

Mechanical behavior of materials

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Week-7

Lecture-34

Strain Hardening and Dislocations Interactions



Mechanical Behavior of Materials (Hindi)

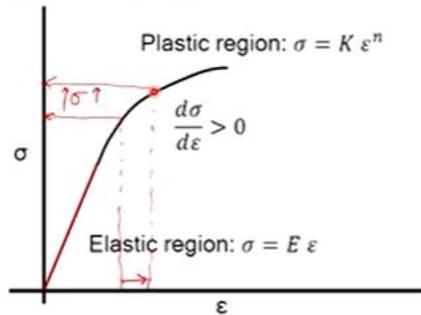
Strain Hardening and Dislocation Interactions

Namaskar phir se swagat karta hoon aapka is course mein mechanical behavior of material jisko hum hindi mein padhenge is part mein hum jaanenge ki dislocation interaction kya hote hain aur uske wajah se strain hardening kaise hoti hai to abhi hum isko step by step samjhenge to strain hardening ke baare mein humne padha tha jab humne stress strain plot kiye the yaani ye mera true stress true strain plot hai aur yahan par humne dekha tha ki ek linear part hai jo elastic region darshata hai jahan par main likh sakta hoon $\sigma \propto E \cdot \epsilon$ yeh strain hai mera aur ek non-linear part hai to yeh mera plastic region hai maine isko likha tha $\sigma = K\epsilon^n$ agar hum dekhenge yeh ab hum plastic region ke baare mein baat karenge jab strain hardening ki baat kar rahe hain tab hum dekhenge ki agar strain jitna badhana hai mere material pe aap dekhenge ki mera flow stress utna badhta hai to isko hi main kehta hoon strain hardening yaani increase in strength with strain to aap kuch is tarah se samajh sakte hain main plastic strain ki baat kar raha hoon agar main plastic strain badha raha hoon to aap dekhenge yahan se mere paas main yahan se strain badha raha hoon to aap dekhenge yahan iske corresponding mere paas stress ki value bhi badh rahi hai to hamara strain

hardening rate tha $d\sigma/d\epsilon$ to strain hardening ki jab hum baat karenge to main keh raha hoon ki $d\sigma/d\epsilon > 0$ hai to ab jahan par strain hardening bhi hoti hai usko main is tarah se kabhi-kabhi books mein likha jaata hai isko work hardening bhi kehte hain work hardening kyunki jab main material par work karta hoon ya deform kar raha hoon mere material par to uski strength badhti hai to kisi kisi case mein yeh $d\sigma/d\epsilon < 0$ ho sakta hai to yaani meri strength decrease ho rahi hai isko main keh sakta hoon strength softening bhi to yeh hum yahan par nahi discuss kar rahe abhi hum strain hardening ke baare mein focus karte hain to mere paas $d\sigma/d\epsilon > 0$ hona chahiye yeh kyun important hai ye isliye important hai jab main applications mein baat karunga jaise twisted bars hai construction industry mein hum dekhenge ki hume strength jyada chahiye to hum usko twist karke rakhte hain twist karke rakhne ke liye rakhne se yaani iska matlab hai ki main uske upar koi plastic strain apply kar raha hoon to plastic strain se meri strength badh jayegi cold rolled sheets bhi hum dekhenge ki jaise agar sheet hai usko main continuously rolling karunga to us sheets ki strength main badha sakta hoon iske ye simple application hai par strength kyun badhti hai yeh sabse important question hai abhi hai na hum jab strain apply kar rahe hain to strength kyun badhni chahiye ye ek isko hum is tarah se jaante hain mechanical behavior mein material ke jab meri strength badhti hai to meri ductility bhi decrease hoti hai to isko main kehta hoon isko text book mein aapne dekha hoga ya kisi research articles mein aap dekhenge isko strength ductility trade off kehte hain yaani jab main strength badhane ki koshish kar raha hoon tab ductility meri decrease hoti hai ductility jab badhane ki koshish karunga to mujhe strength par compromise karna padta hai to abhi hum yahan par dekhenge ki yeh jo strain hardening hai hum dislocation ke point of view mein dekhenge dislocation ke point of view mein is tarah se dekhenge ki hum dislocation jab baat karte hain jab dislocation ki movement hoti hai to ek plastic flow hota hai material mein humne dekha tha ki plastic behavior jo hota hai material ka woh dislocation ke movement se aata hai to humne ek yeh explain kiya tha jab humne theoretical shear strength ki baat ki thi jab hum theoretical shear strength dekha tha ki theoretical shear strength experimentally observed strength se bahut jyada hoti hai aur humne tab baat ki thi ki dislocation responsible hai hamare theoretical shear strength ko kam karne ke liye ya experimental observed shear strength jo hamare theoretical shear strength se kam kyun hai wo dislocation ke presence se to hum dislocation ko is tarah se dekh sakte hain agar main dislocation ki motion agar rok paaunga to to is is tarah se main dekh paaunga ki main strength badha paaunga to ya ulta hum soch sakte hain ki agar main dislocation ki motion aasaan bana dunga to mera jo strength hai ya stress required deformation ke liye wo kam ho jayega to agar main dislocation ki motion impede karunga tab meri strength badhegi to yahan par hum strain hardening ki baat kar rahe aap dekh paa rahenge agar main yahan se is part se yahan tak badh raha hoon main strain increase kar raha hoon strain increase kar raha hoon yaani iska matlab yahan par aap dekhenge ki agar stress increase ho raha hai strain ke saath to main dislocation ki motion is position par thodi kam ho jayegi ab wo jaante kis tarah se kam hogi.



