Mechanical behavior of materials

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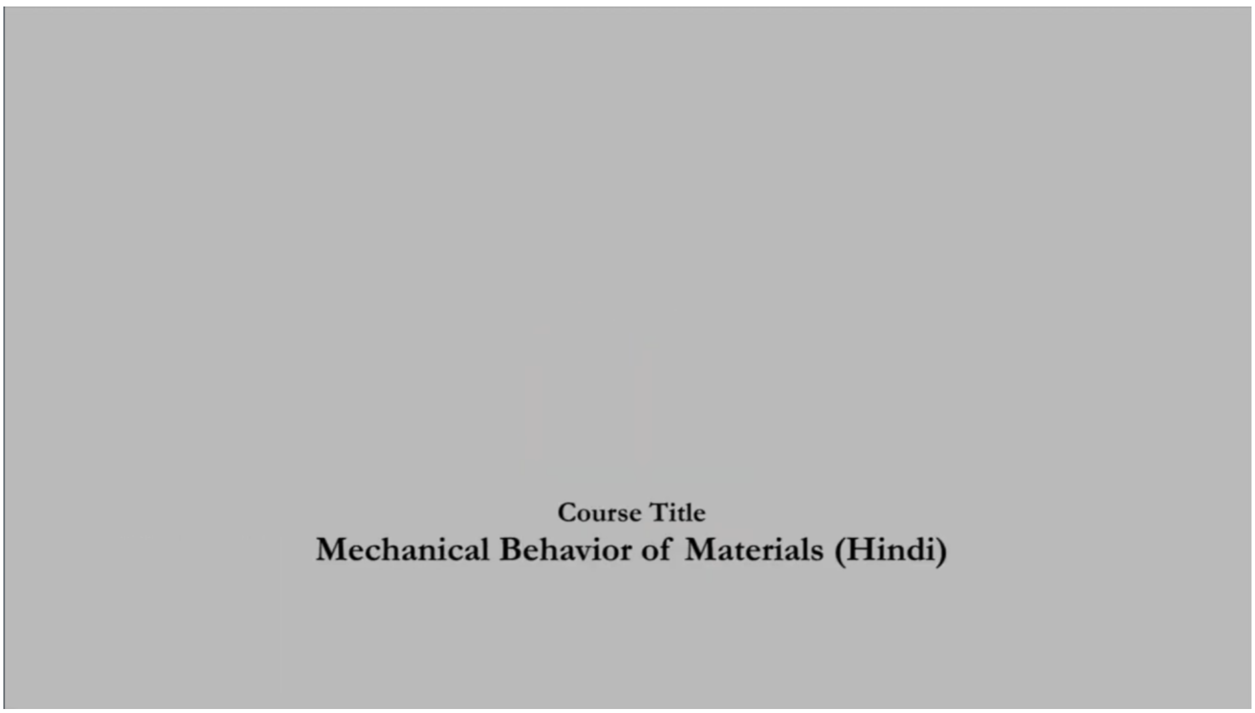
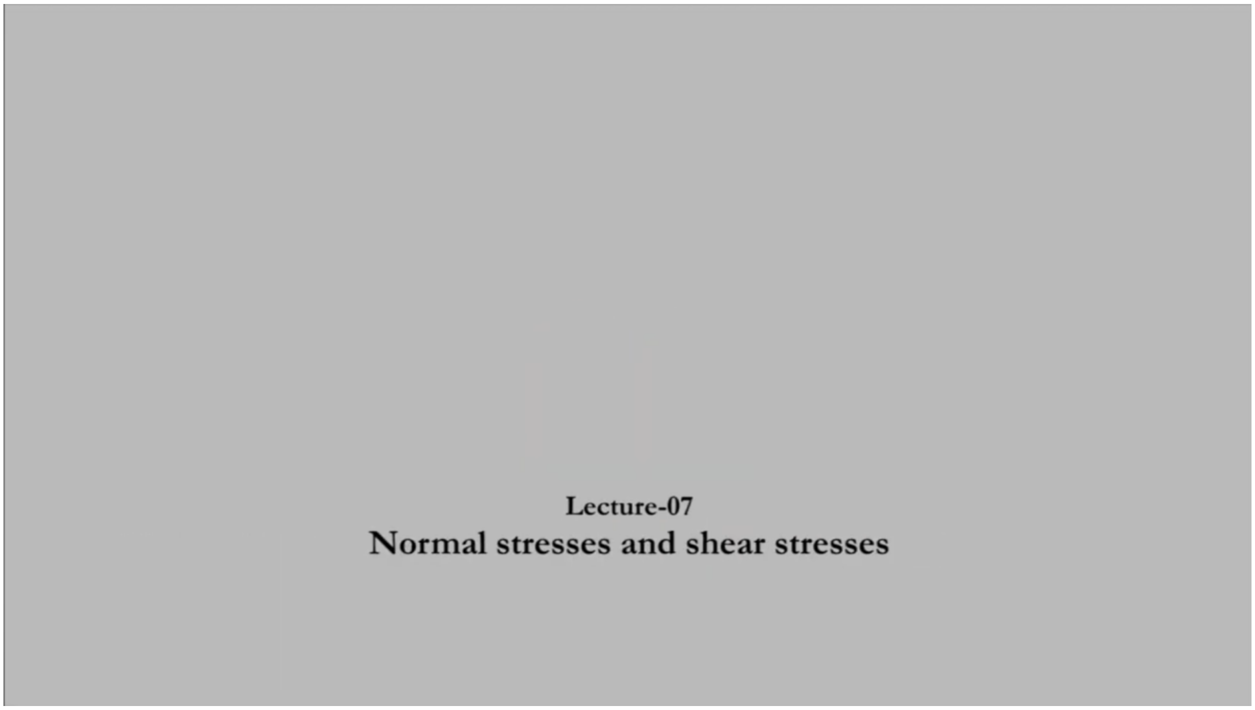
Week-2

