Mechanical behavior of materials

Dr. Niraj Mohan Chawake

Department of Materials Science and Engineering

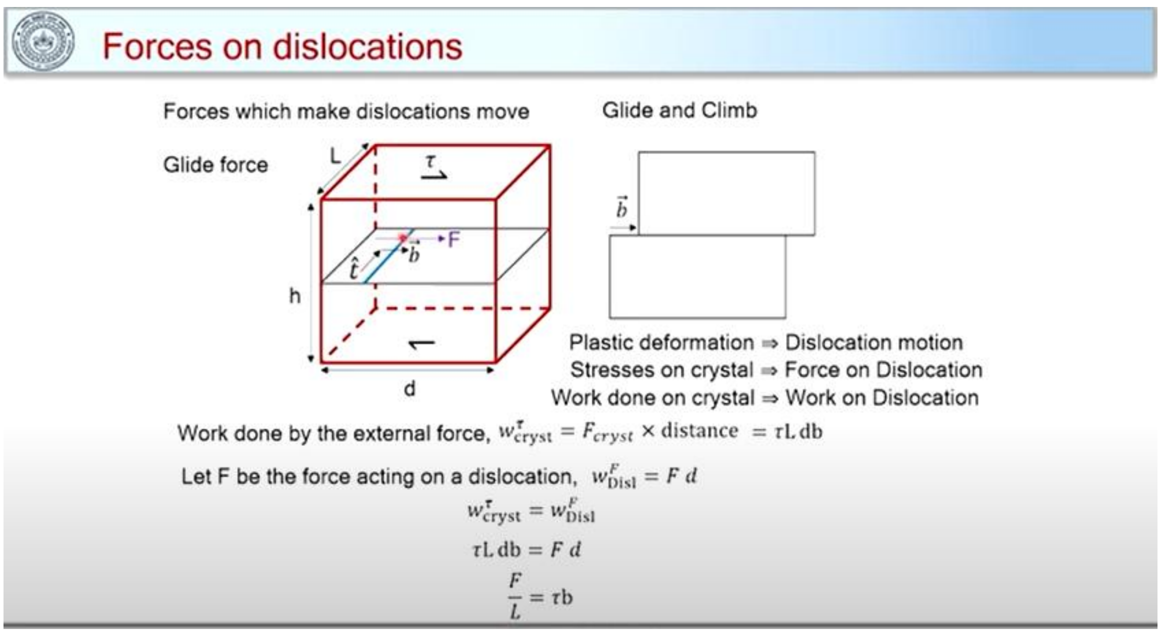
Indian Institute of Technology, Kanpur

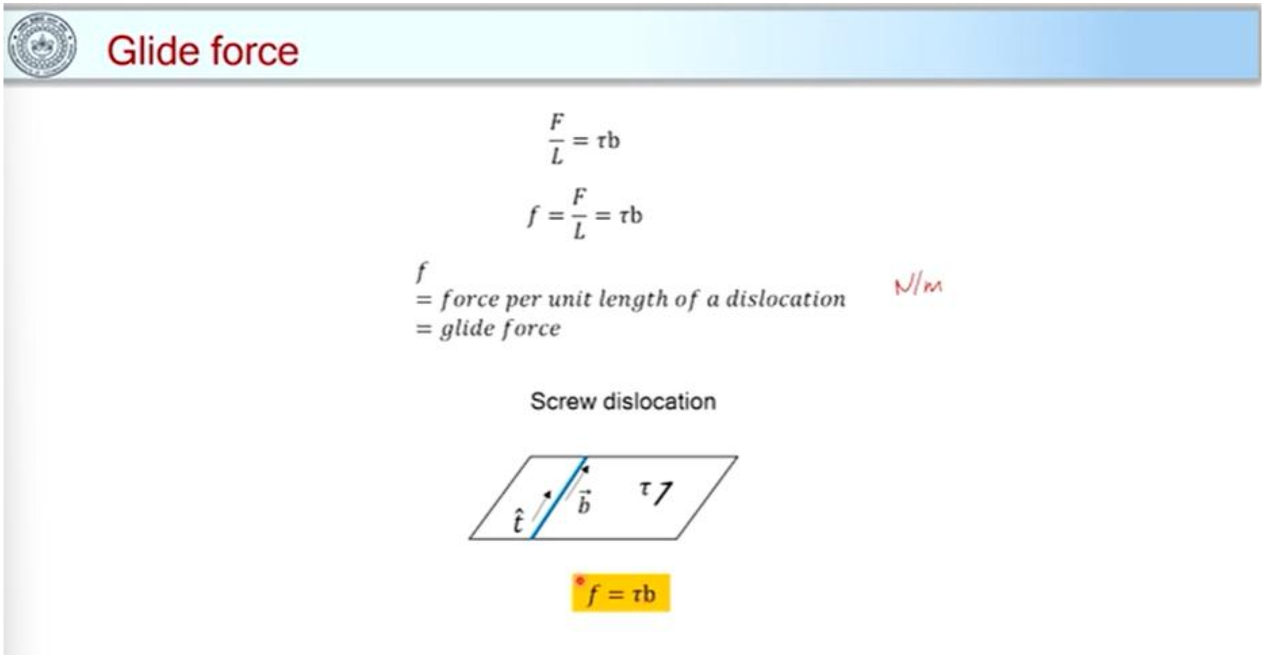
Week-6

Lecture-29

Forces on dislocations: Peach Koehler Equations

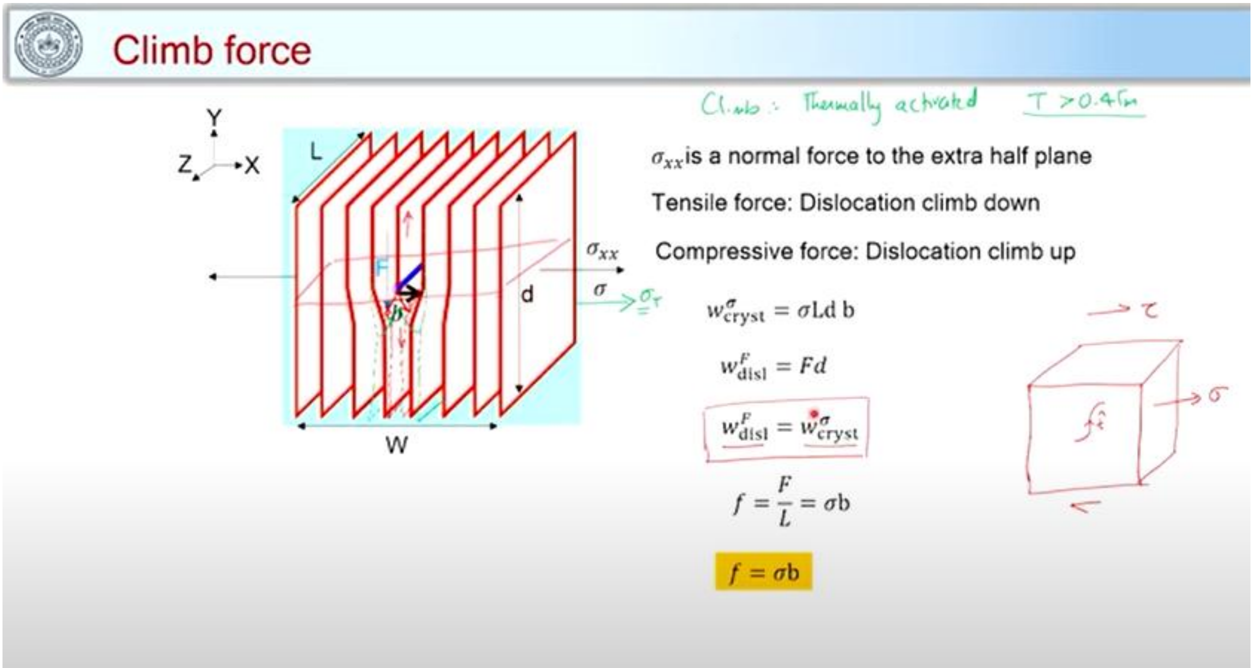
Namaskar phir se swagat karta hoon aapka is course mein mechanical behavior of material jisko hum hindi mein padhenge last part mein humne dekha tha ki dislocation ke motion kya hote hain is part mein hum dekhenge ki Forces on Dislocation kya hote hain to jab hum forces on dislocation ki baat karte to hum dekhte hain ki jo dislocation motion hai jaise glide aur climb ye jo dislocation motion hai ye kis tarah se affect ho sakte hain jab main baat kar raha hoon forces on dislocation to iska matlab ye hai ki mere paas ek crystal hai aur us crystal mein defect hai dislocation agar us crystal ko main deform karna chahta hu to mujhe kis tarah se kis force ki aavashyakta hogi us dislocation ko move karne ke liye to maan lete mere paas ek crystal hai ye crystal jaise maine yahan par maan liya aur mere paas ek dislocation hai yahan par ek slip plane ye jo grey color ka hai plane hai yahan par mark kiya hai aur ek dislocation yahan par maine mark kiya hai uska tangent vector mark kiya hai aur uska Burgers vector mark kiya hai aur aap dekh sakte hain ki ye jo dislocation hai ek slip plane pe hai is crystal mein aur ye jo crystal hai uske dimension yahan par mark kar lete hain hum ye crystal hai jiski dimension hai d h aur l ye teen dimension ho gayi mere crystal ki aur abhi main is crystal par koi ek shear stress apply kar raha hoon maan lete yahan par ye jo shear stress hai τ aur isko is shear stress ko main naam de raha hoon Glide Force kyunki ye jo shear stress hai aap dekhenge ki ye dislocation ko move karega is slip plane pe to jab ye main shear stress apply karta hoon is crystal pe to main iska ek 2D view yahan par dikhana chahta hoon to kya hoga yahan par shearing hoga ye shear stress ki wajah se to yahan par plastic deformation hoga aur jo upper part of crystal hai jo upar ka part hai wo move karega bottom part se by Burgers vector b ya b ke displacement yahan par milega ye mera Burgers vector humne dekha tha jab slip hota hai ya glide hota hai to is tarah se kuch hota hai mechanism to abhi hum dekhenge ki ye jo stress hai ye jo stress ki wajah se mera jo shear deformation aaya yaane plastic deformation hua mere crystal ka Burgers vector b ki wajah se step yahan par create hua is stress ka is dislocation ke moment se kya relation hai yaani main ye jaan sakta hoon ki agar ye shear stress main apply kar raha hu to ye dislocation move hoga maan lete ki is shear stress ki wajah se ek force yahan par lag raha hai is dislocation ke upar aap dekhenge ki yahan par maine ek force mark kiya hai F to ye shear stress ki wajah se ek force yahan par create hoga is dislocation par aur is force ki wajah se ye dislocation move hoga to ye jo shear stress hai ye mera macroscopic stress hai yaani main crystal par apply kar raha hoon aur ye jo macroscopic stress hai ek force create karega mere dislocation ke upar aur woh force ke wajah se mera dislocation yahan par move hoga aur ye jab dislocation move hoga tab mujhe plastic deformation milega to main ye keh sakta hoon ki jab dislocation motion hota hai wo mera plastic deformation govern karega aur jo stresses jo honge crystal pe act kar rahe honge wo stresses forces kuch create karenge dislocation pe aur wo jo forces hai wo dislocation ko dislocation motion ko generate karenge aur phir jab dislocation motion hoga tab mujhe plastic deformation milega to hum jab stresses apply karte hain tab hum ek cheez aur co relate kar sakte hain ki jo yahan par deformation ho raha hai to main bol sakta hoon ki crystal pe ek work ho raha hai yaani agar ye deformation ho raha hai stress ki wajah se to ek crystal pe ek work ho raha hai uski kuch yaani wo work done jo hoga crystal par wo same work done hona chahiye mere dislocation par kyunki agar main ye maan ke chal raha hoon ki dislocation ke motion ki wajah se hi mujhe plastic deformation mil raha hai to jo work done on crystal hoga shear stress ki wajah se wo mera work done on dislocation hoga is force ki wajah se ye hum maan sakte hain to maan lete hain abhi hum ye calculate karte ki jo mera external stress hai external jo force hai main yahan par τ maan