Strain hardening



Strain hardening

Work hardening

$\frac{d\sigma}{d\epsilon} < 0$
 $\sigma \downarrow$ strain softening

Increase in Strength with Strain

$$\text{Strain hardening rate, } \theta = \frac{d\sigma}{d\epsilon}$$

Increase in strength

- Twisted bars (construction industry)
- Cold rolled sheets

Strength-Ductility trade off

Why increase in the strength?

Increase in strength, decrease in ductility

Dislocations causes plastic flow

Explains discrepancy in theoretical shear strength & experimentally observed strength

Easier the motion of dislocations, less amount of stress required

Impede dislocation motion: Increase in strength and decrease in ductility

To humne ek equation dekha tha yeh orowan equation dekha tha tab humne plastic strain ki baat ki thi yeh shear strain jab develop hota hai to humne baat ki thi mobile dislocation density ki baat ki thi uske baad ek burgers vector hai aur ye average dislocation distance traveled hai to humne dekha tha agar main ek maanta hoon ki agar b aur x bar main constant rakh raha hoon agar ye constant rehte to main dekh sakta hoon ki ye jo plastic strain hai directly proportional hoga mere dislocation density ke barabar to yeh ek important relation hum yahan se nikaal sakte hain ki jitni meri mobile dislocation density hogi utni meri plastic strain jyada hogi to jaise jaise dislocation density increase hogi aap dekhenge jaise jaise plastic strain increase ho raha hai waise waise meri dislocation density increase bhi honi chahiye yeh yahan pe hum conclude kar sakte hain ki dislocation density meri badhegi jaise jaise mera plastic strain badh raha hai ab dislocation density jaise jaise badhegi waise waise dislocation ke interactions badhenge agar do yahan pe agar pehle case mein agar do dislocations hai to unme ek do kuch interaction hoga par agar chaar dislocations honge to jyada interactions badhenge to yahan pe hum samajh sakte hain jaise jaise plastic strain badh raha hai waise waise mere dislocation interaction badhenge to hum jaante hain ki agar interaction badhenge to interaction jab main baat kar raha hoon to humne dekha tha ki dislocation ke around ek stress field hoti hai aur yeh jo interaction hai meri dislocation ki stress field ki dusre dislocation ke stress field ke saath yahi cause karegi mere strain hardening ko hum isko aage aur acche se padhenge pehle hum dekhte hain ki yeh interaction hoti kaisi hai to interaction of these stress field hum yahan par hum dekhenge ye affect karegi mere dislocation movement ko ya motion ko to jab main strain hardening ki baat kar raha hoon to main dekh paa raha hoon ki agar mera plastic strain badh raha hai waise mere dislocation ki density badhegi us tarah se mere dislocation ki interaction badhegi aur interaction kyun hogi kyunki unke paas dislocation ke paas ek stress field hoti hai unke around to abhi jaante hain ki dislocation interaction ke baare mein to maan lete mere paas ek simple case main karunga yahan pe ek positive edge dislocation kuch is tarah se hai origin par maine consider kiya hai aur ek other positive edge dislocation kuch is tarah se maine consider kiya hai maan lete hain is dislocation ko naam dete the one is dislocation ko naam dete the two aur inke burgers vector yahan pe $b_1 b_2$ hai simplicity ke liye consider kar lete hain ye jo burgers vector hai dono dislocation ke wo same hai unka magnitude same hai direction

