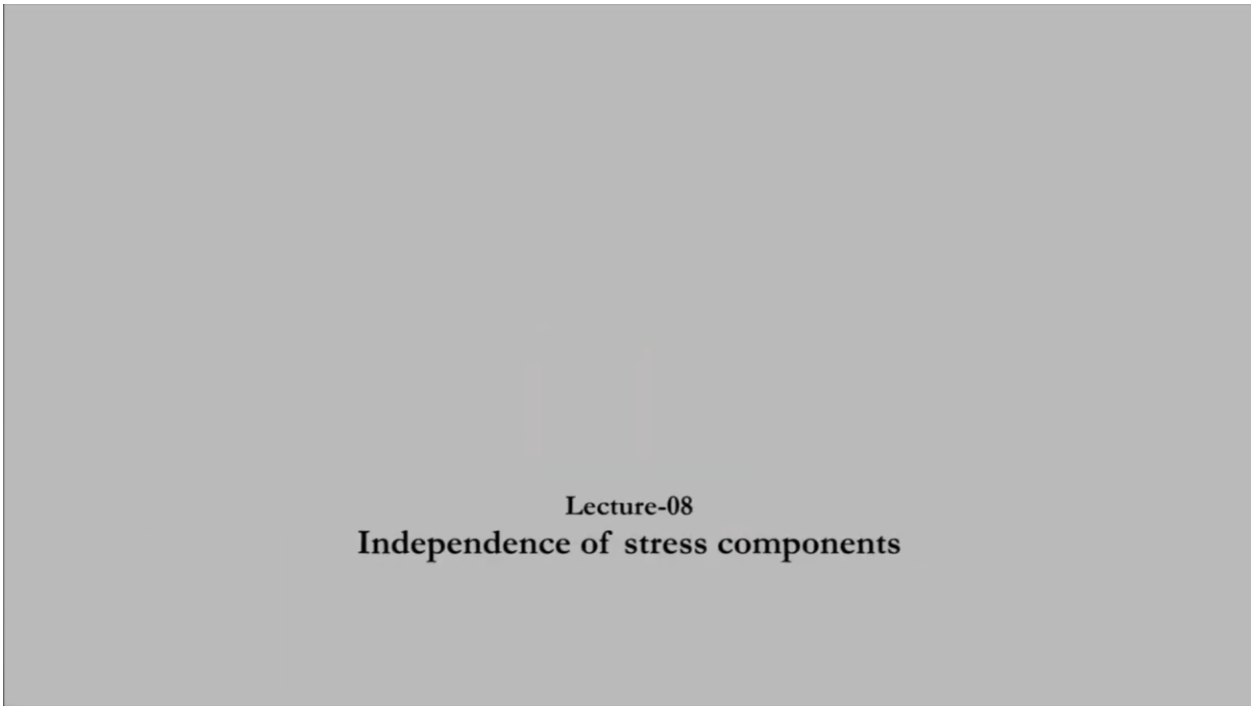
Indian Institute of Technology, Kanpur

Week-2

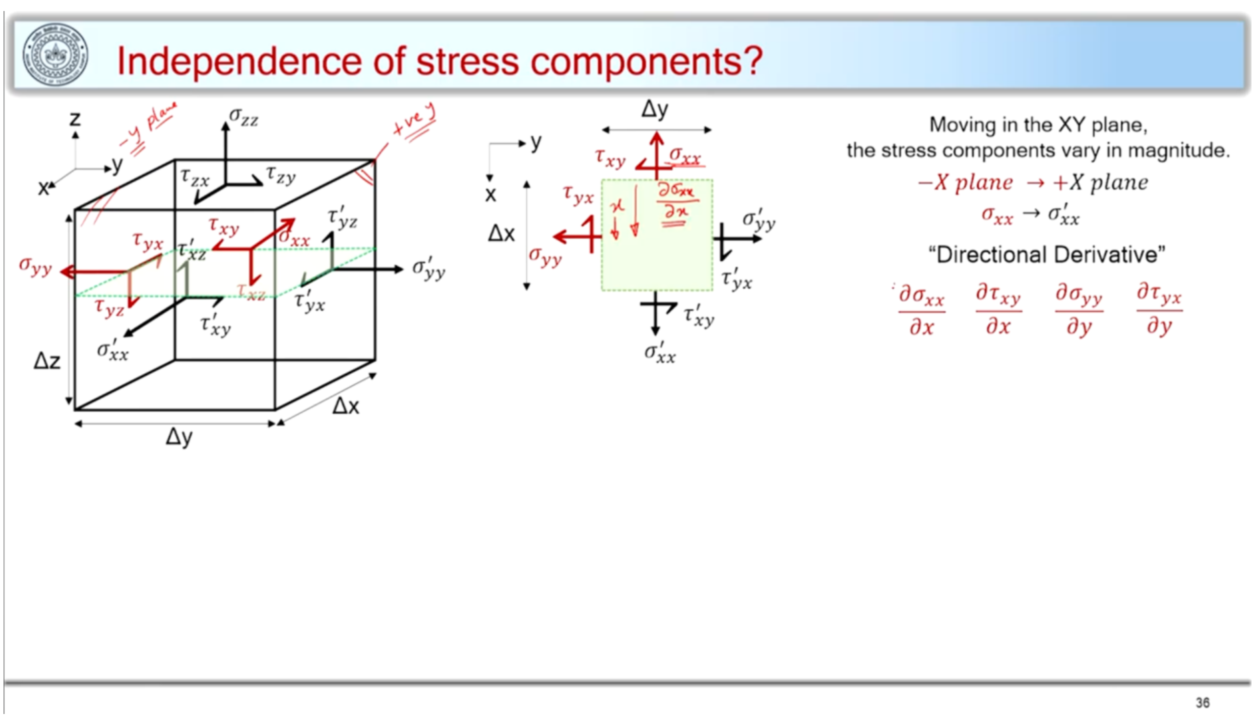
Lecture-8

Independence of stress components





Swagat karta hoon main aapka is course mein jiska naam Mechanical Bar of Material hai jo main Hindi mein padhaunga.
Toh last part mein humne dekha ki jo stress hote hain, jo stress tensor hai, usmein **9 components** hote hain stresses ke, aur unmein se kaun se components independent hain, yeh is part mein aaj dekhenge.



Toh humne last time mein dekha tha ki mera infinitesimal ek cube hai aur iske coordinates maine is tarah se mark karke rakhe, aur yahan par kuch dimensions is infinitesimal cube ke hum mark kar dete jisse yani hum state of stress dekh rahe the material mein.
Toh humne ek point consider kiya tha is cube ko.

Abhi is infinitesimal stress cube ke liye hum iske dimensions mark karte the — x direction mein **Δx**, y direction mein **Δy**, aur z direction mein **Δz**.
Ye iske dimensions ho gaye.

Aur abhi kuch stresses mark karte hain.
Toh yeh jo face hai, yahan par maine kuch stress components mark kiye.
Yeh stress ko main kehta hoon **σ_{yy}**, **τ_{yx}** aur **τ_{yz}**.
Aur isi prakaar yeh jo negative x-plane hai, us par kuch stress components mark kiye — **σ_{xx}**, **τ_{xz}**, **τ_{xy}**.

Ab z-plane par kuch stress components mark kiye — **σ_{zz}**, **τ_{zx}**, **τ_{zy}**.

Abhi is positive y-plane par kuch stress components mark kiye, usko maine likha hai **σ_{yy}'**, **τ_{yz}'**, **τ_{yx}'**.
Isi prakaar positive x-plane par jo kuch components mark kiye hain, unka naam likha hai **σ_{xx}'**, **τ_{xy}'**, **τ_{xz}'**.

Toh aapko toh pata hi hai ki yeh kis tarah se nomenclature hota hai.
Green color ka plane maine mark kiya hai jo ki mera xy-plane hai.
xy-plane yani z-direction isko perpendicular rahegi.
Yeh mera xy-plane ho gaya.
Is xy-plane ko hum z-plane bhi keh sakte hain.

