Mechanical behavior of materials

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Week-6

Lecture-31

Stress field around an edge dislocation

Namaskar aapka swagat karte hain is course mein Mechanical Behavior of Material jo ki hum Hindi mein padhenge. Last part mein humne dekha tha ki Stress Field Around Screw Dislocation kya hai. Is part mein hum dekhenge ki Stress Field Around Edge Dislocation kya hai.To uske liye pehle sabse pehle hum consider karte hain ek Volterra Cut se hum dislocation taiyar karte hain jo ki hamara Edge Dislocation hai. Yahan pe maine coordinate axis mark kar liye x, y aur z aur hum dekhenge ki z jo hai yeh meri axial direction pe hai yani mera jo cylinder ka axis hai wo z direction pe aur yeh r jo hai mere radius hai cylinder ka aur jo hamara Tangent Vector hai usko mark kar lete hain hum us Tangent Vector ko hum z direction pe hi consider karenge to ye ho gaya mera Tangent Vector is direction pe isko hum kahenge Tangent Vector aur aap dekh payenge ki jo mera Burgers Vector hai kuch is tarah se is direction mein to maine Volterra Cut is direction mein yani set ko is perpendicular kiya hai to main dekh paun dekh pa raha hoon ki jo Burgers Vector hai wo perpendicular hai mere Tangent Vector ke saath. To Tangent Vector yahan pe mujhe perpendicular mil raha hai vector ka yeh mere Edge Dislocation ka definition hai.

To Screw Dislocation mein humne kya kiya tha Screw Dislocation mein humne nikale the Displacements kya hai aur un displacement ko usko bhi likh likhte hain humne ek strategy ki thi ki pehle hum Displacements nikalenge Displacement ke baad hum Strains nikalte hain aur Strains ko hum Elastic Stress Strain Relation se hum Stresses nikalte hain is tarah se hum Stress Around Dislocation nikal sakte hain.

To yahan par aap dekhenge jo Volterra Cut se humne displacement taiyar ki hai wo kuch is plane mein hai us plane ko bhi hum mark kar lete hain to us plane ko mark karne ke liye dusra color main istemaal karna chahta hoon to mark kar lete hain hum ye is yeh plane agar aap dekhenge is plane mein mere displacement hai yeh mera plane jo hai yeh mera plane hai xz plane to aap dekhenge ki jo displacement ho rahe hai saare xz plane mein ho rahe hai abhi main kuch displacement yahan par mark kar kar raha hoon haan uske pehle maine baat ki thi ki jo line hai Dislocation Line hai yeh jo line hai along mere axis hai aur z direction ke along hai aur yeh jo line hai wo infinite hai to yeh assumption hum yahan par bhi maan rahe hain to yahan par hum dekhenge ki mere z direction ke along koi displacement nahi hai to w jo hai wo shunya rahega aap dekhenge ki ye jo displacement ho rahe hai yeh mere xz plane mein ho rahe hai aur agar xz plane mein ho rahe hai to mere paas jo displacement aa rahe aayenge wo u aur v rahenge aur wo non-zero rahenge wo shunya nahi rahenge yani humein jo displacement mil rahe sirf xz plane mein mil rahe hai to yeh jo condition hai is condition ko hum kehte hain **Plane Strain Problem** yani yahan par aap dekhenge ki mere jo displacement hai sirf ek hi plane mein hai xz plane mein is type of problem ko kehte hain Plane Strain Problem.

