Mechanical behavior of materials

Dr. Niraj Mohan Chawake

Department of Materials Science and Engineering

Indian Institute of Technology, Kanpur

Week-7

Lecture-35

Dislocations Interactions: Stable configuration & Strain Hardening

Namaskar ka swagat karta hoon is course mein mechanical behavior of material jo ki hum hindi mein padhenge last part mein humne dekha tha ki glide force kya hoti hai do positive edge dislocation aur ek positive aur ek negative dislocation jo ki hamare do parallel slip pe rehte hain.

Is baar hum is dislocation interactions ke wajah se unke kuch stable configuration hogi aur uski wajah se kya strain hardening hogi iske baare mein hum jaanenge to humne last part mein dekha tha ki agar mere paas do positive edge dislocation hai ek ek slip plane pe hai aur ek dusra dusre slip plane pe hai ye do parallel slip planes hai jo h distance apart hai aur in dono ki burgers vector humne b maan ke chali thi to humein jo glide force milega glide force kya hota hai yeh jo glide force hoga mera dislocation one ki jo stress field hogi uske wajah se jo force taiyaar hoga ek glide aur climb force taiyaar hoga to humne yahan par glide force consider kiya tha jo dislocation two par act kar raha hai woh kuch is tarah se milega humein yeh humne dekha tha iska variation jo do

positive edge dislocation ke beech mein hai abhi humne dekha tha ki glide force teen jagah par shunya hai ek yahan hum dekhenge $x = 0$ isko likh lete hain hum ye $x = 0$, $x = h$ aur $x = -h$ to yeh hamari special position ho gayi hai.

To abhi hum dekhenge ki agar main is dislocation ko is glide plane par move karta hoon to mere paas kya glide force kis tarah se change hoga ya yeh jo glide force change ho raha hai wo mere is dislocation ki kaun si stable configuration taiyaar karegi stable yaani inki jo sthiti hogi dono ki position hogi kis sthiti mein ye do stable ho sakte hain yeh hum jaanenge abhi to maan lete hain pehli condition jab mere paas $x = h$ hai yahan pe $x = h$ mera glide force shunya hai aur agar main dekhta hoon glide force shunya yaani yeh maan lete ye dislocation yahan pe hai mera isko likh lete hain is tarah se agar yeh mera dislocation yahan pe hai aur yeh dislocation ko main perturb karta hoon yaani disturb karta hoon maan lete hain main is direction mein disturb kar raha hoon positive x direction par main isko thoda sa move kiya disturb kiya ne is position se yahan par to glide force shunya hai par yahan par agar maine disturb kiya to kya hoga aap dekhenge ki jo glide force hai yahan par is direction par positive hai positive hai na positive x direction pe yaani glide force is direction par lagega mera to kya hoga yeh dislocation jaise hi maine is sthiti se disturb kar diya positive x direction ki taraf to yeh force jo act ho raha hai yeh positive hai aur is dislocation ko is positive x direction par move karega yaani mera dislocation is sthiti se jab disturb ho jayega to ye positive x direction par to is tarah wo move hoga similarly agar mera dislocation yahan par hai minus h pe hai to aap dekhenge ki glide force negative direction mein lagega to agar main isko is direction mein move karunga to mera dislocation is yaani x ke it direction pe move karega.

Abhi hum dekhte hain ki jo mera position hai agar yahan par hai $x = h$ hai is position par aur main isko is direction mein disturb karta hoon maan lete hain ki main is direction mein disturb karunga negative x negative x nahi bolunga yaani yeh jo h hai h se kam ho gaya to aap dekhenge yeh jo glide force hai yeh mera negative hai yahan par is position pe yaani ka matlab hoga ki agar jaise maine dislocation is h position se decrease kiya yaani x ki value yahan par ghat gayi hai to aap dekhenge ki glide force jo negative hai to kya hoga yeh jo glide force mere dislocation ko is direction pe move karega negative yaani is direction par move karega yeh mera glide force is direction mein lagega yahan par glide force is direction mein lag raha tha agar main positive x direction mein move ho raha to yaani x h se agar greater hai to aur x h se chhota hai to aap dekhenge ki glide force mera negative hai aur mera dislocation yahan se disturb hoke is position pe aayega jahan pe phir se glide force ki value shunya hai to yeh hai zero position yaani $x = 0$ position yeh ek condition hai agar main dekhunga $x = -h$ yaani agar main yahan pe consider karta hoon same thing aap yahan pe agar mera dislocation hai yeh dislocation hai to agar yeh dislocation maine disturb ki aur maine jo x ki value hai wo aur negative ho gayi to aap dekhenge ki jo glide force lag raha hai yahan pe ye negative glide force lag raha hai to aap dekhenge jaise maine is dislocation ko is direction mein disturb kiya to aap dekhenge jo glide force hoga woh negative direction mein lag raha hai to mera dislocation is direction mein move hoga is direction mein move hoga aur jaise maine is dislocation ko is direction mein disturb kiya isko bhi likh lete hain is direction mein agar disturb kiya hai yaani towards origin maine disturb kiya to aap dekhenge glide force positive hai to F_{glide} Yahan pe is tarah se lagega to aap dekhenge ki jaise hi F_{glide} Yahan pe lag raha hai aur yeh jo dislocation hai yahan se move hoke yahan tak aa jayega yaani mere origin pe aa jayega $x = 0$.

Ab dekhte hain $x = 0$ pe kya hoga jaise mera dislocation agar is position pe $x = 0$ maan lete hain yeh kuch is tarah se hai yahan par $x = 0$ mera dislocation yahan pe hai to aap dekhenge agar main

