

Mechanical behavior of materials

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Week-12

Lecture-63

Anelastic behaviour and composite materials

Course Title

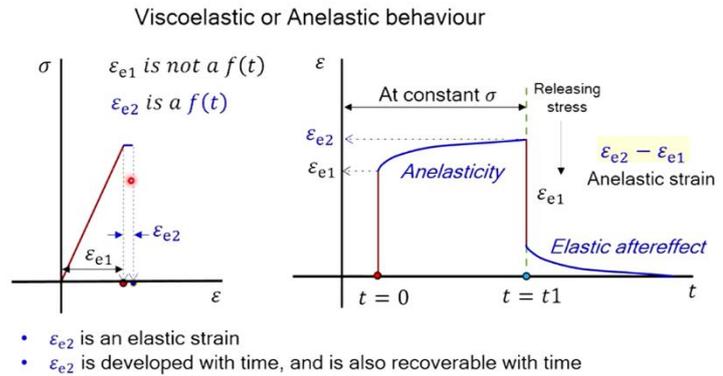
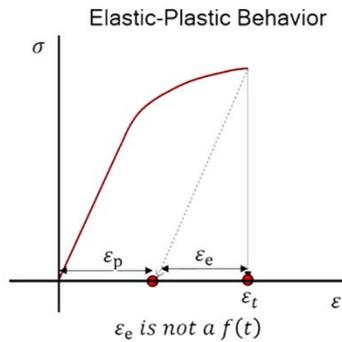
Mechanical Behavior of Materials (Hindi)

Lecture-62
Anelastic behaviour and Composite materials

Namaskar, phir se swagat karta hoon aapka is course mein Mechanical Behavior of Materials jo ki hum Hindi mein padhenge. Is part mein hum ek behavior padhenge material ka jisko hum anelastic behavior kehte hain. Us iske baad hum material jaise composite materials hai iske bare mein jaanenge. To aaiye jaanate hain ki anelastic behavior hai kya? Uske pehle hum jaanate hain ki elastic plastic behavior kya hai? Uska ek revision hum kar lete hain. Jaise elastic plastic behavior mein hum dekhte hain ki hum y axis pe stress (σ) plot karte hain aur x axis pe hum strain (ϵ) plot karte hain. Aur humein kuch aise typical stress strain curve



Anelastic behavior



This is called *viscoelastic* or *anelastic* deformation - Sneek ✓

This viscoelasticity should not be confused with creep, which is time dependent plastic deformation

- Anelastic strains in metals and ceramics are usually so small that they are ignored.
- In many polymers, however, viscoelastic strains can be very significant.

- polymers
- glasses
- amorphous

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milta hai. Jo initial part hai, linear part hai, hum usko elastic behavior kehte hain material ka aur jo non linear part hai usko hum plastic behavior kehte hain. Abhi hum dekhte hain ki agar elastic behavior kuch is tarah se aur plastic behavior kuch is tarah se aur total strain material pe ϵ_t hai to hum elastic strain aur plastic strain kis tarah se nikaal sakte hain. To hum jaanate hain ki yeh mera elastic strain hai aur yahan par abhi hum dekhenge ki agar mein ek line draw kar loon ye initial curve se is tarah se to mujhe kuch elastic jo recoverable part hoga strain ka isko mein elastic strain kahunga aur yeh jo part hoga jo

non recoverable hai isko mein plastic strain kehta hoon. To mere paas kuch elastic aur plastic strain is tarah se aata hai jo ki total strain mein mein in donon se ka contribution nikaal sakta hoon. Abhi aap dekhenge ki ye jo elastic part hai ya plastic part bhi hai yeh time ka function nahi hai. Time ka function nahi hai. Iska matlab yeh hai ki jaise hi stress mein apply kar raha hoon ya stress mere material pe aa raha hai us amount ka strain wo develop ho raha hai. Yahan pe koi time lag nahi hai. Time ke saath yeh strain evolve nahi ho raha hai. To, abhi mein jag elastic plastic behavior ki baat karke anelastic behavior explain karunga, to

