

## Mechanical behavior of materials

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Week-1

Lecture-4

### True Stress – True strain Curve and necking criterion

Namaskar, main aap sabhi ka swagat karta hoon is course mein jo ki Mechanical Behavior of Materials hai, jo main Hindi mein padhaunga. Toh last part mein humne dekha tha ki True Stress aur True Strain kya hai aur True Stress aur True Strain ka relation hai Engineering Stress aur Engineering Strain ke saath. Humne dono plots dekhe the Stress vs. Strain ke, jo Engineering Stress Strain Curve hai aur True Stress Strain Curve hai. Toh aur ek point yahan par main mention karna chahta hoon jo ki is True Stress True Strain Curve ko hum Flow Curve bhi kehte hain. Toh aap kabhi bhi books mein padhenge toh yeh Flow Curve jab aap baat karenge tab hum in True Stress aur True Strain ke beech ka plot ki baat kar rahe hain. Toh abhi humne toh jaana hai ki yaani jaise material deform hota hai, vaise vaise uska dimension badhta hai, jaise length badhti hai aur cross-section area ghatata hai jab hum tensile deformation ki baat karte hain. Agar hum compression ki baat karenge toh mujhe likh dijiye jaise hum compression ki baat karenge tab length ghategi aur jo cross-section area hai, woh badhega. Toh is case mein hum True Stress True Strain bhi isi equation se nikal sakte hain. Same relation, sirf jo values hai yahan par, yeh aapko consider karne padegi negative compression par.

Lecture 04: True Stress - True Strain Curve and Necking Criterion

### True Stress Vs True Strain

- As sample deforms, cross section area decreases
- Stress experienced by sample increases
- Need of instantaneous measurement of stress and strain

$$d\varepsilon = \frac{dL}{L}$$
$$\text{True strain} = \varepsilon = \int_{L_0}^L \frac{dL}{L} = \ln \frac{L}{L_0} = \ln \frac{L_0 + \Delta L}{L_0}$$
$$\varepsilon = \ln(1 + e) \quad \because e = \frac{\Delta L}{L_0}$$
$$\text{True stress} = \sigma = \frac{P}{A}$$
$$\sigma = \frac{P}{A_0} \frac{A_0}{A} \quad \because AL = A_0 L_0$$
$$\sigma = S \frac{L}{L_0}$$
$$\sigma = S(1 + e)$$

Flow curve  
 $\sigma$  vs  $\varepsilon$

Stress

Strain

Compression  $L \downarrow \Delta A \uparrow$   
True stress  
True strain

Par ek question aata hai ki ismein true kya hai, ya hum jab true bolte hain toh true kya hai? Toh isko acche se samjhenge ek example ke zariye. Jaise maan lijiye, hamare paas ek rod hai

jiski length hai L aur isko main deform karna chahta hoon, jaise is tarah se maine ek tensile force apply kiya aur iski length change hoti hai 2L. Jaise main isko deform kiya aur iski length maine badha di 2L kar di. Yeh mera Scenario 1. Yeh mera Initial Condition 1 hai. Yeh main maan leta hoon isko Condition 2. Toh yeh main process manunga 1 to 2. Toh ek se do ki taraf jab main jaata hoon, toh meri length L hai aur yahan par length ho gayi 2L. Aur phir jab yeh condition aati hai tab main usko compress karta hoon is tarah se aur main iski length ko kam karke wapas usi length par le aata hoon L, jo meri initial length hai. Toh yeh meri second condition thi. Toh isko maine wapas is condition mein laya, jahan ki meri length L hai. Toh is condition ko main naam deta hoon 2 to 1. Toh in process ke dauran jo bhi Engineering Strain ya True Strain develop huye, yeh hum abhi janenge. Toh yeh maan lete hain Step 1. Yahan par maine Engineering Strain mark kar liya hai, jo ki delta L yaani change in length upon original length aur True Strain jo humne formula last slide mein dekha tha,  $\text{True Strain} = \ln(1 + e)$ . Toh steps mark kar lete hain yahan par. Step 1 to 2, 2 to 1 aur ek pura step jahan jahan par hum 1 to 2 gaye aur 2 to 1 aaye, toh ek complete step. Inmein kya Engineering Strain aur True Strain aa rahe hain, hum abhi dekhenge.