Lecture-7

Normal stresses and shear Stresses





Namaskar, last part mein humne dekha tha ki state of stress at a point kya ho sakta hai. To humne normal stress aur shear stresses ke baare mein jaana. Humne dekha ki koi bhi traction ya force hai usko hum do stresses mein divide kar sakte hain — ek normal stress aur shear stresses. To humein ek normal stress milta hai aur do shear stresses milte hain. Phir bhi abhi tak hum complete specification nahi de pa rahe state of stress kisi bhi point ke liye, kyunki koi bhi force hum divide karenge to woh orientation par depend karega ya jo area of interest hai, plane of interest hai, us par depend karega. Ye part Main hum dekhenge ki state of stress at a point kya hota hai. To humein chahiye normal aur shear stress components kisi bhi point par. Agar humein define karna hai state of stress, to humein normal aur shear stress components chahiye teen orthogonal planes par.

Is tarah se hum dekhenge Cartesian coordinates mein — ye mere teen coordinates hain: x, y, aur z. Cartesian coordinates mein ye teen planes honge — x-plane, y-plane, aur z-plane.

Plane kya hota hai?
x-plane woh plane hota hai jo perpendicular hota hai x-direction ke; y-plane woh plane hai jo perpendicular hota hai y-direction ke; aur z-plane woh plane hai jo perpendicular hota hai z-direction ke.

To ye mere teen orthogonal planes hue. Ek point par state of stress define karne ke liye humein in tino planes ko consider karna hota hai.

Ab hum dekhte hain — ek plane par humein teen stress components milte hain: ek normal stress aur do shear stresses. To agar teen planes hain, to total humein **9 stress components** chahiye honge.

Yahan par dekhiye, maine teen components diye — ek normal stress hai aur do shear stresses jo y-plane par act kar rahe hain. To agar mujhe kisi bhi point par (maan lijiye point O) state of stress define karna hai, to mujhe in tino planes par stresses ke components pata hone chahiye. Is tarah se mujhe total 9 components milte hain Cartesian coordinates mein, jisse hum state of stress at a point samajh sakte hain.

Ab maan lijiye meri body mein ek small point hai. Us point ko main consider karta hoon ek infinitesimally small cube ke roop mein — ek chhota sa tiny cube. Is cube ko main bada karke yahan dikhata hoon.

To ek body ke ek small point ko agar hum cube maan lein aur us par Cartesian coordinates (x, y, z directions) define kar dein, to ye infinitesimal element ya elemental cube ban jaata hai. Iska mahatva ye hai ki is chhote se area par stress ka variation negligible hota hai — hum ise ek point ki tarah treat kar sakte hain. Lekin mathematics ke liye, stress ko define karne mein, hum isko ek cube element ke roop mein maante hain.

Ab hum define karte hain stresses jaise **σ_{xx}, σ_{yy}, σ_{zz}**, aur shear components jaise **σ_{xy}, σ_{xz}, σ_{yx}**, etc. Yahan par **σ** denote karta hai normal stress ko. Pehla suffix batata hai ki stress kis plane par act kar raha hai, aur doosra suffix batata hai ki stress kis direction mein act kar raha hai.

Humne dekha tha ki ek plane par ek normal aur do shear stresses hote hain, to is nomenclature ke through hum complete stress components define kar sakte hain — jaise σyy, σyz, σyx, etc.

