

Mechanical behavior of materials

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Week-3

Lecture-52

Fracture & Theoretical Cohesive Strength of Materials

Course Title

Mechanical Behavior of Materials (Hindi)

Lecture-51
Fracture & Theoretical Cohesive Strength of Materials

Namaskar phir se swagat karta hoon aapka is course mein Mechanical Behavior of Materials jo ki hum Hindi mein padhenge. Last part mein humne fracture ke baare mein baat ki thi. Is part mein hum fracture thoda aur aage jaakar padhenge. Last time last part mein humne fracture brittle materials ke dekha tha. Abhi hum fracture jab brittle materials ki baat karenge tab hum Griffith theory ke baare mein padhenge. Kuch terms aapko main introduce karunga. Jaise strain energy release rate aur fracture toughness. Toh last part mein humne jab brittle materials ka fracture dekha tha. Tab humne ek theoretical cohesive strength ki baat ki thi.

Aur uska formula kuch is tarah se humne derive kiya tha $\sigma_c = \sqrt{E\gamma / x_0}$ aur humne ye dekha tha ki ye jo value aati hai theoretical cohesive strength ye do se chaar order magnitude higher aati hai hamare experimental values ke. Ye humne silica ke liye dekha tha aur agar hum baaki ke ceramic materials ya jo brittle materials hain jo brittle tareeke se fail hote hain un material mein dekhenge toh yeh value humein hamesha do se chaar order higher milegi experimental value se. Yeh kyon multi hai ye abhi hum jaanenge. Toh humne ek dislocation ke baare mein jab baat ki thi tab humne ye bhi dekha tha ki actual jo strength hoti hai material ki wo do se chaar order higher hoti hai hamare shear strength jo experimental observe shear strength se.

Toh usmein humne bola tha ki dislocation jo hote hain ya defects jo hote hain material mein wo hamare jo theoretical shear strength ka jo higher values hai uske liye zimmedar hote hain. Yaani theoretical shear strength humein zyada milti hai. Wo kyon milti hai? Kyonki hum material defect free maan ke chalte hain. Par jab material mein defect hote hain jaise ki dislocation hote hain usmein humein milega ki jo value hai theoretical shear strength ki wo ghat jayegi aur hamari value experimental values jo hai wo do se chaar order kam milegi theoretical shear strength. Usi analogy ko hum leke ismein bhi consider kar sakte hain.

Ki theoretical cohesive strength hai material ki, wo practical values jo hai experimental values jo hai usse zyada kyon milti hai? Kyonki hamare material mein defects hote hain. Wahan pe humne dislocations ki baat ki thi. Yahan pe is defects ko hum cracks ya 3D defects ke baare mein baat karenge. Jaise materials mein kuch defects hote hain. Cracks inherently present hote hain. Aur ye jo cracks hain wo hamari theoretical cohesive strength kam karenge ya jo hamari practical values hain iske wajah se humein theoretical cohesive strength se humein jo experimental value hai kam milti hai.

Toh yeh sabse pehle iske cracks ke baare mein jab baat ki thi tab Griffith ne ek theory propose ki thi brittle materials ke liye 1920 mein wo abhi bhi valid hai yaani reasonably valid hai brittle materials ke liye. Toh unhone ye kaha tha ki materials mein inherently cracks ya defects present hote hain. Jiski wajah se jo cohesive strength of materials hai wo humein high nahi milti aur experimental values hamesha kam rehti hain theoretical values se. Tab jab hum brittle materials ki baat karenge, tab humein pata hai ki brittle materials elastic behavior dikhaate hain. Yaani jaise ki elastic behavior se hum plastic behavior mein transition karenge, tab material fail ho jaate hain.

Toh, maan lete hain mere paas kuch ek material hai jisko maine ek stress apply kiya hai. Tensile stress apply kiya σ . Aur maan lete hain is material mein ab ek crack present hai. Ek defect present hai. Isko main maan leta hoon surface crack. Is tarah se isko through thickness crack bhi kehte hain. Kyonki yeh jo crack hai wo through out thickness present hai. Yaani is direction mein through out present hai. Is thickness direction mein. Isko hum through thickness crack bhi kehte hain. Yeh jo crack hai yeh surface crack hai aur mostly surface cracks kuch processing ya kisi manufacturing technique ke dwara produce hoti hain surfaces pe.

Toh ye maan ke chal sakte hain ki surface cracks hamesha rahengy materials ke upar. Aur ye surface crack hamari sharp crack hai. Abhi yahan pe main sharp crack kis crack ko bol raha hoon. Toh main agar sharp crack bolunga toh yahan pe agar main is tarah se structure dekhoonga toh ye radius of curvature hai yahan pe sharp crack ka wo equivalent hoga mere ek equilibrium spacing of atoms iske barabar. Toh yahan pe isko hum atomistically sharp crack is tarah se hum dekh sakte hain ki is tarah se crack present hai mere material ke surface pe. Abhi main uske upar ek stress apply kar raha hoon. Uske wajah se kya hoga? Yahan pe main is crack ki length abhi maan raha hoon a.

Aur abhi aap dekhengy is crack ki wajah se jo stress distribution hai material mein kuch is tarah se change hoga. Yahan pe aap dekh rahe honge ki yahan pe ek stress concentration ban raha hai crack ke tip ke saamne. Aur yahan pe yahan par koi stress concentration nahi hai kyonki yahan pe jo material hai wo koi stress carry nahi karega is crack ke abov. Uske wajah se kya hoga? Yahan par is yeh jo area abhi mark kiya hai, yeh jo area hai, yeh koi crack ya yeh jo volume hai, agar main thickness ki baat karunga, toh yeh jo volume hai, yeh koi stress carry nahi karega. Aur iski wajah se main keh sakta hoon ki yahan pe jo strain energy hai, yahan pe koi strain energy available nahi hogi.

Toh, agar main iske dimensions maan ke chalta hoon, yeh maan ke chalta hoon. Iske dimensions hain β . Abhi hum dekhengy is crack ke presence ki wajah se ek strain energy release hogi material mein aur yaani us strain energy ko main is tarah se bhi likh sakta hoon ki yahan pe is region mein ek reduction hogi ya relaxation hoga strain energy mein. Jab main strain energy ki baat karta hoon tab main ek total energy ki baat ki thi humne. Yeh is tarah se main likh sakta hoon $\text{Energy} = 1/2 (\sigma * \epsilon)$ kyonki hum brittle materials ki baat kar rahe hain toh wo elastic behavior dikhaengy.