Toh pehle step mein, jab hum dekhenge Engineering Strain jab main calculate kar raha hoon, toh meri final length 2L hai. Yeh yahan par 2L hai aur initial length meri L hai. Toh change in length meri hogi  $2L - L$  aur usko main divide karunga meri initial length se, jo ki meri L hai is case mein. Toh mujhe Engineering Strain milega 1. True Strain nikalne ke liye, yeh jo value hai, yeh main is formula mein rakhunga. Toh mujhe True Strain milega yahan pe agar maine  $\ln(1 + 1)$ , yeh 1 engineering strain hai, toh mujhe  $\ln(2)$  milega. Toh yeh meri True Strain ki value ho gayi. Jab 2 to 1 agar main baat karunga, toh meri initial length ho gayi 2L aur final length hogi meri L. Toh jab main Engineering Strain nikalunga is process ke dauran aur is step ke dauran, toh meri change in length honi chahiye  $L - 2L$ , jo meri final length L thi is is case mein aur initial length 2L divided by original length. Original length iski is step mein 2L thi. Toh maine 2L se divide kiya, toh Engineering Strain mujhe milega -0.5. Is step mein True Strain agar nikalunga, toh  $\ln(1 + (-0.5))$ , yeh jo ki meri Engineering Strain hai is step mein, toh mujhe milega  $\ln(0.5)$  ya  $-\ln(2)$ . Agar hum is pure step mein, yaani 1 to 2 to 1, toh meri initial length L hogi aur final length meri L hai. Toh hum abhi Engineering Strain nikalenge. Toh meri final length L hai, meri initial length L hai aur initial length se main divide karunga, toh mujhe value milegi 0. Agar True Strain nikalunga, toh yeh value aayegi  $\ln(1 + 0)$  jo ki  $\ln(1)$ , phir se mujhe value milegi 0.

Toh aap yahan par jaan sakte hain ki is step mein main initially chalu kiya tha L length se, phir maine tensile force apply karke isko iski length badha di thi jo 2L ho gayi thi. Uske baad usko wapas maine compress kiya aur wapas length aayi L. Toh yeh jo change hoga strain yahan pe, toh aap jaan sakte hain, initial length L hai, final length L hai. Is step mein mujhe Engineering Strain 0 milna chahiye aur True Strain bhi mera 0 hona chahiye kyunki koi strain hai nahi material pe yahan pe, koi length change ho nahi raha hai yahan pe is pure process ke dauran. Toh mujhe length change yahan pe 0 milta hai, jo in teeno step mein milta hai mujhe. Abhi agar aap dekhenge, in dono step ko agar main add karta hoon, toh kya aata hai? Toh maan lijiye, maine step 1 aur step 2 ke Engineering Strains ko add kar diya. Yahan pe strain mera Engineering Strain 1 hai aur yahan pe Engineering Strain mera -0.5 hai. Toh agar in dono ko maine add kiya, toh mujhe Engineering Strain milta hai 0.5. True Strain ko agar hum dekhenge, toh yahan pe  $\ln(2)$  tha aur yahan pe  $-\ln(2)$  tha. Toh mujhe summation milta hai, in dono ka addition milta hai 0. Toh yahan pe aap dekhenge, yeh step, yeh jo step hai, yeh is step ke equal