To humein jo agar Stress Field nikalni hai to humein yeh Plane Strain Problem solve karna padega to is course ke liye jo Elasticity Theory chahiye Plane Strain Problem ko solve karne ke liye yeh is course ke scope ke bahar hai to yahan pe main direct answers likh raha hoon ki jo Stress Stresses jo aayenge hum Displacement nahi humne Displacement likh liye par hum abhi Strain Fields nahi likh rahe hum direct Stress Fields likhenge is Dislocation ke liye kyunki yeh jo Linear Elasticity Theory jo chahiye yeh problem solve karne ke liye yeh jis type ka mathematical analysis chahiye hamare scope ke bahar hai out of scope of this course kyunki elementary course hai to aapko higher level ka Elasticity Theory chahiye isko samajhne ke liye agar aap interested hai to kisi bhi standard text book ko aap refer karenge to aapko ye Strain Fields bhi mil jayenge. To yahan pe hamara interest find out karna hai ki Stress Field kya hai Edge Dislocation ke around aur ek cheez main bolna chahta hoon yahan pe jo Stress Field hai wo hum Core of the Dislocation nahi nikal sakte yeh ek point aur aapko yaad rakhna hai ki hum Away From Core of Dislocation nikalte hain kyunki jo Elasticity Theory hai wo yahan pe applicable nahi hoti to aap dekh payenge ki jo Stress Field hum nikalenge yeh away from the core hoga yani Dislocation ye jo along z axis iske away hum nikalenge.

To yahan pe abhi Stress Field hum nikalte hain to σ_{xx} ko main kuch is tarah se mujhe relation milenge x aur y ke isme to:

$$\sigma_{xx} = -\frac{Gb}{2\pi(1-v)}\frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$

Phir σ_{yy} kuch is tarah se mujhe milega:

$$\sigma_{yy} = \frac{Gb}{2\pi(1-v)}\frac{y(x^2-y^2)}{(x^2+y^2)^2}$$

Yahan pe dekhenge ki yeh jo term hai yeh change ho rahi hai $x^2 - y^2$ aa rahi hai yahan par aur ye sign change ho raha hai yahan par minus tha yahan par plus hai. σ_{zz} mujhe kuch is tarah se milega yeh isliye milega kyunki ϵ_{zz} yahan par shunya hai kyunki w jo hai yahan par zero hai to main σ_{zz} ko kuch is tarah se likh sakta hoon kyunki aap ϵ_{zz} ko is tarah se likh sakte $1/E$ humne ye relation dekhe the:

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$$

To agar ϵ_{zz} shunya hai to hamare paas yeh identity aa jayegi aur hum ϵ_{zz} ko agar hum dono ko is tarah se likhenge to mere paas σ_{zz} bhi aa jayega. Shear Stresses ke liye likhte hain hum τ_{xy}, τ_{yx} yeh hum likhenge to mere paas yeh τ_{xy} aur τ_{yx} bhi value aa jayengi aur yeh jo values hai $\tau_{xz}, \tau_{yz}, \tau_{zx}, \tau_{zy}$ yeh shunya rahegi.

To hum Stress Tensor likh sakte hain σ Stress Field Around Edge Dislocation mere paas σ_{xx} hai σ_{yy} hai mere paas σ_{zz} hai aur mere paas τ_{xy} hai mere paas τ_{yx} hai aur mere paas τ_{xz} hai jo shunya hai τ_{zx} hai ye shunya hai τ_{yz} ye shunya hai τ_{zy} ye shunya hai ye mere Stress Field ho gayi Edge Dislocation ki. To aap dekh pa rahe honge yahan par mere paas Normal Stresses hai aur Shear Stresses bhi hai to Shear Stresses aur Normal Stresses hai to mere paas jo Edge Dislocation hai uske around kaun si Stress Stresses rahenge main dono dono Stresses reh sakte hain Hydrostatic Stresses bhi reh sakte hain aur Deviatoric Stress Field bhi reh sakti hai. To ab dekhenge hum humne Screw ke liye likha tha humne Screw ke liye likha tha mere paas sirf Deviatoric rehti hai Hydrostatic nahi rehti hai to aap iska implication hum dekhenge ki jab ye Screw aur Edge kisi Stress Field ke saath ya ek dusre ke saath interact karte to hum iska istemaal karenge.