Toh main yeh jo energy hai total energy per unit volume jo hai yeh valid reh sakti hai aur is energy se main ek release in elastic strain energy nikaal sakta hoon. Toh main isko sirf agar stress ke under consider karunga toh main isko kuch is tarah se likh sakta hoon: $U = 1/2 (\sigma^2 / E)$. Abhi hum dekhengy ki ye reduction in relaxation of strain energy kis tarah se hui hai. Isko main maan leta hoon ek U_0 hai. Aur ye hogi ye jo value hai $\sigma^2 * 2 / 2 * \text{Volume}$ agar main yahan pe jo volume jo release karega kyonki ye jo energy hai ye per unit volume thi. Agar main isko multiply karunga is volume se toh mujhe pata chal jayegi ki kitni strain energy yahan pe release hui hai.

Toh is region ka agar main volume nikaalunga toh yahan pe βa hai. Ye crack length agar a maan ke chal raha hoon aur thickness agar main w maan ke chal raha hoon toh ye ho jayega mera volume. Aur ye do regions hain mere paas aise toh yeh total volume mere paas aa jayega $\beta * a * w$ agar main w yahan par thickness of the plate hai. Agar main kuch is tarah se likhunga reduction in strain energy per unit thickness yaani main w ko is side le aaunga toh main is identity ko kuch is tarah se likh paunga: $U_0 = -(\pi\sigma^2 / 2E) * a^2$. Abhi kuch careful studies ke baad hum β ko π ki tarah dikha sakte hain.

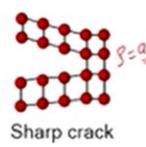


Griffith theory: Role of cracks in fracture

$$\sigma_c = \sqrt{\frac{E\gamma}{x_0}} \quad \text{2 to 4 orders of magnitude higher than the experimental values}$$

Defects: Cracks present in materials

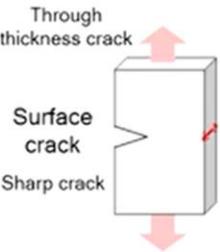
Griffith theory (1920)- Brittle Materials



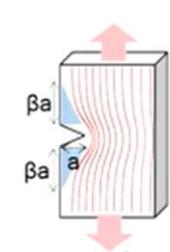
Sharp crack



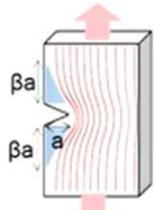
Through thickness crack



Surface crack



Sharp crack



βa
 βa
 a

Reduction or relaxation of strain energy

Reduction in strain energy/thickness

$U_0 = -\frac{\sigma^2}{2E} \beta a a w$

$U_0 = -\frac{\sigma^2}{2E} \beta a^2$

$\beta = \pi$



Alan Arnold Griffith
13 June 1893-13 October 1963
<https://www.ngg.org.uk/collections/search/person/mp96193/alan-arnold-griffith>

$$U = \frac{1}{2} \sigma \epsilon$$

$$U = \frac{1}{2} \frac{\sigma^2}{E}$$

w is the thickness of plate

Ye β ko π hum prove kar sakte hain. Par ye hamare course ke scope ke baahar hai. Toh yahan pe hum maan ke chalte hain ki β ki value π hai. Aur yeh β ki value π yahan pe hum rakhengy toh hamare paas ek reduction in strain energy per unit thickness aa jayegi. Abhi hum dekhte hain jab crack form hote hain toh wahan pe do naye surfaces taiyaar hote hain. Aur wo hamare system ki energy increase karengey. Toh agar main surface energy ki baat karunga toh mere paas do surfaces honge aur main us surface energy ko kuch is tarah se likh paunga: $U_s = 2a * \gamma$. γ meri surface energy per unit area hai. Toh ye jo energy hai ye per unit thickness main likh raha hoon yahan pe.

Abhi ek important sentence hum padhengy jo Griffith ne diya hai. Toh Griffith kehte hain ki crack propagate hogi, jab meri decrease in elastic strain energy yeh equal hogi mere energy required to

create new crack surface. Yaani humne dekha ki jo crack ki wajah se reduction hoti hai elastic strain energy aur crack form hone se naye surface area bante hain usse meri energy increase hogi. Toh Griffith kehte hain crack kab propagate hoga? Jab meri decrease in elastic energy is equal honi chahiye kiske barabar? Yeh increase in the energy due to new crack surface.

Isko samajhte hain. Agar mere paas kuch is tarah se crack hai humne kaha tha yahan pe ek elastic strain energy release hogi is tarah se is area mein aur yeh jo area taiyaar hua hai yahan par uske wajah se kuch surface area naya surface area create hua hai uske wajah se ek energy ho gayi surface energy isko hum U_s maan ke chal rahe hain aur isko maan ke elastic energy U_e . Toh ye jo U_e hai elastic strain energy jo reduction hai wo atleast equal hona chahiye mere U_s se. Toh isko mathematically hum is tarah se likh sakte hain ki total change in the energy $U_t = U_e + U_s$. Humne dekha tha ki U_e mein maine negative sign likha tha kyonki ab wahan pe wo jo energy hai wo reduction hai.

isliye usko negative likha tha. Aur U_s jo energy hai wo increase kar rahi hai isliye yeh positive sign hai. Agar main total energy likhunga mere system ki toh main kuch is tarah se likh paunga. Agar isko main graphically represent karunga toh ye total energy yahan pe main U maan ke chal raha hoon. Agar crack ki tarah yaani meri crack length is tarah se main agar consider karunga a toh $2a\gamma$ ye positive hai. Toh kuch is tarah se aayega. Ye linearly change ho raha hai mere crack length ke saath. Aur yahan par aap dekhenge ki ye jo U_e hai ye kuch is tarah se yaani aap dekhenge $-(\pi\sigma^2 / 2E) * a^2$. Toh yahan pe ye dependent hai a^2 ke hisaab se.

toh humein U_e ka nature kuch is tarah se milega. Agar main U_t plot karunga toh in dono ko agar main add karunga toh mere paas U_t ki value kuch is tarah se vary hogi. Ye meri total energy hogi system ki. Abhi aap dekh paa rahe honge ki ek critical crack length yahan par aap consider kar sakte hain. Iske pehle agar yaani ye critical crack agar length main agar consider kar raha hoon toh iske pehle aap dekhenge ki meri U_t jo total energy hai wo badh rahi hai. Iske baad ye energy ghat rahi hai. Iska matlab yeh hai ki agar meri crack length iske niche hai toh wo crack propagate nahi hogi. Agar iske upar hai toh wo crack propagate hogi.

Aur yahi graphically ye represent kar raha hai. Ye sentence ko hi ye graphically hum yahan par represent kar rahe hain. Abhi hum ye critical crack length kis tarah se nikaalenge? Agar hum

dekhengy is U_t ko agar main differentiate karta hoon a se aur ya ye point pe agar main isko agar differentiate karunga toh yahan pe slope zero aayega. Agar yahan pe main isko shunya maan ke chalunga toh main ek critical crack length ki value nikaal sakta hoon. Toh U yaani $dU_t/da = 0$ agar consider karunga toh ye value kuch is tarah se main likh paunga. Is equation ko maine a se differentiate kiya hai. Mere paas ye kuch aise identity aayegi aur σ ki value main kuch is tarah se likh paunga.