nahi hai, jo ki equal milni chahiye thi. Toh Engineering Strain mein hamein yeh step equal nahi milti hai, par True Strain ke case mein hamein agar dekhenge 1 to 2 to 1 yeh agar humne ek saath nikala aur ya steps mein nikala, toh hamein yeh equal milta hai. Toh yahi Engineering Strain ki kami hai jo hamein nahi bata pati hai ki agar main steps mein jaa raha hoon aur un steps ko add karta hoon toh mujhe jo final strain milta hai, woh equal ho bhi sakta hai, nahi bhi ho sakta hai. Par True Strain mein agar hum steps mein nikale ya total nikale, ek saath nikale, toh hamein woh value, final jo value hai, woh milegi milegi. Jaise ki aur ek example dena chahta hoon, jaise maana ki main, mere paas bahut saare steps hai. Maine step 1 se nikala, step 2 tak gaya, phir third step mein gaya, phir fourth step mein gaya, aise bahut saare steps ya n step tak aa gaya main. Toh agar maine yahan par strains nikale, different strains, in True Strain aur Engineering Strain, yahan pe agar maine nikale aise, toh aap yeh janenge ki agar aur nikala maine yahan se yahan tak direct n tak, ek Engineering Strain nikala aur ek True Strain nikala, toh hum True Strain ke liye toh likh sakte hain ki yeh steps ka addition hum likh sakte hain aise n step, par is case mein hum nahi likh sakte hain. Yeh summation possible nahi hai. n-1 steps ho, toh ek conclusion hum draw kar sakte hain is observation se ki True Strains are additive par Engineering Strains are not additive hain. Aur yahi hamara ek advantage hai True Strains ka. True Strains hum add kar pate hain, agar hum steps mein bhi le toh hum usko add kar pate hain. Par Engineering Strains hum add nahi kar pate. Toh yahan True Strains ki agar hum utility dekhenge, toh True Strains hum bahut saare processing dekhte hain, jaise rolling hai, forging hai, hum ek step mein toh koi product nahi bana pate. Hum bahut saare steps dete hain. Toh un steps ke dauran jo bhi strain develop ho raha hai material mein, woh hum add kar payenge True Strains ke liye, but Engineering Strains ke liye hum woh add nahi kar payenge. Toh True jo shabd hai, yaani woh sachai dikha raha hai ek step ki. Toh yahi, isiliye hum usko True kehte hain. Yaani woh sach hai har steps ke liye. Toh yahi hum same analysis hum True Stress ke liye bhi dikha sakte hain.

Lecture 04: True Stress - True Strain Curve and Necking Criterion

## Why True Strain?

Handwritten notes:  $\epsilon_{1-2} = \epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_n$ ,  $\epsilon_{1-2} \neq \epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_n$

| Step      | Engineering Strain, $e = \frac{\Delta L}{L_0}$ | True Strain, $\epsilon = \ln(1 + e)$               |
|-----------|--|--|
| 1 - 2     | $e_{1-2} = \frac{2L - L}{L} = 1$               | $\epsilon_{1-2} = \ln(1 + 1) = \ln 2$              |
| 2 - 1     | $e_{2-1} = \frac{L - 2L}{2L} = -0.5$           | $\epsilon_{2-1} = \ln(1 - 0.5) = \ln 0.5 = -\ln 2$ |
| 1 - 2 - 1 | $e_{1-2-1} = \frac{L - L}{L} = 0$              | $\epsilon_{1-2-1} = \ln(1 + 0) = \ln 1 = 0$        |

Handwritten notes:  $1-2-1 \neq (1-2) + (2-1)$ ,  $(1) + (-0.5) = 0.5$ ,  $(\ln 2) + (-\ln 2) = 0$

True strains are additive while engineering strains are not.  
 True strains utility: Processing such as rolling, forging etc.