ke chal raha hoon to yahan par ye kuch kaam karega crystal par to ye maan ke chal raha hoon ki ye jo work done hai crystal par because of stress τ shear stress τ ye is tarah se kuch hoga force acting on crystal ye force jo act ho raha hai is crystal par total crystal par into distance yaani kitna displacement ho raha hai upper part ko move karne ke liye to force into displacement mujhe kuch ek work done milega crystal ke upar because of this shear stress abhi ye force main is tarah se likh sakta hoon agar mujhe shear stress pata hai aur ye shear stress kis plane pe act kar raha hai ye act kar raha hai is top plane pe to aap dekhenge ye is plane pe act kar raha hai to ye plane ki dimension hai l aur d to agar main iska area nikaal lunga is plane ka aur usko is stress se multiply karunga to mujhe ek force milega to ye hi yahan par likha hua hai to $\tau \times l \times d$ ye ho gaya area is plane ka aur ye jo displacement hai distance kya hoga yahan par mera upper part jo move ho raha hai crystal ka Burgers vector b se ya ye jo displacement vector hai b yahan par is tarah se main mark kiya to ye ho gaya mera total work done by external shear stress ye aa jayega $\tau l d$ aur b abhi dekhte hain ki is force ke wajah se dislocation par kya work done ho raha hai to maan lete hain jo force act kar raha hai is dislocation ke upar kyunki hum yahan par shear stress apply kar rahe to ek force yahan par generate hoga dislocation ke upar usko main F maan ke chal raha hoon uske wajah se dislocation ki motion hogi to us ye dislocation ke upar bhi ek work done hoga usko hum calculate karte hain isko main bol raha hoon Work Done on Dislocation Due to Force F yaani force F ki wajah se jo dislocation ke upar work ho raha hai wo is tarah se main represent kar raha hoon aur wo main is tarah se likh sakta hoon $F \times d$ aap maan ke chaliye jaise ye dislocation hai agar ye pura distance traverse karta hai pura distance move hota hai yahan se ye pura d agar distance move ho raha hai tabhi jaake humein ek displacement b milega to ye force into ye jo distance hai ye mujhe de raha hai work done on displacement due to force F ye $F \times d$ aa jayega abhi aap dekhenge ye agar ye work done aur ye work done same hai kyunki jab main b ko pura distance d traverse karke launga yahan se is is end se is end tak tab mujhe displacement b milega ye pure crystal ke liye upper part ke crystal ke liye to ye main equate kar sakta hoon to agar main ye equate karunga to mujhe kuch is tarah se milega yaani ye do quantity ko main equate kar raha hoon aur mujhe main is tarah se arrange kar raha hoon $F/l = \tau \times b$ ab dekhenge ki jo l hai yaani crystal ka l yahan pe maine mark kiya ye l jo hai ye length of dislocation bhi hai yahan pe kyunki hum yahan pe ek straight dislocation hi consider kar rahe hain to ye jo l hai ye mera length of dislocation hai aur ye jo force hai ye force acting on a dislocation hai to ye jo part ho jayega ye Force upon length ho jayega aur ye milega τ yaani external stress hai yahan par maine apply kiya into Burgers vector jo dislocation ka Burgers vector hai b to mere paas kuch is tarah se aa jayega aur is force ko main kehta hoon glide force aur isko main is tarah se likh sakta hoon small f yaani ye ho jayega iska ye ho jayega Force Per Unit Length of a Dislocation ye aapko yaad rakhna hai jab main small f likh raha hoon to ye force per unit length of a dislocation hai to iska unit ho jayega Newton per meter aur ye aa jayega mera stress aur ye aa jayega mere mera Burgers vector yaani dislocation ka Burgers vector to ye external stress hai aur ye hai Burgers vector to isko main glide force karunga glide force isliye keh raha hoon kyunki dislocation us plane par move ho raha tha to screw dislocation ke liye bhi hum ye prove kar sakte hain hamare paas yahi relation aayega agar screw dislocation ka bhi case consider karenge aur screw dislocation glide ho raha hai is tarah se to mere paas kuch force per unit length of dislocation ye aayega $\tau \times b$ ye mera glide force hoga kyunki jab main external stress apply kar raha hoon to ye force dislocation par generate hoga aap dekhenge ki yahan par maine τ jo nikaala hai wo Burgers vector ke direction par nikaala hai humne dekha tha ki dislocation kab move hote hai dislocation move hote hai jab jo stress hai wo Burgers vector ko parallel hote hai tabhi dislocation move hote hai ye aap dekh aapne dekha tha pure aur