Is tarah hum ek point par state of stress ko define karte hain Cartesian coordinate system mein Direction mein act kar raha hai to ye y-direction mein act kar raha hai, ye mera sigma yy. To agar hum stress dekhenge to stress ko ek direction mein agar main sigma yy maanun, to ye mera positive y-plane hoga aur ye mera negative y-plane agar main maanta hoon. To is tarah se hum denote karenge. Agar ye is direction mein hai to ye is direction mein hona chahiye, to ye mera sigma yy ho gaya.

Ab shear stress ke baare mein jaante hain. Shear stress ko main denote karta hoon τ (tau) se. To tau denotes shear stress. Ismein bhi pehla suffix denote karta hai ki kis plane ke along ye act kar raha hai, aur doosra suffix denote karta hai ki kis direction ke along act kar raha hai.

To ek example lete hain τ_{yx} ka. τ_{yx} yaani ye mera shear stress hai. Shear stress yaani ye is plane mein act hoga. To mera y-plane ye hai, to ye is plane mein act ho raha hai. Aur kis direction mein hai? Ye x-direction mein hai. To ye mera τ_{yx} x-direction mein act karega.

Similarly, agar hum τ_{yz} dekhen to τ_{yz} ka matlab hai ye shear stress y-plane par act kar raha hai aur z-direction mein act kar raha hai.

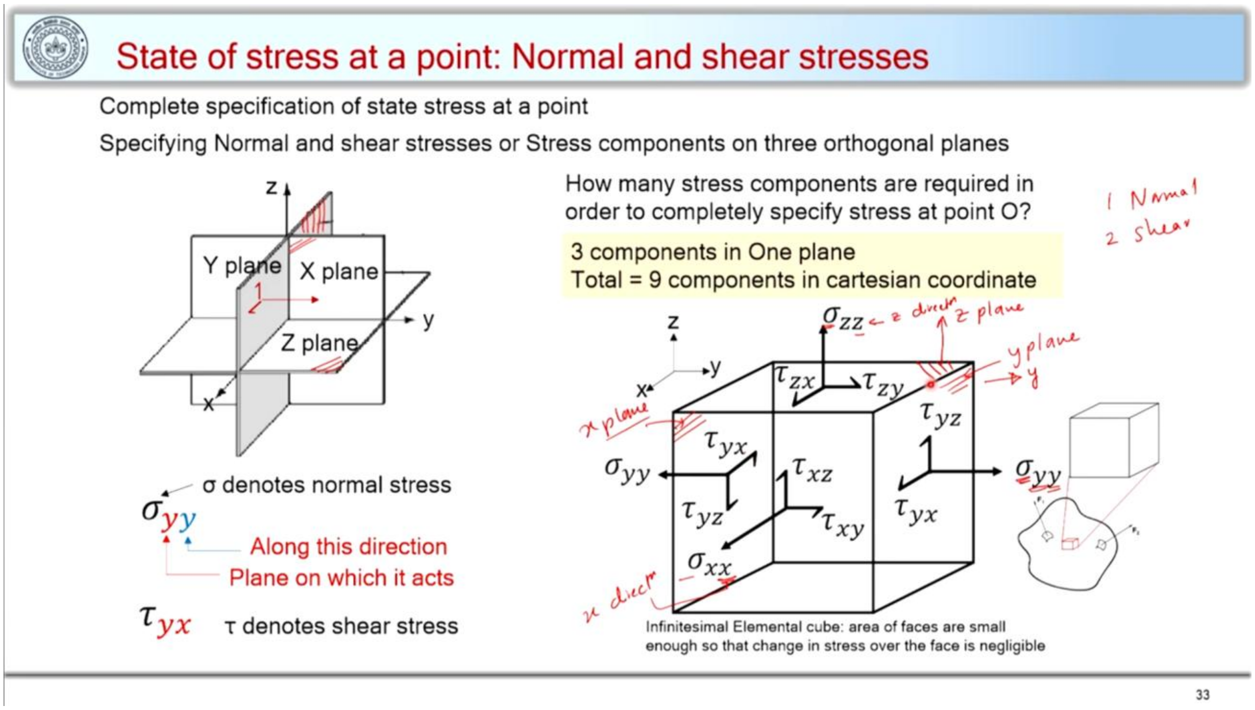
Ab agar main do aur stresses dekhun — jaise σ_{xx} aur σ_{zz} — to ye dono normal stresses hain.
σ_{xx} yaani stress jo x-plane par act kar raha hai. X-plane ka matlab hai plane jo x-direction ke perpendicular hai.
Similarly, σ_{zz} act kar raha hai z-plane par, jo z-direction ke perpendicular hai.