hum dekhenge ki is anelastic behavior mein strain jo develop hoga wo time ke saath develop hoga. Kuch is tarah se agar mein stress strain plot consider kar raha hoon aur maine kuch stress apply kar diya yahan tak. To aap dekhenge ki yeh elastic strain yahan par develop hua hai. Hum sirf elastic behavior mein abhi baat kar rahe hain. Yahan par elastic strain ye develop hua hai. Aur abhi stress pe mein apne material ko usi stress pe rakhunga. Yaani ye stress mein abhi

change nahi karunga. Yaani state of stress yahan pe change nahi ho raha hai. Aur samay ke saath dekhte hain ki strain kaise evolve ho raha hai material pe. To aap dekhenge ki kuch small strain yahan pe

evolve hoga. Time ke saath yeh jo strain hai yeh time ke saath evolve ho raha hai. To hum kahenge jo ϵ_1 hai jo pehla strain tha develop hua tha wo time ka function nahi tha. Yaani jaise maine stress apply kiya us tarah se wo strain develop ho gaya. Par yeh jo strain hai ϵ_2 ye jo elastic strain hai yeh samay ke saath develop ho raha hai. Yeh time ka function hai. Agar mein dekhunga strain yahan pe maine y axis pe plot kiya hai aur x axis pe maine time plot kiya hai. To kis tarah se is scenario ko mein dekh sakta hoon. To yahan par $t = 0$ agar condition mein consider karunga yaani suddenly agar maine stress apply kiya

to ϵ_1 ye elastic strain yahan pe develop hoga aur time ke saath maan lete hain $t = t_1$ ke saath aap dekhenge ki small strain yahan pe develop hoga aur wo strain hai yahan pe ϵ_{e2} ye small elastic strain yahan pe develop ho raha hai. Ye time ke saath develop ho raha hai. Dekhenge ki ye time ke saath develop ho raha hai. Iska matlab maine ye stress yahan pe constant rakha hai. Ye stress maine yahan pe constant chhod diya tha aur dekh raha tha ki material kis tarah se deform ho raha hai. To is stress pe time ke saath material pe strain develop hota hai. Aur yahi behavior hai isko hum kehte hain viscoelastic ya anelastic

behavior. Agar mein is stress par jaake yaani is time par jaake agar yeh stress drop kar doon yaani mein stress agar release kar doon to aap dekhenge ye jo strain hai ϵ_{e1} ye elastic strain jo time ka function nahi tha wo suddenly release ho jayega ya regain ho jayega aur yahan par aap dekhenge ek sudden drop aapko milega aur yeh jo strain hai ϵ_{e2} yeh phir time ke saath decay hoga kuch is tarah se aur aage jaake ye decay ho ke ya recover ho ke yeh jo strain hai material pe yeh shunya aa jayega zero ho jayega to hum dekhenge ki viscoelastic ya anelastic behavior mein

yeh jo elastic strain hai yeh time ke saath develop ho raha hai. To pehli baat to yeh hai ki ϵ_{e2} yeh elastic strain hai. Agar yeh elastic strain hai to hum dekhenge ki elastic behavior mein yeh jo strain hai yeh recoverable hoga aur yeh time ke saath recover hoga. Ye develop bhi time ke saath hota hai aur recover bhi time ke saath hota hai. Yahi elastic deformation ki definition hai ki hum original dimension recover kar sakte hain. Par ye jo strain develop ho raha hai, sirf difference ye hai ki ye strain time ke saath develop ho raha hai. ϵ_{e1} aur ϵ_{e2} mein yahi difference hai. Ye donon elastic strain hai. But ye time ka function nahi hai aur ye time ka