Abhi jante hain ki ye Stress Field Edge Dislocation around kaise hoti hai to ye mere paas equations ho gaye ye humne dekhe maan lete hain mere paas ye xy coordinate maine likh liya yahan pe aur ek dislocation maante hain ki origin pe yahan pe center pe yahan pe to abhi main is quadrant isme quadrant ko divide karta hoon further jahan pe main mark kar raha hoon $x = y$ aur $x = -y$ iska

reason bhi aapko pata chal jayega. Abhi hum mark karenge kuch elements yahan pe aur us elements pe state of stress kya ho sakti hai yeh dekhte hain ye humne ek plane mark kiya hai xy plane is xy plane pe kya state of stress hai ye dekhenge aap dekhenge ki ye mere paas ek Positive Edge Dislocation hai aur is Positive Edge Dislocation ke around kya stress field hai yeh hum dekhenge abhi. To ye maine is tarah se kuch elements mark kar liye yeh mere aath elements is tarah se mark kar liye abhi hum one by one dekhte hain ki meri stress field kya hai.

To sabse pehle hum mark karte σ_{xx} σ_{xx} ke baare mein hum dekhenge to yahan pe hum dekhenge ki ye σ_{xx} ka sign kis pe depend hona chahiye yahan pe aap dekhenge yahan pe x^2 ka term hai yahan pe positive hai aur y^2 ka term hai yahan pe $(x^2 + y^2)^2$ term hai to mera jo sign rahega σ_{xx} ka wo depend karega y ke sign pe to agar mera y positive hai ya agar mera y negative hai yahan pe abhi maine mark kiya agar mera y negative hai yani is direction pe isko quadrant pe aap dekhenge to ye negative aur negative positive ho jayega jo σ_{xx} aayega σ_{xx} aayega mera positive yani wo tensile nature aayega to hum yahan pe ek humne sign conventions likhe the Normal Stresses ye agar is direction mein hai tensile ye positive direction pe hai to hum isko positive kehte hain aur ye tensile stress ho gaya ye hamara sign convention hai. To hum yahan maine yahan par jo chaar elements hai kyunki y negative hai is side par to yahan par ye jo σ_{xx} aa raha hai ye positive aa raha hai tensile aa raha hai usi tarah se aap dekhenge ki agar y positive hai to yahan pe y positive ho jayega aur x aur y ka koi koi bhi sign ho to aap dekhenge ki y ka sign dominate karega yani y ka sign decide karega σ_{xx} ki value agar y ka sign positive hai to negative into positive yeh negative ho jayega to aap dekhenge yahan par jo stresses aayenge ye compressive stresses aayenge to is tarah se maine compressive stresses mark kar liye ye σ_{xx} kyunki ye plane mera x plane hai aur is direction pe normal stress jo hai wo σ_{xx} rahega ye maine σ_{xx} is tarah se mark kar liya mere paas upar mein jo stress hai wo compressive hai neeche mein hai wo tensile stress hai σ_{xx} ki value.