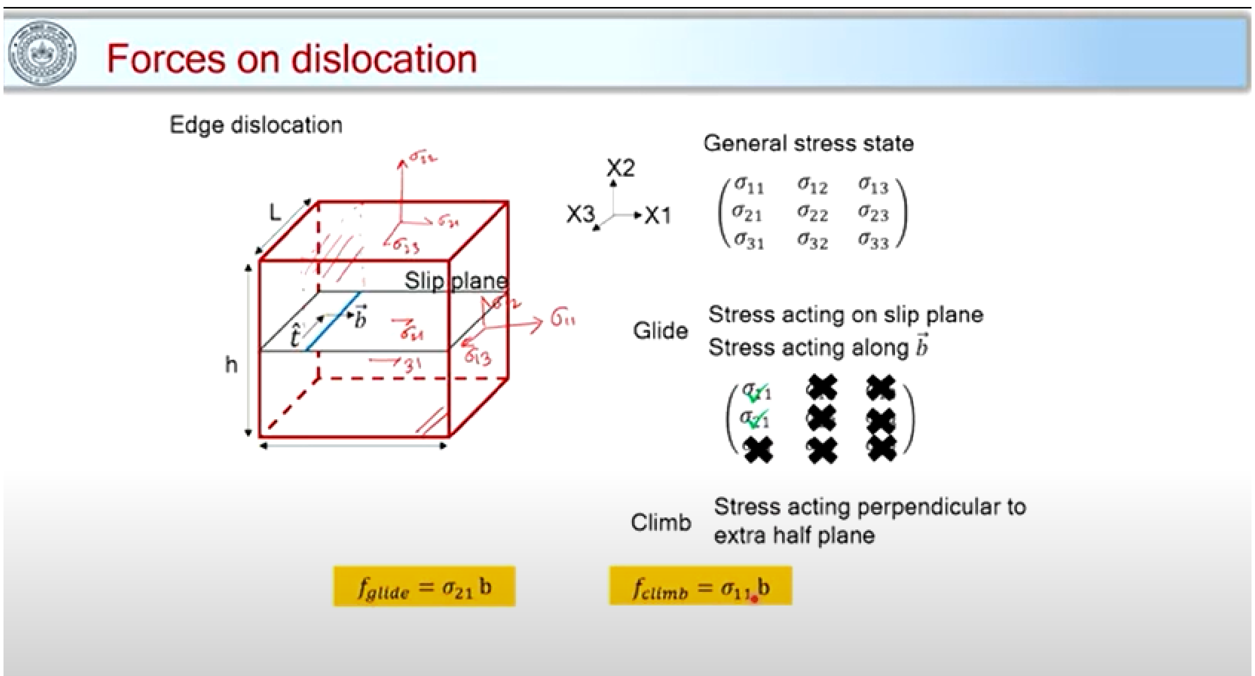


edge screw dislocation ke liye.

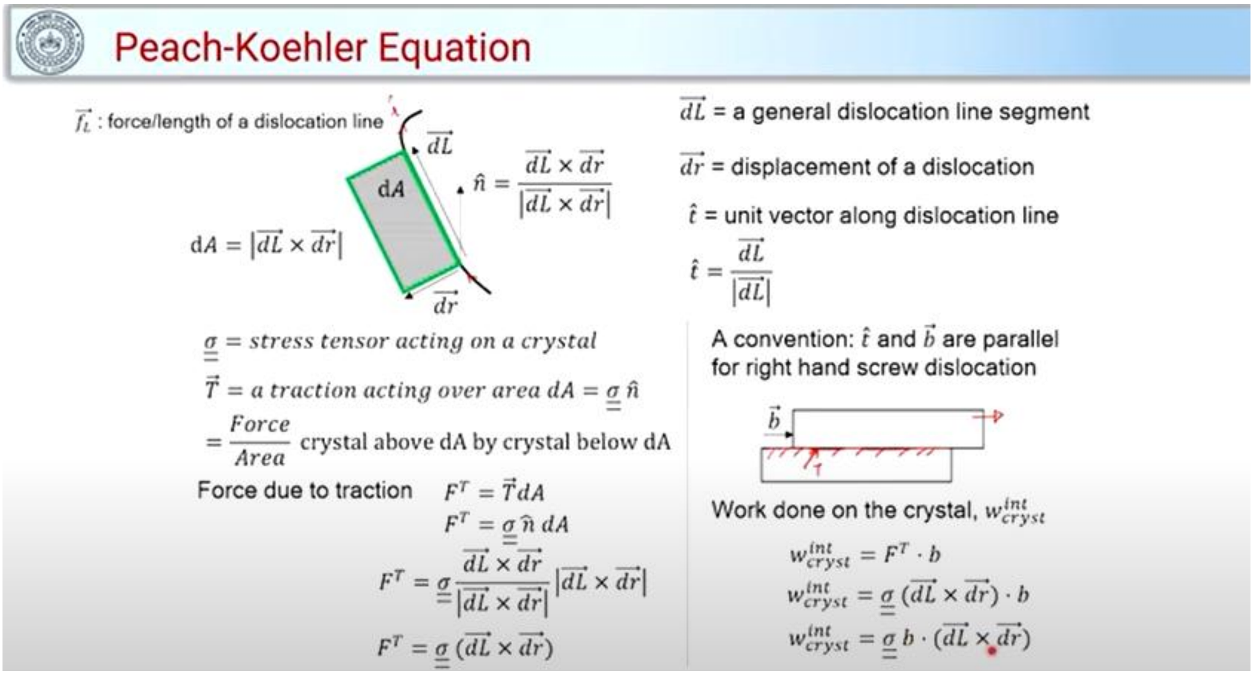
To abhi dekhte hain ki Climb Force kya aata hai humne glide force to abhi dekh liya abhi climb force ke baare mein discussion kar lete hain to climb humne dekha tha ki ye hota hai hamare edge dislocation ke liye to yahan pe ek dislocation mark kiya aur ek pura crystal main yahan pe mark kiya hu to yahan par hum dekhenge ye maine Burgers vector mark kiya ye extra half plane hai aur ye jo blue color ki line hai jo andar ja rahi hai ye meri dislocation line vector hai abhi hum dekhenge iske dimension mark kar lete hain ye mark kar lete w d aur l ye teen dimension maine mark kar liye mere crystal ke aur kuch coordinate axis bhi mark kar lete hain to hum dekhenge ki climb kab hota hai jab main Normal Stress ko apply karta hoon mere crystal ko tab jaake climb hota hai mere edge dislocation ka to ye maine σ_{xx} Ek normal stress mark kar kyunki ye jo stress hai ye perpendicular hai mere x plane ko ya ye along x direction hai isliye isko σ_{xx} Ek normal stress hai aur isko main σ likh raha hoon kyunki ek hi stress yahan par present hai to isko main σ mark kar raha hoon yahan par par aap dhyaan rakhiye kyunki ye jo stress hai normal stress hai to σ_{xx} Ek normal stress hai wo jo act ho raha mere extra half plane pe aap dekhiyega ki ye jo extra half plane hai is par act ho raha hai mera σ jo stress normal stress hai kyunki ye jo plane hai ye perpendicular hai is is stress ko aur iske wajah se ek force act hoga is dislocation line pe wo kuch maine is tarah se mark kiye F kyunki humein pata hai hum climb force nikaal rahe hain to climb mera dislocation ka kya hoga ye mera glide plane ho jayega jo along x direction hoga ye mera glide plane ho jayega humne dekha tha ye aise kuch glide plane hoga aur mera dislocation climb up hoga ya climb down hoga to ek force maine mark kiya is tarah se y direction par to jab main tensile force rehta yaani σ jo main apply kar raha hoon jo stress hai wo tensile force create karega tab jaake wo dislocation climb down hoga agar wo tensile nature hai aur compressive force hai to dislocation climb up hoga isko is tarah se samajhte hain hum jab main jab main kuch is tarah se mark kar raha hoon planes ko agar maine tensile force apply kiya ye σ agar maine is tarah se mark kiya to aap dekhenge ki ye jo planes hai dislocation ke around ye stretch honge ye is tarah se kuch st stretch honge aur humne dekha tha ki climb jo process hai wo Thermally Activated process hai yahan par ye bhi likh lete ki climb thermally activated hai yaani ye jo process hai ye hoti hai greater than $0.4T_m$ Ye humne ye bhi part yahan par dekha tha tab jab ye main tensile stress apply kar raha hoon yahan pe to ye planes jo hai wo elongate ho gaye aur aap dekhenge ki ye jo ye jo dislocation climb down hoga yaani iska matlab hoga ki yahan se jo atoms hai wo yahan par accommodate honge yahan par space create hui hai to yahan se jo atoms hai yahan par yahan par is tarah se abhi maan lete hain ki mera jo force hai wo compressive hai tab kya hoga ye jo planes hai ye jo yahan par maine mark kiye ye planes compress ho jayenge isko bhi is tarah se mark kar lete hain hum ye agar planes compress ho rahe hai to main kuch is tarah se mark kar raha hoon is planes ko ye mere extra plane ki taraf move ho gaye aur aap dekhenge jab extra plane ki taraf move honge tab ye jo ye jo atoms hai yahan par wo opposite direction pe move hone chalu honge kyunki planes jo atoms hai yahan par ye compress ho raha hai to ab dekhenge ki jab main compress kar raha hoon ya compressive stress apply kar raha hoon extra plane ko to mera climb up hoga to ye do points aap yaad rakhein par hum ek general derivation karein forces jo dislocation pe act ho rahe phir aapko pata chalega ki climb up aur climb down kyun ho raha hai to yahan abhi ke liye hum ye jaan lete hain ki jab main tensional force apply karta hoon to dislocation climb down hota hai aur compressive force apply karta hoon to dislocation climb up hota hai to yahan par bhi hum ek work done likhte hain jo normal stress maine yahan par apply kiya yahan par main agar ye initial condition yahan par main consider kar raha hoon to work done jo ho raha hai yahan par work done on crystal σ jo normal stress hai ye main is tarah se likh sakta hoon ye σ kis plane pe act ho raha hai ye σ is plane pe act ho raha hai crystal ke to iske dimension hai d aur l ye jo dimension hai ye l hai to main l aur d se multiply kar raha hoon is stress ko to mujhe force mil jayega aur force into displacement mujhe work done mil jayega crystal ka to displacement kya hoga jab crystal shear hoga to mujhe b milega displacement to ye jo aa gaya ye mera work done aayega usi tarah se jo force act ho raha hai dislocation line par main is tarah se likh sakta hoon ki agar ye pura distance move hota hai tab jaake mujhe ek displacement b milega to ye force acting on dislocation into d mujhe work done on a dislocation by force F aise milega yaani force ki wajah se jo dislocation ke upar work done ho raha hai ye milega agar ye dono equal hai equal main kar sakta hoon jab ye b yaani dislocation pura move hoga ye pura length tab ke mujhe ek step b milegi us condition mein main dono ko compare kar sakta hoon is tarah se aur main phir se kuch is tarah se likh sakta hoon f ko small f ko $F/l = \sigma \times b$ to ye ho gaya mera climb force yahan par jo stress lag raha hai ye mera tensile ya compressive nature hai ya normal stress hai to main f ko is tarah se likh sakta hoon to mere paas climb force aur glide force is tarah se ho gaye aap dekhenge ki ye jo force yahan par stress ye bhi normal stress yahan pe lag raha hai ye along the direction of b hai ya along the Burgers vector direction hai par ye ho gaye mere climb aur glide force ye mere pure edge aur pure screw ke liye ho gaye