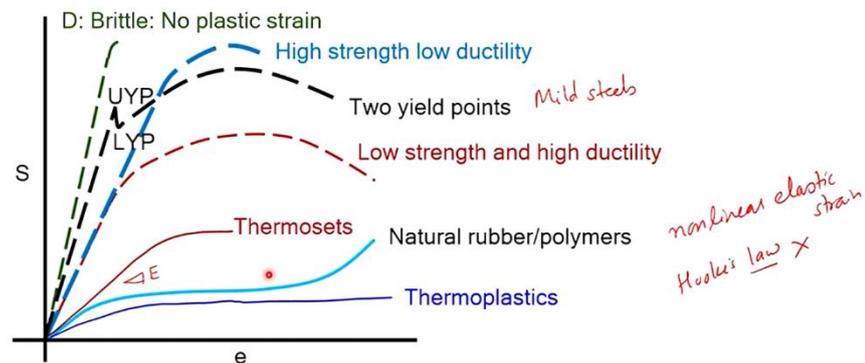
function hai. To ϵ_2 ke baare mein humne itni baat nahi ki thi metals aur ceramics ke baare mein. Yeh jaanenge kyunki ϵ_{e2} jo ye jo strain develop hota hai ye bahut small hota hai. Ye jo behavior hai abhi ye ϵ_2 jo develop hota hai samay ke saath isko hum kehte hain viscoelastic aur anelastic behavior. Isko aap iske saath confuse na kariye. Creep ke saath jaise creep mein humne bhi dekha tha ki ek strain develop hota hai material ke saath. Par wah plastic strain hota hai. Yaani wah recoverable strain nahi hai. Wah permanent deformation hai. Wah plastic deformation hai. To creep mein humein plastic

strains develop hote hain ya plastic deformation hota hai creep mein. Par viscoelastic aur anelastic deformation jo mein baat kar raha hoon wo saare elastic strain hai aur wo recoverable strains hain. To ye jo ϵ_{e2} yahan pe maine baat ki hai ye elastic strains hai aur ye metals aur ceramics mein bahut small hote hain aur isliye hum inko ignore karte hain. Abhi tak ke saare analysis mein hum in strains ko ignore karte hain. Par kuch materials hain jaise ki polymers hai ya glasses hai ya amorphous materials hai. Ismein ye viscoelastic strains bahut significant role play karte hain. Isliye ye behavior ko in materials mein aapko samajhna

padega. Jaise yahan pe hum likh lete hain. Jaise polymers yahan pe humne likh liya polymers glasses ya amorphous materials. Ismein yeh strains anelastic strains develop hote hain aur ek significant contribution karte hain. Humne ek Snoek effect padha tha. Iske baare mein bhi yahan pe mein mention kar deta hoon. Humne Snoek effect padha tha. Snoek effect bhi ek manifestation hai anelastic behavior ya viscoelastic behavior ka. Abhi hum dekhenge ki anelastic strain kya hai? To ye jo difference hai $\epsilon_{e2} - \epsilon_{e1}$ ye hai anelastic strain aur ye jo behavior hai isko mein kahunga anelasticity. Jab ye



Stress-Strain curves



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stress yahan pe maine constant rakha hai aur time ke saath ye strain develop ho raha hai. Isko hum kehte hain anelastic strain. Aur ye behavior ko hum kehte hain anelasticity. Yahan pe humne stress release kar diya. To, yahan pe sudden drop mila recoverable strain ka ϵ_{e1} ka. But doosra jo time ke saath jo strain recover hoga ϵ_2 yeh effect hum kahenge elastic after effect. Donon same phenomena hai par ye samay ke saath develop ho raha hai aur ye samay ke saath ghat raha hai. To isko anelasticity kahenge isko elastic after effect ho gaya. To ye ho gaya introduction mere viscoelastic aur anelastic behavior ka.

To abhi hum aage badhte hain aur kuch stress strain curves dekhte hain. To humne hamare course mein mechanical behavior padha. To humne elastic behavior padha, phir plastic behavior padha. Humne ye bhi jaana ki ye jo strength yahan pe develop hota hai, ye kis-kis contribution se aate hain. Humne kuch strengthening mechanisms bhi padhe. To, yeh typical curves is slide mein maine yahan pe represent kiye. To yahan par aap dekhenge ki yahan par ek brittle material hai. Yahan par koi plastic strain nahi hai. Yahan par humein sirf fracture strength milega. Aur yahan pe humne dikhaya hai two yield points. Ye mild steels mein humein milta hai. Humne dekha tha. Yahan pe likh lete

hain isko. Mild steels mein humein ye behavior milta hai ya low carbon steels mein. Aur yahan pe aap dekhenge ki high strength low ductility steels ya high strength low ductility materials aur low strength high ductility materials. Kuch is tarah se unke stress strain curve humne plot kiye. Abhi kuch polymers ke hum plot kar lete hain. Jaise natural rubber ya polymers ke stress

strain curve kuch is tarah se behave karte hain. Aur thermosets kuch is tarah se humein stress strain curves dikhate hain. To agar hum dekhenge polymers ya thermosets ya natural rubbers ya thermoplastics mein yeh jo elastic strain hota hai woh kaafi zyada hota hai.