Is equations ko main agar solve karunga toh mere paas σ ki value kuch is tarah se aayegi. Abhi hum a agar a_c ki value leta hai toh yeh σ_c ho jayega. Toh main σ_c ko kuch is tarah se likh paunga. Yaani jab a agar a_c hai toh ye σ mera σ_c yaani critical stress ho jayega. Aur iski value aa jayegi $\sqrt{(2E\gamma / \pi a c)}$. Toh ye mere paas ek identity aa jayegi aur ye critical stress main nikaal paunga is energy balance se. Toh main a_c ko bhi nikaal sakta hoon. Agar mujhe critical crack length nikaalni hai toh agar main equation is tarah se rearrange karunga toh main a_c ki value kuch is tarah se likh paunga. Toh agar a_c ko main is tarah se plot karunga σ_c se.

toh a_c ka variation kuch is tarah se milega mujhe. Aapko yahan pe dhyaan dena hai ki σ_c aur a_c ye inversely proportional hain ek doosre ke. Yaani a_c is inversely proportional to σ_c^2 . Toh a_c ka variation kuch is tarah se milega. Isko samajhte hain. Agar mere paas kuch is tarah se crack length hai. Maan lete hain mere paas kuch is tarah se stress hai mere paas aur mere paas kuch is tarah se crack length aa jayegi a_1 . Toh agar mere material mein ye crack length available hai, ek critical stage pe rahega. Agar mere material mein crack length ye hogi is stress pe, ye stress maine apply kiya mere material pe, aur ye agar crack length hai a_2 toh mera material fail hoga.

hoga is crack length pe. Toh aap is tarah se samajh sakte hain. Toh ye jo region hai yahan pe ye safe region hai. Ye jo region hai mera unsafe region hai. Agar ulta agar hum consider karengy agar mere paas kuch is tarah se crack length hai a_3 aur main material ko ye stress apply kar raha hoon, isko maan lete hain σ_3 . Yeh stress agar main apply kar raha hoon toh material fail nahi hoga. Agar material pe yaani ye crack length hai aur material pe main ye stress apply kar raha hoon, isko maan lete hain σ_4 toh material yahan pe fail hoga. Toh hum dekh sakte hain ki is equation se hum bata sakte hain ki kitna stress main apply kar sakta hoon.



Griffith theory: Role of cracks in fracture

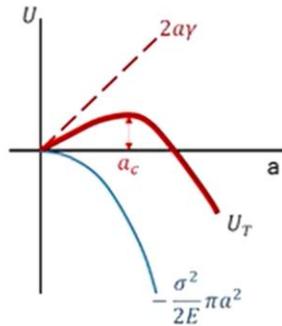
Formation of two new surfaces Increase in the energy of the system

$$U_s = 2a\gamma$$



A crack will propagate when the decrease in elastic strain energy is at least equal to the energy required to create the new crack surface

Total change in the energy $U_T = U_E + U_s$



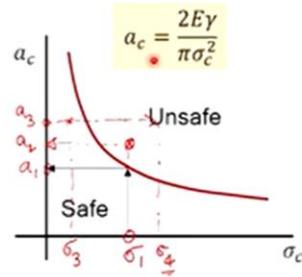
$$U_T = -\frac{\sigma^2}{2E}\pi a^2 + 2a\gamma$$

$$\frac{dU}{da} = 0$$

$$0 = -\frac{\sigma^2}{2E}\pi 2a + 2\gamma$$

$$\sigma = \left(\frac{2E\gamma}{\pi a}\right)^{1/2}$$

$$\sigma_c = \left(\frac{2E\gamma}{\pi a_c}\right)^{1/2}$$



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agar material mein mere paas cracks is tarah se present hain aur agar material mein cracks is tarah se present hain toh kitna maximum stress main apply kar sakta hoon material pe. Toh ye Griffith theory ka ek advantage hai hamare paas. Toh yahan pe yeh jo boundary dikha rahi hai yeh ek unstable region dikha rahi hai jo safe to unsafe region mein transit karti hai. Aur jab hum cracks aur fracture stress ki baat karenge tab yeh meri surface cracks hai. Yahan pe humne dekha ye surface pe present hai. Aur iski length hum consider karte hain a . Agar material mein agar crack inside present hai yaani yeh bhi through thickness crack hai agar yeh inside present hai iski length hum $2a$ consider karte hain hamesha.

Inko hum kehte hain internal cracks. Aur ye jo equation humne derive kiya hai, yahan pe ye equation hamesha valid rahega. Aapko hamesha agar surface crack di hai toh uski length humein aapko a consider karni hai. Agar internal cracks di gayi hain toh ye jo length hai aapko hamesha us crack ki consider karni hai $2a$. Ye jo equation hai wo dono ke liye valid hai. Abhi humne silica ka example diya tha last class mein. Toh humne silica ke liye kuch ye parameters liye the E , γ , σ_0 aur humne ek stress nikaala tha jo humein mila hai isko hum kahe the cohesive strength of a material toh ye without cracks hai yaani without any defects toh humein ye value mili thi 24.4 GPa.

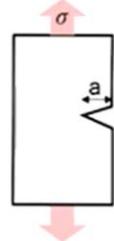
aur humne kaha tha yeh jo value hai wo 2 to 4 orders higher hai. Abhi hum Griffith theory se ek value nikaalengy ki kab material mera fail ho sakta hai silica ke liye. Agar main crack length maan ke chalta hoon 10 microns ya 0.01 mm ye typical crack length ho sakti hai mere silica materials mein. Toh ye practically humein dekha gaya hai ki crack length is length ki hoti hai. Toh agar ye values main yahan pe mere paas E hai, γ ki value hai aur a ki value hai agar main sigma nikaalne ki koshish karunga toh ye fracture stress kuch is tarah se aayega. Toh fracture stress ki value aayegi ye saari values maine yahan pe plug in kar di hain. Aur yeh jo fracture stress ki value aayegi.

0.025 GPa. Yeh meri with cracks hai. Aap dekh paa rahe honge ki yeh jo value hai yeh much much smaller hai mere theoretical cohesive strength se aur yeh just small nahi hai, yeh kam se kam do se chaar order small rehti hai most of the materials ke liye. Toh aap yahan pe aap dekh sakte hain ki jo Griffith theory hai kuch valid hai brittle materials ke liye. Aur ye practically isi stress pe material mera break hota hai ya fracture hota hai. Toh Griffith theory ke kuch advantages hum dekh sakte hain. Toh Griffith theory ke baare mein kuch sentences maine yahan likhe hain. Toh Griffith theory ne jo discrepancy humein mili thi theoretical cohesive strength aur experimental observed values ki jo.