Handwritten note: True strain  $1-2-1 = (1-2) + (2-1)$

Toh abhi main aage badhta hoon. Toh hum, maine Flow Curve ki baat ki thi jab maine True Stress aur True Strain ki jab baat ki thi tab, agar main Flow Curve draw karta hoon toh Y-axis pe mera aayega  $\sigma$  aur X-axis pe aayega mera True Strain  $\epsilon$ . Toh yeh agar plot kiya maine, yeh mera linear part hai, yeh elastic region hai jahan par stress is proportional to strain through a

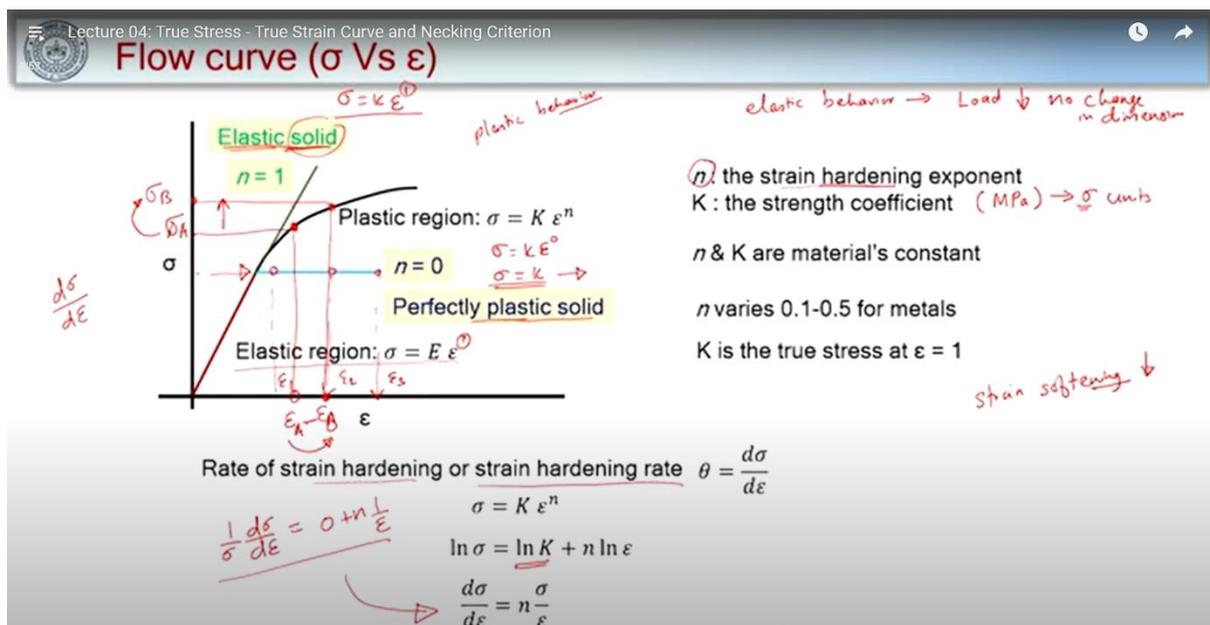
material constant which is your Young's Modulus. Aur yeh mera non-linear part hai which is my plastic part in case of metals and alloys. Yaani yeh metals aur alloys ke case mein yeh mera non-linear part hai. Toh isko main plastic region kehta hoon aur yeh iska equation main dunga  $\sigma = K\epsilon^n$ . Toh yeh ek simple relation hai jo main istemal kar raha hoon is part ko batane ke liye. Toh yeh jo part hai n, isko hum kehte hain Strain Hardening Exponent kyunki yeh epsilon  $\epsilon$  hai, iska power hai, toh power ko hamesha hum exponent kehte hain. Isliye hum isko Strain Hardening isko Strain Hardening kyun kehte hain? Abhi thodi der mein hum dekhenge. Aur yeh jo K hai, yeh Strength Coefficient hai. Is strain ka pehla prefactor hai. Toh isko hum isko coefficient kahenge. Isliye humko Strength Coefficient kehte hain. Toh n aur yeh jo K hai, yeh material constants hai. Yaani kisi particular material pe depend karenge. Toh n ki value agar 1 hai, toh isko hum kehte hain Elastic Solid. Agar n ki value 0 hai, isko hum kehte hain Perfectly Plastic Solid. Isko thoda hum samjhenge. Jaise hum is plastic region mein jab hum sigma n ki value agar 1 hai, toh yeh simple relation aayega, ek linear relation aayega, isliye hum isko Elastic Solid kehte hain. Is case mein, agar n ki value 0 hai, toh hamein  $\sigma = K$ . Toh yeh ho gaya mera Perfectly Plastic Solid. Isse hum ek bhi ek cheez infer kar sakte hain ki Strength Coefficient ka unit bhi hamara MPa hona chahiye. Yeh jo stress ka unit hona chahiye. Agar hum dekhenge is case mein Elastic Solid aur Plastic Solid ki baat kar rahe hain hum. Toh Elastic Solid hum Elastic Solid isliye kehte hain, yeh Elastic Solid aap Elastic Behavior se compare na