Ab bhi kuch shear stresses dekhen: τ_{xz} aur τ_{xy}.
τ_{xz} yaani shear stress jo x-plane par act kar raha hai z-direction mein. Z-direction upar hai to τ_{xz} ka direction upward hoga.
τ_{xy} yaani shear stress jo x-plane par act kar raha hai y-direction mein.

Similarly, τ_{zx} act karega z-plane par along x-direction, aur τ_{zy} act karega z-plane par along y-direction..

Waise hi tau yx, tau yx kar act karega y-plane mein, to ye mera y-plane hai aur along x-direction.
Aur tau yz act karega y-plane mein along z-direction.
To yahan par humne dekha ki ye tau yx aur ye tau xy dono hi is direction mein act ho rahe hain — y-direction mein act ho rahe hain.

Aur tau yz bhi aap dekhenge ki ye is direction mein act ho raha hai aur ye is direction mein act ho raha hai.



Yahan par change kya hai?
Ye maine maana tha ki ye mera positive y-plane hai, ye mera negative y-plane hai.
To ye kis plane par act ho raha hai mere reference point ke hisaab se,
us hisaab se bhi mere nomenclature change honge ya unke magnitude — jo bhi direction hai — wo change honge.
Wo hum dekhenge next slide mein.

To yeh ho gaya mera *state of stress* maine define kiya — to mujhe kitne component chahiye?
Mujhe **nine components** chahiye.
To maine kaha tha ki teen *orthogonal directions* hain — to yeh 3 × 3 ho gaye, **nine components**.

To abhi hum **sign convention** dekhenge — *sign convention* **normal stresses** ke liye.
To simple hai — jab agar yeh is plane ke against act kar raha hai, to hum usko kahenge **compression** ya **negative stress**.

To maan lijiye yeh mera ho gaya *z-plane*.

Y *z-plane* kyon hai? Kyonki *z-direction* ko perpendicular hai.

To iska agar normal main maan leta hoon is plane ka is direction mein, to agar mera stress yahan par **σ_{zz}** (sigma zz) agar aap dekhenge, to yeh against act kar raha hai normal ke — yaani **σ_{zz}** ka direction is taraf hai.

To yeh mera normal hai, to isko main kahunga **compression**.

To agar along act karega, to wo **tension** hoga.

Agar mera **σ_{zz}** ka direction is taraf rehta, along rehta, to yeh ho jaata mera **tensile stress**.

To yeh mera simple **sign convention** hai.

To *normal stress* mein hum **tension ko denote karte hain positive se**, aur **compression ko denote karte hain negative se**.

To iska magnitude — yeh act ho raha hai mera (mark kar lete hain) —

Yeh plane jo hai mera, yeh mera **positive y-plane** hai,

Aur yeh kis direction mein act ho raha hai?

Yeh act ho raha hai mera **negative x-direction** mein.

Yeh jo stress hai — negative x-direction mein.

To yeh jo stress ho gaya, pehle isko main nomenclature se likh deta hoon —

Yeh ho gaya mera **τ_{yx}**, y-plane par act ho raha hai along x-direction.

To yeh ho jaayega mera **$-\tau_{yx}$**.

Ab iska convention hum dekhte hain —

To yeh ho jaayega **τ**, isko maine negative yahan par mark kiya hai —

Iska sign convention:

Yeh jo abhi **τ_{yz}** hum mark karenge, to **τ_{yz}** yaani kya hoga?

Yeh act ho raha hai mera **y-plane par**, *positive y-plane* par, par **negative z-direction** mein.

To yeh jo stress hoga, yeh jo shear stress hoga, yeh bhi **negative** hoga.

Shear stress ka convention simple hai —

Agar mera shear stress **along positive direction on positive plane** act kar raha hai, to yeh

positive kahenge.

Agar **along negative direction on positive plane**, to yeh **negative** hoga.

To agar hum dekhenge **τyx**, τyx kya ho raha hai?

Act ho raha hai hamara *positive y-plane* par par *negative x-direction* mein.

To positive plane, negative direction — to yeh ho gaya hamara **negative**.

Similarly, **τyz** — y-plane positive hai aur direction negative hai, to yeh ho jaayega **negative shear stress**.

To hamare paas **nine components** of stress hain.

To unmein se agar hum simple example dekhen —

Jaise abhi main kuch *shear stresses* mark karta hoon yahan pe —

Yeh ek mark karte hain shear stress —

Yeh shear stress kya hoga?

Yeh act ho raha hai mera **negative y-plane** pe, aur **negative x-direction** mein.