Ya agar hum kahenge Youngs modulus iska kaafi kam hota hai. Agar aap dekhenge iska slope agar mein dekhunga ye slope ki agar mein baat karoon to ye slope kaafi kam hota hai. To yaani elastic modulus in case mein kaafi kam hota hai. Isliye humein elastic strain bhi zyada milenge. Aur yahan pe aap dekhenge ki elastic strain ye linear nahi hai. Yaani non linear elastic strains humein milte hain. Yaani yeh Hookes law follow nahi karte hain. Jahan par non linearity nahi hai. To yahan pe yeh kuch stress strain curve humne summarize kiye saare materials ke liye. Abhi jab hum

Composite materials

Isostrain model $\epsilon_c = \epsilon_m = \epsilon_f$

$$F_c = F_m + F_f \Rightarrow \frac{F_c}{A_c} = \frac{F_m}{A_c} + \frac{F_f}{A_c}$$

$$\Rightarrow \frac{F_c}{A_c} = \frac{F_m A_m}{A_m A_c} + \frac{F_f A_f}{A_f A_c} \Rightarrow \sigma_c = \sigma_m V_m + \sigma_f V_f$$

$$\Rightarrow \epsilon_c E_c = \epsilon_m E_m V_m + \epsilon_f E_f V_f$$

Addition: series $\Rightarrow E_c = E_m V_m + E_f V_f$

$V_m + V_f = 1$

$$P_c = \sum_{i=1}^n V_i P_i$$

Isostress model $\sigma_c = \sigma_m = \sigma_f$

$$\Delta L_c = \Delta L_m + \Delta L_f \Rightarrow \frac{\Delta L_c}{L_c} = \frac{\Delta L_m}{L_c} + \frac{\Delta L_f}{L_c}$$

$$\Rightarrow \frac{\Delta L_c}{L_c} = \frac{\Delta L_m L_m}{L_m L_c} + \frac{\Delta L_f L_f}{L_f L_m} \Rightarrow \epsilon_c = \epsilon_m V_m + \epsilon_f V_f$$

$$\Rightarrow \frac{\sigma_c}{E_c} = \frac{\sigma_m}{E_m} V_m + \frac{\sigma_f}{E_f} V_f \Rightarrow \frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_f}{E_f}$$

Parallel

$$\frac{1}{P_c} = \sum_{i=1}^n \frac{V_i}{P_i}$$

materials ki baat kar rahe hain tabhi hum jaanate hain ki composite materials hote kya hai? To uske liye hum kuch simple case study consider karenge jaise composite materials mein. To maan lete hain mere paas kuch is tarah se matrix hai aur mein ek foreign material ya reinforcement usko mein kahunga aur yahan par simple understanding ke liye yahan pe mein fibers consider kar raha hoon. To yahan pe kuch fibers maine is matrix mein reinforce kar diye hain. Kuch is tarah se ye long fibers hain to aap dekhenge ki kuch is tarah se maine long fibers is matrix mein reinforce kar diye. Abhi hum inka property agar mujhe determination karni hai jaise elastic modulus determine karna

hai to mein kya case consider kar sakta hoon? Uske liye kuch models yahan pe diye hain. Maan lete hain agar mein ek force is matrix aur reinforced yaani is composite ko apply kar raha hoon. Maan lete hain F_c isko mein maan leta hoon ki ek force ek total composite pe mein apply kar raha hoon. Abhi aap dekhenge ki yahan par mein isostrain model apply karunga. Aap turant hi samjhenge ki jab mein is force ko apply karunga is composite par tab is matrix ka bhi deformation hoga aur is fibers ka bhi deformation hoga. Agar aap dekhenge ki is composite ko intact rehna hai ek geometrical compatibility agar maintain karni hai fibers aur matrix mein ya reinforce aur matrix mein, to hum