Phir abhi hum dekhenge σ_{yy} ki value usi tarah se abhi aap dekhenge ki yahan pe to positive sign hai aur yeh jo value hai y ki yeh abhi yeh bhi decide karegi aapki σ_{yy} ki value aur yeh jo term hai $x^2 - y^2$ ye iski jo value hogi yeh bhi decide karegi ki aapka σ_{yy} kis nature ka hoga maan lete hain mera x greater hai y se yani wo kis case mein hoga x greater is case mein is is part mein jo hoga yahan par x bada hoga agar x y se bada hai to yeh jo value hai yeh value positive hogi aur y yahan par bada hai yani y agar positive hai yani main is quadrant ki baat kar raha hoon yahan is side ki baat kar raha hoon to y positive hai aur x bada hai y se to mere paas jo σ_{yy} aayega wo aayega tensile nature ka yani uski value positive rahegi to yahan par pehle main mark kar leta hoon yahan par aap dekhenge ye ye kuch aur ek cheez maine mark kar li yahan pe σ_{xx}, σ_{yy} aur σ_{zz} jo hai wo shunya rahenge kyunki ye line pe aap dekhenge $x = 0$ hai aur $y = 0$ yahan pe aayega theek hai to yahan pe hum dekhenge jahan pe $\sigma_{yy} < 0$ tha to maine likha tha $\sigma_{xx} > 0$ aur $y > 0$ hai to $\sigma_{xx} < 0$ aayega yani yahan pe compressive nature hai aur σ_{xx} ka yahan pe tensile nature hai ya below this side aur negative side of y yahan par maine dekha yeh jo value hai abhi humne discuss ki thi x agar bada hai yeh agar hum equation consider kar rahe x agar bada hai y se aur y positive hai to mere paas jo value aayegi wo tensile aayegi humne just now abhi discuss kiya abhi hum dekhenge ki ye side par bhi x bada hai aur yahan par x^2 ki value ho rahi hai to agar main is side pe bhi hoon to yeh element pe bhi jo stress aayega wo bhi tensile aayega agar aap yahan par main is side hoon yeh negative side hai x ka par yahan par x ka square ho raha hai to is part mein mere x ki value badi rahegi y se to yahan par bhi mujhe jo stress milega wo tensile milega to mere paas σ_{yy} ki values maine abhi mark kar liye abhi dekhte hain is case mein to is case mein aap dekhenge ki y ki value negative hai to yahan par agar mujhe tensile mil raha tha to yahan par mujhe compression

milega similarly yahan par mujhe tensile mil raha tha yahan par mujhe compression milega kyunki yahan par y ki value negative hai is is part mein is side par baat kar rahe hai.

To abhi hamare paas ye do elements reh gaye aur ye do elements reh gaye inke baare mein dekhte hain yahan par hum dekhenge ki x ki value yahan par hum dekhenge y ki value badi hai x se to $x^2 - y^2$ kya hoga ye negative ho jayega to yahan par jo value milegi aur y positive hai yahan par jo value aayegi wo negative aayegi to humein yahan par compression milega aur is case mein humein tension milega abhi hum dekhenge yahan par σ_{zz} ki value hum plot nahi kar sakte kyunki z perpendicular hai is direction pe par aap in dono ko add karke iski value nikal sakte jab hum agar z direction bhi dikhayenge to hum iski bhi value plot kar sakte hain.

Abhi hum Shear Stresses ki value dekhte Shear Stresses mein aap dekhenge ki $x^2 - y^2$ agar ye x agar bada hai y se to ye jo term hai yeh positive ho jayegi aur main positive side pe hoon x ke is side pe hoon to mere paas jo value aayegi τ_{xy} aur τ_{yx} ki wo positive aayegi aur positive Shear Stress hum is tarah se likhte hain agar mere paas kuch element hai is tarah se to hum positive Shear Stress ka convention yeh hai ki agar stress positive plane pe lag raha hai aur positive direction lag raha hai positive plane aur positive direction to wo Shear Stress hamara positive rehta hai wo Shear Stress positive rehta hai ya negative plane aur negative direction par lag raha hai to Shear Stress hamara positive rehta hai to yahi yahan par hum mark karenge positive Shear Stress τ_{xy} ki value to kuch pehle yeh bhi mark kar lete x aur y ki x aur y agar saman hai to mere paas σ_{yy} ki value shunya aayegi yahan pe aap dekhenge ki x aur y equal ho jayenge to σ_{yy} ki value shunya aayegi aur τ_{xy} ki value bhi yahan pe shunya aayegi similarly is condition pe is line pe meri σ_{yy} aur τ'_{xy} yahan pe likha hai kyunki ye rotated hai 90 degree se to isliye τ'_{xy} likha hai par ye τ_{xy} ya τ_{yx} ye yahan pe value is line pe jahan pe $x = y$ ki values hai is line pe hamare paas σ_{yy} aur τ_{xy} ki value shunya aayegi.