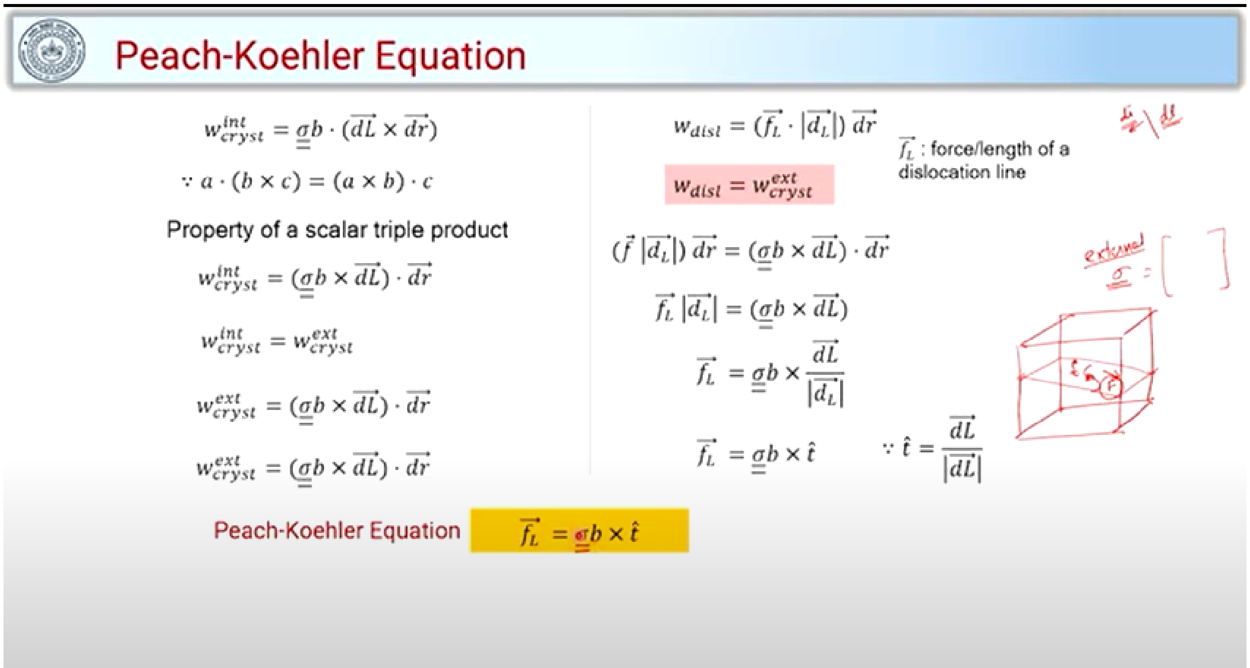
Abhi humein dekhna hai ki agar main general case agar consider karta hoon to kaun sa force uspe act karega yaani agar mujhe mere paas ek state of stress hai agar main aise kuch aapke paas ek crystal de doon is tarah se samajhiye ek random crystal hai mere paas aur yahan par koi bhi dislocation yahan pe padi hui hai is crystal mein uska tangent vector kisi bhi tarah se aur ek main stress apply karta hoon is a crystal ko wo shear bhi ho sakta hai wo tensile bhi ho sakta hai is tarah se agar combination hai to ye combination of jo stresses hai in combination ke wajah se is dislocation pe kya force act hota hai ye abhi humein dekhna hai ye to humein ek conceptual dekha tha yahan par ek concept aapko yaad rakhni hai ki jo force done hai force acting on dislocation aur force aur work done on dislocation due to force F ya work done on crystal due to stress σ ye dono equal hai yahi concept hum aage jaake bhi istemal karenge ye tha mera climb force abhi hum ek example dekhte hain ki forces on dislocation hote kya hai to maan lete hain mera ek crystal hai aur ye maine edge dislocation yahan pe consider kiya ye ho gaya $x_1 x_2 x_3$ Ye maine direction mark kar liye mere aur yahan par ek maine general state of stress consider kiya hai to ye general stress state hai mere paas to yahan par agar hum dekhenge to σ_{11} Kis direction mein hona chahiye aap abhi hum phir se isko likh lete hain to σ_{11} Mera is direction mein hona chahiye humne dekha tha ki σ_{11} Kya represent karega along 1 direct one acting on one plane along one direction σ_{12} Is tarah se act hoga aap dekhenge σ_{12} Mere one plane pe along two direction to ye mera σ_{12} Hoga usi tarah σ_{13} Is tarah se hoga usi tarah se hum baaki ke stresses hum likh sakte hain to yahan pe main do hi stress mark karunga kyunki yahan pe last part mein maine last slide mein maine de bola tha ki jo stress along b direction par kaun se act ho rahe hain aapko wo dekhne hain yahan par kaun se stress act ho rahenge along b direction ye plane mera kaun sa hai ye plane mera hai y plane ya x_2 Plane ya plane 2 hai yahan par kyunki ye x_2 Direction ko perpendicular hai to mera x_2 Plane hai to yahan par jo stresses lagenge to ek jo stress lag raha hai yahan par ye lag raha hai σ_{11} Kyunki yahan par agar main dekhunga ye edge dislocation hai ye edge dis to iska half plane is tarah se hoga aur ye jo normal stress lag raha hai σ_{11} Ye is half plane ke half plane ko normal hai to ye ek plane lag raha hai agar shear stress ki baat karunga to shear stress mera jo plane 2 pe lag raha hai ye ye mera two plane tha aur along one direction ye mera σ_{21} Ho jayega agar main is plane pe baat karunga to ye mera σ_{22} Ho jayega aur ye ho jayega mera σ_{21} Aur ye ho jayega σ_{23} Usi tarah se baaki aap jo stress component hai ye mark kar sakte hain par aap dekhenge ki yahan par jo stresses hai do hi stress Burgers vector ke direction par hai wo hai σ_{21} Aur σ_{11} To hum glide force jab baat karte hain tab glide force kya karega mere is dislocation ko glide karega is plane pe ye slip plane pe to stress acting along b kaun sa hai yahan pe glide ke liye to glide ke liye stress acting along b humne dekha tha ki shear stress jo act kar raha hai wo glide karega mere is dislocation ko to σ_{21} Jo hai wo shear stress hai wo is dislocation ko glide karega is plane pe aur isi plane pe wo lag raha hai aur climb jo rahega wo stress acting perpendicular to extra half plane to humne extra half plane yahan pe nikaala tha aur ye jo σ_{11} Hai wo is plane ko perpendicular hai to hum dekhenge ki agar main ye state of stress likhta hoon to kaun se kaun se stress mere dislocation ko move karenge yaani is state of stress mein kaun se components hai jo mere dislocation ko move kar sakte hain to σ_{12} To nahi hoga σ_{11} Zarur hoga kyunki yahan pe humne dekha σ_{11} Move karega mere dislocation ko ye extra half plane ko perpendicular hai σ_{13} Nahi karega kyunki is plane mein nahi act ho raha hai kyunki ye σ_{13} Aap dekhenge ki ye is direction mein act ho raha hai σ_{21} σ_{21} Is plane mein act ho raha hai along direction b to σ_{21} Mere dislocation ko move karega σ_{23} σ_{23} Yahan par aap dekhenge ki ye is direction mein act kar raha hai to ye bhi nahi hoga σ_{31} σ_{31} Kahan pe hai σ_{31} Bhi mark kar lete ki σ_{31} Ye jo plane hai ye plane ye jo plane hoga direction mein σ_3 Along one direction to is plane pe act ho raha hai aur one direction pe act ho raha hai to aap dekhenge ki ye ye plane pe act nahi ho raha hai is plane pe act nahi ho raha hai par ye along one direction act ho raha hai to slip plane pe nahi act ho raha hai kyunki slip plane to mera perpendicular hai is plane ko to aap dekhenge ki σ_{31} Ye plane hai aur σ_{21} Hi isi plane par act ho raha hai to σ usi tarah se aap σ_{32} Ko dekh sakte hain ya σ_{33} Ko dekh sakte hain ye dono bhi act nahi karenge σ_{22} Bhi act nahi karega to hamare paas do components hai stress state ke jo is dislocation ko move kar sakte hain aur ek karega glide jo glide karega wo rahega σ_{21} Kyunki ye shear stress hai aur climb karega ye σ_{11} Ye rahega mera kyunki ye normal stress hai aur ye mere stress ye mere dislocation ko climb karega to ye ho gaye mere forces on dislocations