Cracks and fracture stress

Surface crack



e.g. Silica

$E = 95 \text{ GPa}$
 $\gamma = 1 \text{ J/m}^2$
 $x_0 = 0.16 \text{ nm}$
 $a = 10 \mu\text{m} = 0.01 \text{ mm}$

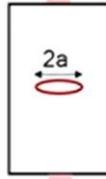
$\sigma = \left(\frac{2E\gamma}{\pi a} \right)^{1/2}$

$\Rightarrow \sigma_c = 24.4 \text{ GPa}$ Without cracks

$\sigma_f = \left(\frac{2 \times 95 \times 10^9 \times 1}{\pi \times 0.01 \times 10^{-3}} \right)^{1/2} \times 10^{-9} \text{ GPa}$

$\sigma_f = 0.025 \text{ GPa}$ With crack

Internal crack



Griffith theory:

- Solves discrepancy between Theoretical Cohesive strength and experimentally observed fracture strength
- Fracture is always considered to be propagation- (rather than nucleation-) controlled; this is a serious short-coming of the theory
- Not applicable for ductile materials

fracture strength hai unmein, ki wo discrepancy wo hataane mein humein madad ki hai. Aur yeh jo hai kuch Griffith theory ke baare mein humne yahan par dekha tha ki Griffith theory predict kar sakta hai ki crack kab propagate hoga. Par crack kab nucleate hoga ye is theory se hum bata nahi paate. Toh ye ek shortcoming hai is theory ki. Aur ye jo theory hai humne dekha tha ki humne ye develop ki thi brittle materials ke liye. Ye ductile materials ke liye valid nahi hai. Toh ye ho gayi hamari Griffith theory aur iske advantages aur kuch disadvantages also. Toh jab hum baat karte hain materials jo plastically deform karte hain, ab uske.

liye hum kuch terminologies introduce karenge. Jaise strain energy release rate. Jo materials deform hote hain plastically. Toh agar hum ek scenario consider karte, humne consider karte ek through thickness crack, sharp crack yahan pe hum consider kar rahe hain aur humne dekha tha ki yahan pe stress concentration taiyaar hoga. Kyonki yahan pe ek crack hai uske tip ke aage aur yahan pe strain energy release hogi aur yahan pe mera stress concentration hoga yaani stress badhega. Toh agar mera material plastic hai, agar yahan pe tip ke agar stress yield strength se zyada ho gaya toh kya hoga? Yahan pe jo material hai wo deform hoga. Agar.

stress yield strength se zyada hai toh is zone ko hum kya kahenge? Jahan pe material mera plastically deform ho raha hai. Us zone ko hum kehte hain plastic deformation zone. Abhi hum plastic deformation zone ki jab baat kar rahe hain tab hum Griffith theory ko kuch is tarah se modify kar sakte hain. Toh Griffith theory ko modify karne mein Orowan aur Irwin ne bahut accha kaam kiya hai. Hum iske baare mein abhi zyada detail mein nahi jayenge. Par ye equation humne dekha tha aur isko humne thoda sa modify kiya hai. Yahan pe humne ek term istemaal ki hai γ_p jo ki plastic work hai. Agar mera brittle material hai toh mere paas sirf surface energy rahegi. γ_p wahan pe shunya.

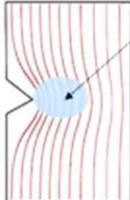
20:34 - 21:26 rahega. Toh, yeh jo relation hai, humne kuch is tarah se hai jo hum plastic materials ke liye yaani jo materials plastic behavior ya plastically deform hote hain unke liye hum likh sakte hain. Toh, yahan pe ye jo term hai $2\gamma_s + \gamma_p$ isko main ek replace kar raha hoon ek naye word se jisko main keh raha hoon G_c . Isko hum kehte hain critical strain energy release rate. Yeh meri value hai critical strain energy release rate ko. Iske kuch units dekh lete hain. Iske units hain Joules / m². Aur is Joules ko main agar is tarah se likhunga Newton-meter. Toh aur iske unit dekhoonga toh ye aa jayega Newton / m. Toh ye jo term hai critical strain energy.

release rate. Isko hum crack extension force bhi kehte hain. Toh aapko literature mein ya books mein yeh do term milengy isi identity ko ya is quantity ko bataate hue yaani crack extension force aur critical strain energy release rate. Abhi hum G_c ki jab baat karenge tab G ye jo hum is tarah se nikaal sakte hain. G ye jo hai change in meri elastic energy, change in energy of the material with respect to extension in the crack. Kuch is tarah se hum consider kar sakte hain. Ye meri strain energy hai. Toh agar main material consider kar raha hoon kuch is tarah se, surface crack hai yahan pe aur ek unit thickness hai, toh.

main G kuch is tarah se consider karunga. Yeh jo G mera energy release per unit increase in crack length. Yaani jaise-jaise meri crack length badhti hai, us tarah se jo energy release hogi, usko hum kehte hain critical strain energy release rate. Iske naam se hi aapko pata chalega ki ek strain energy release rate hai. Yeh rate main baat karunga jab meri crack length badhegi ya mere crack propagate hogi material mein. Toh main is equation ko kuch is tarah se likh paunga: $\sigma = \sqrt{(E * G_c / \pi a)}$. Aur humein G_c nikaalni hai toh humein γ_s aur γ_p nikaalne ki zaroorat padegi. Jaise γ_s yaani surface energy humein pata honi chahiye aur γ_p yaani plastic work.