Abhi hum is element ki baat kar rahe hain aur ye equation consider kar rahe to humne discussion kiya tha ki $x^2 - y^2$ jo value hai yeh yahan par positive hai aur x ki value bhi positive hai to yeh jo Shear Stress hai yeh positive aayega to hum kuch is tarah se mark kar sakte hain yahan par τ_{yx} ki value shunya aayegi to yeh is tarah se maine Shear Stresses ki value mark kar li to mere paas ye positive Shear Stresses hai us tarah se hum main dekh paunga ki yahan par bhi main Shear Stresses ki value mark kar sakta hoon yahan pe $x > y$ hai yeh jo value hai ye positive rahegi aur x ki value yahan pe mark kar sakte x ki value agar negative hai yahan pe is element pe to jo Shear Stress aayega wo negative aayega to wo positive plane pe negative direction pe act karega kuch is tarah se agar maine ye consider karte hain ye element agar main consider kar raha hoon to aap dekhenge ki $x^2 - y^2$ ki value yahan pe y greater hai to yeh jo element aayega yahan pe x ki value positive hai to yeh jo element aayega yahan pe Shear Stress negative lag rahe hai similarly yahan pe hum dekhenge ye ye element pe ye element pe aap dekhenge ki x ki value greater hai y^2 se to aapke paas is tarah se kuch element aayenge usi tarah se main ye mark kar sakta hoon is tarah se yeh jo element hai aap dekhna hai ki $x^2\ y^2$ ki value kya hai aur x ki value kya hai usi tarah se main saare elements ki values mark kar sakta hoon to yahan par main dekhunga ki x ki value positive hai is side par x ki value positive hai to yeh jo element hai yeh element ki value positive aayegi Shear Stress ki value yahan par x ki value negative thi to yeh jo elements hai saare elements pe value jo aayegi wo negative aayegi kyunki x bada hai yahan par yahan par x chhota hai to yahan par saari value positive aayegi to isi tarah se main Shear Stress aur Normal Stresses ki values nikal sakta hoon.

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-v)}y\frac{3x^2+y^2}{(x^2+y^2)^2}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-v)}y\frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\sigma_{zz} = v(\sigma_{xx}+\sigma_{yy})$$

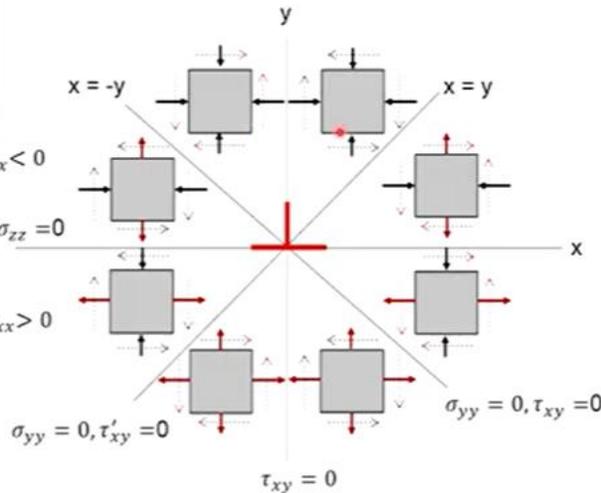$$\tau_{xy} = \tau_{yx} = \frac{Gb}{2\pi(1-v)}x\frac{x^2-y^2}{(x^2+y^2)^2}$$