Abhi hum general state state dekhte hain Peach Koehler Equation to hum maine pehle baat ki thi agar mere paas general stress state hai aur mere paas ek random dislocation hai wo us dislocation pe kaun se forces act honge pehle hum uska derivation dekhenge phir uski application dekhenge is equation to maan lete ki mere paas ek dislocation hai ye ek dislocation hai ek random dislocation hai mere paas maine yahan par kuch define nahi kiya hai aur iske ek dislocation line segment hai dl ye maan maan ke chal raha hoon ek dislocation line segment hai uska ek small part main consider kar raha hoon is dislocation line segment ka is tarah se kuch to agar main is tarah se mark kar raha hoon to agar main dislocation line isko maan ke chal raha hoon to ye tangent vector hai mera to ek mera tangent vector is tarah se hoga to agar hum is line segment ko maan ke chal rahe aur agar main koi external stress apply karta hoon koi bhi stress state apply karta hoon mere crystal ko wo crystal pe jo apply hua hai jo stress wo is dislocation ko dislocation par ek force act karega aur uski wajah se ye dislocation move hoga ye humne dekha hai glide hoga ya climb hoga humein pata nahi par humein dekhna hai ki wo force act karega aur is dislocation ko move karega to maan lete kuch is direction mein move kar raha hai yaani mera dislocation dr displacement ho raha hai is direction mein to kuch is tarah se agar mera dislocation ka movement hua ye small segment hi badha yahan pe yahi is tarah se hua aur ek area ek traverse karega wo dA isko main maan leta hoon small area dA to agar is kaafi chhota area hai dA to main kuch is tarah se likh sakta hoon dA ko $dl \times dr$ yaani agar ek vector hai line vector hai aur wo dr se move hota hai to is tarah se maan ke chalenge ki ek cross section area jo ban raha hai wo rectangular cross section area hai to ye $dl \times dr$ ho jayega mere paas jo area aayega aur agar main is plane ko consider karunga jab ye plane jab ki main baat karunga to ye iska main ek normal nikaal sakta hoon normal is tarah se likh sakta hoon ki agar mere paas cross product pata hai to mujhe ek vector mil jayega aur uska magnitude $dl \times dr$ ho jayega to dA vector hai to n jo hai ye normal vector hai is area ka ya surface ka to ye is tarah se main likh sakta hoon $dl \times dr$ to ye aa gaya mere paas normal vector ya iska magnitude bhi main le sakta hoon kyunki ye n unit vector hai to ye mere paas $dl\ dr$ iska magnitude ho jayega agar main isko define kar raha hoon area ko to is plane ki location bhi main bata sakta hoon ye normal ke direction se jo perpendicular rahega wo mera area ka location rahega abhi t agar main maan ke chalta hoon t unit vector hai along dislocation line to iska isko main is tarah se likh sakta hoon ye mere paas small length maine dl mark mark ki thi to ye divided by iska magnitude dl ye aa jayega unit vector along t maan ke chalte hain ki ek force per length of dislocation hai is iske wajah se ye movement hui hai dislocation line segment ki ye hai F_l Ek line segment pe act kar raha hai isliye main isko F_l Maan ke chal raha hoon abhi agar ek stress hai jo crystal pe act kar raha hai ye humne dekha tha ki agar ek crystal stress tensor jo hai wo ek crystal pe act kar raha hai wo ek force create karega ye stress ki wajah se ek Traction taiyaar hoga agar main dekhunga agar crystal movement dekhunga to ek traction taiyaar hoga traction kaise taiyaar hoga ek traction taiyaar hoga jo crystal ke niche ka area hai wo ek traction force taiyaar karega ye above area pe upar aur us traction ko main is tarah se kuch likh sakta hoon stress aur n ye jo n hai ye hai mera is area ka normal to agar ye mujhe area ka normal pata hai to main ya surface ka normal pata hai to main traction nikaal sakta hoon us surface par kitna lag raha hai to isko is tarah se mathematically bhi likh sakte hain ye hota hai force upon area isko main maan ke chalta hoon conceptually agar aap dekhenge ki ye jo force upon area jo act ho raha hai stress act ho raha hai ye crystal niche wale jo crystal ki wajah se upar wale crystal par jo stress lag raha hai isko hi main traction keh sakta aur usko main mathematically is tarah se kuch likh sakta hoon isko bhi hum samjhenge abhi thodi der mein to main likh sakta hoon ki force due to traction kuch is tarah se to force due to traction main likh sakta hoon ye traction lag raha hai is area par to $t \times dA$ kyunki area mujhe pata hai kya hai to ye jo traction hai main yaani force hai traction ke dwara ye is tarah se likh sakta hoon $t \times dA$ to traction ki value agar main yahan par rakhta hoon to ye ho jayega σn yahan se main replace kar raha hoon ye t ko to ye yahan par mere paas aa jayegi force due to traction aur jab yahan par main n ki value replace karunga ye humne n nikaala tha kuch is tarah se aur area ki value bhi yahan par nikaali thi kuch is tarah se main agar replace karta hoon to mere paas ek identity aayegi aur wo identity kuch is tarah se rahegi F_T Yaani force due to traction mere paas external stress hai σ aur $dl \times dr$ dl hai meri dislocation line segment dr hai meri displacement of dislocation due to force acting on a dislocation because of external stress σ to ye mere paas ek identity aa gayi agar hum dekhenge yahan par ye crystal hai ya humne dekha tha ki agar crystal displace ho raha hai b displacement vector se ye mera Burgers vector hai to maan lete hain Convention humein maan ke chalna padega jab hum ye consider kar rahe ki traction jab act kar raha hai tab hum dekhenge ki mera slip is plane pe ho raha hai to main displacement sirf consider karta kar raha hoon above part ka kyunki agar main ek part ka displacement consider kar raha hoon to bottom yaani b agar main total displacement consider kar raha hoon but aap dekhenge ki ye to dono relatively move ho rahe hai to main maan ke chal raha hoon ki mera sirf top part move ho raha hai bottom part ki wajah se to isliye main jo traction consider karunga wo niche part ki wajah se upper part ke liye yaani agar mein traction is tarah se nikaalunga to ye traction is tarah se act hoga niche part ki wajah se a above crystal pe to agar main dono consider karunga to kyunki ye agar is direction mein move ho raha hai aur is direction mein move ho raha hai to mujhe work done zero milega jo ki main balance ho jayega yahan par but agar main work done stress ki wajah se agar dekhunga to main above part ko hi consider kar raha hoon ki wo is direction mein move ho raha hai aur uske liye convention hai wo convention kya hai agar mere paas t aur b parallel hai yaani mere paas right hand yaani mere paas screw dislocation hai aur wo right hand screw hai to is direction mein move hoga wo Burgers vector ke direction mein move hoga ye main convention maan ke chal raha hoon to ye convention aapko yaad bhi rakhna hai agar kuch derivations karne hain dislocation movement ke liye to abhi jaante hain ki simple work done on a crystal kya hoga ye jo internal work done on crystal main is tarah se baat kar raha hoon kyunki main above part ko consider kar raha hoon ek hi part ko to work done on a crystal kuch is tarah se likh sakta hoon ye force mere paas aa gaya force into b ye force move karega mere crystal ko ab above crystal ko is tarah se ye mera aa jayega Work Internal Work Done on a Crystal $F_T \times b$ to main F_T Ki value yahan par likh lu likh leta hoon to ye kuch is tarah se likh sakta hoon abhi main ye jo b hai wo is tarah se le aa idhar le aaunga kyunki ye dot product hai to main work internal work done is tarah se likh paunga $\sigma \cdot b(dl \times dr)$