Toh ek z-plane ya xy-plane — toh main abhi ke liye xy-plane mark kar bol raha hoon isse.
Toh yeh green plane agar main upar se dekhunga, yani z-direction se dekhunga, toh mujhe kuch is tarah se dikhayega.

Toh uske dimensions maine mark kar liye — yeh mera x-direction ho gaya, yeh mera y-direction ho gaya, **Δx** iski dimension x-direction mein, aur **Δy** y-direction mein.

Aur stresses maine mark kar liye jo ki negative planes par the — yeh mera negative y-plane tha aur yeh mera negative x-plane tha.
Toh **σ_{xx}**, **τ_{xy}**; **σ_{yy}**, **τ_{yx}** — yeh mark kar liye.

Aur usi tarah se positive x aur positive y planes par jo stress components act ho rahe, unko maine mark kar liya hai — **σ_{xx}'**, **τ_{xy}'**, **τ_{yx}'**, **σ_{yy}'**.

Abhi yani hum upar se dekhenge toh mujhe yahi stress components dikhte hain is plane mein jo act ho rahe hain.

Abhi yani humne kuch stress components mark kiye hain jaise maine **σ_{yy}** aur **σ_{yy}'** mark kiya hai.
Toh yeh isliye mark kiya hai kyunki jaise agar main cube consider karta hoon toh main xy-plane mein move kar raha hoon, toh mere stresses change hone chahiye ya stresses badalte hain.

Yeh kisi bhi point par agar main move kar raha hoon, toh yeh maine grihit dhara hua hai.
Toh jaise main negative x-plane se positive x-plane tak ja raha hoon — jaise negative x-plane yeh mera negative x-plane hai aur yeh positive x-plane hai — toh **σ_{xx}** change ho raha hai mera **σ_{xx}'**.

Toh yeh jo variation hai isko hum directional derivative se samajh sakte hain.

Directional derivative kya hota hai — jaise yeh stress mein kya change aa raha hai jaise main x-direction mein move ho raha hoon.
Isko hum samajhte hain — jaise main is direction se negative x-plane se positive x-plane ki taraf move ho raha hoon, toh mera jo **σ_{xx}** hai kaise change ho raha hai.
Jaise main x-direction mein badh raha hoon aage, toh main bol sakta hoon ki small change jo ho raha hai **σ_{xx}** mein with respect to x-direction, is tarah se main is directional derivative ko likh sakta hoon.

Usi tarah main baaki ke stress components ka change kaise ho raha hai us direction ke along likh sakta hoon, jaise **τ_{xy}** mein kya change ho raha hai — jaise **τ_{xy}** hai negative x-plane pe aur **τ_{xy}'** likha hai toh yeh is tarah se kaise change ho raha hai yeh humne yahan par likha hai.

Toh main **σ_{xx}'** ko is tarah se likh sakta hoon:
$\sigma_{xx}' = \sigma_{xx} + (\Delta\sigma_{xx}/\Delta x) \times \Delta x$

Toh yeh mujhe **σ_{xx}'** ki value dega.

Usi tarah **τ_{xy}'** ki value mujhe milegi:
$\tau_{xy}' = \tau_{xy} + (\Delta\tau_{xy}/\Delta x) \times \Delta x$

Similarly, **σ_{yy}'** ko main is tarah se likh sakta hoon:
$\sigma_{yy}' = \sigma_{yy} + (\Delta\sigma_{yy}/\Delta y) \times \Delta y$

Usi tarah **τ_{yx}'** bhi likh sakta hoon:
$\tau_{yx}' = \tau_{yx} + (\Delta\tau_{yx}/\Delta y) \times \Delta y$