To yeh jo stress hoga, yeh hamara ho jaayega **τyx**, par iska sign hoga **positive**,

Kyonki yeh *negative y-plane* pe act ho raha hai aur *negative x-direction* mein act ho raha hai.

Agar main yeh stress maanta hoon is direction mein,

To yeh bhi y-plane pe act ho raha hai,

Y-plane pe — kaunsa y-plane pe act ho raha hai?

Yeh *negative y-plane* pe act ho raha hai, aur kis direction mein act ho raha hai?

Negative z-direction mein.

To yeh jo stress ho jaayega, yeh bhi mera **τyz** hoga, par yeh hoga **positive**.

Maan lo yahan par ek stress hai is direction mein —

Yeh kaunsa plane hai mera?

Yeh plane hai mera *z-plane*.

Kaunsa z-plane hai?

Agar main maan chalu ki yeh *negative z-plane* hai —

Kyonki z-direction to upar hai, to yeh *positive z-plane* hoga aur neeche *negative z-plane*.

To yeh *negative z-plane* hoga,

Aur yeh kis direction mein hai?
Yeh *negative y-direction* mein.
To yeh ho jaayega mera **τzy**, aur iska sign rahega **positive**.

Similarly agar main is direction mein mark karta hoon —
To yeh jo plane hai, yeh plane hai mera *z-plane* — *positive z-plane*,
Aur direction hai *negative x-direction*.
To yeh jo stress ho jaayega, yeh ho jaayega **τzx**,
Par iska sign rahega **negative**,
Kyonki yeh *positive plane* pe act ho raha hai par *negative direction* mein.

To isi tarah se baaki ke aap **nine components** ka sign nikaal sakte hain.

Humne **sign convention** yahan likha hai —
Normal stress ke liye simple hai — agar wo tension along normal direction act kar raha hai, to hum usko **tension** kahenge,
Agar *opposite to normal direction* act kar raha hai, to hum usko **compression** kahenge.
Aur *compression* ko hum **negative sign** se,
Aur *tension* ko hum **positive sign** se denote karte hain.

Shear stresses mein agar wo *positive direction on positive plane* pe hai to **positive shear stress**,
Agar *negative direction on positive plane* pe hai (ya vice versa), to **negative shear stress**.

Hamare paas **nine components** hain stress ke —
To yeh ho gaye hamare:
σxx, **τxy**, **τxz**,
τyx, **σyy**, **τyz**,
τzx, **τzy**, **σzz**.

Yeh hamare **nine components** ho gaye.
To unmein se aap dekhenge to yeh jo teen components hain diagonal par —
Yeh hamare **normal stresses** hain.

Aur yeh jo *off-diagonal components* hain —
Yeh hamare **shear stresses** hain.

To stress ko hum kisi bhi point par is form mein represent karte hain.
Kabhi-kabhi aap dekhenge kuch textbooks mein **τ_{xy}** ko **σ_{xy}** likhte hain,
τ_{xz} ko **σ_{xz}** likhte hain —
To τ ki jagah σ ka symbol istemaal karte hain.
To wahan aapko confuse nahi hona hai.

To hum **σ_{ij}** is tarah se define karte hain —
Agar yeh mere suffix hain —
Jab $i = j$ rahega, wo mera **normal stress** hoga.
Aur jab $i \neq j$ rahega, wo mera **shear stress** hoga.