dekhenge ki yeh jo strain develop hoga in donon mein woh same hona chahiye. To kuch is tarah se mein consider kar sakta hoon ki agar total strain develop ho raha hai is composite mein usko mein ϵ_c maan ke chal raha hoon to wo equal hona chahiye jo matrix phase mein jo strain develop hoga wo equal hona chahiye jo fiber phase mein jo strain develop hoga yaani teeno strain same hone chahiye tabhi jaake ye composite uniformly deform karega aur fibers aur matrix ke beech mein kuch geometrical compatibility rahegi. To ye condition ho gayi yahan pe. Abhi hum dekhte hain ki force aap dekhenge ki ek composite pe maine apply kiya. Ye share hoga mere matrix aur reinforcement ke saath. To

yahan pe ye yahan pe mein reinforcement consider kar raha hoon. To ye matrix aur reinforcement ke saath share hoga. To yahan pe mein consider kar sakta hoon. Agar mein maan loon yahan par yahan par kuch is tarah se mein isko C ki jagah F likh dete hain. To yahan par aap dekhenge agar mein cross section area consider karunga aur divide karunga is composite ke cross section area se to ye ho jayega F_c/A_c , F_m/A_c aur F_f/A_c agar mein is tarah se kuch rearrangement karunga yahan dekhenge is part mein to maine F_m/A_m yaani area of matrix se divide kiya hai aur area of

matrix se multiply kiya hai. Yahan pe humne wahi kiya hai. Yahan pe maine area of fiber se divide kiya hai aur area of fiber se multiply kiya hai. To aap dekhenge ye jo part hai ye ho jayega mera composite pe upar jo stress lag raha hai wo part ho jayega. Ye part mera matrix pe jo stress aayega ya matrix pe stress develop hoga wo hoga. Aur A_m/A_c ko mein likh sakta hoon volume fraction of my matrix (V_m). Similarly aap ye part agar aap dekhenge to ye ho jayega fiber ke upar jo stress aa raha hai ya fiber jo stress experience kar raha hai ye ho jayega kyunki ye force / area hai aur ye jo part hai

A_f/A_c yeh ho jayega volume fraction of fibers (V_f). Kyunki hum is direction mein dimensions ko constant maan ke chal rahe hain. To yeh mein simplify karke likh sakta hoon. To yahan pe aap dekhenge ki agar mein E_c se multiply karta hoon yaani is strain se agar mein multiply karunga through out ya mein is tarah se bhi likh sakta hoon ye σ_c ko mein likh sakta hoon strain in composite multiply by Youngs modulus of composite. Similarly ye jo part hai σ_m ko mein $\epsilon_m \cdot E_m$ yaani Youngs modulus of matrix yahan pe σ_f ko likh sakta hoon ϵ_f into modulus of fiber to mein is tarah se kuch likh sakta hoon aur abhi humein pata hai ki $\epsilon_c = \epsilon_m$ aur ϵ_f same hai to kyunki hum isostrain consider kar rahe hain to yahan pe ye teeno cancel out ho jayenge aur mein kuch is tarah se relation likh sakta hoon. To ye ho gaya Youngs modulus of composite. Mein likh sakta hoon Youngs modulus of composite $E_c = E_m \cdot V_m + E_f \cdot V_f$. Aur ye hum keh sakte hain ki ek series mein addition hum is tarah se consider kar sakte hain. Jab mein isostrain model ki baat kar raha hoon. To mein kisi bhi property ko kuch is tarah se likh sakta hoon ki agar mere paas different reinforcements hain aur mein isostrain model consider kar raha hoon to average