$y > 0, \Rightarrow \sigma_{xx} < 0$

$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$

$y < 0, \Rightarrow \sigma_{xx} > 0$

$x = -y$

$x = y$

$\sigma_{yy} = 0, \tau'_{xy} = 0$

$\sigma_{yy} = 0, \tau_{xy} = 0$
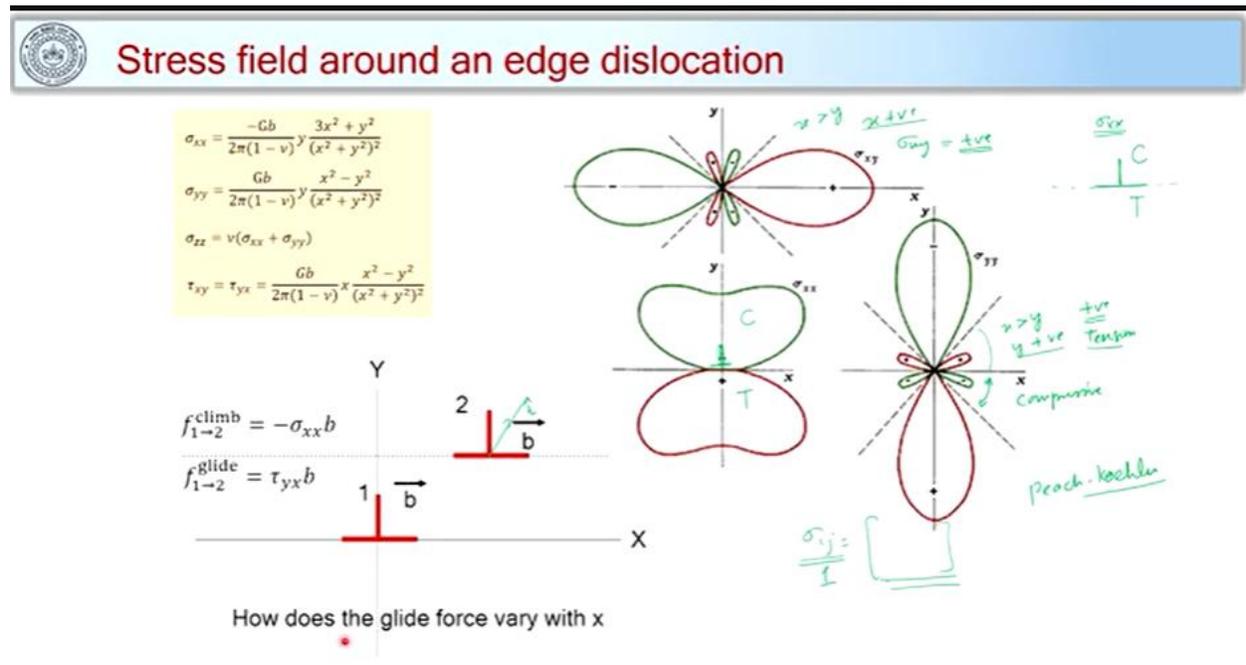
$\tau_{xy} = 0$

+ve Tensile

τ = +ve

+ve plane +ve direction

To mere paas kuch important observations hai yahan par mere paas Positive Edge Dislocation is tarah se to mere paas jo nature mil raha hai wo yahan par Normal Stress ka nature jo mil raha hai wo mere paas compressive mil raha hai above aur below jo mil raha hai wo tension mil raha hai abhi hum isko aur dekhenge jab hum Contour Plot plot karenge tab to ye mere paas jo equations hai abhi main Contour Plot plot kar raha hoon aur xy plane hi consider kiya yahan par maine to aap dekhenge ki σ_{xx} ki value maine plot ki hai to aap dekhenge ki yeh jo nature hai σ_{xx} ka yeh compressive hai yahan par aur yahan par jo nature hai yeh jo red color pe dikha hai yeh positive hai to yahan par mere paas agar yeh mera dislocation hai kuch is tarah se to upar mein compressive nature hai aur neeche mein tensile nature to ya slip plane ke above mujhe mil raha hai slip plane ke upar mujhe mil raha hai compressive nature aur below mil raha hai mujhe tensile nature similarly σ_{yy} ki value aap dekhenge yahan par to aap yahan par dekhenge σ_{yy} ki value bhi yahan par compressive hai above slip plane aur below slip plane hai tension σ_{xy} ki value aap dekhenge σ_{xy} ki value yahan par aap dekhenge jo x yahan par is quadrant mein x bada tha $x > y$ aur σ_{xy} ki value yani yahan pe τ_{xy} τ_{xy} to ye determine karegi x se determine karegi to yahan pe agar x positive hai to yahan par x positive hai to jo τ_{xy} σ_{xy} aayega wo positive aayega nahi positive shear rahega usi tarah se yeh jo value hai yahan par negative value aayegi σ_{xy} ki aap dekh payenge yahan par bhi σ_{yy} mein ye ye relation tha yahan par yahan par agar aap dekhenge is is part mein $x > y$ hai aur y positive hai to ye jo value aayegi σ_{yy} ki ye positive aayegi yani tension aayegi kyunki normal stress hai aur isme aapki value aayegi compressive to yeh hogi meri Stress Field Around Edge Dislocation.