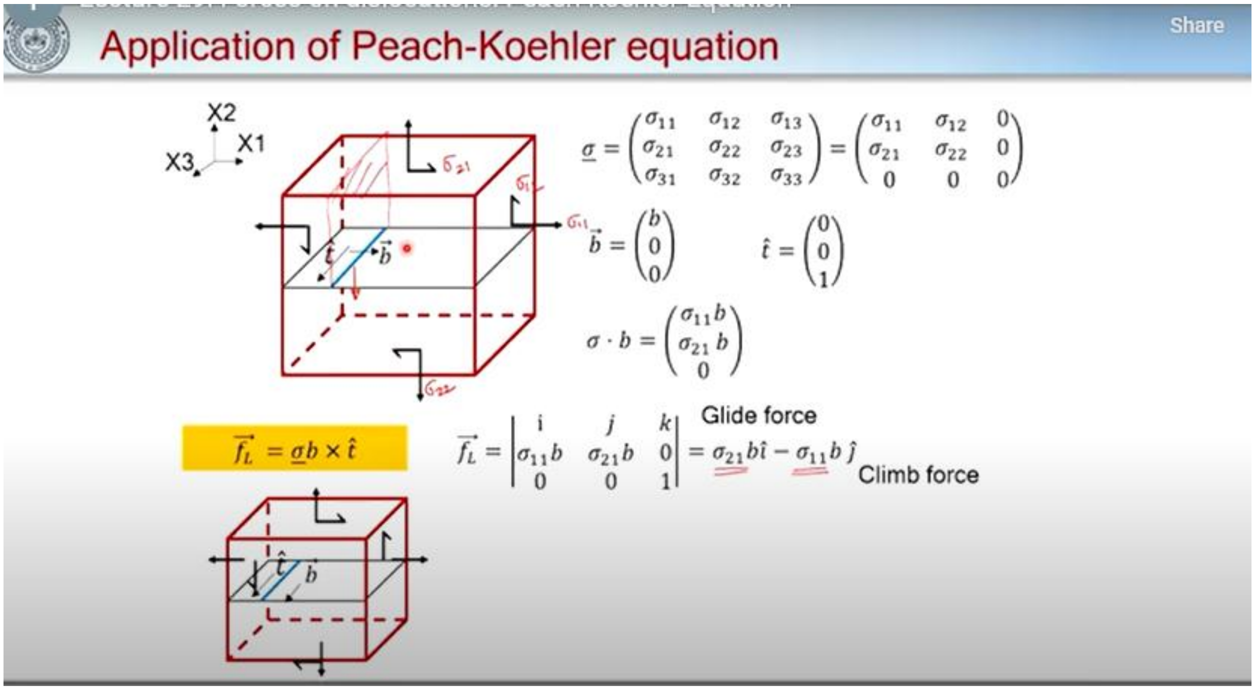


To ye internal work done ho gaya crystal ka abhi main ek property istemal karunga scalar triple product agar mere paas kuch is tarah se $A \cdot (B \times C)$ hai to main usko is tarah se likh sakta hoon $(A \times B) \cdot C$ to main ye term ko is tarah se rearrange kar raha hoon aap dekhiyega ye $\sigma \cdot (dl \times dr)$ tha to main isko likhunga $(\sigma \cdot b) \times dl \cdot dr$ agar main agar ye dekhunga ki jo work done hai agar equilibrium consideration karunga to ye balance hona chahiye work internal work done by crystal is equal to external work on a crystal to agar ye same hai to ye external work kuch is tarah se aa jayega mere paas ye iski value same rahegi aur ye agar same hai to ek main abhi work done on a crystal maine nikaala abhi work done on a dislocation nikalenge to mere paas a force acting on a dislocation hai fourth force per unit length of a dislocation hai kuch is tarah se aur dislocation line segment mere paas kuch is tarah se hai aur mujhe pata hai ki ye dislocation move ho raha hai dr se to humne kuch is tarah se likha tha ye jo dl tha ye move ho raha hai kuch dr se to mere paas ye displacement hai aur mujhe pata hai ki force is dislocation pe kya lag rahi hai ye maine maan ke chala tha ki F_l Hai to work done on a dislocation is equal to work external work on a crystal yaani σ jo external stress act ho raha hai crystal pe ye ho rahi hai saman hona chahiye agar ye saman hai to main ye jo identity hai isko aur is identity ko equate kar sakta hoon agar main isko equate kar raha hoon to mere paas kuch is tarah se relation aa jayega agar hum dekhenge to yahan pe aur dr kyunki ye displacement hai isko main cancel out kar sakta hoon aur ye non-zero hoga to isliye cut ho jayega aur aapke paas kuch is tarah se identity aayegi agar main isko dl ko is tarah se bhejta hoon is right hand side pe ye denominator mein aa jayegi to ye jo term hai ye meri tangent vector hai humne define kiya tha tangent vector ko is tarah se to ye aa jayega mera force acting on a dislocation $(\sigma \cdot b) \times t$ to yahan par ye jo relation hai isko hum kehte hain Peach Koehler Equation ye equation ye darshata hai agar maine abhi do teen slides pehle kuch is tarah se drawing ki thi agar mere paas koi bhi ek crystal hai random crystal hai aur koi bhi ek random plane hai kahin par bhi ek random plane hai maan lete ek plane hai mere paas aur iske upar koi bhi dislocation hai aur main koi bhi state of stress apply kar raha hoon yaane isko main σ maan ke chal raha hoon ismein nine component honge yaane ek state of stress maine yahan par apply ki ye mujhe pata chalegi external ye jo mera hai ye external stress hai iske wajah se kya force act hoga is plane par ye humein ye Peach Koehler equation se pata chalta hai to mujhe kya pata hona chahiye agar mere paas Burgers vector hai dislocation ka aur mere paas tangent vector hai mere dislocation ka to main force nikaal sakta hoon jo main agar stress act kar raha hoon is crystal par to main ek force nikaal sakta hoon to yahan par aap dekhenge ye jo hai maine double line nikaali thi ye darshata hai mera second rank tensor jo σ hai wo mera second rank tensor hai to ye ho gaya mera Peach Koehler equation.



