Mechanical behavior of materials

Dr. Niraj Mohan Chawake

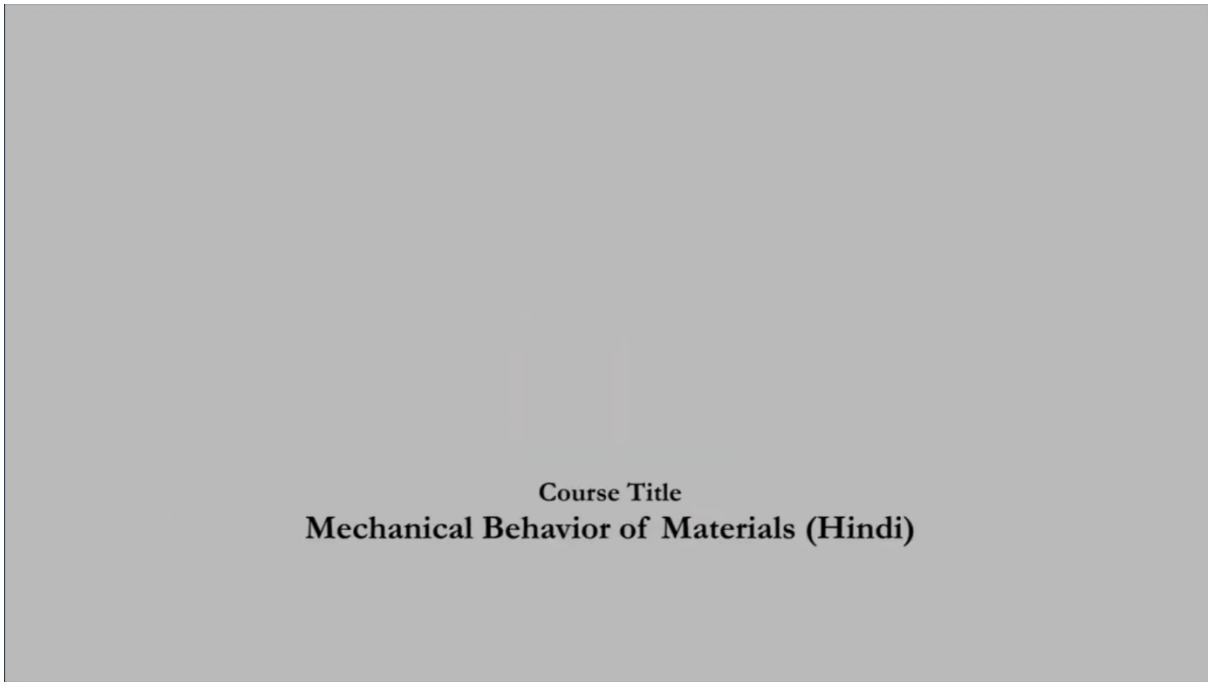
Department of Materials Science and Engineering