isko is direction mein move karunga yaani disturb kar raha hoon to aap dekhenge ki yeh jo glide force hai yahan pe yeh negative glide force hai to jaise main is dislocation ko is direction mein move karne ki koshish karunga to glide force negative yaani glide force mujhe mere dislocation ko is direction mein dhakelega yaani opposite direction mein dhakelega ya allow nahi karega is direction mein move karne ke liye to jaise maine is direction mein disturb kiya positive x direction pe to dislocation phir se wapas apne original position pe aayega similarly agar main dislocation ko is direction mein move karunga to aap dekhenge ki jo glide force hai wo positive hai positive yaani is yahan pe hai ye positive glide force lag raha hai to positive yaani wo dislocation ko positive direction pe move karega to jaise hi maine dislocation ko is direction mein move karne ki koshish ki to glide force opposite direction mein lagra aur dislocation mera is position par aayega to aap dekhenge ki yeh jo position hai $x = 0$ yeh meri stable equilibrium hai jaise main isko disturb karta hoon positive x ya negative x ki taraf yeh wapas usi position par aa raha hai agar hum main yahan par hoon $x = h$ ya $x = -h$ agar main usko origin ki taraf disturb kar raha hoon to wo origin ki taraf move ho raha hai mera dislocation aur origin se away agar main disturb kar raha hoon yaani origin se dur agar disturb kar raha hoon to woh h se infinite ki taraf move hoga to isliye in do positions mein yeh jo position hai $x = h$ aur $x = -h$ ye jo positions hai ye meri unstable equilibrium hai equilibrium isliye keh rahe hai kyunki yahan pe forces zero hai isliye ye equilibrium hai par yeh jo position hai yeh meri unstable equilibrium hai jaise maine isko disturb kiya ye equilibrium maintain nahi kar payega yaani $x = h$ aur $x = -h$ pe mera dislocation nahi reh payega par stable equilibrium jo hai woh mera position $x = 0$ hai jaise main isko yahan pe disturb karne ki koshish karunga aap dekhenge ki dislocation wapas mere original position pe aayega to jab mere paas do positive edge dislocations hai is tarah se to aap dekhenge ki $x = 0$ yeh meri stable equilibrium position hai yeh meri stable equilibrium position hai to mere dislocation chahenge ki yeh ek dusre ke upar rahe aise is tarah se yeh hai isko kehte hain configuration aur yeh jo hai is configuration ko hum kahenge stable configuration jab mere paas do positive edge dislocation hai do parallel slip planes pe jo ki h distance apart hai.

Yeh is tarah se bhi aap dekh sakte hain humne dekha tha ki dislocations ke around stress field hoti hai to aap dekhenge ki humne dekha tha ki positive edge dislocation hai to iske slip plane ke upar jo hai wo compressive stress field rehti hai aur net jo tensile stress field rehti hai wo slip plane ke neeche rehti hai wo dono ke liye same hogi aap dekhenge ki yeh jo configuration hai aap dekhenge ki iski tensile stress field aur iski compressive stress field ek dusre ko attract karegi is tarah se aur dono ek dusre ke upar rahenge yeh jo position milegi mujhe dislocations ki isko hum kehte hain stable configuration dislocations ki usi tarah se hum dekhenge ki mere paas ek positive aur ek negative edge dislocation hai kuch is tarah se dono parallel slip plane pe to humne iska bhi glide force nikaala tha to glide force kuch is tarah se humein mila tha to mera glide force hai abhi hum jaante hain ki in dono mein kya stable equilibrium position ho sakti hai to maan lete hain pehli condition $x = h$ yahan pe bhi hum teen positions mark kar lete hain $0, h$ aur $-h$ ye stable ya ye equilibrium position isliye hai kyunki yahan pe glide force ki value shunya hai ab in teenon mein se kaun si stable hai ye humein dekhna hai to maan lete hain mere paas $x = h$ hai to mere paas kuch aisa dislocation hai ye negative dislocation is tarah se aur is dislocation ko main $x = h$ pe agar disturb karta hoon maan lete hain ki main isko origin ki taraf dhakelne ki koshish kar raha hoon is direction pe to aap dekhenge ki jo glide force lag raha hai glide force ye yahan pe positive glide force hai to jaise main is dislocation ko is side dhakelne ki koshish karunga ya push karne ki ya disturb karne ki koshish karunga to glide force positive hai to wo mujhe h yaani is is position par wapas layega to glide force is direction mein lag raha hai aap dekhenge ki agar main is $h = 0$ se is direction mein move karunga yaani away from origin origin se dur agar dhakelne ki koshish

karunga to yahan par glide force kya lag raha hai negative direction pe lag raha hai to main dislocation ko agar is direction mein force push kar raha hoon to glide force mera kuch is direction mein lag raha hai to aap dekhenge ki yeh glide force mera is position par is dislocation ko wapas layega $h = 0$ pe similarly agar is condition mein dekhenge $-h$ ki position par dekhenge agar yahan par mera dislocation hai yahan pe agar mera dislocation hai to is position pe agar main isko origin ki taraf lane ki koshish karunga to aap dekhenge ki ye jo glide force hai glide force mera negative glide force hai jo magnitude hai wo negative hai to ye dislocation ko origin se away lane ki koshish karega to ye wapas $h = 0$ position pe aayega agar is direction mein move karenge yaani minus x direction pe agar hum move karenge is direction mein move kar raha hoon to aap dekhenge jo glide force hai glide force mera positive glide force hai is tarah se to jo glide force mera is direction mein act karega F_{glide} Jo hai is direction mein act karega aur aap dekhenge ki jo dislocation hai wo $-h$ position pe aake ruk jayega abhi $x = 0$ pe dekhte hain $x = -h$ humne dekha aur $x = 0$ dekhte hain to $x = 0$ position pe agar usko hum disturb karenge ya h plus minus δx karne ki koshish karenge to aap dekhenge ye $x = 0$ jo position hai isko aap main agar disturb kar raha hoon positive direction pe to yahan pe jo agar mera dislocation yahan pe hai maan lete isko bhi likh lete hain agar mera dislocation origin pe hai yahan pe hai aur main isko disturb kar raha hoon is tarah se to aap dekhenge ki jo glide force lag raha hai wo mera positive glide force lag raha hai to dislocation yahan se move hoke mere h position pe aake ruk jayega agar main is direction mein move karunga is direction mein move karunga mere dislocation ko disturb karne ki koshish karunga to aap dekhenge ki yahan par negative glide force lag raha hai is direction par to yeh dislocation yahan se aake move hoke mere minus h position par stable hoga to aap dekh paa rahe hain ki yeh jo position hai yahan par $x = 0$ yeh meri unstable position hai aur yeh jo position hai $x = h$ aur $x = -h$ yeh meri stable equilibrium hai to yeh hogi meri stable equilibrium jab mere paas do hi dislocation hai positive dislocation se ek positive dislocation hai aur ek negative dislocation hai edge dislocation to tab jaake $x = h$ ye jo position hai ye meri stable position rahegi to $x = h$ agar main consider karunga ye meri stable equilibrium position rahegi to ye jo distance hoga yaani $x = h$ yaani ye ye jo distance hai yahan se ye h ho gaya yaani $x = h$ yahan pe ho gaya to agar hum dekhenge $y = h$ bhi hai kyunki to do plane ka distance hai aur $x = h$ ye meri stable position hai to aap dekhenge ki ye jo angle hai dono ke beech ka yeh 45 degree aap nikaal sakte ho yaani aap dekhenge ki ye jo stable equilibrium configuration hai mere do dislocations ke ek positive dislocation aur ek negative dislocation on parallel slip plane ye $\theta = 45°$ Rahega in dono ke beech ka angle hamesha 45 rahega.