kariye. Hamara Elastic Behavior yaani hota hai ki jab hum material ko load karte hain aur unload karte hain, toh material ke dimensions mein koi change nahi aata hai. Yeh hamara ho gaya Elastic Behavior. Par is case mein yeh yahan par agar Elastic Region dekha jaaye, toh yahan par power jo hai, woh 1 hai aur yahan par bhi power 1 hai. Toh isliye hum isko Elastic Solid kehte hain, na ki yeh yahan par yeh plastic bhi ho sakta hai. Yaani plastic behavior dikha sakta hai par isko hum kahenge Elastic Solid kyunki iska power 1 hai, bas yahi reason hai. Perfectly Plastic Solid mein yeh hota hai ki aapko jaise koi bhi plastic strain aap material pe dalenge, ek stress par hi woh deform ho sakta hai. Toh mujhe koi bhi stress nahi chahiye. Agar mujhe material ko deform karna hai, jaise yeh different strains maine plot kiye, toh is strains ko mujhe laane ke liye material par ek hi stress ki, stress ki ek hi stress ki avashyakta hai, isliye hum kehte hain isko Perfectly Plastic Solid. Aur yeh jo behavior hai, yeh plastic behavior. Toh yeh thoda sa dhyan rakhiyega. Toh n ki value hum dekhenge, jo bhi Strain Hardening Exponent hai, iski value hoti hai metals aur alloys ke liye 0.1 se 0.5 ke beech mein. Aur yeh jo K ki value hai, yeh True Stress hai at  $\epsilon = 1$ . Abhi hum dekhenge yeh, yeh exactly hai kya yeh cheez. Toh abhi ek, ek value aur ek cheez main define karna chahta hoon, Rate of Strain Hardening or Strain Hardening Rate. Toh isko hum define karte hain plastic region mein. Yeh plastic region mein hum define karte hain theta se isko sambodhit kiya jaata hai,  $\theta = d\sigma / d\epsilon$ . Yaani kitna stress change ho raha hai mera strain ke hisab se. Agar mera itna strain mujhe change chahiye, toh mujhe kitna stress change karna padega? Toh yahi hamara Rate of Strain Hardening hai ya Strain Hardening Rate hai. Toh agar hum yeh equation dekhte hain,  $\sigma = K\epsilon^n$ , yeh jo humne define kiya yahan par plastic region mein. Agar hum isko differentiate karen,  $\epsilon$  se, True Strain se. Toh, ya main pehle isko ln ke roop mein likh leta hoon,  $\ln\sigma = \ln(K) + n\ln\epsilon$  which is Strain, jo ki mera Strain hai. Toh, toh agar jab main isko differentiate karunga, toh