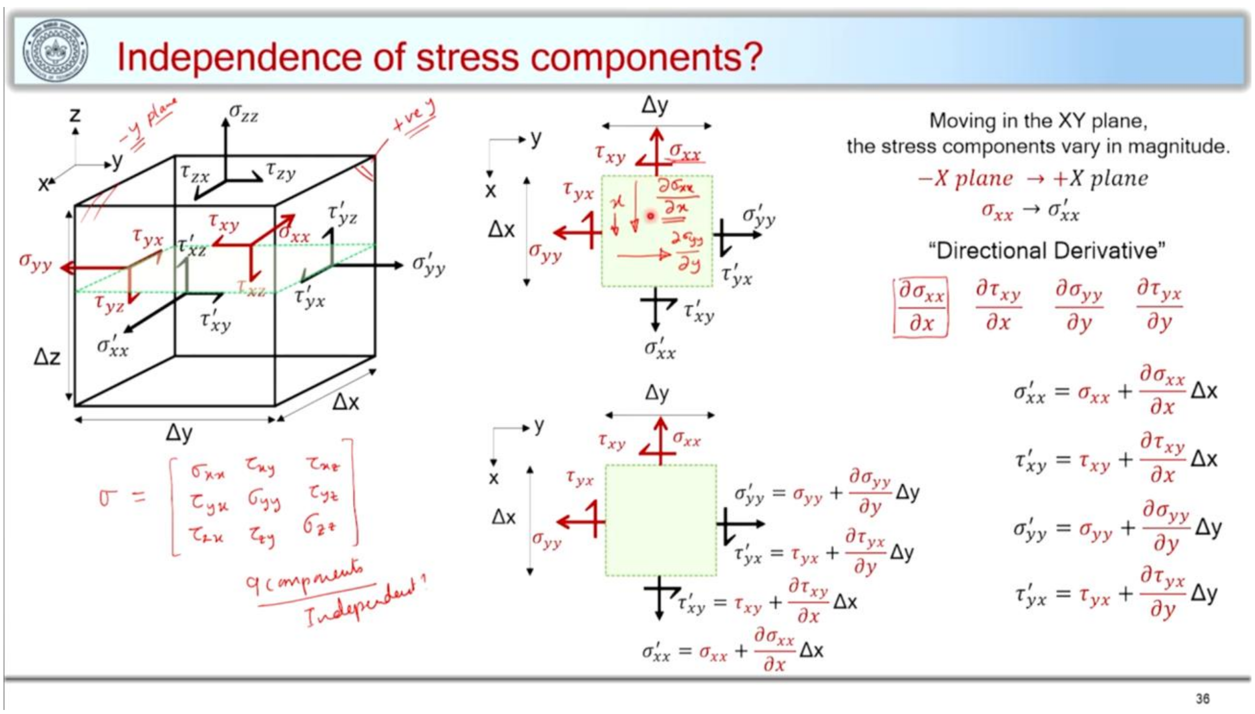
Toh mere saare components maine likh liye.
Toh is tarah se yeh negative faces par mere components the, positive faces par mere components is tarah se mil jaayenge jo ki maine directional derivative ke hisaab se likhe.

Abhi hume dekhna hai ki stress components — yani hamara ek question hai: jaise mere paas stress tensor hai.
Stress tensor humne likha tha:

$$\begin{bmatrix} \sigma xx & \tau xy & \tau xz \\ \tau yx & \sigma yy & \tau yz \\ \tau zx & \tau zy & \sigma zz \end{bmatrix}$$

Toh inme se yeh 9 components ho gaye mere.
Toh inme se kaun se components mere independent hain — yeh hi aaj hume dekhna hai.

Toh isko dekhne ke liye humne yeh ek plane consider kiya tha.
Is plane ke around jo bhi maine stress components hain, woh sab mark kar liye the.



Abhi yeh jo mere mark components hain, toh humne ek baat ki thi ki hamare paas yeh jo member hai aur jo plane hai, ismein bhi jo bhi components aayenge aur jo is cube mein jo bhi components aayenge, woh sab static equilibrium mein rahenge.

Toh hum static equilibrium ki baat karte toh hum baat karte hain: **ΣF = 0** aur **ΣM = 0**.
Toh saare forces ka summation zero hona chahiye, saare moments ka summation zero hona chahiye.