Dislocation interactions: Stable configurations
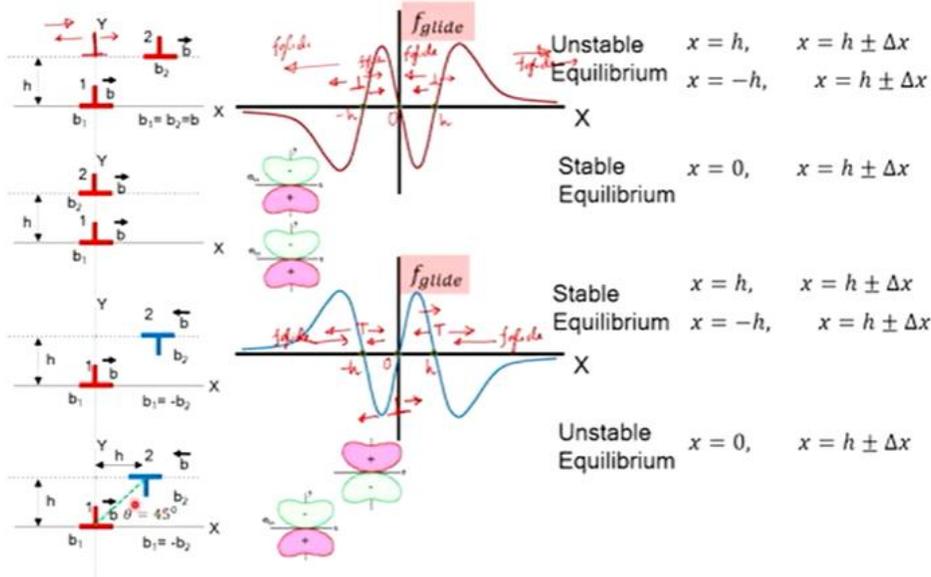
To kuch is tarah se samajh sakte hain aap dekhenge ki ye jo dislocations hai is pe agar ye positive dislocation agar main consider karunga to slip plane yahan pe is yeh hai mera slip plane to slip plane ke above mera compressive stress field rehta hai aur neeche tensile stress field rehta hai is dislocation ke liye ye mera slip plane hai aur slip plane ka above mere compressive stress field rahegi aur slip plane ke neeche ye neeche wala part ho gaya ye yahan pe tensile stress field rahegi aur ye stress fields kuch is tarah se act karegi yaani aap dekhenge ki yahan pe compressive yahan pe compressive aur ek dusre ke prati distance maintain karne ki koshish karegi to aap dekhenge ki yahan par jo angle hoga wo hamesha 45 degree rahega do pure edge dislocations ek positive hai aur ek negative hai to yeh ho gaye mere stable configuration aaiye abhi jaante hain iska mahatva kya hai strain hardening ke prati to mere paas kuch stable configuration ho gaye to aap dekhenge ki do positive dislocation hai aur do same nature ke dislocation hai to unme jo angle rahega 90 degree rahega yaani do ek dusre ke upar rahenge aur agar ek positive ek negative hai to dekhenge 45 degree hai ya ek negative hai ek positive hai yahan par bhi humein 45 degree ek configuration milega stable configuration milega.
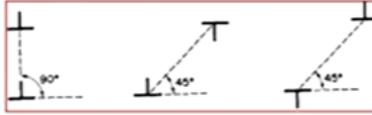
To abhi hum dekhenge ki humne edge dislocation ke liye dekha tha what happens when you have two screw dislocations yaani jab do screw dislocation rahenge tab aapke paas kya kya condition aati hai to yeh aapko solve karna hai iske liye same method hum dekhenge to screw dislocation ke liye stress field hum dekhenge screw ke liye humne kuch is tarah se mark ki thi stress field hamare paas yeh jo components the yeh shunya the aur hamare paas sirf $\sigma_{xz}\sigma_{yz}$ Aur σ_{zx} Aur σ_{zy} Sirf shear components se ye stress field hogi aapki burgers vector aap kuch is tarah se consider kar sakte hain kisi bhi direction mein burgers vector aap consider kar sakte parallel so burgers vector aap is direction mein consider kar sakte hain [001] aur tangent vector yahan pe b se multiply karna padega ye mera burgers vector hoga tangent vector aayega unit vector wo jo [001] dono agar aap consider karenge to burgers vector aur tangent vector dono parallel rahenge to ye stress field hai aur aap peach koehler equation istemal kar sakte hain aur force nikaal ke aap nikaal sakte ho ki dono ke beech mein kya interaction rahenge to answer simple hai yahan par maine direct result

diye ki do dislocation jo opposite sign ke hain wo attract karenge aur same sign ke woh repel karenge abhi hum dekhte hain ki strain hardening ke prati kya iska importance hai to humne F_{glide} Nikaala tha agar mere paas ek do positive dislocation se slip plane ke liye to mere paas glide force kuch is tarah se aa raha hai aur yeh mera glide force hai abhi aap dekh paa rahenge ki ye jo glide force hai iska variation kuch is tarah se aur yahan pe kuch maximum values mil rahi hai mere paas glide force ki yahan pe ek maximum value hai yahan pe ek maximum value hai yaani magnitude hum baat kar rahe hain yahan pe magnitude iska maximum hai nature aap iska plot karoge glide force ka to aap dekhenge ki ye jo magnitude hai glide force ka ye maximum magnitude same magnitude hai yaani aap dekhenge ye exactly same glide force yahan pe act kar raha hai to agar main isko is tarah se kuch dekhun mathematically agar main iska treatment loon agar main is glide force ka derivative leta hoon is agar ye mera nature hai aur iska main derivative lunga aur yahan pe maximum values pe mujhe shunya milna chahiye to yeh maximum glide force ki value hogi to agar main iska derivative leke isko zero equate karunga is term ka derivative leke x se to mere paas kuch is tarah se equation aayega $x^4 - 6x^2h^2 + h^4$ Aur iska agar main root lunga yaani x^2 Ki value kuch is tarah se aayegi aur x ki value main is tarah se lunga to mere paas chaar values aani chahiye ek plus minus $\sqrt{3 \pm 2\sqrt{2}}h$ to yahan par aap dekhenge ki chaar roots mujhe milne chahiye to ek plus ho gaya ek minus ho gaya yahan par bhi ek plus ho gaya ek minus ho gaya to ye chaar roots mujhe kuch is tarah se milenge ye chaar roots ek do teen aur yeh chaar yahan par meri F_{glide} Ki value maximum rahegi to usko main is tarah se nikaal sakta hoon $F_{glide_{max}}$ Isko nikaalne ke liye kya karunga main yeh jo value hai x ki value ismein rakh dunga agar main is equation mein rakh dunga to mere paas $F_{glide_{max}}$ Ki value kuch is tarah se aayegi aur agar aap isko solve karenge to aap dekhenge ki yeh jo term hai yeh $1/h$ ho jayegi aur main kuch is tarah se likh paunga yaani agar main ye value yahan par rakhta hoon to yeh jo value hai aap solve karke dekhenge to main kuch is tarah se likh paunga yeh aati hai $0.25/h$ kuch is tarah se approximately to yeh 0.25 ko yahan par $1/4$ likh sakta hoon to $1/4$ agar main multiply karunga 2 se to 8π aa jayegi to mere paas ek glide maximum ki ya dislocation per unit length jo force acting per unit length of a dislocation aur uski jo maximum value hai kuch is tarah se aayegi to ye mere paas ek maximum glide force ki value ho gayi ab iska hum istemal karenge strain hardening nikaalne ke liy.