$$\sigma = K \epsilon^n$$

$$\ln \sigma = \ln K + n \ln \epsilon$$

$$\frac{d\sigma}{d\epsilon} = n \frac{\sigma}{\epsilon}$$

ln(K) ka yeh toh constant hai material ke liye, toh yeh 0 ho jaayega aur ln(epsilon) yeh aayega. Toh isi se hum derive karenge yeh yeh relation. Toh hamein  $d\sigma / d\epsilon = n \sigma / \epsilon$ . Toh yeh mera ek relation hoga, yeh yahi mera Strain Hardening Rate hai. Abhi isko main Strain Hardening kyun keh raha hoon? Toh agar aap dekhenge isko acche se agar yeh plastic region mein dekh raha hoon, toh main maanta hoon ki agar yeh mera strain hai koi bhi ek strain hai, main isko maan leta hoon Strain A. Yeh mera, iske liye mujhe lag raha hai Stress A. Agar main uska strain banana chahta hoon, jaise maine strain badha diya aur yeh aa gaya mera Strain B. Toh mujhe chahiye, itna stress mera badhega material ko deform karne ke liye. Toh yeh mera change in stress hoga  $d\sigma$  aur yeh mera hoga change in strain. In dono ka difference. Toh yeh ho jaayega change in strain. Toh mujhe lag, mujhe Strain A se Strain B jaane ke liye, sigma A se sigma B tak ka stress badhana padega. Toh yeh badhana isko hi kehte hain hardening, yaani material hard ho gaya kyunki main mera material is strain ke liye is stress par deform ho raha tha aur Strain B ke liye sigma B jo ki sigma A se higher hai, isliye woh material hard ho gaya. Toh isliye hum isko Strain Hardening Rate ya Strain Hardening Coefficient batate hain aur isiliye is exponent ko, jo bhi yahan pe hai, isko hum Strain Hardening Exponent kehte hain. Ek aur cheez hoti hai Strain Softening bhi hota hai. Yaani jab material soft hota hai strain ke hisab se. Agar main strain badha raha hoon material ka aur material soft ho raha hai, toh usko Strain Softening bhi kehte hain, jo hum dekhenge baad mein is class mein.



Abhi ek Necking Criteria aapko main samjhana chahta hoon. Toh Necking Criteria yeh books mein isko kehte hain Considere Criterion. Toh ek statement hai jo aap books mein padhenge.

Toh yahan pe angrezi mein usko main angrezi mein padhta hoon. Toh yahan pe likha hai ki Necking begins when the increase in stress due to decrease in the cross-sectional area is greater than the increase in load bearing capacity of the specimen due to work hardening. Toh humne jo Strain Hardening abhi dekha hai, isko bhi Work Hardening bhi kehte hain. Toh yeh Strain Hardening aur Work Hardening, yeh dono ek hi term hai, ek hi terminology hai, kyunki hum jab material ko deform karte hain, toh hum uske upar work kar rahe hain, isliye hum usko Work Hardening bhi kehte hain. Toh yahan pe likha hai Necking, toh humne dekha tha ki Necking kya hota hai? Jaise jaise hamara material deform hota hai, tensile deformation ki baat karenge hum. Hum tensile deformation jab baat karte hain  $\sigma$  toh hamara change in area kya hota hai? Jaise jaise deformation badhta hai, toh area ghatate jaata hai material ka. Cross-section area ghata hai. Toh isliye hum usko wahan par  $\Delta A$  hamara decrease hoga. Toh yahan par likha hai ki Necking begins when increase in stress due to decrease in the cross-sectional area. Toh hamara jab cross-sectional area decrease hoga, toh isko hum ek aur terminology aap books mein dekhenge, isko kehte hain Geometric Softening bhi kehte hain, kyunki hamara cross-section area ghat raha hai. Toh hamara cross-section area hamara material ki geometry bata raha hai, isliye hum isko Geometric Softening bhi kehte hain. Softening isliye kyunki jaise-jaise cross-section area ghatega, toh uska load bearing capacity hamara ghatega. Toh iska matlab yeh hota hai ki jab hamara Geometric Softening dominate karega Work Hardening ko, tab Necking chalu hogi. Toh isko dekhte hain hum mathematically kaise likh pate hain. Toh humne Necking ki jab baat ki thi, tab humne bataya tha ki Necking ek instability region hoti hai. Vahan par hamein koi load ko badhana nahi padta hai ki strain ko develop karne ke liye. Humne Engineering Stress-Strain curve ke through dekha tha yeh ki material mein strain apne apne badhta hai agar main above chala jaata hoon, upar chala jaata hoon. Toh Force ko hum likhenge