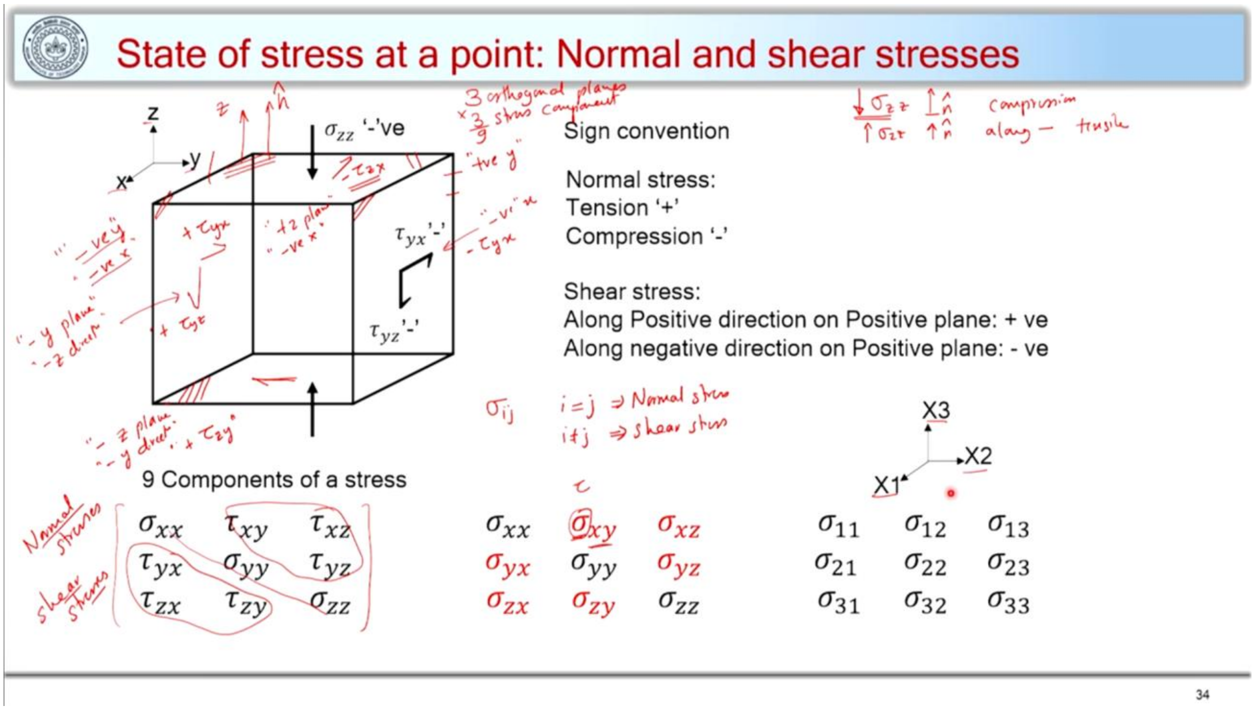
To hum dekhenge ki agar $i \neq j$, to mera shear stress hona chahiye.
To yeh symbol kabhi-kabhi — yaani kuch textbooks mein **τ** likhte hain, kuch mein **σ** likhte hain.

Agar maine coordinate axes **x_1, x_2, x_3** kar di —
Is case mein x, y, z the —
To maine **σ_{xy}**, **τ_{xy}** aise likha tha.
Agar mere paas axes hain **x_1, x_2, x_3**,
To isko hum likhenge —
σ_{11}, σ_{12}, σ_{13},
σ_{21}, σ_{22}, σ_{23},
σ_{31}, σ_{32}, σ_{33}.

Dhyan dena hai ki yeh jo *diagonal components* hain, wo **normal stresses** hain,
Aur *off-diagonal components* **shear stresses**.

To is form mein hum stress tensor represent karte hain —
Aur stress ko samajhne ke liye kisi bhi given point par humein chahiye:
3 *normal stresses* aur 6 *shear stresses*, total 9 components.

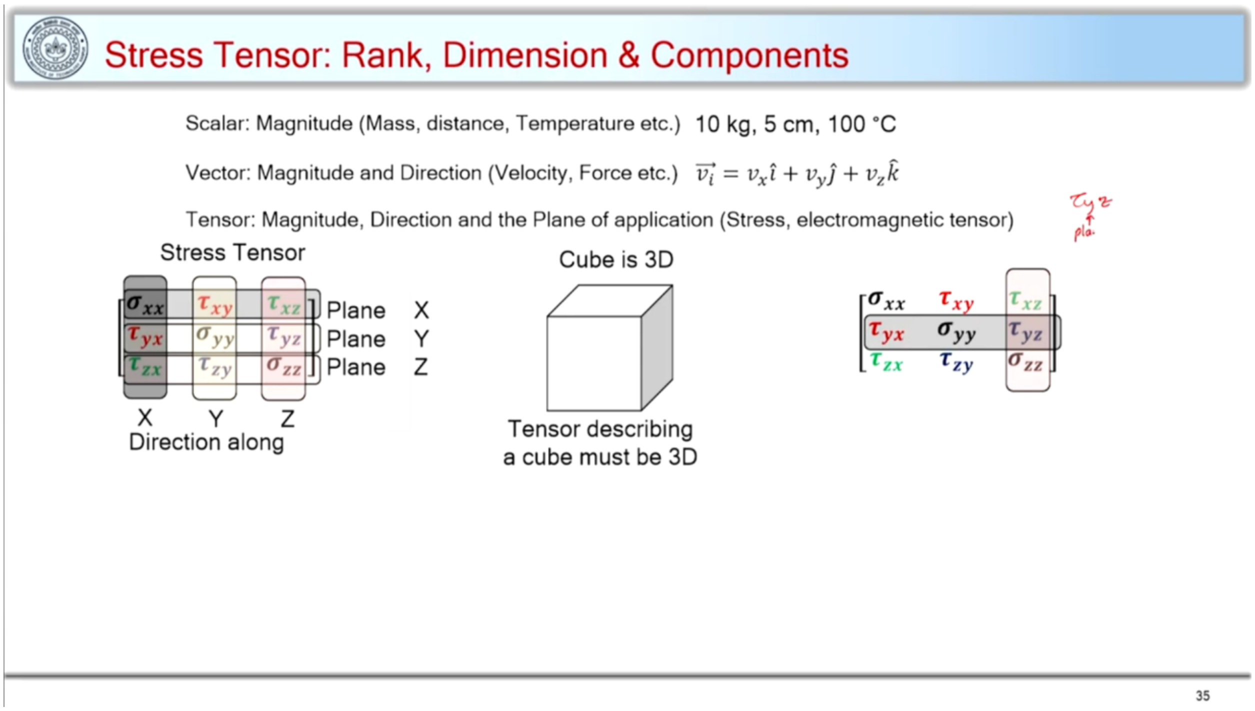
Ab hum dekhte hain —
Jo humne representation kiya tha stress ka, usko hum kehte hain **tensor**.