Strain energy release rate, G

Materials: Deform plastically



Plastic Deformation Zone

$$\sigma = \left(\frac{2E(\gamma_s + \gamma_p)}{\pi a} \right)^{1/2}$$

γ_s : Surface energy
 γ_p : Plastic work

$2(\gamma_s + \gamma_p) = G_c$ Critical strain energy release rate
J/m²

$N \text{ m/m}^2 = N/m$

Crack extension force

$G = \frac{dU_0}{da}$ Strain energy

Energy release per unit increase in crack length

We need to find out γ_s and γ_p

$$\sigma = \left(\frac{E G_c}{\pi a} \right)^{1/2}$$



Unit thickness

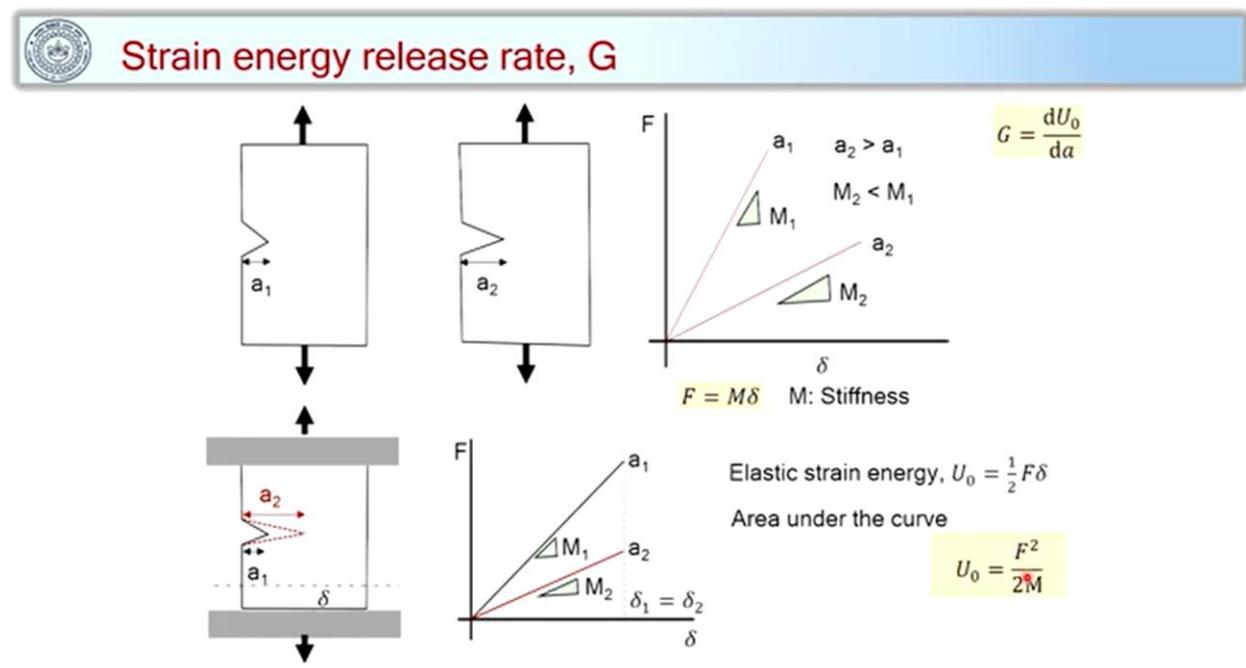
kya hoga? Ye bhi humein pata hona chahiye. Toh abhi hum G_c ya critical strain energy release rate isko kis tarah se nikaalte hain ye dekhengy. Toh humne jaana hai ki is slide se ki hum jo Griffith theory hai usko modify karke hamare plastic material ke liye bhi likh sakte hain. Toh G ko humne dekha tha ki ye strain energy release rate hai per unit crack length kuch is tarah se. Toh agar maan lete hain mere ek material hai aur usmein crack lengths alag-alag hain. Surface crack lengths yahan par a_1 hai, yahan pe a_2 hai. Aur main kuch force is tarah se apply kar raha hoon. Tensile force apply kar raha hoon aur isko main force versus displacement likh.

sakta hoon. Yaani kitni meri crack propagate hogi ya kitna displacement milega. Kuch is tarah se agar main displacement ki baat karunga is tarah se. Toh is material mein kis tarah se displacement hoga ye hum yahan pe plot karenge. Toh jab main force apply kar raha hoon agar mere material mein crack length a_1 hai toh mujhe kuch is tarah se deflection milega. Agar material mein crack length a_2 hai toh mujhe kuch is tarah se deflection milega. Aap dekh paa rahe honge ki is geometry se jo agar main slope nikaalunga toh yahan par slope main maan le raha hoon ki jab F versus δ ka slope jab crack length a_1 hai toh slope yahan.

pe M_1 hai aur F versus δ jab crack length a_2 hai uska slope M_2 hai. Is geometry se hum dekh paa rahe hain ki jo a_2 hai wo badi hai crack length mere a_1 se. Toh yahan par naturally aap dekh paa rahe honge ki ye jo slope hai M_2 ye small rahega M_1 se. Toh ye jo force versus displacement jab plot kar rahe hain aap dekh paa rahe honge jahan pe crack length meri badi hai wahan pe mera slope kam hai. Yaani mujhe agar crack badi hai toh mujhe force kam lagega same displacement laane ke liye. Agar crack length chhoti hai toh force mujhe zyada lagega same displacement laane ke liye. Ye jo M hai isko hum kehte hain stiffness of material jab mere material mein koi crack present.

hai aur ek simple relation yahan pe likh sakta hoon $F = M * \delta$. Toh maan lete hain kuch ek scenario yahan pe consider karte hain crack length hai mere paas a_1 aur ye jo material hai yahan pe kuch is tarah se hinged hai aur yahan pe main isko ek tensile force apply kar raha hoon. Abhi maan lete hain ki yeh jo material hai aur hum dekhengy ki iska jo displacement hai yeh ek δ hum fix kar dete hain. Toh agar main force versus δ ka plot karunga toh material mere a_1 ke liye mujhe kuch is tarah se force milega. Agar main δ yahan pe fix kar diya, yeh jo deflection hai agar main isko fix kar doonga toh yeh mujhe force is tarah se milega.

Agar material mein crack length a_2 hai toh aur mujhe same displacement chahiye toh mujhe kuch is tarah se itna force mujhe apply karna padega aur wo mujhe pata chalega ek material ke stiffness value se jo mujhe ek stiffness hai mere material ka M_2 jab mere material mein crack length a_2 present hai. Abhi aap dekh paa rahe honge ki mujhe G nikaalna hai aur ye jo G hai ye main abhi is condition mein nikaalne ki koshish karunga jahan pe $\delta_1 = \delta_2$ yaani jo displacement hai mere material ka do material mein mere material mein a_1 aur a_2 cracks hain tab main force nikaal raha hoon separate forces nikaal raha hoon yahan pe aur main nikaalne ki koshish kar raha hoon ki kitni.



strain energy release hogi yahan pe. Toh elastic strain energy ki jab baat karengy hum toh elastic strain energy mere paas main is tarah se likh paunga $U_0 = 1/2 (F * \delta)$. Ye ye jo value hai ye value hogi meri is area under this curve. Toh ye hoga area under the curve. Agar main ye F ki value ya δ ki value yahan se replace karta hoon toh mere paas kuch is tarah se value aayegi U_0 ki yaani elastic strain energy ki value aayegi $U_0 = F^2 / 2M$. Toh hamare paas U_0 ki ek value aa jayegi jo ki is area under the curve rahega. Abhi hamare paas U_0 hai aur humein G nikaalna hai. Ab hum dekhengy ki isko kis tarah.

se aage badhengy. Toh mere paas G ki value hai, U_0 ki value hai $F^2 / 2M$. Mere paas ye kuch condition hai. Abhi main $\delta_1 = \delta_2$ consider kar raha hoon. Tab yeh satisfy hoga jab mera $F_1/M_1 = F_2/M_2$. Ye ratio constant rahega ya same rahega. Toh mere paas agar teesra force main agar consider kar raha hoon ya teesri crack length hai yahan pe material mein present mere toh wo jo ho jayega F_3/M_3 . Toh ye jo ratio hai F/M ye mera constant hai different crack lengths ke liye. Toh abhi mere paas F/M constant hai. Agar main isko differentiate karta hoon a se, partial differentiation leta hoon main a se toh mere paas kuch is tarah se ek relation.