$$F = \sigma A$$

$$dF = A d\sigma + \sigma dA$$

$$\text{At necking, } dF = A d\sigma + \sigma dA = 0$$

Aur at Necking maine bola hai ki No increase in load, toh main maanta hoon ki change in force 0 hona chahiye. Toh is is term ko main shunya ke saath equate kar deta hoon. Toh agar isko main solve karunga, toh mujhe milega  $d\sigma / \sigma = -dA / A$ . Toh agar hum dekhenge, just Necking se pehle, humne ek baat dekhi thi ki constancy of volume hota hai plastic deformation mein Necking se pehle. Yeh constancy of volume agar maante hain, toh hum yeh likh sakte hain jaise hum tensile deformation mein change in length upon original length hamara hota hai  $dL$ , jo hamara strain hai. Toh hum isko is tarah se bhi likh sakte hain  $-dA / A$  kyunki hamara cross-section area ghat raha hai. Toh yeh hum isko cross-section area upon instantaneous area ko hum change in strain likh sakte hain. Toh yahan par agar hum yeh rearrange karte hain yeh term, toh hamein milega ki yeh

$$\therefore -\frac{dA}{A} = \frac{dL}{L} = d\epsilon$$

$$\frac{d\sigma}{d\epsilon} = \sigma$$

Yeh hamara Strain Hardening Rate hai. Toh hum Necking ke, jab Necking ki jab hum baat karenge, toh yeh do statement hum is mathematical derivation se baat kar sakte hain. Se Necking begins at point where Strain Rate of Strain Hardening is equal to Stress. Toh humne term define kiya tha, yeh Rate of Strain Hardening hai aur yeh Stress hai. Jab yeh equal honge tab hamari Necking chalu hogi, kyunki yeh derivation hamara usi se aaya hai. Toh in terms of engineering values,  $dS / d(\epsilon)$ , yeh engineering stress hai with respect to engineering strain, yeh 0 hona chahiye.