property mein kuch is tarah se likh sakta hoon. Ye ho jayenge inki summation series mein ek volume fraction ho jayega us ith phase ka ith reinforcement ka into property of ith reinforcement. To aap is tarah se property of composite determine kar sakte hain. To yeh ho gaya ek ek scenario. Abhi hum doosra scenario consider karte hain jahan par matrix kuch is tarah se hai aur mere fiber kuch is tarah se aligned hai aur mein force F_c kuch is tarah se apply kar raha hoon. To aap dekhenge ki ye isostress model hai. Is isostress model isliye hai kyunki yahan pe aap dekhenge ki jo fiber hai aur

matrix hai yeh donon same stress experience kar rahe hain. Aap dekhenge iske geometry ya alignment ki wajah se yeh jo matrix hai iska area A hai aur sigma ye jo fiber hai iska bhi area A hai aur aap dekhenge ki stress experience by matrix aur stress experience by fiber donon same rahenge. To agar mein ye jo force apply karunga tab wahan pe kuch elongation hoga. Yeh composite deform hoga. To mein change in length (ΔL) likh sakta hoon. To mein change in length is tarah se likh sakta hoon ki total length change jo composite ka hai wo ho jayega change in length of matrix phase plus change in length of fiber phase. Agar

mein initial length se divide karunga. Kuch is tarah se L_c se agar mein yaani length of composite mein kuch is tarah se consider karunga. Yeh maan lete hain mera length of composite hai kuch is tarah se. To aap agar isse divide karenge to aapko ek identity milegi.

Abhi hum kuch is tarah se isko rearrange kar sakte hain. Agar mein is term ko kuch is tarah se likh sakta hoon. Yaani mein L_m yaani length of my matrix se divide karunga aur length of matrix se multiply karunga. Similarly ye jo part hai yahan pe mein length of fiber se divide karunga aur length of fiber se multiply karunga.

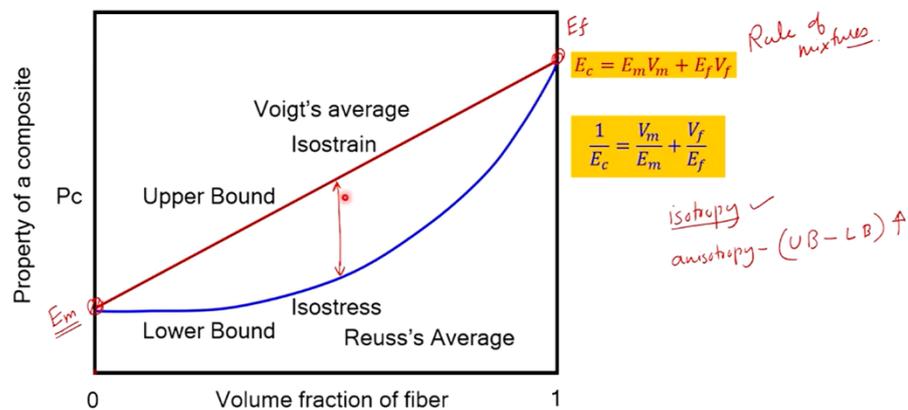
To aap dekhenge ki yeh jo part hai $\Delta L/L$ yeh ϵ ho jayega kyunki yeh definition ho jayegi mere strain ki aur ye jo definition hai ye bhi strain ho jayega mere matrix phase mein aur yahan pe mein consider kar sakta hoon ki area constant hai. To ye jo part hai ye represent karega mere volume fraction ko. L_m/L represent karega V_m aur L_f/L represent karega V_f volume fraction of fiber. Aur yeh jo part hai $\Delta L_f/L_f$ yeh represent karega epsilon yaani strain in fiber. Agar mein is ϵ ko kuch is tarah se likh sakta hoon. σ/E yaani Youngs modulus of composite is equal to sigma

divided by Youngs modulus of matrix aur is epsilon ko likh sakta hoon σ_f by Youngs modulus of fiber abhi humein pata hai ki σ , σ_m aur σ_f equal hai kyunki ek isostress condition mein hai to mein kuch is tarah se likh sakta hoon yaani ye composite ka Youngs modulus ho jayega aur mein likh raha hoon isko mere matrix aur fiber ke Youngs modulus ke dwara aur volume fraction of matrix aur volume fraction of fibers ke dwara. To mein kisi bhi property ko kuch is tarah se likh sakta hoon. $1/P = \sum (V_i/P_i)$ ye jo hai property of individual phase ya yahan pe matrix aur reinforcement ki jab mein baat karunga to ye property ho