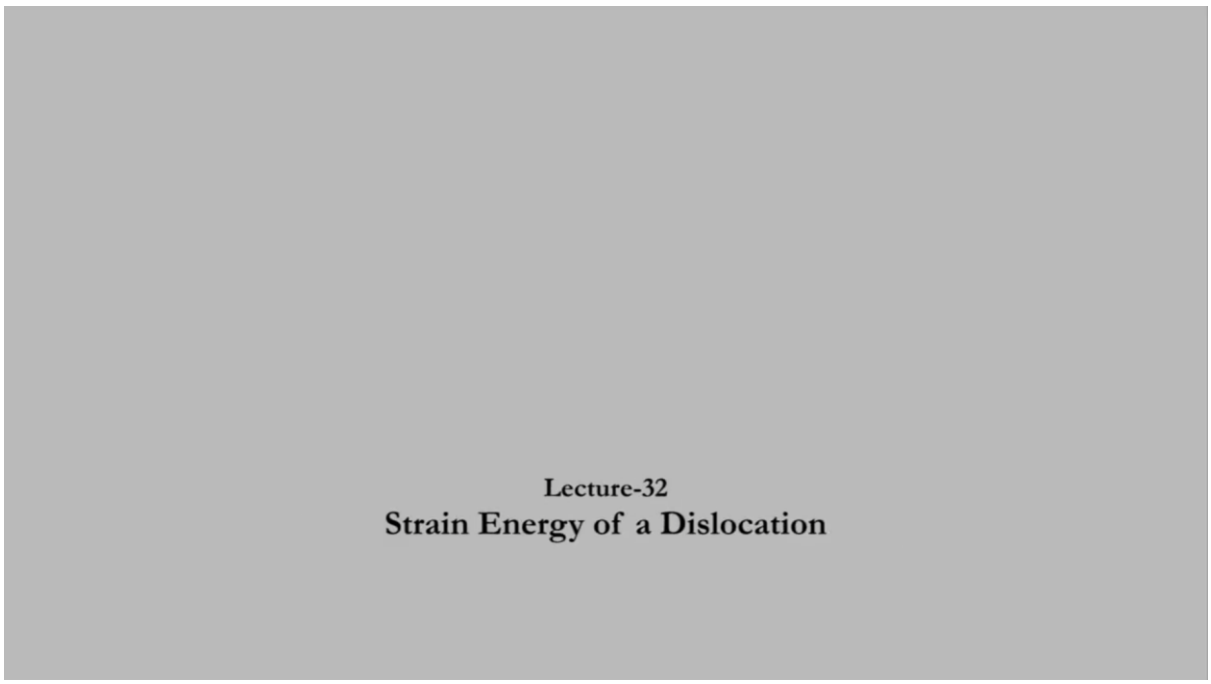
Indian Institute of Technology, Kanpur

Week-2

Lecture-32

Strain Energy of a Dislocation





Namaskar phir se swagat karta hoon aapka is course mein mechanical behaviour of material Hindi mein hum padhenge isko to last part mein humne dekha tha ki stress fields kya hoti hai edge aur screw dislocation ke liye is part mein hum jaanenge ki strain energy of dislocations kya hoti hai to humne strain energy dekhi thi elastic strain energy ki main baat kar raha hoon abhi is part mein aur is slide mein hum dekhenge elastic strain energy of a screw dislocation kya hoti hai to yeh humne dekha tha humne ek Volterra cut se ek screw dislocation tayar kiya tha yahan pe mere paas coordinate axis hai aur yeh jo. Dislocation line hai yeh along z axis hai yaani is cylinder ke axis ke around along hai aur yeh jo displacement hai yeh parallel hai mere yeh is axis se to yeh jo yahan par Burgers vector hai aur yeh Burgers vector yeh jo hai parallel hai is line ke liye to yeh mera Burgers vector ho gaya abhi hum dekhenge ki yeh jo line tha yeh infinite bhi tha aur humne dekha tha ki jo displacement se agar main x plus y plane mein move hota hoon to mere displacement z plane mein badhte hain yeh aur z direction mein badhte hain to yeh humne dekha tha. Abhi hum dekhe the ki iski strains kya hai strain tensor kya hai is screw dislocation ka humne stress tensor bhi dekha tha abhi hum iski elastic strain energy is tarah se define kar sakte hain to agar humein koi bhi stress state pata hai kisi bhi point par ismein to main iski energy nikal sakta hoon energy kis tarah se nikalenge to main elastic stored energy per unit volume nikal sakta hoon humne derive kiya tha aur kuch is tarah se likha tha yeh simplified ya expanded form hai to main elastic strain energy kuch is tarah se likh sakta hoon 1/2 of σij εkl main kuch is tarah se likh sakta hoon. To agar mere paas ek generalized relation hai aur humne is tarah se isko derive kiya tha main stresses ke terms mein likh sakta hoon agar mujhe stress field pata hai to main sirf stress mein hi energy likh sakta hoon to humne is tarah se dekha tha ki main isko sirf is tarah se convert kar sakta hoon $1/2E\ \sigma ij^2$ to ya agar shear stresses hai to main kuch is tarah se likh paoonga so yeh hum yeh tab kar