$$dS / d(\epsilon) = 0$$

Isko bhi hum dekhenge at maximum value of S. Is term ko hum abhi shortly dekhenge. Toh hamare paas yeh toh relation aa gaya. Toh main yeh Stress vs. Strain plot karta hoon. Yeh plastic region mein maine plot kiya sigma vs. epsilon, True Stress True Strain, ya yeh mera Flow Curve hai. Aur yeh maine  $d(\text{sigma}) / d(\text{epsilon})$  jab main plot karunga strain ke versus, toh mujhe aisa curve milega, kyunki yeh mera Flow Curve hai, agar isko main differentiate karunga, toh yeh  $d(\text{sigma}) / d(\text{epsilon})$  mera decrease ho raha hai, toh hamein slope decrease aise milega.  $d(\text{sigma}) / d(\text{epsilon})$  yeh agar main plot karunga, yeh jahan par equal hoga, yaani yahan par jo intersection hoga, yaani yeh mera hoga  $d(\text{sigma}) / d(\text{epsilon})$  sigma ke barabar ho jaayega aur yahan par hi mera Necking shuru hogi. Toh yeh jo strain hai, agar main strain is point ke corresponding mein  $\epsilon_u$  maanta hoon, yaani Uniform Strain. Jab tak material uniformly deform hota hai, toh hum jo strain ki baat karenge vahan pe, hum uniform strain ki baat karenge. Toh yahi hamara uniform strain hai aur isko hum aise likh sakte hain. Yeh  $d\sigma / d\epsilon$  aur humne yahi previous slide mein agar humne yeh derivation kiya tha,  $d\sigma / d\epsilon = n * \sigma / \epsilon$ . Yahan par main  $\epsilon$  ko  $\epsilon_u$  ki baat karunga, Necking ke, Necking ke point par. Toh agar yeh main relation istemal karoon aur yeh value yahan par put kar doon Necking ke liye, toh mujhe milega  $\epsilon_u$  yaani yeh True Uniform Strain equal to Strain Hardening Exponent yaani mera n ke value ki barabar aa jayega. Toh main Necking pe jab meri Necking hogi ya UTS pe, True Strain Strain Hardening Exponent ke barabar hoga. Toh isko kehte hain hum  $\epsilon_u$  ko True Uniform Strain. Abhi is point ko hum jante hain jaise humne Strain Stress vs. Strain yaani Engineering Stress Strain plot kiya. Toh main yeh linear region hai aur yeh mera plastic region hai. Toh humne baat ki thi ki UTS is region ke dauran agar hum differentiate karte hain  $dS / d\epsilon$ , toh yeh 0 milta hai.  $dS / d\epsilon$  kaise aayega yahan pe? Yeh mera stress decrease ho raha hai, jaise-jaise mera strain badh raha hai. Toh agar hum is is region ko differentiate karenge, toh yeh slope decrease ho raha hai. Toh iska differentiation jab hum karenge, toh yeh point pe humko 0 milega kyunki yeh maximum value pahunch gayi aur aapko pata hai maxima ya minima pe differentiation 0 hota hai. Aur iske baad yeh decrease ho raha hai, toh yeh negative value mein aayegi. Toh yeh meri value 0 rahegi. Toh jab Necking jab hum baat karenge, toh  $dS / d\epsilon = 0$ . Yeh hamari Necking hai. Yeh value hamari 0 rahegi strain ke hisab se plot kare toh. Toh yahan par hum dekhenge, agar yeh curve hum dekhenge, toh yahan par hamara Work Hardening ya Strain Hardening dominate karega aur is region mein hamara Geometric Softening dominate karega. Toh yeh jo, yeh jo region aata hai hamara non-linear region aur plastic region, yeh hamara dono ki contribution se aata hai, Geometric Softening aur Work Hardening ya Strain Hardening ke. Is region mein jahan par uniform deformation hai, is region mein hamara Work Hardening dominate karta hai. Aur is region mein hamara Geometric Softening dominate karta hai. Toh yahi hamara Necking ka Considere Criterion hai. Yeh simple Necking Criteria hai. Yahan par main rukta hoon. Phir baaki ki cheeze jaise ki Elastic Modulus ka origin kya hai, yeh hum next bhag mein dekhenge.

*Temporally ΔA & geometrical softening*

"Necking begins when the increase in stress due to decrease in the cross-sectional area is greater than the increase in load bearing capacity of the specimen due to work hardening."

Instability:  
No need to increase load to develop further strain

$$F = \sigma A$$

$$dF = A d\sigma + \sigma dA$$

At necking,  $dF = A d\sigma + \sigma dA = 0$

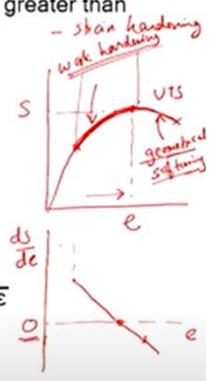
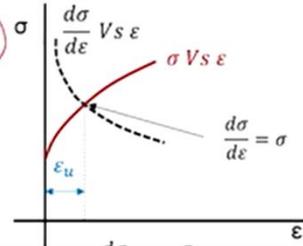
$$\frac{d\sigma}{\sigma} = -\frac{dA}{A}$$

Constancy of volume

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

$$\therefore -\frac{dA}{A} = \frac{dL}{L} = d\varepsilon$$

*UTS (before uniformly uniform strain)*



$$\frac{d\sigma}{d\varepsilon} = n \frac{\sigma}{\varepsilon_u}$$

$$\varepsilon_u = n$$

True uniform strain

- Necking begins at a point where rate of strain hardening is equal to the stress.
- In terms of engineering values  $\frac{dS}{de} = 0$  at max. value of S!!

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