Stable configurations

$$\text{Screw} \quad \sigma_{ij} = \begin{bmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & 0 \end{bmatrix} \quad \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} b \quad \hat{l} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

What happens when you have two screw dislocations?

Two dislocations with opposite sign will always attract each other and with same sign will always repel
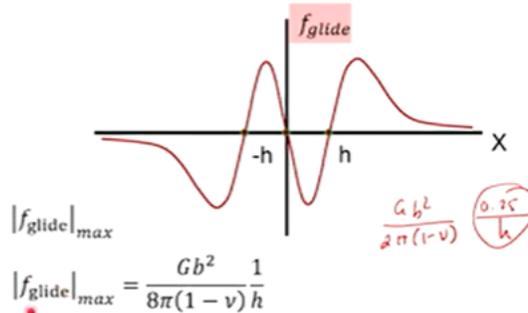
$$f_{glide} = \frac{Gb^2}{2\pi(1-v)} x \frac{x^2 - h^2}{(x^2 + h^2)^2}$$

$$\frac{df_{glide}}{dx} = 0$$

$$x^4 - 6x^2 h^2 + h^4 = 0$$

$$x^2 = (3 \pm 2\sqrt{2})h^2$$

$$x = \pm\sqrt{3 \pm 2\sqrt{2}}\, h$$

$$|f_{glide}|_{max}$$

$$|f_{glide}|_{max} = \frac{Gb^2}{8\pi(1-v)}\frac{1}{h}$$

$$\frac{Gb^2}{2\pi(1-v)} \quad \boxed{\frac{0.25}{h}}$$

To yeh mere paas ek value aa gayi maximum glide force ki abhi hum dekhte hain kuch dislocation configuration abhi hum stable configuration nikaalne ki koshish karenge yahan pe maine dislocation maan liya hai ek aur ek dusre slip plane pe ek dislocation maan leta hoon to yeh negative dislocation maan leta hoon yahan par aur stable configuration ye kab aayegi jab $x = h$ rahega aur planar distance jo hai wo h hai to yaani in dono ke beech ka angle rahega 45 to isi tarah se main kuch ek ek lattice taiyaar kar sakta hoon dislocation ka to maine dekha ki agar mere paas bahut saare dislocation hai aur unki stable configuration kuch is tarah se main create kar sakta hoon to isko kehte hain taylor lattice isko hum taylor lattice bhi kehte hain ya taylor dislocation lattice bhi kehte hain ye taylor lattice ho gayi taylor dislocation lattice ho gayi abhi hum consider karenge kuch is tarah se main abhi dislocation density nikaalne ki koshish karunga mere paas ek crystal hai yahan pe bahut saare slip planes hai aur wahan pe bahut saare dislocations hai jo ki stable configuration mein arrange hai abhi hum dislocation density nikaalne ki koshish karenge to dislocation density main kuch is tarah se define karta hoon ρ se main likhta hoon isko yaani $\rho =$ Length/Volume to total length of dislocation upon total volume to main agar kuch L agar consider karunga yeh maine x aur y lattice consider kiye the to z direction mein uski length rahegi kyunki hum pure edge dislocation consider karenge to jo length of crystal rahega wahi mera length of dislocation rahega to ye L ko main maan leta hoon length of a dislocation aur agar main yeh lattice consider karunga ye taylor lattice agar main consider karunga to aap dekhenge ki is lattice mein ye 2D lattice mein mere paas kitne dislocations hai ek do teen chaar ye chaar dislocations ka contribution is lattice mein kya hoga ye mere paas chaar dislocations hai to 1/4 contribution hai in dislocation ka to yeh ho gaye mere paas number of dislocations main nikaal raha hoon abhi 1/4 contribution ho gaya aur ye center wala jo hai wo ek 1 ho gaya to mere paas do dislocation ho gaye is lattice mein is configuration mein aap dekhenge ki do dislocations ho gaye to ye jo dislocation hai wo shared rahega chaar lattice mein to isliye 1/4 ho gaya to aapke paas length kitni aayegi ek lattice mein 2L to length aa gayi meri 2L yahan pe kyunki number of dislocations do hai aur volume hoga yahan pe aap dekhenge ki yeh meri length hogi ye width hogi aur height us tarah se aayegi to yahan par aap dekhenge ki ye do dimensions se h aur h, 2h ho jayenge ye aur h aur h,

$2h$ to ye iska area ho jayega $(2h)^2$ Into L jo L hoga x z direction mein z direction mein maana tha ki jo dislocation length hai L hai mere paas volume kuch is tarah se aayega aur main isko solve karunga to dislocation density ko main likh sakta hoon $1/2h^2$ To ek relation ho gaya mera dislocation density ka $1/2h^2$.