bhi same hai to ye mera configuration ho gaya abhi dekhenge ki ye dislocation one mera dislocation two ke saath kis tarah se interact karega to abhi dislocation two ki main position fix kar leta hoon maan leta hoon ki ye jo dislocation two hai ye just above ek slip plane hai uske upar hai aur uska distance hai h yaani is position se is dislocation ke position se ye jo position hai ye h hai ye ek aur slip plane hai to jab ek ke paas jo stress field rahegi humne dekha tha ki ye positive dislocation hai yahan par ek stress field rahegi iski ye stress field mere dusre dislocation ke saath kis tarah se interact karegi ye hum jaanenge abhi to yaani hum ye keh sakte hain ki ye stress field ek force create karegi is dislocation ke upar dislocation two ke upar ye humne dekha tha ki glide aur climb force act hota hai humne jaana tha jab stress field ke baare mein padha tha ye configuration aur ye question maine bhi aapko pucha tha to yahan par hum dekhenge ki ye one jo dislocation hai wo two pe kaun se forces taiyaar karta hai aur wo taiyaar karne ke liye humein jaanna padega ye hum jaan sakte hain peach koehler equation se to hamare paas ek stress field hai dislocation one ki main yahan pe mark kar raha hoon to hamare paas $\sigma_{xx}\sigma_{yy}\sigma_{zz}\tau_{xy}\tau_{yx}$ kuch is tarah se stress field hogi dislocation one ki ye ye humne equations dekhe the aur yahan pe main burgers vector mark kar raha hoon to maine bola tha ki burgers vector jo hai kuch positive x direction mein to dono ke burgers vector $b[100]$ is tarah se likh sakta hoon aur tangent vector jo hai wo along z direction hai jo ki perpendicular hai is plane ko to tangent vector main kuch is tarah se mark kar sakta hoon $t[001]$ is tarah se mere paas ek burgers vector aa gaya tangent vector aa gaya aur abhi mere paas peach koehler equation hai to us tarah se main nikaal sakta hoon ki dislocation one ki stress field ye agar one ki main stress field maan ke chal raha hoon ye kaun se forces act karegi dislocation two pe to peach koehler analysis hum karte hain.

Strain hardening and dislocation interaction

$b, \bar{x} = \text{constant}$

$\gamma = \rho_m b \bar{x}$

Dislocation density increases

Dislocations have stress fields around them

Interaction of these stress fields Can affect the motion/movement of dislocations

Strain hardening in materials

Dislocation interaction

$b_1 = b_2 = b$

$$\vec{b} = b \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \hat{t} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-\nu)y} \frac{3x^2 + y^2}{(x^2 + y^2)^2}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-\nu)y} \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$$

$$\tau_{xy} = \tau_{yx} = \frac{Gb}{2\pi(1-\nu)x} \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

Abhi ye stress field hogi dislocation one ki abhi hum jaanenge ki forces kya ho sakte hain force per unit length of dislocation ye aayega F/L ye aayega σ ye jo stress field hai dislocation one ki b jo hoga ye mere dislocation two ka burgers vector hoga aur t jo hoga ye ye bhi dislocation two ki tangent vector hogi to ye humne dekha tha yahan pe force per unit length of dislocation to isko nikaalte hain abhi to $\sigma \cdot b$ pehle consider karte hain to mere paas σ_{xx} hai τ_{xy} hai τ_{xz} shunya hai