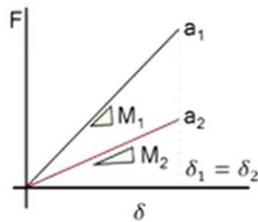
aayega $(1/M) * (dF/da) + F * (d(1/M)/da) = 0$ kyonki ye toh constant hai toh iska derivation shunya aayega. Toh yeh mere paas kuch is tarah se term aayega aur abhi main yahan se change in force per unit length kuch is tarah se nikaalunga is relation se. Toh ye ek relation mere paas aa jayega. Abhi main strain energy release rate nikaalne ki koshish karunga. Toh yahan par mere paas G ki value ye hai dU_0/da aur yahan pe U_0 ki value agar main is tarah se consider karunga $F^2 / 2M$ isko main differentiate karunga toh ye kuch is tarah se mere paas ek relation aayega. Abhi ye dF/da ki value mere paas yeh hai, yeh value agar main yahan pe insert karunga toh G ki.

value kuch is tarah se aayegi: $-1/2 F^2 * d(1/M)/da$. Toh ye mere paas ek strain energy release rate ki value aa gayi. Abhi is strain energy release rate ki value ko main critical strain energy release rate kab hoga? Ye jab F ki value F_c jayegi ya critical value of force main jab apply karunga ya jab crack length a ye ho jayegi a_c yaani critical crack length ke barabar. Toh main critical strain energy release rate ki value kuch is tarah se likh paunga: $G_c = -1/2 F_c^2 * d(1/M)/da$ jab crack length meri a_c ho jayegi. Yaani main kuch is tarah se mathematically likhunga: jab $F \rightarrow F_c$ ya $a \rightarrow a_c$ critical values tab G .

will tend to G_c . Toh yahan pe humne mathematically is tarah se likha hai. Abhi hum isko kis tarah se nikaal sakte hain? Agar main $1/M$ agar mujhe stiffness values pata hain toh main $1/M$ versus a agar plot karunga toh mujhe kuch is tarah se relation milta hai experimentally. Agar hum ek critical value ki baat karengy yahan pe agar main slope loonga iska $d(1/M)/da$, yeh ek value main consider kar raha hoon aur isko main critical value maan ke chal raha hoon. Toh ye slope yahan pe mujhe mil jayega aur isse main G_c ki value nikaal paunga. Yeh jo a_c hai kuch is tarah se main baat karunga. Toh yahan tak agar meri crack.



Strain energy release rate, G



$$\delta_1 = \delta_2 = \frac{F_1}{M_1} = \frac{F_2}{M_2}$$

$$\frac{F}{M} = \text{constant} \quad \text{For different crack lengths}$$

$$\Rightarrow \frac{1}{M} \frac{\partial F}{\partial a} + F \frac{\partial(1/M)}{\partial a} = 0 \quad \Rightarrow \frac{\partial F}{\partial a} = -FM \frac{\partial(1/M)}{\partial a}$$

$$G = \frac{dU_0}{da}$$

$$U_0 = \frac{F^2}{2M}$$

Strain energy release rate,

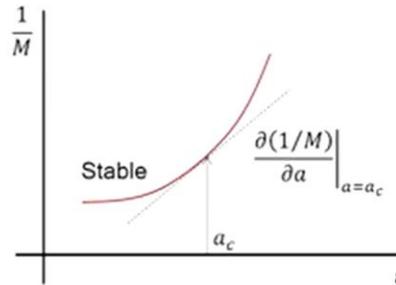
$$G = \frac{\partial U_0}{\partial a} = \frac{1}{2} \left[2F \frac{\partial F}{\partial a} + F^2 \frac{\partial(1/M)}{\partial a} \right]$$

$$\Rightarrow G = -\frac{1}{2} F^2 \frac{\partial(1/M)}{\partial a}$$

Critical Strain energy release rate,

$$G_c = -\frac{1}{2} F_c^2 \frac{\partial(1/M)}{\partial a} \Big|_{a=a_c}$$

*F → F_c
or a → a_c
G → G_c*



length hai yahan is region se pehle a_c se pehle agar hai toh meri crack stable hai. Iske upar crack catastrophically badhegi ya mera material catastrophically fail hoga. Toh yahan tak main stable crack regime mein hoon aur iske above main ek unstable crack regime mein hoon. Toh is tarah se hum critical strain energy release rate nikaal sakte hain materials ke liye. Ye methodology humne is tarah se dekhi. Abhi ek aur ek terminology main aapko introduce kar sakta hoon jab karna chahta hoon jab hum fracture ki baat karte hain toh uske liye hum ek case study consider karte hain de Havilland Comet crash ki. Toh yeh jo de Havilland jo tha yeh ek airplane operator.

tha, company thi jo airplanes ko operate karti thi. Toh inka ek plane January 10th 1954 ko Rome se London ja raha tha tab wo mid-air mein hi uska accident ho gaya aur wahan pe 56 passengers ki maut ho gayi. Phir unhone bahut saare considerations kiye, design considerations kiye aur saare safety values ko consider karke phir se ek operation chalu kiya aur April 8 1954 mein phir se unhone operation chalu kiya aur wo plane ja raha tha Rome se Cairo tak. Phir se yeh uska accident ho gaya jaise hi yaani 30 minute ke andar hi take-off lene ke baad aur wahan pe bhi jo bhi saare passengers thae unki maut ho gayi. Toh yahan pe saare design consideration, saare safety.

values, safety considerations karne ke baad bhi ek accident kyon hua? Yeh case study is fracture of materials ko importance bataati hai. Toh yahan pe jo aeroplane humne istemaal kiya tha wo

aluminium alloy se bana hua tha yaani jo fuselage tha wo aluminium alloy se bana tha. Toh fuselage jab hum baat karengy toh main usko cylinder karke yahan par consider kar raha hoon. Toh yahan pe jab aluminium ki baat karunga toh G_c jo critical strain energy release rate hai ya crack extension force hai material liye wo 53 N/mm hai aur elastic modulus yahan pe 76000 N/mm² toh aur hoop stress ki value.