To aapko question yani question is tarah se pooche jate hain exam mein agar mere paas ek Positive Edge Dislocation hai to slip plane ke upar mere paas jo nature aata hai wo compressive aata hai hamesha aur jo below slip plane aata hai wo tensile aata hai yeh mostly σ_{xx} ke liye main bol raha hoon to yeh aapko yaad rakhna hai to abhi mere paas kuch dislocation configuration hai agar mere paas ye kuch aise is tarah se dislocation hai aur maan lete hain kuch aur ek slip plane pe ek dusra dislocation hai to ye mera ek dislocation hai 1 dislocation yeh mera dusra dislocation hai dono ka

Burgers Vector agar same hai to jo Climb Force lagega jaise mere paas ek Stress Field hogi humne to Stress Matrix likhi thi yahan par σ_{ij} agar main 1 dislocation ke liye consider kar raha hoon ye 1 dislocation ke consider kar raha hoon to mere paas ek Stress Field aa jayegi mujhe iska Burgers Vector pata hai aur mujhe iska Tangent Vector bhi pata hai to main Climb Force ya Glide Force nikal sakta hoon to Climb Force mera kuch is tarah se aayega aur Glide Force mera kuch is tarah se aayega yeh humein milega Peach-Koehler Relation se hum isko dekhenge aapko isko aap isko solve kar sakte hain iska ye values aapke paas hai Stress Matrix hai aapke paas aapke paas Burgers Vector ki direction hai aur Dislocation Tangent Vector ki yahan pe Tangent Vector iska is direction mein hoga to agar mere paas kuch iski Stress Field available hai aur ye jo Dislocation hai ispe kya force lagega is Stress Field se ya aap Peach-Koehler Equation se nikal sakte hain to mere paas ek Glide Force lagega aur ek Climb Force bhi lagega par abhi ek interesting cheez aap kar sakte hain ki agar ye Dislocation main move karta hoon is slip plane pe is Dislocation ke relatively to kya force change hoga yeh aapko dhoondhna hai to aapko dekhna hai ki How does this Glide Force vary with x aapko dekhna hai ki agar main Dislocation move kar raha hoon is direction pe is slip plane pe to yeh Glide Force mera kaise change hoga isko hum dekhenge agle part hamare aane wale parts mein par us tab tak aap isko solve karne ki koshish kariyega.

Stress field around an edge dislocation

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-v)} y \frac{3x^2 + y^2}{(x^2 + y^2)^2}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-v)} y \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$\sigma_{zz} = v(\sigma_{xx} + \sigma_{yy})$$

$$\tau_{xy} = \tau_{yx} = \frac{Gb}{2\pi(1-v)} x \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$f_{1\to2}^{climb} = -\sigma_{xx}b$$

$$f_{1\to2}^{glide} = \tau_{yx}b$$

How does the glide force vary with x

To is part mein humne dekha ki is part mein humne dekha hai ki Stress Field jo hai Dislocation ki Edge Dislocation ki yani agar mere paas Positive Edge Dislocation hai uske slip plane ke upar Compressive Nature rehta hai aur uske neeche Tensile Nature rehta hai to abhi ke liye main yahan par rukta hoon next part mein hum janenge jo Dislocation se dono Dislocation se unki Energy kya hoti hai. Dhanyavad.