Toh ek aisi equilibrium condition hum apply karte hain — jaise moment along z-direction.
Yani moment along z-direction hum define karenge aur isko is tarah se likhenge:
Σ(Mz) = 0

Toh moment aapko pata hai — moment = force × perpendicular distance.
Toh is case mein hum abhi moment nikaalenge z-direction ke along.

Toh hamari z-direction kahan par hai?
Hamari z-direction kyunki xy-plane hai, toh z-direction is plane ko perpendicular hai.
Toh hum yeh point O maan lete hain jo ki along z-direction hai.
Aur is point O ke around hum moment nikaalne ki koshish karenge.

Toh jab yeh point maine centre pe maan liya, toh yeh distances ho jaayenge — yahan se is plane ka negative y-plane ka distance ho jaayega **Δy / 2**, aur positive y-plane ka distance ho jaayega **Δy / 2**.
Similarly, negative x aur positive x-plane bhi equal distance **Δx / 2** apart hain point O se.

Toh 3D mein hum is tarah se dekhenge, toh 3D mein hume dikhega ki yeh jo mere planes hain, yeh mera — yeh jo mere force components hain, toh hum moment jab nikaalenge toh moment is tarah se nikaalte hain — ek counter-clockwise.

Moment aayega point O ke around aur ek clockwise moment aayega.
Toh counter-clockwise moment kaunsa hoga ismein?
Yeh jo forces hain — yeh jo forces hain **τxy** aur **τxy′** — yeh karenge hum.
Yeh mujhe is point ke around ek counter-clockwise moment denge,
kyunki agar main force is tarah se apply kar raha hoon,
toh yeh forces move karenge is direction mein — ek counter-clockwise motion de rahe hain is point O ke around.
Isiliye hum isko counter-clockwise moment kahenge.

Toh abhi hume moment jab nikalna hai toh hume force chahiye.
Toh hamare paas toh yeh stress components hain.
Aur ek cheez yahan par main bolun — jab hum shear stresses nikalte hain
toh shear stresses ko is symbol se denote karte hain — yeh half-arrow ki tarah se.

Toh hamare paas abhi force nikalna hai.
Hamare paas toh stress component hai, toh stress component — yeh **τxy** kis plane par act kar raha hai?
Yeh act kar raha hai mere negative y-plane par.

Toh negative y-plane yahan par kahan par hai?
Negative y-plane yeh plane hai.
Toh mere paas stress hai, toh stress ko main force mein jab convert karunga,
toh stress ko likhte hain: force upon area.
Toh force jab nikalne ki koshish karunga, toh force aayega: **stress** × **area**.

Toh yeh negative y-plane ka area kya hoga?
Iske dimensions ho gaye mere **Δz** × **Δy**.
Toh yeh mera **τxy** hai.
Aur yeh **τxy** jo negative y-plane par act ho raha hai,
uska area hai **Δy** × **Δz**,
aur point O se kitna distance apart hai?
Yani **Δx / 2**.

Toh yeh perpendicular distance ho gaya is force ka jo acting point hai.
Toh yeh ho jaayega $\mathbf{\Delta x / 2}$.

Toh yeh mera moment aayega — complete moment aayega because of $\boldsymbol{\tau_{xy}}$.

Similarly, $\boldsymbol{\tau_{xy}'}$ se kya aayega?
Toh $\boldsymbol{\tau_{xy}'}$ act kar raha hai mere positive y-plane ke plane pe,
jiska area hoga $\mathbf{\Delta y \times \Delta z}$,
aur moment arm hoga iska $\mathbf{\Delta x / 2}$.
Yeh ho gaye mere counter-clockwise moments.

Toh inka summation aana chahiye mere clockwise moments pe.
Toh yeh clockwise moments kaunse hain?
Yahan pe kaunse forces act kar rahe hain?

Toh mere paas yeh do clockwise moments hain.
Aur yeh jo forces hain jaise normal stresses $\boldsymbol{\sigma_{xx}}$ aur $\boldsymbol{\sigma_{yy}}$,
yeh point O se pass ho rahe hain,
toh yeh koi moment create nahi karenge along this point O.