sakte hain jab mera material isotropic hai to main material ko mera isotropic maan raha hoon humne yeh jo derivations kiye the screw dislocation ke liye isotropic material ke liye kiye the. To mere paas ek kuch is tarah se energy aa gayi yeh jo hogi meri elastic strain energy jo ki aa rahi hai normal stresses ki wajah se aur yeh jo aa rahi hai meri shear stresses ki wajah se abhi humne dekha tha ki jab screw dislocation ki baat ki thi to mere paas yeh do stresses the τxz aur τyz aur yeh derivations mein humne dekha tha yeh is tarah se aa rahi hai agar main inka square kar loon yahan pe to yeh jo value hai yeh zero ho jaayegi kyunki mere paas yeh koi stresses hai hi nahi available hi nahi hai. To main iska stress tensor bhi likh leta hoon to mere paas kuch is tarah se stresses the humne dekha tha yeh thi τxz aur τyz to yeh yahan par zero hai yeh τzx aur τzy mere paas yeh kuch is tarah se stress field thi is screw dislocation ke around to yeh jo terms hai yeh zero ho jaayegi yahan se iski wajah se koi contribution nahi hai last energy inko square karke main add karta hoon to mere paas kuch is tarah se value aayegi jo elastic strain hai per unit volume to aapko yaad rakhna hai ki elastic stored energy hai per unit volume to yeh equation aa jaayega mere paas. Isko main solve karta hoon to kuch is tarah se aayega yeh jo yeh hai $\{(Gb^2)/8\pi^2\}\{1/(x^2 + y^2)\}$to yeh jo hai yeh meri elastic stored energy per unit volume hai to abhi main isko is tarah se bhi likh sakta hoon $1/(x^2 + y^2)$ ko is tarah se bhi likh sakta hoon $1/r^2$ to humne yeh bhi dekha tha ki agar main kuch is tarah se likh raha hoon mera theta hai aur to yeh y ho jaayega yeh x ho jaayega aur yeh r tha $r^2 = x^2 + y^2$ yeh bhi humne dekha tha. Abhi main kya kar raha hoon yeh meri total energy hai elastic stored energy per unit volume abhi main ek annular ring consider kar raha hoon is dislocation pe kuch is tarah se main usko mark kar leta hoon yeh jo hai ek kuch annular ring mein yahan pe consider karoonga jiski thickness rahegi dr yeh jo thickness hai yeh rahegi dr aur yeh jo annular ring hai isko main ek hollow cylinder mein bhi consider kar sakta hoon annular nahi chhota sa part mein yahan par consider kar raha hoon yahan par yeh maan lijiye throughout hai yahan par aur yeh cylinder ho gaya mera.