Agar main is lattice ko ek shear stress apply karta hoon abhi yahan par maine τ apply kiya aur is shear stress mein is dislocation jo lattice hai isko disturb karne ki koshish karta hoon mere paas kuch force lagega mujhe yeh jo force hai main is tarah se contribute karta hoon ek dislocation pair ko main agar disturb karna chahta hoon ya move karna chahta hoon to to main kuch is tarah se likh sakta hoon $\tau \cdot b$ jo ki is dislocation ka burgers vector hai kuch is tarah se to $\tau \cdot b$ aur isko agar mujhe move karna hai to mujhe maximum glide force apply karna padega kuch is tarah se to ye maximum glide force kyunki humne maximum glide force nikaala tha do dislocation ke pair ke liye to isliye mujhe ye maximum glide force apply karna padega to $\tau b = F_{glide_{max}}$ Isko main agar compare karta hoon to τ ki value aur ye jo $F_{glide_{max}}$ Yahan pe main replace karta hoon to τ ki value jo aayegi $Gb/8\pi(1-\mu) \cdot 1/h$ abhi ye $1/h$ ki value main kuch is dislocation density dwara replace kar sakta hoon to τ_c Ki value yaani yahan par critical shear stress jo lag raha hai yahan par agar main likh sakta hoon τ_c Ko τ τ_c Ko kuch is tarah se likh sakta hoon $K \cdot \tau$ jo meri critical value hai ek constant hai ye K to $K \cdot \tau$ is tarah se likh sakta hoon aur yeh $1/h$ ko main $\sqrt{2} \cdot \sqrt{\rho}$ Is tarah se is equation se replace kar sakta hoon to τ_c Ek critical shear stress ki value aayegi yeh shear stress kaun sa hai critical shear stress is dislocation ke pair ko move karne ke liye jo critical shear stress lagega uski ek value hai aap dekhenge to ye τ_c Ko main yeh jo term hai is term ko ye pure term ko main ye constant hai isko main replace kar sakta hoon α se to agar main dekhunga τ_c Ko kuch is tarah se likhunga $\alpha Gb\sqrt{\rho}$ Aur α ki jo value hai wo 0.5 to 1 ke beech mein vary hoti hai isko kehte hain hum Taylor hardening equation ye mere Taylor hardening equation hai textbook mein aap isko is tarah se refer karenge Taylor hardening equation to ye meri yahan par agar aap dekhenge ki critical shear stress jo hai wo proportional kiske saath hai yahan par aap dekhenge ki wo dislocation density ke saath proportional hai to yahan par humne dekha tha ki agar stable kuch configuration ho rahi hai dislocations ki wo mujhe kya indicate karegi ki ek dislocation density ke saath meri shear stress ki value kaise related hai to aap dekh sakte ho ki dislocation density jaise jaise badhti hai to critical stress to move dislocation should increase yaani dislocation move karne ke liye jo critical stress rahega wo badhega jaise jaise dislocation density badhegi abhi humne dekha tha ye equation ya Taylor hardening equation humne dekha abhi hum isko agar plot karenge τ_c Versus ρ to hamare paas ek perfect ek neat linear relation milega to aap dekhenge ki jaise jaise dislocation density badh rahi hai waise waise mera critical shear stress kya critical stress hai ye dislocation ko move karne ka critical stress wo badhega yahan se mere strain hardening ki origin hai yahi plot mera strain hardening ka origin ha.

$$|f_{glide}|_{max} = \frac{Gb^2}{8\pi(1-v)}\frac{1}{h}$$

Density of dislocation, $\rho = \frac{Length}{Volume}$

$$\rho = \frac{2L}{(2h)^2 L} = \frac{1}{2h^2}$$
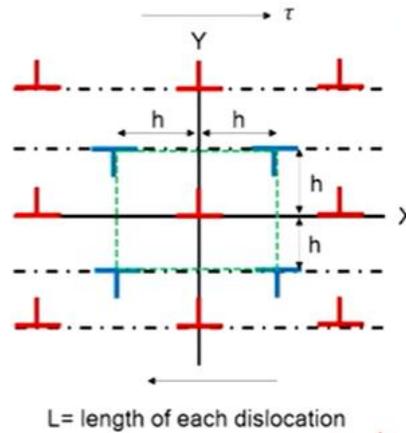
For a dislocation pair

$$\tau b = |f_{glide}|_{max}$$

$$\tau = \frac{Gb}{8\pi(1-v)}\frac{1}{h}$$

$$\tau_c = k\frac{Gb}{8\pi(1-v)}\sqrt{2}\sqrt{\rho}$$

$$\tau_c = \alpha Gb\sqrt{\rho} \qquad \alpha \sim 0.5 - 1.0$$

$$\tau_c = \boxed{k}^c$$

Taylor lattice
Taylor dislocation lattice

$= \frac{1}{4} \times 4 + 1$

$= \underline{\underline{2}}$

L= length of each dislocation

Taylor hardening equation

As dislocation density increase, critical stress required to move the dislocation increases

Abhi hum isko strain ke saath kaise correlate karenge ye humne Orowan equation dekha tha $\gamma = \rho_m b\bar{x}$ Aur humne maan ke chala tha agar b aur $\bar{x}$ Agar mere constant hai to aap dekhenge ki ye jo γ hai wo directly proportional kya hai mere mobile dislocation density ke saath aur jaise jaise γ badhega waise waise meri mobile dislocation density badhegi aur jaise jaise meri mobile dislocation density badhegi to agar main ye equation istemal kar raha hoon to jaise ρ_m Badhega waise waise mera critical stress jo rahega deformation ke liye wo badhega to main dekh sakta hoon ki jaise jaise plastic strain badhega us tarah se mera jo stress hai flow stress hai woh bhi badhna chahiye to yeh practically bhi dekha gaya hai.

To yahan par kuch schematic mein main yahan par dikha raha hoon aap dekhenge ki yahan pe aap dekh rahe hain ki stress strain curves diye hai to yahan par aap dekhenge ki strain jaise jaise strain badh raha hai waise waise mera strain hardening to ho hi raha hai yahan pe aur kuch aap kar sakte ho jaise yeh mera material tha yahan par percentage cold work yaani main kuch deform nahi tha initially kuch deform nahi tha jaise jaise main usko deform kar raha hoon percentage cold work uspe badha raha hoon to aap dekhenge ki jo yield strength ki value hai wo badh rahi hai yahan pe yahan pe aap dekhenge ki yield strength ki value badh rahi hai jaise jaise mera percentage cold work badh raha hai percentage cold work badh raha hai yaani kya ho raha hai mera plastic strain badh raha hai material ke upar jo initial plastic strain hai wo badh raha hai aur uske wajah se meri dislocation density badh sakti hai aur yeh agar dislocation density badh rahi hai to yeh stress mera badhega to aap dekh sakte ho ki jaise plastic strain aur percentage cold work badh raha hai waise waise yield strength badh raha hai aur ductility yahan pe ghat rahi hai to ye iska explanation yeh relation se hum explain kar sakte hain aur ek case study dekhte hain low carbon steel pe yahan par bhi aap dekhenge ki percentage cold work yahan par main badha raha hoon pehle jo green curve hai blue curve hai aur red curve hai to yahan par dekhenge yahan se 0 percent 4 aur 24 percent cold work maine kiya hai aur aap dekhenge jo yield strength hai yahan par yeh badh raha hai to aap dekhenge ki percentage cold work jaise jaise isko bhi likh lete hain percentage cold work jaise badh raha hai waise mera yield strength badh raha hai yaani critical stress to deform wo badh raha hai yahan pe to ye ek case study ho gayi yahan se hum yahi hamara origin hai strain hardening ka