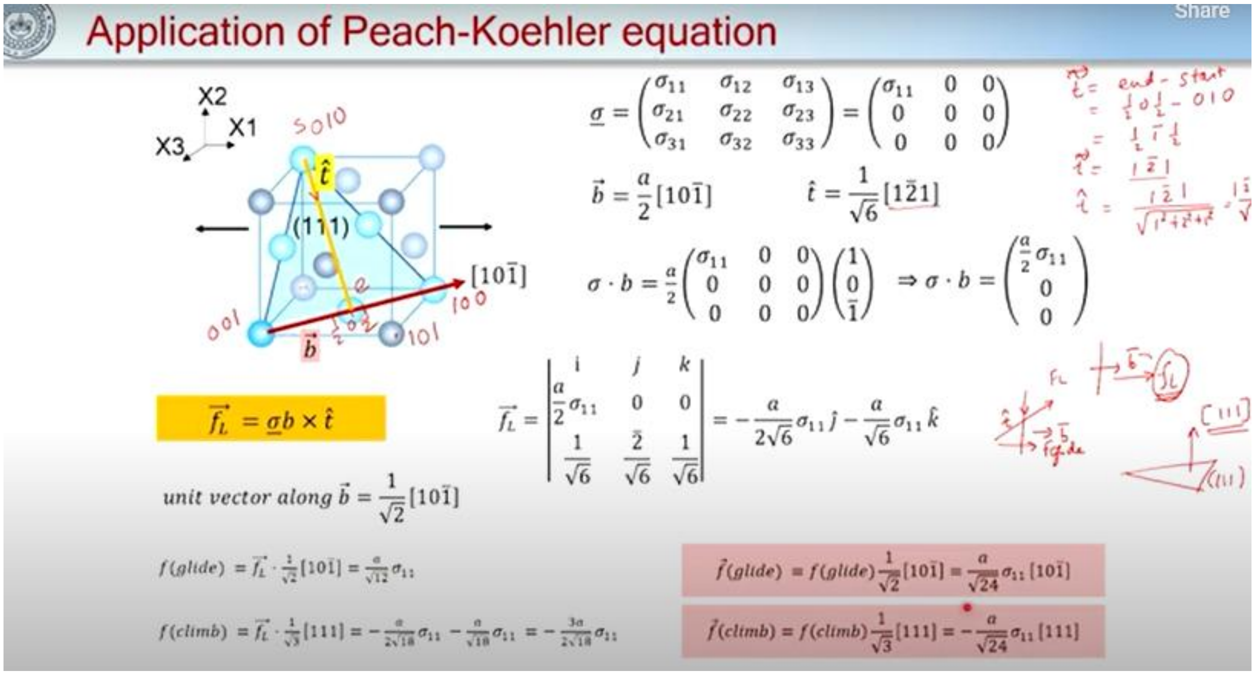
Aaiye jaante iski applications kya hai Peach Koehler equation ki to ye mere paas ek maine ek crystal nikaala hai aur yahan par ek slip plane hai is slip plane par edge dislocation hai is tarah se ya tangent vector perpendicular hai mere Burgers vector ko aur iski coordinate axis maine mark kar li hai ab ek state of stress apply karte random state of stress yahan par aap dekhenge ki maine sirf shear stresses yahan par mark kiye aur yahan par maine stress state likh li yahan par kuch is tarah se aap dekhenge ki agar ye mere coordinate axis hai to mere paas normal stresses yahan par shunya hai aur jo shear stresses hai to shear stresses mere paas σ_{12} Hai σ_{13} Hai σ_{21} Hai aur σ_{31} Hai to yahi mere paas shear stresses available hai agar aap inko identify karna chahte hai to aap us method se kar sakte hain jo humne padhi thi to abhi humein glide force nikaalna hai to glide force ke liye hum pehle dekh chuke hain ki glide force kya hai stress state hai jo along b direction hai to ye stress state hai jo plane 2 par lag raha hai aur one direction mein to σ_{21} To main directly nikaal sakta hoon ki σ_{21} Hi mera glide karega is dislocation ko is plane pe kyunki normal nahi hai to normal stress nahi hai shear stress hai to ye glide karega par abhi hum Peach Koehler ke equation se nikalenge ki sach mein humein glide force milta hai ki nahi milta hai to maine b mark kar liya to b mera x direction ke along hai to main kuch is tarah se likh sakta hoon $b[b00]$ aur t jo hai tangent vector mera ye hai x3 direction pe to isko main likh sakta hoon $t[001]$ ye unit vector hai to main main yahan par one likh sakta hoon yahan par b ye jo hai ye magnitude hai mere Burgers vector ka to b aur t maine yahan par likh liya hai to $\sigma \cdot b$ nikaalta hoon agar main iska product lunga b aur σ ka to mere paas kuch is tarah se aayega $[0 \quad \sigma_{21}b \quad \sigma_{31}b]$ abhi main Peach Koehler equation apply karta hoon to Peach Koehler equation kya hai $F_l = (\sigma \cdot b) \times t$ to main iska aur iska agar cross product nikaalunga to kuch main is tarah se cross product likh sakta hoon aur aap dekhenge ye mere paas jo answer aaya hai ye $\sigma_{21}bi$ to ab dekhenge ye jo i hai ye bata raha hai ki jo force lag raha hai mere ye dislocation pe ye state of stress ki wajah se ye along i direction lag raha hai agar i direction yaani x1 direction pe aap dekhenge ki ye dislocation is direction mein move ho raha hai aur kaun sa component isko move kar raha hai ye move kar raha hai mera ye component σ_{21} Aur humne dekha tha ki σ_{21} Hi ek hi component hai is b a yaani jo along Burgers vector hai is dislocation ke Burgers vector ke along hai ye σ_{21} Hi hai to humne dekha tha ye jo mera stress lag raha hai ye force on dislocation per unit length hai to humne Peach Koehler ke equation ke dwara bhi nikaala hai ki $\sigma_{21}bi$ ye mera glide force yahan par nikalta hai abhi aur ek example lete hain maan lete mere paas ek edge dislocation hai aur yahan par hum apply karte hain normal stresses bhi yaani humne yahan par kuch shear stresses apply kiye aur kuch normal stresses apply kiye to agar humne kuch concept dekhe the isi class mein to ye mera extra half plane ho jayega is dislocation ka aur is dislocation ka extra half plane jo hai yahan pe ye jo normal stress lag raha hai yahan pe to hum dekhenge ki ye dislocation pe climb force act karega aur ye jo shear stress hai yahan pe ye shear stress is dislocation ke upar glide force apply karega to isko samajhte hain agar maine stress state is tarah se likha hai agar jo stress state mere paas ye available hai agar main inko nikaalta hoon to mere paas hai $\sigma_{11}\sigma_{12}\sigma_{21}$ Aur σ_{22} Aap isko nikaal sakte ho jaise ye hai ye jo stress hai ye σ_{11} Hai agar ye nikaalu to ye ye σ_{22} Ho jayega ye agar stress hai to ye σ_{2} Plane pe lag raha hai along one direction to ye σ_{21} Ho gaya isi tarah se ye shear stress aap nikaal sakte ho ye mera one plane par lag raha hai isko bhi likh lete hain hum to ye ho jayega σ ye one plane pe lag raha hai aur along two direction to ye σ_{12} Ho jayega to is tarah se mere paas kuch stress state hai abhi main b ko likh sakta hoon b ko kuch main is tarah se likh sakta hoon $b[b00]$ kyunki along x1 direction hai aur t mera x3 direction par to ye unit vector hoga $[001]$ to agar main $\sigma \cdot b$ nikaalta hoon to mere paas kuch is tarah se aayega $[\sigma_{11}b \quad \sigma_{21}b \quad 0]$ aur agar main Peach Koehler equation nikaalta hoon yaani is dislocation par kaun se forces lag rahe hain is state of stress ki wajah se to main agar ek cross product leta hoon mere paas kuch is tarah se aayega $\sigma_{21}bi - \sigma_{11}bj$ agar aap dekhenge to humne pehle bataya tha ki ye σ_{21} Hai yahi b ke along lag raha tha to mere paas ek glide force aayega is shear stress ki wajah se aur ek climb force aayega is stress ki wajah se jo perpendicular hai mere a ha extra half plane ko to ye mere paas ek climb force aayega aur ye along j direction hai j yaani x2 direction mein to aap dekhenge ki ye jo dislocation hai climb yaani ye force is tarah se act hoga along j direction aur ye negative sign hai to ye climb down hoga yahan pe is tarah se mera force lag raha hai is dislocation pe climb force to hamare paas do component aa gaye glide force aur climb force to main aapko aur ek example dena chahta hoon aap is tarah se ye agar mere paas same stress state hai aur ye screw dislocation hai yahan par to aap kya force lag rahenge iske upar glide force ya climb force ye aapko nikaalne hai aap dekhenge ki humne glide force ke liye nikaala hai ki mere paas shear stress hai shear stress ka ek component hai aur ye normal stress hai to humne dekha tha ki normal stress agar positive hai to climb down hota hai dislocation ka to aap dekhenge ki yahan par negative sign hai to yahan par climb down ho raha hai.

Abhi ye to simple case thi yahan par abhi yaani ye to humne kis direction mein lag raha hai Burgers vector humein pata hai abhi hum crystal structure agar consider karenge to Peach Koehler equation ka kya utility hai ye dekhte hain ki yahan pe humein directly pata nahi chal sakta to aap ye solve kariye screw dislocation ke liye



Aur ek example hum lete hain maan lete hain mere paas ek FCC crystal structure hai to humein pata hai ki ye (111) plane hai ye mera glide plane hai aur ye jo direction hai ye meri Burgers vector ki direction hai ya a slip direction hai ye humne dekha tha aur is crystal ke hum maan lete hain ye coordinates hai yaani ye ye reference axis to is tarah se hum is crystal ko consider karenge ye mera (111) plane hai aur ye jo direction hai ye meri $[10\bar{1}]$ direction hai aapko Miller indices dhundhne padenge yaani kisi physical metallurgy book ko aap refer kariye aur is direction ya plane ko kis tarah se represent karte hain Miller indices se ye padh sakte hain to par ye jo direction hai yahan pe maine $[10\bar{1}]$ direction consider ki hai aur ye jo direction hai ye half direction jo rahegi ye humne dekha tha ki ye mera Burgers vector rahega is dislocation to half aur ye jo ye jo direction hai yahan par yellow jo maine mark ki ye meri tangent vector hai to ye agar main tangent vector consider kar raha hoon aur ye Burgers vector consider kar raha hoon to mere paas ek stress maine apply kiya x1 direction par maan lete hain ye jo mera stress hai x1 direction pe abhi aap dekh payenge ki yahan pe main pata nahi kar paa raha hoon ki ye stress is Burgers vector ko parallel hai ya nahi hai ya is tangent vector ya extra half plane ko perpendicular hai ya nahi hai to is condition mein mere paas Peach Koehler jo equation hai wo uski utility kaafi badh jaati hai to maan lete hain maine stress state likha hai aur yahan pe maine stress state likha hai yahan pe ek uniaxial stress hi apply kar raha hoon x1 direction pe to baaki ke saare stress components hai wo shunya hai to mere paas ek σ_{11} Hi rahega aur b jo hai wo is tarah se maine likha hai $a/2[10\bar{1}]$ to ye jo b rahega yahi di jo shortest lattice translation hai ye mera Burgers vector rahega aur ye rahega $a/2[10\bar{1}]$ aur jo tangent vector hai main kuch is tarah se likh sakta hoon $a/\sqrt{6}[1\bar{2}1]$ to ye kis tarah se nikaalenge dekhte hain to agar ye point agar main consider kar raha hoon to agar main tangent vector kuch is tarah se consider kar raha hoon to aap dekhenge ye jo point hai ye mera x2 direction par hai to ye mera $[010]$ ho jayega aur ye jo point hai ye point ho jayega ye mera x3 aur x1 plane par hai to mera x2 yahan par shunya hai ye one ho jayega aur ye ho jayega one to ye mere paas ek direction ye do coordinates se agar main tangent vector ko is tarah se define kar raha hoon yaani aap vector ko nikaalte hain tab end minus beginning is tarah se nikaalenge to mere paas ye $[101] - [010]$ ye mera start hai aur ye mera end hai to main is tarah se likhunga aur ye jo aayega mere paas one