To pehle samajhte hain — *tensor* kya hota hai? tensor jaanane se pehle hum jaante hain ki pehle scalars kya hain. To scalars ko hum jaante hain ki scalars ko define karte hain magnitude se — jaise distance ka hai, ye temperature ka unit hai — isme sirf hume magnitude pata hota hai. Uske baad humne padha hoga vectors, jiske liye hume magnitude ke saath-saath direction bhi pata hona chahiye — x direction me iska component kya hai, y direction me kya component hai, z direction me kya component hai — ye hume pata hona chahiye, tab hum us vector ko define kar sakte hain. To kuch quantities hain jaise velocity, force — jinme magnitude aur direction donon hote hain.

Humne stress ko jana tha ki hume teen cheezein chahiye — magnitude, direction aur plane. To abhi hum stress tensor ke baare me jaante hain. Humne dekha tha ki humne ye jo state of stress hai kisi bhi point pe, ye ek cube ke around define kiya tha — yaani ek infinitesimal small cube, ek tiny cube ko define kiya tha. Aur ye tiny cube hamara 3D me hai — Cartesian coordinate me three dimension hai. To jo tensor isko represent karega wo must be in 3D — yaani three-dimension me hona chahiye.

To hum stress tensor ko is tarah se represent karte hain: ye hamare nine components ho gaye stress tensor ke ya stress ke. Agar aap dekhenge to ye pehla part — ye jo part hai — ye represent karta hai hamare stresses ko jo x-plane ke perpendicular hai. Dusra aur teesra jo hai — wo determine karte hain ki kaunse components y-plane aur z-plane ke perpendicular hain.



Agar hum dekhen column-wise to columns me jo stresses hain — wo x-direction ke along hain, dusre column me y-direction ke along aur teesre column me z-direction ke along act karte hain.

Abhi hum tensors ke baare me jab jaante hain — to agar mujhe jaise $\tau_\gamma z$ jana hai — to $\tau_\gamma z$ yaani kya hona chahiye? Humne dekha tha ki ye mera plane batata hai kis plane pe act kar raha hai — ye mera y-plane pe act kar raha hai aur ye mera z-direction me act kar raha hai. To hume $\tau_\gamma z$ agar pata karna hai to do cheezon ki zarurat hai — kaunsa plane aur kaunsi direction.