Strain energy of a screw dislocation

We can evaluate if a Stress state at a point is known

Elastic stored energy /volume

$$U = \frac{1}{2E}(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2) + \frac{1}{2G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)$$

$$\tau_{xz} = -\frac{Gb}{2\pi}\frac{y}{x^2+y^2}$$

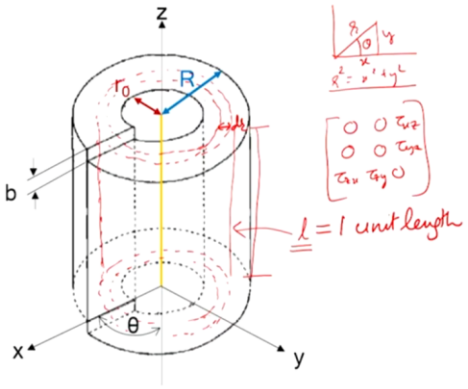
$$\tau_{yz} = \frac{Gb}{2\pi}\frac{x}{x^2+y^2}$$

$$U(x,y) = \frac{1}{2G}\left[\left(\frac{Gb}{2\pi}\right)^2\left(\frac{x^2}{(x^2+y^2)^2} + \frac{y^2}{(x^2+y^2)^2}\right)\right]$$

$$U = \frac{Gb^2}{8\pi^2}\frac{1}{x^2+y^2}$$

$$\frac{U}{V} = U_r = \frac{Gb^2}{8\pi^2}\frac{1}{r^2}$$

$$U = \frac{Gb^2}{8\pi^2}\frac{1}{r^2}2\pi r dr(l)$$



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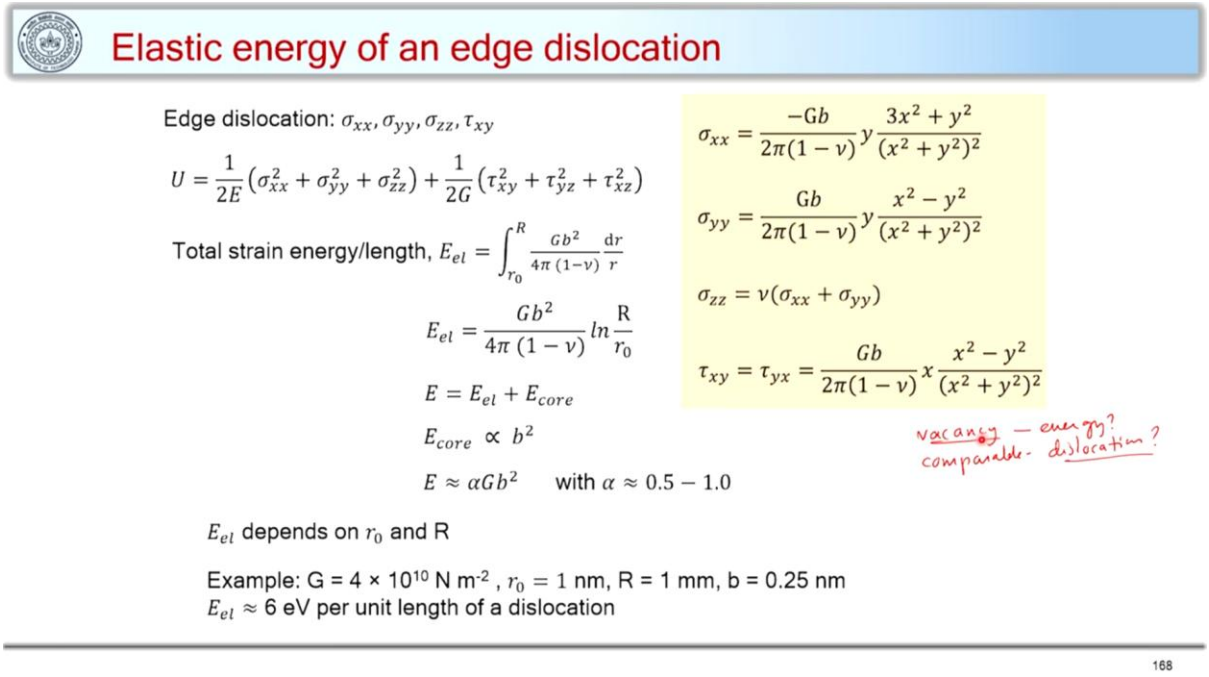
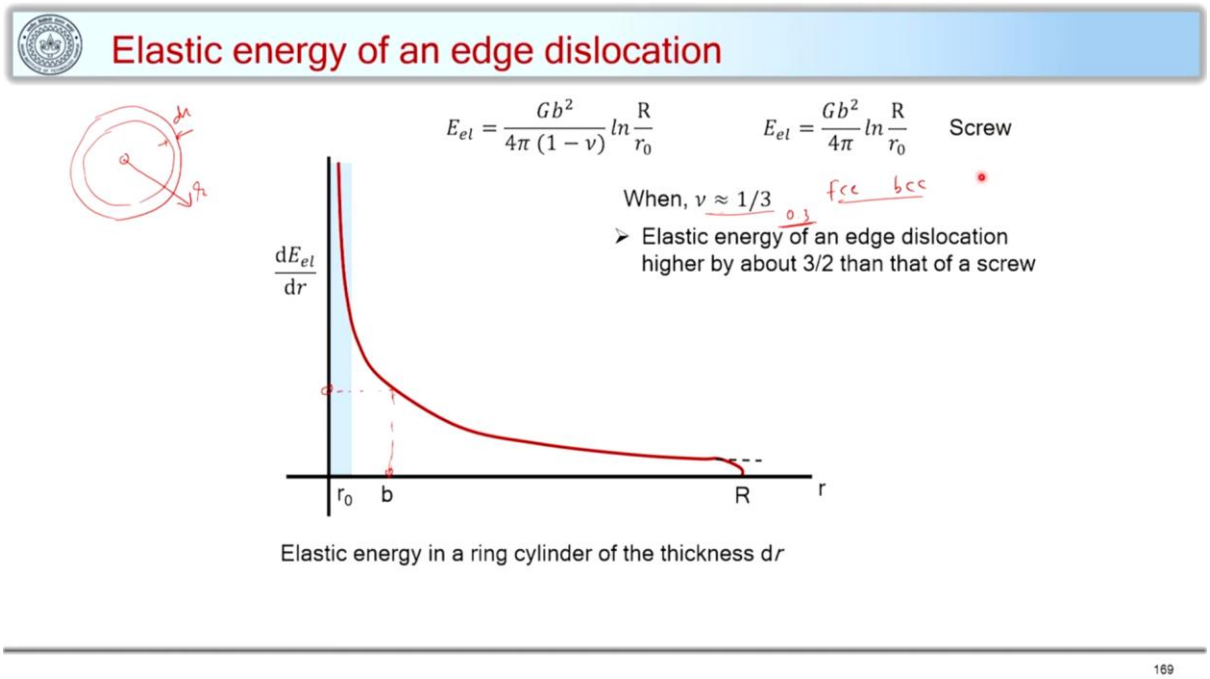
main usko solve karta hoon to mere paas yeh kuch is tarah se relation aayega to yeh jo hai agar main l ko is tarah se likhoonga total strain energy per unit length mere paas kuch is tarah se aayegi jab main consider karoonga jaise humne ek annular ring consider ki thi abhi yaani humne dekha tha yahan pe ek r_0 tha aur yeh R tha aur humne consider kiya tha small annular ring ke liye yahan pe dr annular ring ke liye dr is tarah se consider kiya tha. Agar main isko integrate karta hoon to mere paas kuch total strain energy per unit length aayegi aur main kuch isko r_0 to R yeh limits rakh sakta hoon kyunki yahan par hum dekhenge ki elasticity theory valid nahi hai to hum yeh core of dislocation maan lenge r_0 to R aur yeh jo aayegi isko agar main integrate karta hoon is poore r direction pe to yeh jo term hai isko mujhe integrate karna hoga to mere paas kuch values is tarah se aayegi yeh hogi meri total strain energy per unit length of a dislocation to yeh jo energy hai yeh per unit length hai yeh aapko yaad bhi rakhna hai. To yeh $\{(Gb^2)/ 