Toh yeh ho jaayega mera $\boldsymbol{\tau_{yx}}$.
Agar main clockwise moment nikal raha hoon,
$\boldsymbol{\tau_{yx}}$ yeh act kar raha hai mere negative x-plane pe.

Pehle maine jo bola tha — yeh jo plane hai mera negative x-plane hai,
aur yeh jo mera hai negative y-plane hai.
Toh $\boldsymbol{\tau_{yx}}$ act kar raha hai mere negative y-plane pe.
Toh iska area hai — yeh plane ka area hoga $\mathbf{\Delta z \times \Delta x}$.

Toh mere paas aa jaayega — yeh ho jaayega $\mathbf{\Delta x \times \Delta z}$,
kyunki iska area hoga — yeh x hai is direction mein,
toh yeh $\mathbf{\Delta x \times \Delta z}$.
So yeh jo area hoga is plane ka, yeh $\mathbf{\Delta x \times \Delta z}$ hoga,
jo yahan par mark kiya hua hai,
aur iska moment arm hoga $\mathbf{\Delta y / 2}$.

Similarly, yeh jo $\boldsymbol{\tau_{yx}'}$ hai,
yeh act kar raha hai mere positive y-plane pe.
Toh positive y-plane ka area hoga $\mathbf{\Delta x \times \Delta z}$,
aur iska moment arm hai $\mathbf{\Delta y / 2}$.

Toh mere paas dono moment aa gaye —
counter-clockwise moment balance ho jaayega clockwise moment se.

Toh jab yeh balance hoga,
tab hamara summation zero aana chahiye.

Ab aap dekhenge ki in teeno ka product —
jaise **$\Delta y \times \Delta z \times \Delta x$**,
$\Delta y \times \Delta z \times \Delta x$,
yahan par bhi **$\Delta y \times \Delta z \times \Delta x$**,
toh yeh volume ho jaayega is infinitesimal cube ka,
aur yeh constant hai,
toh hum ise cancel out kar sakte hain.

Toh hamare paas yeh terms baaki rahenge.
Aur humne yahan par abhi **$\tau xy'$** ko replace karenge, **$\tau yx'$** ko replace karenge
directional derivatives ke hisaab se.

Toh hamare paas yeh terms aayenge:
$\tau xy' = \tau xy + (\Delta\tau xy/\Delta x) \times \Delta x$
$\tau yx' = \tau yx + (\Delta\tau yx/\Delta y) \times \Delta y$

Toh agar hum dekhenge,
jaise hum bolenge ki agar yeh area bahut small hai,
toh **$\Delta x \to 0$** aur **$\Delta y \to 0$**,
toh yeh terms jo hain — yeh term zero ho jaayegi,
aur hum is tarah se likh paayenge:
$\tau xy = \tau yx$

Toh hum is tarah se dekhenge ki **$\tau xy = \tau yx$**.

Humne is exercise se yeh dekha ki
yeh jo do components hain **τxy** aur **τyx**,
yeh equal hone chahiye jab hume static equilibrium maintain karna hai kisi material mein.

Toh similarly hum yeh bhi dikha sakte hain ki
jo stress components hain — shear components —
$\tau xz = \tau zx$ aur **$\tau yz = \tau zy$**.

Toh agar hum stress tensor dekhenge,
toh hamare paas yeh toh normal stress components hain,
aur yeh shear stress components hain.

Toh **$\tau xy = \tau yx$**,
$\tau xz = \tau zx$,
aur **$\tau yz = \tau zy$**.

Toh hume **6 independent stress components** milte hain hamare paas.

