Mechanical behavior of materials

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Week-5

Lecture-22

Maximum Shear Strength of Crystalline Material

Namaskar aapka phir se swagat karta hoon Mechanical Behavior of Material is course mein jo hum Hindi mein padhenge. Last part mein humne dekha tha ki plastic deformation ki shuruaat kaise hoti hai aur uske liye humne ek mechanism dekha tha. Humne bola tha ki jo plastic deformation hota hai wah slip ke wajah se hota hai. Is part mein hum slip ko achche se janenge aur jo deformation ke liye shear stress lagta hai, humne bola tha ki shearing se plastic deformation hota hai, to maximum shear strength jo hota hai crystalline material ke liye, yeh is part mein hum janenge.

To pehle jante hain ki slip main jab baat karta hoon to slip kaise hota hai. To maan leta hoon ki mere paas ek do atomic planes hai yahan par, aur yahan par hum hard sphere model ka istemaal karenge. Aur ye jo atomic planes hai ye main maan leta hoon ki mere closed packed planes hai. Humne dekha tha ki slip jo hota hai wo close packed plane aur close packed direction pe hota hai.

To maan lijiye mere paas ye do atomic planes hai aur in dono ke jo plane ka distance hai, to yani center to center agar main maan leta hoon ye ek plane hai, ek center hoga iska aur ek center hoga jo interplanar distance hai, isko main a maan raha hoon. Aur abhi ye jo do distance hai, yani ye do atoms ke beech ka distance hai is plane mein, ye main b maan raha hoon. To mere paas yahan par agar aap dekhenge ki do positions hai atoms ki, ye main equilibrium position maan leta hoon is atoms ki. To ye jo equilibrium distance hai, ye position hai, isko main b maan raha hoon. Maan lete hain ye meri x direction hai aur is direction mein ye jo atomic planes hai ye slip honge. To ya x jo hai ye meri slip direction hai. Agar main ek shear stress apply karta hoon is material pe aur shear stress is level ka badh jata hai to ye atomic jo planes hai ek dusre ke upar slip hogi. Slip yani hum Hindi mein jane the ki ye phislegi ek dusre ke upar.

To yahan par ye agar slip hoti hai to jante ki kaisi hoti hai slip. To main agar shear stress apply karta hoon to aap thoda dhyan dijiye aur ye jo position hai maine equilibrium position se ye maine apni reference position maan li. Agar atom slip hote atomic plane slip hote ek dusre ke upar to ye jo atoms hai ye apne equilibrium position par hi ja sakte yani ek equilibrium position se another equilibrium position ya dusre equilibrium position par hi aani chahiye. Jante slip kaise hoti hai. To aap dekhenge ki atom slip ho gaye yani atom yahan par tha aur ye dusre equilibrium position par aa gaya. Ye atom yahan par tha ye slip hokar yahan par aa gaya. To is tarah se ek atomic plane ka movement hua ek dusre ke upar, isko hi hum slip kehte hain.

Abhi aap janenge ki ye jo configuration hai atoms ka humne mana tha ki yahan par kuch arrangement change hogi jab meri atomic movement hogi ek dusre ke upar to mujhe ek energy chahiye ye atomic planes move karne ke liye. Agar main stress apply kar raha hoon yani iska matlab hai ki mujhe ek energy chahiye jab main displacement kar raha hoon jab Force into Displacement mujhe kya milega energy milegi. To mujhe ek energy chahiye. Agar main energy positions ki baat karunga is atomic ki atomic positions ki to mere paas do equilibrium positions thi, ek yahan pe yahan par sabse energy kam rahegi atoms ki kyunki ek dusre ke upar lie kar rahe is cavity ke upar lie kar rahe to ye yahan par energy kam honi chahiye is do position. To main agar energy variation plot karunga x direction ke hisaab se to mujhe milengi ki ye jo position hai ye jo position maine mark ki thi yahan par mere paas lowest energy aayegi agar mere atom ye exactly is atom ke upar hai to yahan par meri energy highest aayegi, ye aap samajh sakte hain. To mera energy jo variation hoga agar jab slip ho rahi hai to is tarah se hoga kuch.

To agar hum dekhenge ye jo distance hai aur ye jo position hai main ek mark kar leta hoon ek position ye hai atoms ki aur B ye do meri A aur B equilibrium position hai. Aur Q ye meri highest energy position hai kyunki agar atom ek dusre ke upar honge to wo highest energy configuration hoga uska. Aur ye jo distance hai Q aur A aur B ka ye $b/2$ hoga geometry se samajh sakte hain. To ye position meri aur A jo position mein mere atoms ke arrangement is tarah se the ye meri initial position thi aur B meri final position hai. B ye dono equilibrium position hai aap dekhenge ki atoms is cavity ke upar lie kar rahe hain ya equilibrium position par lie kar. Aur Q jo hai wahan par mera atomic arrangement kuch is tarah se hoga ya atoms ek dusre ke upar lie karenge is tarah se aur ye meri highest energy position hai.