$[1\bar{2}1]$ is tarah se aayega to agar aap dekhenge to maaf kariye yahan par galti ho gayi hai to is tarah se isko thoda sa aur acche se samajhte hain isko hum is tarah se likhte hain ye jo point hai ye point hai mera $[010]$ aur ye jo point hai ye mera ho gaya $[001]$ ye ho gaya $[100]$ aur ye jo point hoga ye beech ka hoga is tarah se ye ho jayega ye ye point hoga mera $[011]$ aur sorry ye ho jayega $[101]$ to ye jo point hoga ye ho jayega iska aadha to ye hai $1/2\ 0\ 1/2$ is tarah se theek hai abhi ye mera agar main isko start maan raha hoon aur isko end maan raha hoon to main t vector kuch is tarah se likh paunga end minus start to ye aa jayega $1/2\ 0\ 1/2 - 0\ 1\ 0$ to ye aa jayega $1/2\ -1\ 1/2$ to ab jab hum Miller indices consider karte to isko normalize karte to ye aa jayega $[1\bar{2}1]$ to ye jo direction hai yahan par maine likhi hai $[1\bar{2}1]$ ye tangent vector ho jayegi yahan pe aur iska unit vector humein nikaalna hai to ye t vector ho jayega yahan pe aur iska unit vector nikaalna hai to hum iske magnitude se divide karenge to ye ho jayega $1^2 + (-2)^2 + 1^2$ To ye aa jayega $[1\bar{2}1]/\sqrt{6}$ To ye jo value hai mere paas ye t vector ke aa gayi is tarah se main kuch tangent vector nikaal sakta hoon mere crystal mein to mere paas Burgers vector hai mere paas tangent vector hai aur mere paas stress hai abhi to abhi hum Peach Koehler equation apply karte hain is condition mein to main $\sigma \cdot b$ pehle consider karunga to $\sigma \cdot b$ maine consider kiya to yahan par maine bola tha ki main $1/2$ nikaal sakta hoon bahar to ye $a/2$ sorry $a/2$ bahar nikaal sakta hoon to main agar main $\sigma \cdot b$ nikaalu to main in dono ka product consider karunga aur mere paas kuch answer is tarah se aayega $\sigma \cdot b$ ka $a/2[\sigma_{11}\ 0\ 0]$ aap inka product le sakte hain aur abhi hum dekhenge ki agar main Peach Koehler equation apply kar raha hoon to mujhe abhi inka cross product nikaalna hai iska aur iska to main cross product kuch is tarah se nikaalunga to mere paas ye ho jayega $\sigma \cdot b$ aur ye mera tangent vector agar main nikaalu to mere paas kuch ye product aa raha hai yahan par aap dekhenge ye jo product aa raha hai ye ho gaya mera $-a^2/2\sqrt{6}\sigma_{11}j - a^2/\sqrt{6}\sigma_{11}k$ to agar unit vector hum nikalenge Burgers vector ke along to hum dekhenge ki agar glide ho raha hai to ye jo unit vector hai ye $a/\sqrt{2}[10\bar{1}]$ aayega agar main iske magnitude se divide karunga isko b ko magnitude se divide karunga to ye mere paas $a/\sqrt{2}[10\bar{1}]$ ye aa jayega mera unit vector a along Burgers vector to F_{glide} Kya hoga F_{glide} Mera ye hoga ki agar mere paas F_l Hai to main ye jo ye mere paas ek force upon unit length of dislocation aa gaya aur isko main along Burgers vector nikaalunga ye mera ho jayega glide force ka component to isliye main ye jo force hai iska dot product lunga along b to humne dekha tha ki agar mere paas koi dislocation hai aur uska Burgers vector is direction pe to glide force mera isi direction mein hai theek hai to isliye main jo F_l Jo aa raha hai iska agar dot product lunga isse to mujhe is F_l Ka ek component mil jayega aur jo component rahega mera glide component rahega to aap is tarah se bhi samajh sakte hain main thoda aur simplify kar dekhta hoon agar mere paas kuch is tarah se hai aur Burgers vector mera is tarah se is direction mein hai maan lijiye mera tangent vector hai to aur mera F is direction mein aa raha hai to main F_l Ka ek component lunga is direction par jo mujhe dega glide component to isliye main yahan par ek dot product le raha hoon aur mujhe ek magnitude milega glide force ka to magnitude mujhe kuch is tarah se milega $a^2/2\sqrt{12}\sigma_{11}$ Agar mujhe vector aur glide component nikaalna hai aur glide force ka direction nikaalna hai to main main ye jo magnitude mila usko unit vector se multiply karunga to ye F_{glide} Mera aa jayega ye magnitude se maine ye Burgers vector se maine unit Burgers vector se maine multiply kiya to ye aa gaya mere paas $a^2/\sqrt{24}\sigma_{11}[10\bar{1}]$ ye aa gaya mera vector form glide force ka vector form abhi main climb force usi tarah se nikaalunga to yahan par aap ye dekhiye jo F_l Hai to hum dekhenge ki glide force jo hai wo extra half plane ke around hona chahiye to mera ye jo plane hai ye (111) plane hai isko is tarah se samajhiye mere paas ye (111) plane hai iska normal jo hoga wo mera $[111]$ hi hoga ye cubic system ke liye valid hai ye humne FCC ke liye consider kiya to mujhe glide force ka agar component nikaalna hai to main is direction main uska dot product lunga aur yahan iska unit vector jo aayega unit vector mere paas $1/\sqrt{3}[111]$ hai to ye hoga $F_l \cdot$ ye jo unit vector hai along one one direction one one one direction kyun kyunki mera extra half plane is tarah se hoga perpendicular hoga is (111) plane ko isliye humne ye direction jo consider ki hai wo $[111]$ direction consider kiye to main ye agar consider karunga to mere paas magnitude aa gaya climb force ka to ye mere paas ho jayega climb force agar climb force ki direction mujhe nikaalni hai to mujhe sirf uske unit vector se multiply karna hai to agar main isko multiply karunga unit vector se to ye aa jayega mera $\sigma_{11} a^2/\sqrt{18}[111]$ to ye aap dekh payenge ye mere paas kuch climb force ke component hai aur sorry glide force ke component hai aur climb force ke component hai ye humne kis tarah se nikaale pehle humne nikaala tha Burgers vector phir uske baad nikaale the humne tangent vector tangent vector aap nikaal sakte hain ki agar humein coordinates pata hai to hum vector nikaal sakte hain aur hum tangent vector ka humne pehle uske baad unit vector nikaala tha is is tarah se phir humne Peach Koehler equation apply kiye the a yaane $\sigma \cdot b$ pehle product nikaala phir $\sigma \cdot b$ ko cross kiya t se to humein hamare paas kuch ye relation aaye phir humein ye to total force hai yahan par hum bata nahi sakte ki kaun sa mera climb component hai aur glide component hai to ye jo force aaya mera dislocation ke upar usko maine consider kiya along Burgers vector direction aur along extra half plane to jab main Burgers vector ke direction along consider karta hoon to mere paas ek magnitude aayega glide force ka aur jab main along extra half plane consider karta hoon to mere paas ek magnitude aayega climb force ka aur main agar us magnitude ko unke unit vector se multiply karunga to mere paas uske directions bhi aayenge to agar main agar ye consider karunga to mere paas glide jo force hai wo $[10\bar{1}]$ direction mein rahega aur climb force jo rahega wo mere $[111]$ direction mein rahega to ye aap dekhenge ki yahan par ek simple case nahi tha ye yaani aap yahan par intuitively nahi bata payenge ki ek jo force lag raha tha wo mere Burgers vector ke along tha ya nahi tha humein ye deduce karna pada yaani nikaalna pada ki kaun sa mera climb component tha aur kaun sa mera glide component tha to abhi ke liye main yahan par rukta hoon to aaj ke part mein humne dekha tha ki jo force lag rahe hai isko thoda summarize karte force lag rahe hai dislocation ke upar wo hum nikaal sakte hain Peach Koehler equation se ye important relation hai aur Peach Koehler jo equation milega jo force upon per unit length of dislocation rahega $(\sigma \cdot b) \times t$ is tarah se aayega to ye force hai to ye is tarah se humein milega to humein dekhna hai ki jab glide aur climb nikaalna hai force ka component glide aur climb forces ke component humein nikaalna hai



Tab humein dekhna hai ki ye jo glide force hai ye main along mere Burgers vector nikaalu aur climb jo component hai ye mere along extra half plane nikaalu extra half plane ke liye aapko us plane ka normal consider karna hai sirf to is part mein humne Peach Koehler equation ke baare mein padha ye equation bahut hi important hai kyunki aage jaake hum dekhenge ki yahi equation hum istemal karenge kisi do dislocation ke beech mein jo force hai wo nikaalne ke liye abhi ke liye yahan par dhanyavad