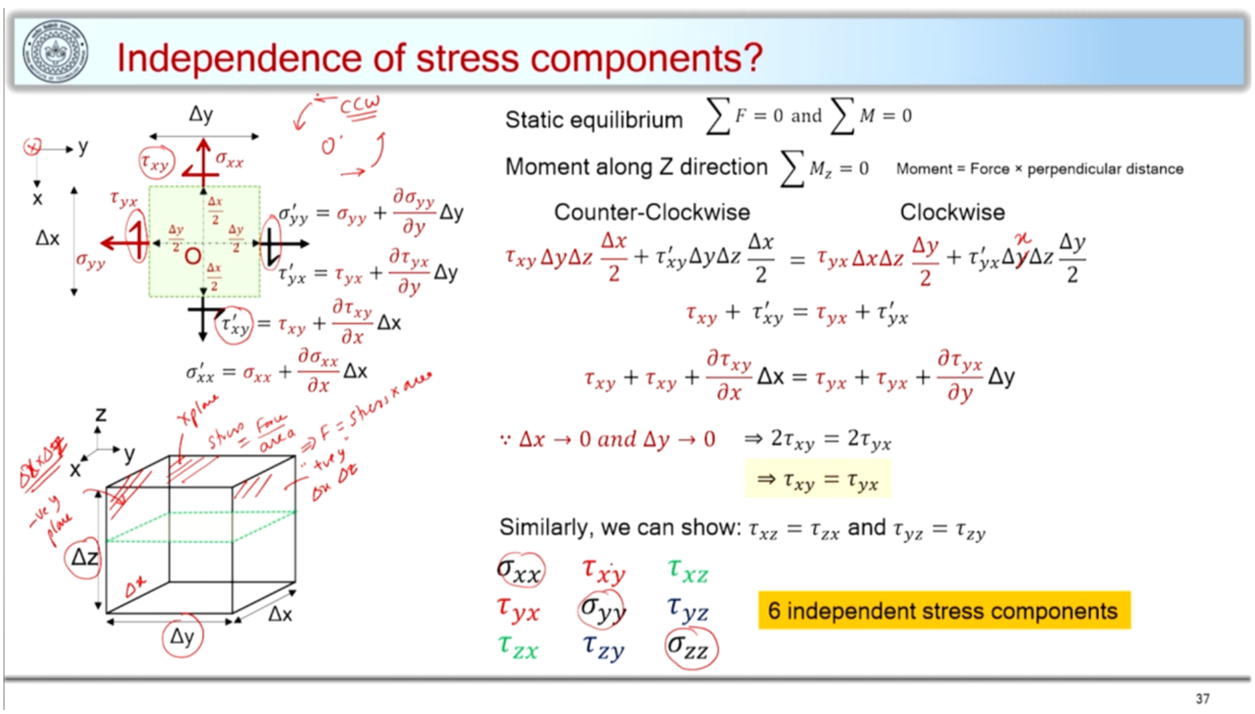
Toh aap agar dekhenge —
yeh ek mera independent component hai (**σxx**),
yeh ek mera independent component hai (**σyy**),

yeh ek mera independent component hai (**σzz**) —
teen ho gaye normal stresses.

Phir **τxy** agar mujhe pata hai toh mujhe **τyx** pata hai,
toh mujhe aur ek component milega jo independent rahega — **τxz**,
agar mujhe pata hai toh mujhe **τzx** ke baare mein pata chalega,
kyunki dono same hain.
Toh ek independent component ho gaya **τyz**,
agar mujhe pata hai toh **τzy** mujhe pata chal jaayega.

Toh ek independent.
Toh mere paas yeh teen aise independent components aur teen shear stress components.
Toh mere paas kul milake **6 independent stress components** aa jaayenge.

Toh hamare paas stress tensor hai,
aur jab hum static equilibrium ki baat karte hain,
toh hume sirf **6 independent stress components** chahiye.
Baaki ke stress components equal ho jaayenge
kyunki yahan par moment balance hona chahiye.



Toh agli class mein hum dekhenge,
aur agli video mein hum dekhenge
ki stress jo matrix hai,
iske kuch special conditions —
jaise plane stress condition —
wo hum agle bhaag mein dekhenge.

Abhi ke liye main yahan par hi rokta hoon.
Dhanyavaad.