yahi reason hai hamara strain hardening ka isko hi hum kehte hain strain hardening ya work hardening isko bhi likh lete hain yahi mera reason hai strain hardening aur work hardening ka to this is the origin of strain hardening.
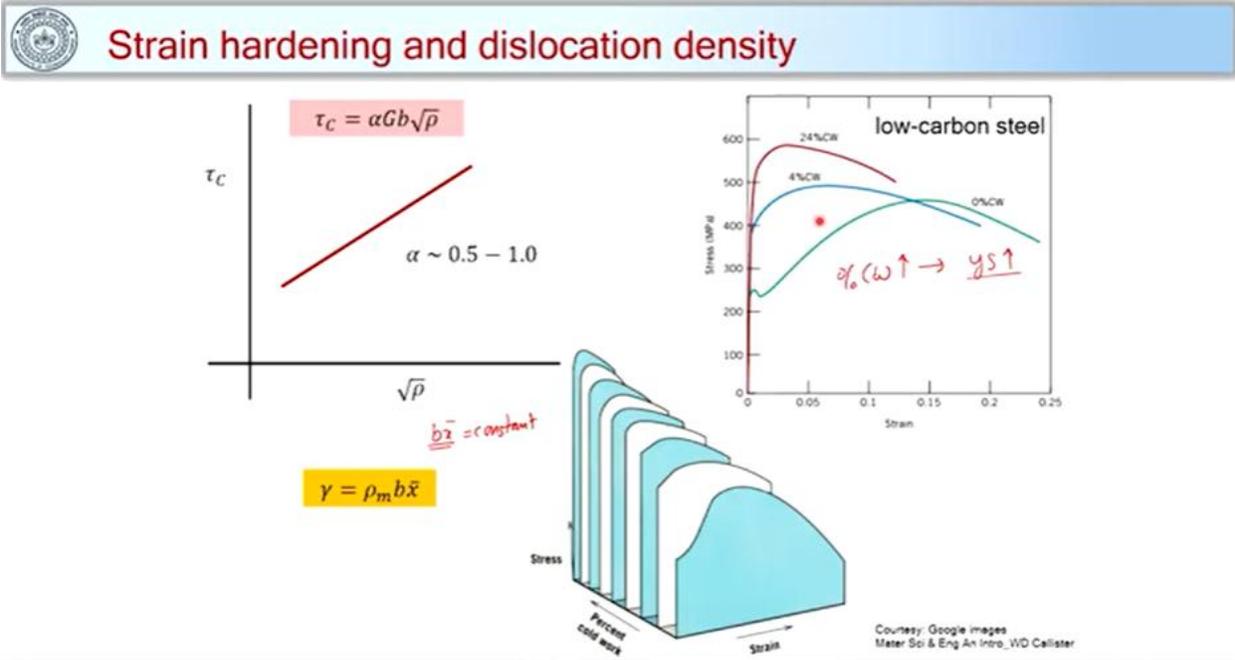
Abhi hum dekhte hain ki yeh yeh jo equation hai yeh mera Taylor hardening equation hai ab hum kuch condition dekhte hain ki jab dislocation interaction hote to kuch annihilation bhi hote hain annihilation kaise hota hai ye bhi dekhte hain agar mere paas do dislocation hai do positive dislocation hai ek hi slip plane pe hai to yeh kya hoga dono repel karenge to in dono ke beech mein repulsion hoga kyunki inki stress field jo hai interact karegi dono ki stress field saman hai to dusron ko ek dusre ko repel karegi hum agar in dono dislocation ko ek maan lete hain isko hum is tarah se consider karte hain $2b$ consider karte hain hum is tarah se bhi isko samajh sakte hain iski energy kya hogi humne dekha tha ki agar in agar ye dono dislocation ek ho jaate hain to inki energy ho jayegi yahan pe b ki jagah main $2b$ likhunga to yeh $4\alpha Gb^2$ Ho jayega aur is case mein aap dekhenge ki dono ki energy kya hogi $\alpha Gb^2 + \alpha Gb^2$ Yaani $2\alpha Gb^2$ To yahan pe aap dekhenge ki ye configuration mein meri energy badh rahi hai to isliye ye dono dislocation kabhi paas nahi aayenge ye dono dislocation ek dusre ko repel karenge kyunki jo individual energy unka addition hai yeh jo energy mil rahi hai is energy se kam hai similarly agar mere paas do dislocations hai ek positive hai ek negative dislocation hai dono ek hi slip plane pe to ye dusre ko attract karenge to yahan par attraction hoga aur is attraction ke wajah se kya hoga agar mere paas is tarah se kuch dislocation hai to ye dislocations ek dusre ko attract karenge aur yahan par aap dekhenge ki ye dono dislocation attract karke ek dusre ko annihilate karenge annihilate karenge yaani nasht karenge annihilation ka matlab hindi mein nasht hota hai to ek dusre ko nasht kar denge to usi tarah se agar mere paas do screw dislocation hai to agar dono ek ek right hand screw hai ek left hand screw to aap dekhenge ki dono ek dusre ko attract karenge aur annihilate karenge.
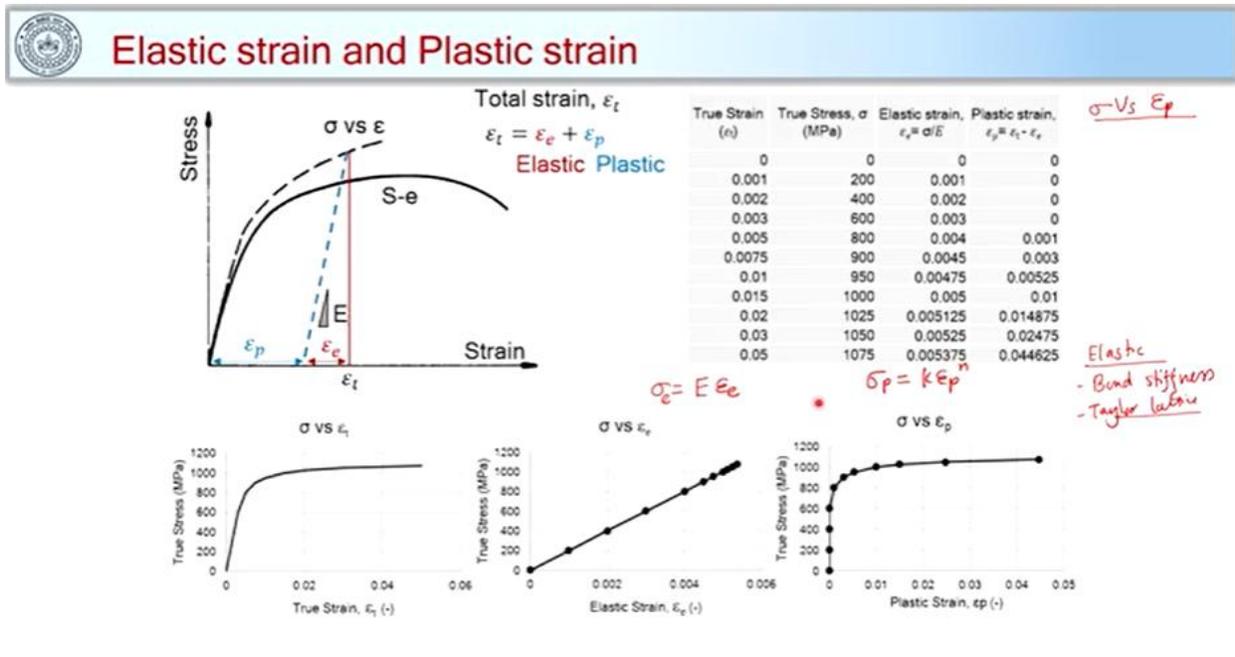
To yahan par aap dekh paayenge ki agar isko is tarah se samjhe ki mere paas do agar mere paas edge dislocation hai is tarah se yeh positive hai edge dislocation aur negative dislocation hai agar yeh dono ek dusre ko attract karenge ye negative dislocation hai to aap dekhenge ki mera product kya hoga yahan par agar attract hone ke baad ye do half plane dusre se merge ho jayenge aur mere