τ_{yx} hai σ_{yy} hai σ_{yz} zero hai usi tarah se σ_{zx} σ_{zy} zero hai aur mere paas σ_{zz} hai aur ye ho gaya mera burgers vector to agar main inka multiplication karunga to mere paas $\sigma_{xx}b$, $\tau_{yx}b$ aur shunya kuch is tarah se aayega abhi hum force nikaalenge force per unit length of a dislocation to main cross product lunga iska aur humne tangent vector nikaala tha tangent vector hai mera 0 0 1 yahan pe to main kuch is tarah se likhunga aur mere paas kuch relation is tarah se aayega f_l ye hai mera force per unit length of a dislocation $\tau_{yx}b - \sigma_{xx}bj$ to ye aa jayega mera force per unit length of a dislocation ye kaun sa force hai ye force humne nikaala hai dislocation one ke stress field se jo generate hoga mere dislocation two ke upar to ye force act ho raha hai mere dislocation two ke upar jo ki generate ho raha hai mere dislocation one ke stress field ke dwara to main f_l ko kuch is tarah se likh paunga mere paas τ_{yx} hai to main τ_{yx} ki value yahan se likh sakta hoon aur σ_{xx} ki value maine yahan se likh li hai to mere paas ye σ_{xx} ki value maine yahan se replace kar li abhi aap jaanenge yahan par y ki jagah maine h ki value replace ki hai to aap dekhenge ki ye jo y hai yahan par maine consider kiya kyunki y direction pe to ye h maine yahan pe consider kiya hai yaani maine y ko h ki tarah likha hai yahan pe is relations mein ab dekhenge yahan pe ye jo value hai ye i hai yahan par ye i direction pe i direction meri x direction pe hogi yahan pe yahan pe x direction hogi yahan par ye i lag raha hai aur ye yahan par j ye unit vectors hai mere x aur y direction pe to agar yahan pe ek i term hai to main keh sakta hoon ki ye jo force lag raha hai ye wala force lag raha hai ye is direction mein lag raha hai to ye jo force hoga ye mera glide force hoga aur ye jo j hai ye mera y direction par lag raha hai to ye hoga mera climb force to mark kar lete hain ye mera glide force hai aur ye mera climb force hai to aap dekh paa rahenge ki is dislocation ke stress field ki wajah se is dislocation par do forces lag rahe hain ek glide force aur ek climb force to inko aur detail mein hum dekhte hain.



Dislocation interactions

Peach-Koehler analysis:

$$\vec{f}_L = \underline{\sigma} b \times \hat{i}$$

$$\sigma \cdot b = b \begin{pmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sigma_{xx} b \\ \tau_{yx} b \\ 0 \end{pmatrix}$$

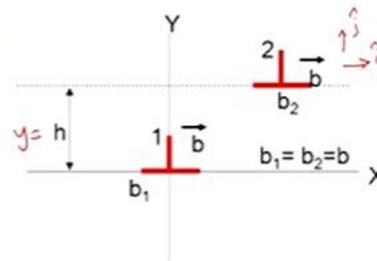
$$\vec{f}_L = \begin{vmatrix} i & j & k \\ \sigma_{xx} b & \tau_{yx} b & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\vec{f}_L = \tau_{yx} b \hat{i} - \sigma_{xx} b \hat{j}$$

$$\vec{f}_L = \frac{Gb^2}{2\pi(1-\nu)} x \frac{x^2 - h^2}{(x^2 + h^2)^2} + \frac{Gb^2}{2\pi(1-\nu)} h \frac{3x^2 + h^2}{(x^2 + h^2)^2}$$

Glide force

Climb force



$$\sigma_{xx} = \frac{-Gb}{2\pi(1-\nu)y} \frac{3x^2 + y^2}{(x^2 + y^2)^2}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-\nu)y} \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$$

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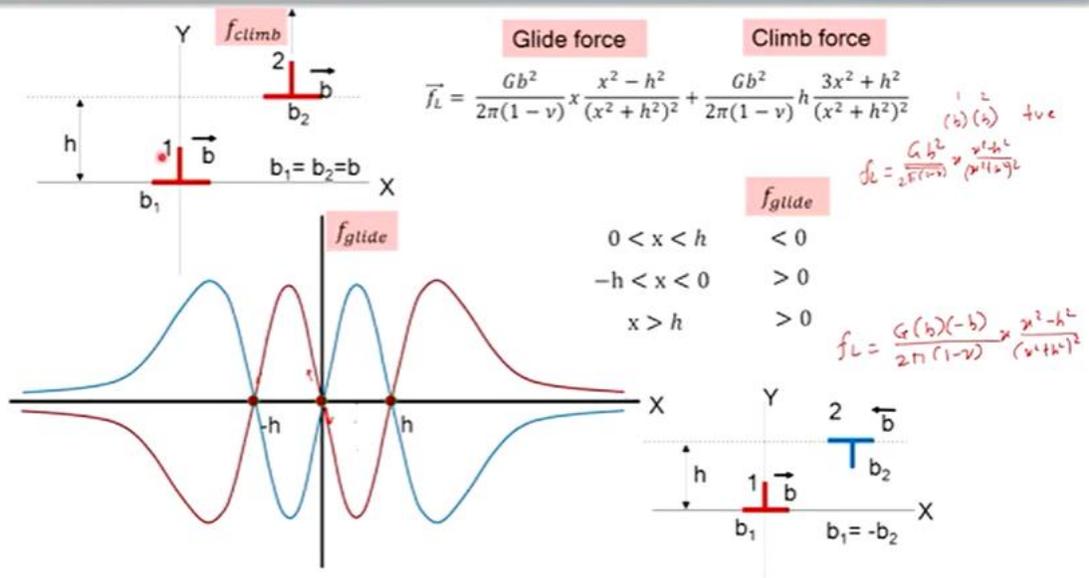
To mere paas ye configuration hai aur humne forces nikaale forces kaun se stress field jo hai dislocation one ke dwara two pe upar jo lag rahe hai forces unko to hum glide aur climb force likh lete is tarah se to yahan par abhi hum dekhenge ki climb force ek dash lag raha yaani is direction mein lag raha hai par hum jab glide karenge is dislocation ko is slip ke upar to force ka nature kis