4\pi\}\{\ln R /r_0\}$ ek important result hai dislocations ke energy ke liye yeh agar mere paas relation hai yeh mera relation agar main agar total energy dekhoonga dislocation ki to yeh jo hai yeh dislocation ki energy yeh energy jo hai dislocation ki energy plus core ki energy jisko hum calculate nahi kar paate elasticity theory se yeh equal to elastic energy ho jaayegi aur yeh jo E_{core} hai yaani core of dislocation yeh aur dislocation ke mil total energy ho jaayegi mere dislocation ki. Aur agar main yahi identity use karta hoon total energy per unit volume ke liye to mere paas σxx σyy σzz hai aur τxy hai yahan par to agar yeh values main yahan par put karta hoon to main kuch is tarah se nikal paoonga total strain energy per unit length wahi method hum employ karenge r_0 to R yeh jo value aayegi add karke yeh aayegi $\{Gb^2 /4\pi(1 – \nu)\}\ln (R/r)$ isko hum integrate karenge to mere paas kuch is tarah se relation aayega to yeh relation hum phir se use kar sakte hain aur yahan par hi sirf difference hai edge dislocation ke liye yahan pe bottom mein denominator mein $4\pi \times(1 – \nu)$ hai. Aur screw dislocation ke liye bhi hum phir se wahi likh sakte hain ki elastic energy jo yeh energy hai plus uske core ki energy aur core ki energy humne abhi dekhi thi ki wo directly proportional to b square hai agar screw bhi hai ya edge bhi hai to E_{core} ki value hum directly proportional to b square likh sakte hain aur agar hum total energy likhenge to total energy hum kuch is tarah se likh sakte hain αGb^2 yahan pe bhi aap dekhenge jo elastic energy b square se proportional hai directly proportional to b square to main kuch is tarah se likh sakta hoon αGb^2 agar main in dono ko add karta hoon to to yeh jo α ki value hai. For most of the metals and alloys bahut saare metals aur alloys ke liye point five to one ke beech mein vary karti hai yahan par G jo hai