paas lattice mein koi dislocation nahi rahega ek half plane do half plane merge hoke ek defect free crystal taiyaar karenge to dislocation annihilation bhi hota hai based on dislocation ke natures aur dislocation ke position to jab hum dekhenge ki yeh dono process annihilation aur interaction ye strain hardening ko originate karte hain yahan par aap yeh baat samajhie strain hardening yaani yahan par main generic term istemal kar raha hoon strain hardening ki kyunki maine bola tha ki strain softening bhi hota hai to annihilation mera strain softening ko lead karega but yeh strain hardening yahan par main general term istemal kar raha hoon to yeh jo interaction hai mere dislocation ke strain hardening ko generate karte hain to annihilation of dislocation strain softening ko result karenge.

Dislocation interactions & Annihilation
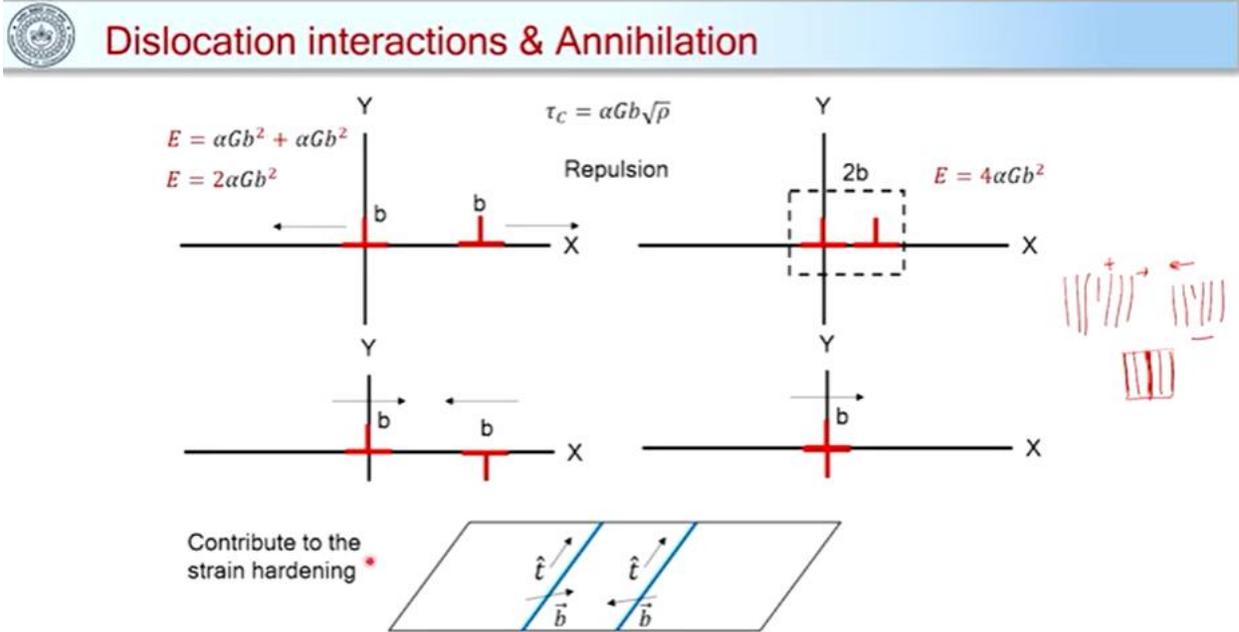
Abhi maine ek question pucha tha aapko kuch parts mein ki mere paas ek stress strain curve hai is tarah se maine stress plot kiya strain plot kiya yeh mera engineering stress strain curve hai aur yeh true stress true strain curve hai aur humne dekha tha ki koi bhi total strain hai mere paas ek point par yeh total strain hai usmein kya components hai plastic strain aur elastic strain ke maine find out karne ke liye bola tha aur maine bola tha ki aapko stress versus plastic strain nikaalna hai ye plot nikaalne ke liye bola tha main sochta hoon ki aapne kiya hoga exercise par abhi nahi kiya hoga to ab yahan pe samajhie ek elastic part hai aur ek plastic part hai to hum elastic plot kis tarah se nikaalte hain hum is tarah se nikaalte hain ki yahan pe hum ek parallel line draw karte hain aur parallel line draw karke jahan pe wo intersect karegi x axis pe jo recovered part rahega ye part jo rahega ye mera strain rahega elastic strain aur jo remaining part rahega wo mera plastic strain rahega jo ki permanent deformation generate karega yahan pe kuch raw data maine diya hai aur is raw data se aap dekhenge ki mere paas total true strain hai aur true stress hai is tarah se ab yahan par maine elastic strain nikaalne ki koshish ki σ/E to is tarah se mujhe kuch elastic strain mil raha hai aur plastic strain main is tarah se nikaalunga is equation se ye $\epsilon_p = \epsilon_t - \epsilon_e$ So aap ye dekhenge ki plastic strain main sirf subtract kar raha total strain se elastic part to aapke paas plastic strain aayega aur main agar ye plot dekhunga ye true stress versus true strain plot hai yaani true strain aur true stress ye plot hai aur yeh hai σ versus ϵ_e ϵ_e Yaani mera elastic part aur ye jo hai σ versus ϵ_p To ye maine pucha tha aapko plot karne ke liye abhi important point yahan pe kya hai aap