tarah se change hoga hum dekhenge to hum dekhenge climb force mera is direction mein lag raha hai abhi hum yahan par consider nahi karenge climb force ab hum dekhte hain ki glide force kaise vary ho raha hai to main F_{glide} yahan par mark kar raha hoon aur main kya karunga main is dislocation ko is plane par move karne ki koshish karunga aap dekhenge ki is dislocation ko main is plane pe move karne ki koshish kar raha hoon tab jaake dekhte hain ki glide force kaise vary ho raha hai to mere paas kuch conditions hai to agar main glide force nikaal raha hoon to aap dekh paayenge jo glide force hoga wo x jab value shunya rahegi yaani is position pe aayega tab x ki value agar main shunya likhta hoon yahan pe to ye jo glide force hai wo shunya ho jayega ye yahi term main consider kar raha hoon abhi ye jo term hai wo shunya ho jayegi to glide force mera zero rahega jab x ki value shunya rahegi abhi hum dekhte hain ki yaani x ki value shunya ya ye jo dislocation hai wo is position par aayega tab x ki value shunya ho jayegi yaani yahan se main x yahan par zero hai x ki value abhi hum dekhenge aur kuch conditions par ki jab x ki value h ho jayegi yaani x agar ho jayega $x^2 - h^2$ agar x ki value h ho gayi to ye jo term hai ye phir se shunya ho jayegi to mere paas ek aur ek point aa gaya yahan par glide force shunya ho jayega yahan usi tarah se agar x ki value $-h$ hogi to bhi ye shunya hoga mere paas glide force ki kuch is tarah se values aa gayi teen values aa gayi abhi hum dekhte hain kuch aur condition to hum F_{glide} yahan par mark karenge aur hum ek condition consider kar rahe ki jab x ki value meri shunya aur h ke beech mein rahegi h kya hai mera interplanar distance hai yaani do glide plane ke beech ka ek distance hai to x ki value agar shunya se badi hai shunya se badi hai aur h se choti hai to aap dekhenge ye jo term hai ye term negative ho jayegi aur x mera positive hai is side hai agar main is side hoon to agar x positive hai to ye jo value hai glide force mera negative ho jayega mere paas glide force kuch is tarah se milega abhi hum dekhenge ki is condition mein agar mera x minus h se bada hai aur shunya se chota hai yaani mera x is side hai to aap dekhenge ki ye x ki value negative hai yahan pe aur ye jo value hai $x^2 - h^2$ ye bhi value negative aayegi to negative aur negative banke positive ho jayega to glide force mere paas aa jayega ye greater than shunya aayega abhi hum dekhte hain ki x ki value h se badi hai aur x positive hai to hum dekhenge ki x ki value h se badi hai to $x^2 - h^2$ kya aayega ye aa jayega positive aur x agar positive hai yaani main is side jab baat karunga to ye positive aur positive to glide force mere paas aa jayega positive greater than zero abhi hum ye agar relation acche se dekhenge $x^2 - h^2$ aur in x to aap dekhenge ye jo value hai wo ek maximum reach karegi yahan par agar ye domain main consider kar raha hoon agar x ki value shunya aur h ke beech mein hai to glide force to mera negative hai negative yaani main is direction mein jaunga kuch is direction mein jaunga to aap dekhenge ki kahin to bhi meri ek ye jo glide force hai wo maximum value reach karegi negative side par yaani kahin to bhi yahan par wo maximum value reach karegi kisi h value ke liye aur yahan par aap dekhenge ki agar x h se bada hai to glide force positive hai to bhi aap dekhenge ki ye jo value hai wo kisi ek value ke liye maximum reach karegi mera jo glide force hai uska nature kuch is tarah se aayega aap dekh paa rahenge humne dekha ki kisi particular h ke liye ek mujhe maximum value mil rahi hai is glide force ke liye aur jab x h se bhi bada hai to main particular h ke liye mujh mere paas ye value milegi x ki uske liye glide force maximum rahega usi tarah se hum dekhenge agar is tarah se to humne dekha tha ki agar x minus h se bada hai aur shunya se chota hai to glide force ki value positive hai to yahan pe bhi hum dekh paa rahenge ki glide force kuch is tarah se jayega aur ek maximum value yahan pe milegi kahin to h ki value aur yahan pe ye phir se yahan pe zero aa rahi hai to kuch glide force is tarah se badhega phir ghatega phir wo negative h ke liye aur negative x ke liye negative rahega to aap dekh paa rahenge jo glide force ka variation hai mera kuch is tarah se ho raha hai to ye jo ho gaya mera glide force kis tarah se vary ho raha hai jab agar mera dislocation yaani maine yahan pe dislocation is tarah se mark kiya tha agar is plane pe main glide