maximum hoop stress ki value yahan pe 138 N/mm² thi. Jab main fuselage ki baat kar raha hoon toh yahan par main baat kar raha hoon yeh aeroplane ki body hai aur isko main cylinder karke consider kar raha hoon. Toh yahan par jo hoop stress ki baat kar raha σ wo maximum operating hoop stress yahan pe wo 125 MPa hai. Toh ye hoop stress ye jo value di thi usse bhi kaafi kam tha. Abhi aap dekhengy ki yahan pe jo windows bani hain, is plane ki jo windows bani thi, woh square shape ki thi. Aur is windows ke bagal mein kuch rivets kiye thae. Us rivets ki wajah se kuch crack introduce hue thae material mein aur uske wajah se kuch stress concentration badhi thi windows ke around. Ye ab agar aap dekhengy windows ke around kuch stress concentration badhi thi kuch is tarah se agar yahan pe cracks hain toh kuch is tarah se aap dekh paa rahe honge ki ek stress concentration yahan pe badhi hui thi aur jo crack ki origin hui thi yahan par windows ke paas jahan par rivet joint hue thae aur wo join ho ke yahan pe windows ki ye jo boundary hai yahan pe attach ho gaye. Yaani crack propagate ho ke yahan pe attach ho gaye. Phir yahan se kya hua? Ye jo poori length hai window ki ye consider ho ke poori ek total crack length ban gayi. Abhi hum kuch calculations karke dekhte hain. Toh hum agar ye values hamare paas hain toh ek ac ki value hum is tarah se nikaal sakte hain.

Critical crack length kitni ho sakti hai material mein maximum? Toh ye jo crack length hai wo maximum crack length hai 67 mm. Toh materials mein iske jo inside thae wo itni badi crack nahi thi. 67 mm bahut badi crack length is tarah se yaani ek hum conservative design consider karke chal rahe thae. Par jab crack hui toh ye crack is windows ko join hui ya aur phir ye window length plus crack length milke ek badi crack yahan pe consider hui. Toh yahan pe kuch is tarah se main is crack ko is tarah se consider karunga. Toh yeh meri total crack length yahan pe ban gayi jo window ki size ki thi aur wo is value se kaafi zyada thi. Isliye yahan pe ek fracture catastrophically propagate.



Fracture toughness

The DeHavilland Comet Crash

1. January 10, 1954: Rome Ciampino to London Heathrow, was lost on (56 Passenger died)
2. April 8, 1954: Rome Ciampino to Cairo, again crashed into the Mediterranean sea within 30 minutes of take-off

Al alloy fuselage $G_c = 53 \text{ N/mm}$, $E = 76000 \text{ N/mm}^2$

Fuselage = cylinder

σ_{tang} or $\sigma_{\text{hoop}} = 138 \text{ N/mm}^2$

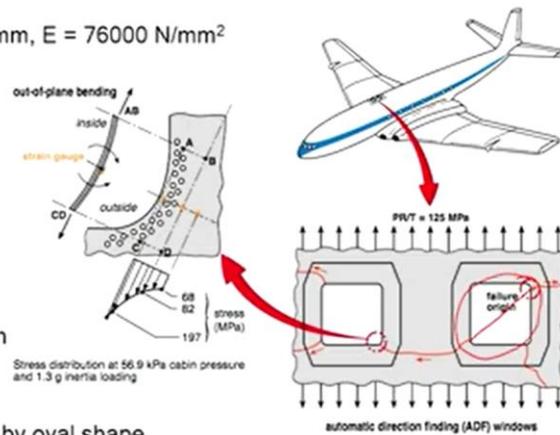
$$\sigma = \left(\frac{EG_c}{\pi a} \right)^{1/2} \quad a_c = \frac{EG_c}{\pi \sigma^2}$$

$$\Rightarrow a_c = \frac{53 \times 76000}{\pi \times (138)^2} = 67 \text{ mm}$$

- Cracks ending at windows
- Window length + Crack length

Aloha aircraft disaster

Square windows were replaced by oval shape



<https://aerospaceengineeringblog.com/dehavilland-comet-crash/>

Images are for educational and teaching purpose only

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hua. Toh iski wajah se abhi aap dekhengy ki ye jo windows hain wo ab planes mein yeh square shape ki ya sharp corners ki nahi rehti hain, yeh hoti hain hamesha oval shape ki. Aur similar ek example humne dekha tha Aloha aircraft disaster ka jab humne fracture ki baat chaalu ki thi. Similar incidence ya similar case study aap isse predict kar sakte hain ki Aloha aircraft disaster kyon hua tha. Toh ye jo fracture toughness hai abhi hum isko samjhengy. Fracture toughness hota kya hai? Toh humne ek ye relation dekha tha. Abhi hum is relation ko kuch is tarah se rearrange karte hain: $\sigma * \sqrt{(\pi * a)} = \sqrt{(E * G_c)}$.

Agar hum ye do term yahan pe dekhengy, ye dono term mere materials properties hain, E aur G_c . Abhi is is dono material property ko main ek term se replace kar raha hoon. Isko main keh raha hoon K_c aur yahi mera fracture toughness hai aur isko bhi hum kehte hain critical stress intensity factor. Toh ye ho gayi meri new terminology fracture ke corresponding, isko hum kehte hain fracture toughness. Toh main $\sigma * \sqrt{(\pi * a)} = K_c$ likh sakta hoon kuch is tarah se. Agar main general case ki baat kar raha hoon toh $K = \sigma * \sqrt{(\pi * a)}$. Yeh meri ho jayegi stress intensity.

factor. Yaani jab K ki value K_c kab banegi? Jab a ki value jab a_c reach hogi, $a \rightarrow a_c$ ya $\sigma \rightarrow \sigma_c$ critical value. Tab jaake yeh jo stress intensity factor hoga wo critical value pahunchega aur usko hum kehte hain critical stress intensity factor ya isko hi hum kehte hain fracture toughness. Jab $a \rightarrow a_c$ ya $\sigma \rightarrow \sigma_c$ toh ye jo K ki value hai ye banegi meri fracture toughness ya critical stress intensity factor. Iska units kuch is tarah se hai: $\text{MPa} \cdot \text{m}^{1/2}$ aur $\text{MN} \cdot \text{m}^{-3/2}$ ye dono units books mein ye hamesha istemaal karte hain. Aapko.

careful rehna hai jab hum kuch problem solve karenge is units ke baare mein. Toh fracture kab hoga? Jab fracture hoga jab K ki value K_c ke barabar hogi. Tab jaake hamare material ka fracture hoga. Toh in general case hum is tarah se likh sakte hain $K = \alpha \cdot \sigma \cdot \sqrt{\pi \cdot a}$. Ye jo α hai ye depend karega mere specimen aur crack geometry pe. Isko hum kehte hain geometric constant yaani meri crack kahan pe present hai, mera specimen kis tarah se hai, kis tarah se loading stress hai us pe ye α ki value depend karegi. Alpha ki value aapko kisi bhi ek standard text book mein mil jayegi.