dekhenge ki mere engineering stress strain curve mein throughout engineering stress strain curve yeh jo part hai elastic part ye hamesha rehta hi hai aap dekhiye ki linear vary ho raha hai yaani hamesha ek elastic part jo rahega is true stress strain curve mein rahega agar plastic part ki baat karenge ya plastic region ki baat karenge to ye yield stress tak nahi rehta yahan par aap dekhenge ki plastic strain yahan tak shunya tha 600 tak yahan pe shunya tha yahan par aap dekhenge 600 tak shunya tha aur yahan se woh badh raha hai to aap dekhenge plastic part mera kuch stress tak nahi rahega par elastic part hamesha hi rehta hai yeh iska implication hai is is analysis ka yeh implication hai ki elastic part hamesha hi true stress true strain curve mein present rehta hai aur woh yeh bhi rehta hai humne elastic origin is tarah se bhi diya tha elasticity ka origin humne diya tha bond stiffness interatomic bonding nature likha tha yeh bond stiffness ki baat ki thi taylor lattice isko bhi main disturb jab karunga to iske liye bhi mujhe kuch ek critical stress ki zarurat hoti hai yahan par bhi jo taylor lattice yaani dislocation lattice jo taiyaar hue the us woh bhi elastically bhi behave kar sakta hai ye iska ek implication hai abhi hum dekhenge aur abhi main aur ek exercise aapko dena chahta hoon to aap is pure stress strain curve ko humne isko is tarah se likha hai σ ko yaani elastic part mein is tarah se likha hai $E \cdot \epsilon$ aur yeh jo hai plastic part main kuch $K\epsilon_p^n$ Is tarah se likh sakta hoon yahan par strain hardening exponent use karke yahan par yeh jo true stress true strain curve hai isko likhna hai yaani kis tarah se aap likh sakte hain ye aapko mathematically likhna hai is equation ko to mere paas complete stress strain ka relation aa jayega elastic behavior mein bhi aur plastic behavior mein bhi aapko ye concept apply karne hain yahan pe.

Elastic strain and Plastic strain

True Strain (n)	True Stress, σ (MPa)	Elastic strain, $\varepsilon_e = \sigma/E$	Plastic strain, $\varepsilon_p = \varepsilon_t - \varepsilon_e$
0	0	0	0
0.001	200	0.001	0
0.002	400	0.002	0
0.003	600	0.003	0
0.005	800	0.004	0.001
0.0075	900	0.0045	0.003
0.01	950	0.00475	0.00525
0.015	1000	0.005	0.01
0.02	1025	0.005125	0.014875
0.03	1050	0.00525	0.02475
0.05	1075	0.005375	0.044625

Abhi humne aur ek cheez dekhi thi yield locus yaani yield surface ki baat ki thi jahan par humne dekha tha ki elastic to plastic behavior change hota hai aur ek locus humne draw kiya tha state of stress se abhi hum isko relate karenge strain hardening ke saath to humne kuch is tarah se plot kiya tha 2D stress state ke liye yahan par σ_1 Plot kiya tha σ_2 Plot kiya tha ye principal stresses hai aur humein kuch is tarah se yield locus mila tha ye 2D stress state ke liye to humne bola tha ki iske andar agar material hai mera state of stress hai to yahan pe koi yielding nahi hogi iske bahar agar ja raha hai to yahan pe yield hogi aur yeh jo loci hai isko hum yield locus kehte hain ye humne dekha tha acche se abhi jab strain hardening hoti hai to humne dekha tha ki yield strength badhta

hai yield strength agar badhta hai to wo kuch is tarah se badhta hai kabhi agar is tarah se badh raha hai yaani mera yield locus is tarah se expand ho raha hai kuch is tarah se expand ho raha hai aap dekhenge ki yeh jo ellipse hai yaani humne ye von mises ellipse draw kiya tha yahan par agar aap bhul gaye honge to aap isko dekh lijiyega revise kar lijiyega agar von mises criteria se humne ellipse plot kiya tha aur ye ellipse jab is tarah se expand hota hai to aap dekhenge isko kehte hain isotropic hardening yaani mera material isotropic harden ho raha hai yaani ab saare direction mein wo equally expand ho raha hai aur iska origin yahan pe stable hai wo change nahi ho raha hai to ellipse expand in symmetric fashion yahan par symmetric wo expand ho raha hai ab yahan par dekhenge ki dusre type ka bhi hardening hota hai to mere paas kuch yield locus is tarah se aur yeh yield locus is tarah se move ho raha hai yahan par aap dekhenge ki iska origin yahan pe tha origin yahan pe shift ho gaya phir is tarah se mera yield locus badh gaya badh raha hai change ho raha hai to is type of hardening mein is type of hardening mein hum kya yahan par aap dekhenge ki iska origin yahan pe shift ho raha hai isko kehte hain kinematic hardening yahan par kya hota hai ellipse just translated yaani ellipse just yahan par translate ho raha hai.

Yield locus and Strain hardening

Isotropic hardening
Ellipse expands
in a symmetric fashion

Kinematic hardening
Ellipse is just
translated
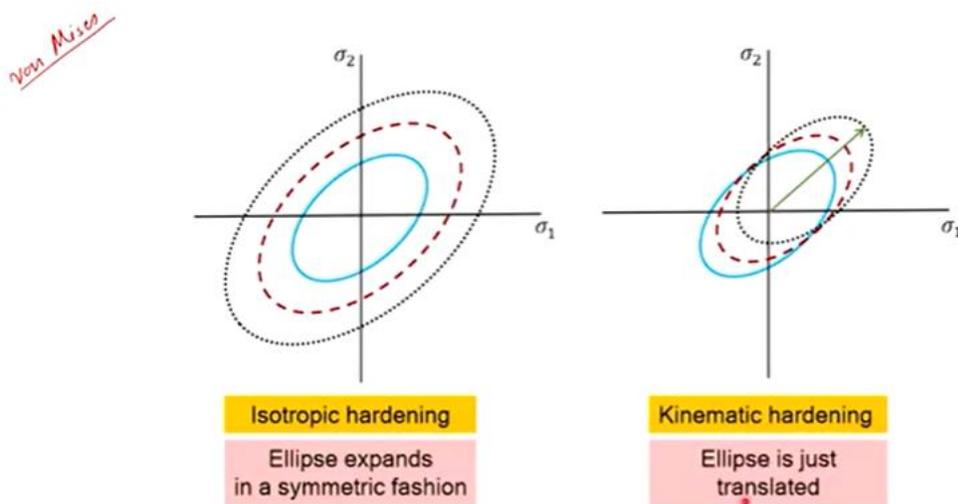
To yahan par is part mein humne dekha tha ki dislocation interactions kya hote hai ek dislocation ki stable configuration kya hoti hai dislocation ke stable configuration se strain hardening kaise generate hota hai aur strain hardening ka material behavior par kya asar hota hai yeh humne dekha tha next part mein hum jaanenge ki dislocation ke interaction ke wajah se dislocation density kaise taiyaar hoti hai aur is dislocation density ko aur acche se analyze karenge agle part mein abhi hum main yahi ke liye rukta hoon dhanyavad