karunga is dislocation ko to ye jo glide force hai kuch is tarah se mujhe milega iska variation kuch is tarah se milega aur ek glide force ki value shunya kab hoti hai teen position par shunya hogi jab $x = 0$ rahega $x = h$ rahega aur $x = -h$ rahega abhi hum dekhte hain agar mera ye dislocation maine to dono dislocation positive dislocation maane the agar mere dono dislocation yaani ek dislocation negative hai yaani ek positive dislocation hai aur ek negative dislocation hai negative dislocation hai agar main isko is tarah se move karunga tab meri iski glide force ki value kis tarah se vary honi chahiye to aap dekh paayenge ki agar main isko plot karunga tab main ye b_1 ki jagah main minus b_2 rakhunga yahan par to jab main is tarah se plot karunga ye b ki jagah main minus b_2 rakhunga to yaani yahan par Gb^2 aa raha hai ye Gb^2 kuch is tarah se aa raha hai aap isko is tarah se samajh sakte hain jo force acting per unit length hai wo Gb^2 main sirf ye term likh raha hoon baaki ki se term to same hogi ye b mere burgers vector one jo dislocation one ka burgers vector tha aur two ka jo vector tha ye rahega is tarah se to isliye ye b^2 aa raha tha agar main ye do jab positive rahenge ab do negative rahenge baaki ki terms same rahegi ye $2\pi(1 - \mu)x(x^2 - h^2)/(x^2 + h^2)^2$ ye same rahega is condition mein glide force main kuch is tarah se likhunga G pehle dislocation ka burgers vector dusre dislocation ka burgers vector minus b upon $2\pi(1 - \mu)x(x^2 - h^2)/(x^2 + h^2)^2$ to aap dekhenge ki ye term jo hai wo negative ho gayi to jo bhi yahan par glide force ka variation mil raha hai ye dono positive hai unke liye mil raha hai agar ye ek positive ek negative hai uske liye mujhe just reverse milega kyunki mera sign sirf change ho raha hai yahan par positive se negative ho jayega to mujhe kuch positive x mein kuch is tarah se variation milega aur negative x mein mujhe kuch is tarah se variation milega to ye humne yahan pe dekha ki glide force kaise vary ho raha hai glide force yaani kaun sa force hai ye force act ho raha hai mere dislocation two ke upar jo mere dislocation one ke stress field ke dwara agar wo positive dono positive hai to wo ye red line ki tarah ye vary hoga agar ye ek positive ek negative hai to ye blue lines ki tarah vary hoga to yahan par bhi aap dekh paa rahenge jo glide force hai wo shunya rahega mere $x = 0$ $x = h$ aur $x = -h$ to yahan par humne dekha ki dislocation ke interaction ke wajah se jo glide force variation hai kis tarah se ho raha hai to ye humne do simple cases yahan par consider kiye hai



Dislocation interactions



Abhi hum dekhenge next part se hum dekhenge ki jab ye interaction hote hain to ye hardening kaise ho sakti hai hum strain hardening ki baat kar rahe hain us context mein us sthiti mein hum dislocation interaction ki baat kar rahe hain to yahan par maine do cases dikhayi hai to next part mein hum jaanenge ki ye dislocation interaction ki wajah se kuch stable configuration banenge dislocation ki to wo stable configuration strain hardening ko kis tarah se generate karti hai kis tarah se affect karti hai ye hum jaanenge agle part se abhi ke liye yahan par rakhta hoon dhanyavad.