jayegi matrix aur reinforcement ki. To kuch is tarah se mein do models se ye composite ke properties ko determine kar sakta hoon. Aur yahan par aapko dhyaan rakhna hai ki volume fraction of matrix plus volume fraction of fiber yeh one hota hai ($V_m + V_f = 1$). Donon ko mila ke value one ho jayegi. To agar mein ye models agar consider karoon to yahan par maine kuch property yahan par likhi hai. Property of composite aur yahan pe volume fraction of fiber yahan pe maine consider kiya hai. To maan lete hain ki yahan pe ye composite ka volume fraction composite mein volume fraction of fiber kuch is tarah se change ho raha hai. To aapko



Composites: Isostress and Isostrain models



isostrain model agar mein consider karunga to ye relation aapko is tarah se milega. To yahan par aap dekhenge ye ho jayegi volume fraction of my matrix. Yahan pe yaani agar mein property ki baat karunga to volume fraction of fiber yahan pe shunya hai to ye jo property ho jayegi ye mere matrix ki ho jayegi aur ye jo property hogi wo ho jayegi. Yaani yahan pe agar mein elastic modulus ki baat karunga to ye ho jayegi E_f . Aur agar mujhe fiber content pata hai to mein us fiber content ke liye us composite ki property yahan pe is tarah se isostrain model se predict kar sakta hoon. To ye ho jayegi $E_c = E_m \cdot V_m + E_f \cdot V_f$.

fraction of matrix + fiber into volume fraction of fiber. Ye ho gaya isostrain model. Aur isostress model agar hum dekhenge to property humein kuch is tarah se milegi. To ye hogi yaani Youngs modulus ki baat kar raha hoon to yahan par mujhe variation kuch is tarah se milega. Abhi hum dekhte hain ki isostrain model agar mein consider kar raha hoon to mujhe higher property mil rahi hai composite ki aur isostress mein mujhe lower mil rahi hai property. To isiliye isostrain model ko kabhi-kabhi upper bound bhi kaha jaata hai aur isostress models ko lower bound models kehte hain. Aur isko isostrain ko Voigt

average bhi kahte hain aur isostress ko hum Reuss average bhi kehte hain. Generally dekha jaata hai ki agar mein koi bhi composite bana raha hoon to aapko properties jo composite ki hai woh in do bounds ke beech mein hi milegi. To yeh ek upper bound aur lower bound ke beech mein milegi. Isko isko rule of mixture bhi kehte hain. Yahan pe likh lete hain. Isko kabhi-kabhi rule of mixtures bhi kehte hain. To ye ho gaye mere composites ke property

determination. Agar mujhe volume fraction pata hai mere matrix aur fibers ke to mein ek property ko predict kar sakta hoon. Kuch is tarah se do bounds ke beech mein. Upper bound property

aur lower bound property. To generally properties aapko kuch is tarah se milegi. In donon ka difference aur ek property aur ek information yahan pe dekh in donon ka jo jitna difference rahega ye mere material mein isotropy ko bhi explain karti hai. Yaani jitna kam difference utni isotropy yahan pe aapko milegi. Aur jitna zyada difference utna material anisotropy. To yahan par likh dete hain anisotropy agar mein baat karunga to upper bound aur lower bound ka jo difference hai wo zyada rahega. To yeh ho gaye kuch composites ki properties aur aaj ke part mein humne ek behavior dekha material ka jisko hum anelastic behavior

kehte hain jo ki time dependent behavior hai aur wo elastic strain develop karta hai jo ki recoverable hota hai. Generally anelastic behavior yeh polymers, glassy materials ya amorphous materials mein dekha jaata hai. Humne uske baad composite materials ke properties ko kis tarah se determine karte hain yeh bhi dekha. To abhi ke liye mein yahin par rukta hoon aur is class ke baad is course ko samapt karta hoon. Dhanyawad.