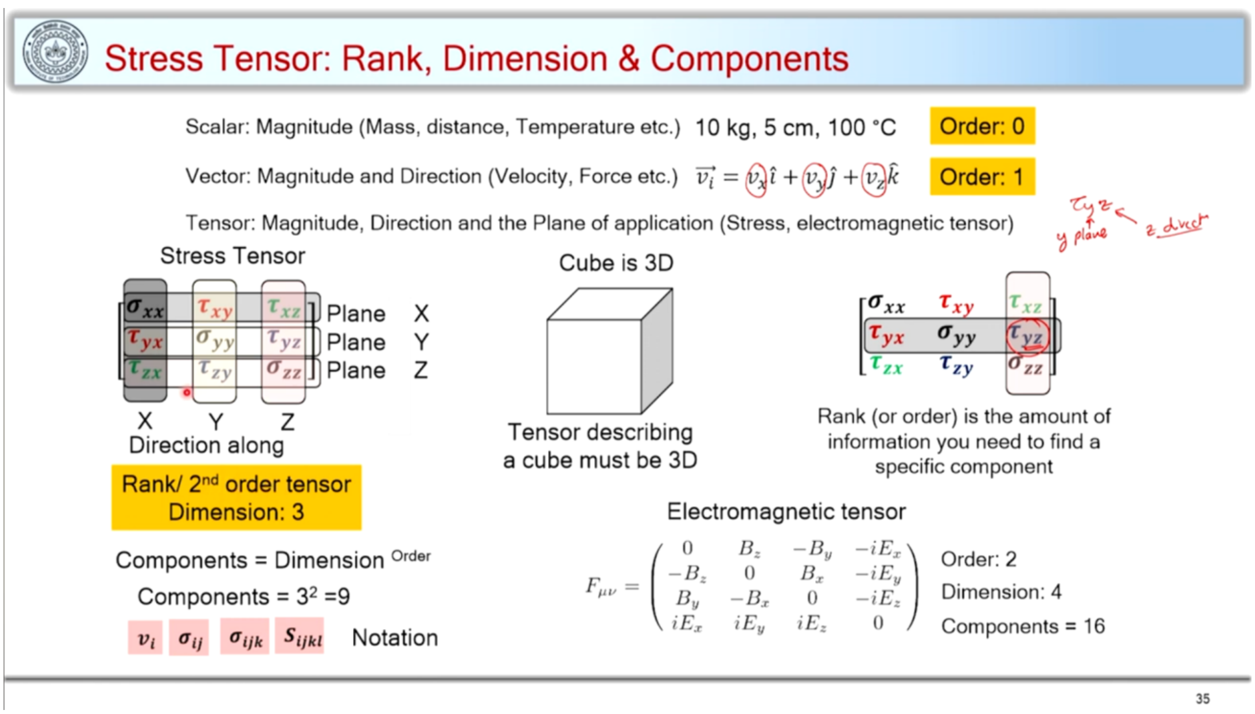
To ye ho gaya hamara tensor ka rank. Rank yaani simple bhaasha me hum ye bata sakte hain ki rank ek information hoti hai jo aapko chahiye to find a specific component. Jaise mujhe is stress tensor me ye component find karna hai to mujhe do cheez ki zarurat hai — ek kis plane pe act kar raha hai, aur ek kis direction me act kar raha hai. To do minimum information chahiye — isliye rank = 2.

To yahan stress tensor ka rank ho jayega 2. Aur iska dimension 3 hai kyonki humne Cartesian coordinates me cube consider kiya hai. To rank = 2 aur dimension = 3.

Ab agar mujhe do information chahiye aur dimension 3 hai, to mere components kitne hone chahiye? Isko hum define karte hain: components = dimension to the power order.

To mera yahan dimension = 3, order = 2 → components = $3^2 = 9$ components. Jo humne pehle bhi dekhe the.

Ab hum dekhte hain ki vectors kaunsa tensor hai. Vector me hume sirf ek component chahiye — x-direction ka ek, y-direction ka ek, z-direction ka ek. To agar ye mujhe pata hai to main vector



define kar sakta hoon. To vector ho gaya tensor of order 1. Rank ko order bhi kahte hain. To vector = tensor of order 1.

Scalar me hume koi direction nahi chahiye → tensor of order 0. Stress = tensor of order 2.

To is tarah hum likh sakte hain:

- Scalar → tensor of order 0

- Vector → tensor of order 1
- Stress → tensor of order 2 (rank 2).

Aur components ko hum likhte hain components = dimension^n. To yahan $3^2 = 9$ components.

Agar hum electromagnetic tensor dekhen, to uska order bhi 2 hai aur dimension 4 hai. To components = $4^2 = 16$ components.

Notation ke liye:

- Order 1 tensor → Vi (ek suffix)
- Order 2 tensor → σ_{ij} (do suffixes)
- Order 3 tensor → T_{ijk} (teen suffixes)
- Order 4 tensor → C_{ijkl} (char suffixes).

Hum fourth-order tensor bhi dekhenge — jo hamara stiffness ya compliance tensor hai.

To is simple way se humne tensors ka introduction kiya jo hum is course me use karenge.

To hamara stress ka rank ya order = 2, dimension = 3, aur components = 9. Humne ye 9 components pehle dekhe the.

Phir se revision ke liye: jaise koi component $\sigma_x z$ ho to ye batata hai kis plane pe act kar raha hai (plane = x), aur kis direction me act kar raha hai (direction = z). Agar x aur z same nahi hain to ye shear component hai. Agar same hote to normal stress hota.

To yahan main rukta hoon, aur next part me hum dekhenge ki stress tensor me kaunse components independent aur dependent hain — force equilibrium aur moment equilibrium ke basis pe — yaani off-diagonal aur diagonal components ka relation.

Dhanyavaad.