mera shear modulus ab jaante hain ki yaani jo elastic energy hai yeh aap is relation se dekh paayenge ki yeh depend karta hai r yaani kis position par aap dekh rahe hain aur r_0 yaani mere core ka jo dimension hai uske upar depend karta hai to ek example lete hain agar mere paas kuch material hai G jiski shear modulus itni hai 4×10^{10} newton per meter square aur r_0 one nanometer hai aur at a distance of r agar main dekh raha hoon yeh at a distance of one millimeter pe dekh raha hoon aur uska Burgers vector 25 hai to us dislocation ki energy. Per unit length kuch is tarah se yeh aayegi six electron volts per unit length of dislocation yaani ek dislocation ke per unit length ki jo energy hogi woh six electron volt hai ab ek exercise mein dekhta hoon ki koi bhi vacancy tayaar karne mein aapko kitni energy chahiye yeh aapko dhoondhna hai aur aapko yeh bhi dhoondhna hai ki yeh comparable kya hogi dislocation ke sath. aap usko dislocation ke saath compare kar sakte hain to aap yahi values aap choose kar sakte ho aur dekh sakte ho ki dislocation ki energy aur vacancy formation ki energy kya kitna antar hai usmein



Abhi hum jaante hain ki elastic energy of dislocation hum is tarah se kuch likhe the edge dislocation ke liye agar main is energy ko wo jo small dr annular ring humne consider ki thi usse agar differentiate karoon energy in annular cylinder karoon thickness dr humne yeh consider kiya tha us se differentiate krta hu to main kuch is tarah se plot kr paunga yaani main elastic energy in a ring cylinder krunga thickness r humne ye consider kiya tha yahan pe yeh main dr hai aur isko agar main differentiate krta hu is energy ko yani ye energy kaise change ho rhi hai karoon agar meri cylinder ki thickness change ho rahi hai to iske hisaab se kaise change ho rahi hai to aap dekh paayenge is direction pe main r plot karoonga aur yahan pe main plot karoonga ki yeh energy r ke hisaab se kis tarah se change ho rahi hai. Yaani is direction mein kis tarah se change ho rahi hai yeh hum dekhenge to humein kuch is tarah se relation milega yaani is tarah se variation milega to aap dekhenge ki jaise jaise mera r badh raha hai yaani main dislocation core se door ja raha hoon yahan par mark kar lete hain hum jitna jitna main dislocation core se door ja raha hoon utna energy jo hai wo decrease ho rahi hai yaani change in energy wo decrease ho rahi hai aur yeh jo energy hai aap dekhenge kuch cases ke liye jab b rahegi us hisaab se main dislocation ki energy consider karoonga yahan par b ki value ke hisaab se main apne dislocation ki energy mark karoonga. Yeh agar meri b hai yaani mera Burgers vector hai uske correspondingly jo value rahegi wo main consider krungaa dislocation ke calculation me. Ye ek important observation hai, yaha pe aap dekhenge is part me r_0 pe jo value hai vo infinity ki taraf ja rahi hai yaani infinity badh rhi hai jo energy hai dislocation ki

is part me define nahi kar sakte, r_0 ke neeche define nhi kr sakte. yahan par bhi aap dekhenge ki jab r equals R yaani small r equal to capital R tab aap dekhenge ki yeh jo value hai wo zero par aani chahiye kyunki hum change dekh rahe hain to change humara zero par aana chahiye waise is tarah se hum dekhenge ki jo energy hai wo ghatti jaayegi jis tarah se main is r direction pe move ho raha hoon. To yeh mere edge dislocation ki energy yahan par maine plot ki thi ki change kaisi ho rahi hai with respect to r screw ke liye main kuch is tarah se likh paoonga yahan par maine bataya tha ki ek hi difference hai yahan par $(1 - \nu)$ jo term hai ν jo hai mera Poisson ratio hai yeh term zyada hai yahan par denominator mein edge dislocation ke liye to screw ke liye yeh formula humne dekha tha agar hum dekhenge ki most of the metals ν ki value one third ke aas paas hoti hai FCC BCC materials ke liye to aap dekhenge ki jo value hai agar main one by three yahan par rakhta hoon to aap jo elastic energy of an edge dislocation hai higher by about three by two than screw dislocation. To kisi bhi material ke liye jaise FCC ya BCC material ke liye yeh value close to 0.33 rehti hai to yeh value jab rahegi to aap dekhenge ki jo edge dislocation ki energy hai wo screw dislocation se zyada rehti hai to is part mein humne dekha ki strain energy humne nikali thi screw dislocation ki aur edge dislocation ki aur yahan par mujhe pata chal raha hai ki yeh jo elastic energy hai edge dislocation ki pure edge dislocation ki zyada rehti hai screw dislocation ki comparison mein to



yahan par hi main rukta hoon is part mein humne dekha ki humne jo stress fields thi edge aur screw dislocation ki usse hum elastic strain energy kaise nikal sakte hain aur humne yeh bhi dekha ki most of the material ke liye jab Poisson ratio ki value positive hoti hai to aap dekhenge ki edge dislocation ki energy screw dislocation se zyada hoti hai aur hum jo yeh jo elastic energy hai hum yeh core of the dislocation pe nahi nikal sakte kyunki wahan par humari elasticity theory fail hoti hai abhi hum jab aage jaayenge next parts mein hum in jo derivations kiye the elastic strain energy ke iska use karenge humare dislocation interactions ke liye abhi ke liye rukta hoon dhanyavaad