Fracture toughness and Stress intensity factor

$$\sigma = \left(\frac{E G_c}{\pi a} \right)^{1/2}$$

$$\Rightarrow \sigma \sqrt{\pi a} = \sqrt{E G_c} \quad E \text{ and } G_c \text{ are material's properties}$$

$$\sqrt{E G_c} = K_c$$

$K_c = \text{Fracture toughness or critical stress intensity factor}$

$$\Rightarrow \sigma \sqrt{\pi a} = K_c$$

$$\frac{K}{a} \rightarrow \frac{K_c}{a_c} \quad \sigma \rightarrow \sigma_c$$

$$K = \sigma \sqrt{\pi a}$$

Stress intensity factor

$\text{MPa m}^{1/2}$ or $\text{MN m}^{-3/2}$

Fracture will occur when $K = K_c$

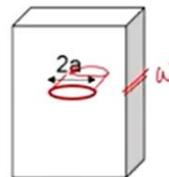
In general, $K = \alpha \sigma \sqrt{\pi a}$

Where α is the geometric constant:
Depends on specimen and crack geometry

e.g., when "a" is a through thickness crack

$$\alpha = \left(\frac{w}{\pi a} \tan \frac{\pi a}{w} \right)^{1/2}$$

For $a \ll w$ then $\alpha = 1$



Isko hum kehte hain geometric constant. Ek example lete hain ki ek through thickness crack ke liye yahan pe mere paas ek internal crack hai. Toh humne dekha tha internal crack jab hum baat karengy toh uski length hum consider karengy $2a$. Yeh through thickness hai yaani ye crack through out thickness hai is tarah se. Through out yeh iske thickness ki. Toh, yeh through thickness crack hai aur α ki value kuch is tarah se aapko milegi: $(W / \pi a * \tan(\pi a/W))^{(1/2)}$. Ab hum dekhengy ki yeh jo W hai yaani yahan pe main kuch is tarah se likhunga ye width hai ye jo width hai wo hamare crack length se kaafi badi hai toh α ki value yahan pe 1 ho jayegi. Toh is tarah se hum α ki value.

jo ki geometric constant hai ismein incorporate kar sakte hain. Ek general case hum is tarah se baat kar sakte hain. Par yahan pe dhyaan dene wali baat hai ki ye jo fracture toughness hai K_c ye mera nothing but $\sqrt{(E * G_c)}$. Toh ek example lete hain. Ek partial through thickness crack ko hum consider karte hain. Aur jab hum isko consider kar rahe hain toh K ki value main is tarah se maan ke chal raha hoon: $\alpha * \sigma * \sqrt{(\pi * a)}$. Aur yahan pe α ki value mujhe is tarah se di hui hai jahan pe a mera depth of penetration hai aur t meri wall thickness hai. Agar hum dekhengy ki K_c ki value yaani meri fracture toughness ki.

value yahan pe hum dekhengy ki ye $24 \text{ MN} * \text{m}^{(-3/2)}$ hai. a ki value 5 mm di hai, thickness ki value yahan pe 12 mm di hai, applied stress yahan pe σ 172 MPa diya hai. Abhi humein bataana hai ki agar yeh conditions humein di hain toh material fail hoga ki nahi hoga. Keh sakte hain ki jab K ki value K_c ke barabar hogi tab material fail hoga. Tab hum abhi α ki value pehle nikaal lete hain kyonki humein K ki value yahan pe dhoondhni hai. Toh K ki value yahan pe... toh α ki value kuch is tarah se aa gayi. Uh yahan pe agar main a ki value 5 mm rakh raha hoon aur t ki value 12 mm rakh raha.

hoon. Toh, yahan pe kuch α ki value 1.26 aayegi. Aur humein yahan pe of agar main nikaalunga, agar K_c ki value nikaalni hai, toh σ ki value is tarah se aayegi $K_c / (\alpha * \sqrt{(\pi * a)})$ kuch is tarah se likhte hain. Main humne kaha tha ki σ jab critical value pahunchega yahan pe fracture stress ki value main consider kar raha hoon yahan pe σ consider kar raha hoon. Toh kuch is tarah se main likh sakta hoon $K_c = \alpha * \sigma * \sqrt{(\pi * a)}$. Toh yahan pe agar main calculate

karunga σ_f ki value toh yahan pe kuch 171 MPa aa rahi hai. Agar ye 171 MPa aa rahi hai toh aap dekh paa rahe.

honge ki ye jo value hai σ_f ki applied stress se kam hai. Toh aap dekh paa rahe honge ki mera applied stress zyada value hai jiski value zyada hai. Toh yahan par mera material fracture hoga. Toh aap ye equations istemaal karke aur mere applications ki kya conditions hain jaise stress ki kya condition hai, crack length kya hai agar ye materials mein mujhe pata hai toh main predict kar paunga ki mera material fail hoga ki nahi hoga. Toh is part mein humne do-teen cheezein seekhi hain. Jaise humne brittle materials ke liye Griffith theory ki baat ki. Griffith theory ko modify karke humne kuch plastic work.



Fracture toughness: solved example

Example: Partial through thickness crack

$$K = \alpha \sigma \sqrt{\pi a}$$

$$\alpha = \sec\left(\frac{\pi a}{2t}\right) \quad a : \text{depth of penetration,} \quad t : \text{wall-thickness}$$

$$K_c = 24 \text{ MN m}^{-3/2} \quad a = 5 \text{ mm and } t = 12 \text{ mm}$$

Applied stress, $\sigma = 172 \text{ MPa}$

$$\alpha = \sec\left(\frac{5\pi}{24}\right) = 1.26$$

$$\sigma_f = \frac{K_c}{\alpha \sqrt{\pi a}} = 171 \text{ MPa}$$

Fracture will take place

$$K = K_c$$

$$\sigma \rightarrow \sigma_c \quad \sigma_f$$

$$K_c = \alpha \sigma_f \sqrt{\pi a}$$

introduce kiya aur critical strain energy release rate ke baare mein baat ki. Humne phir fracture toughness ki baat ki. Aur humne in sabka istemaal karke agar mere material mein kuch critical crack length hai ya applied stress kuch critical stress pe pahunch raha hai toh material fracture hoga ki nahi hoga yeh hum jaan sakte hain. Toh abhi ke liye yahan pe hum rukte hain. Fracture ke baare mein aur padhengy agle part mein. Dhanyavaad.