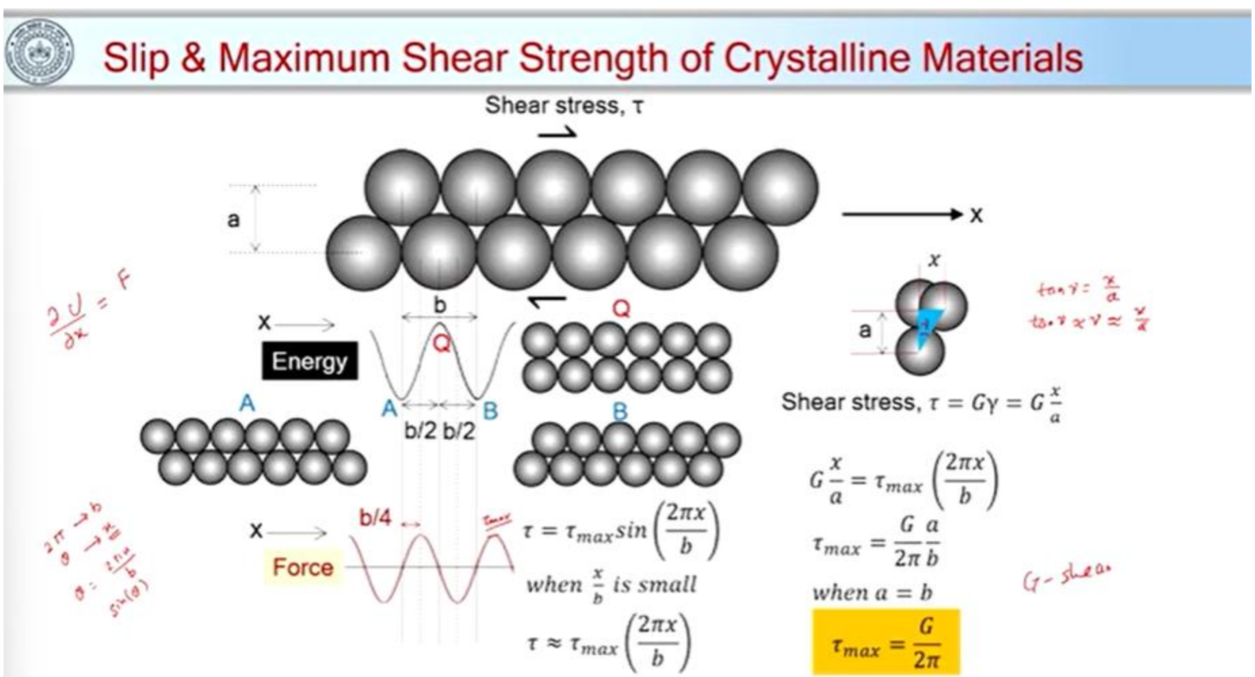
To jab main energy ki baat karunga to main energy se force bhi nikaal sakta hoon. To main force nikaalta hoon ki agar ye energy position is tarah se hai meri ye energy fluctuation hai to mujhe force kitna lagega ya ye atoms ko is tarah se move karne ke liye. To agar mujhe A position se B position tak jaana hai to mujhe ye jo energy barrier hai Q ka ye mujhe cross karna padega aur uske liye mujhe kuch force ki avashyakta hogi. Wahi force hum is part mein nikaalne ki koshish karenge. To humne dekha tha ki energy ka derivative agar lenge agar main U agar ye energy maan raha hoon aur U ka agar main derivative lunga x direction ke hisaab se to mujhe force milta hai. To is tarah se kuch mujhe force aayega humne dekha tha jab humne bond stiffness padhi thi. To agar main force nikalunga to main iska ek derivative lunga aur main derivative is tarah se jab lunga tab mujhe pata chalega ki jo force hai mera shunya rahega equilibrium position ke liye. Agar aap iska derivative loge to aapko is tarah se kuch force milega aur yahan par aapki ek shunya ki value aayegi. To aapke paas teen equilibrium position honge A, B aur Q. Q ye highest energy position ho. Aur yahan par aap dekhenge ki mera force maximum kis point par aa raha hai mera force maximum aa raha hai is point par jo ki ye $b/2$ ka aadha hai ye aayega mera $b/4$.

Agar ye $b/4$ aa raha hai to hum jo shear stress jo lagega kuch is tarah se likh payenge. Agar ye jo stress hai yani shear stress mera x direction ke hisaab se main kis tarah se likh paunga main ek sine wave ki tarah likh paunga kyunki agar main dekhunga ye force variation jo hai ye sinusoidal dikh raha hai lagbhag. To main kuch is tarah se mera shear stress ki value nikaal paunga. Maan lete hain ye jo maximum jo stress lag raha hai shear stress lag raha hai kyunki yahan par hum shear stress apply kar rahe the ye jo maximum stress hai isko main τ_{max} maan raha hoon aur ye jo total distance hai A aur B ka A aur B ke beech ka ye b hai to mera period jo hoga wo b hoga. Aur agar main isko sine wave maan ke chal raha hoon agar main yahan pe 0 degree maan raha hoon to ye π ho jayega 2π position. To ye mera 0 ye $\pi/2$ ye π ye $3\pi/2$ aur ye 2π ho jayega. To hum kuch is tarah se samjhenge agar main 2π move karta hoon to main b distance cross karta hoon. Agar main θ agar koi bhi θ maan ke chal raha hoon agar mujhe x distance cross karna hai aur iske corresponding θ hai to main θ ki value kuch is tarah se nikalunga $2\pi x/b$. To yahi value hai agar hum $\sin\theta$ nikalenge kisi bhi isme position ke saath to is tarah se hum likh sakte hain. To ye ho gaya mera shear stress ka relation ek τ_{max}. τ_{max} yani yahan par likh lete ye jo stress hai maximum stress jo humein milta hai $b/4$ pe milta hai isko main τ_{max} maan ke chal raha hoon.

To ek mere paas ek relation aa gaya ab hum janenge jo displacement hai agar main x/b bahut small agar maan ke chal raha hoon small agar mera small displacement hai x direction par to x/b small hai to main isko thoda simplify kar sakta hoon is tarah se ki main likh sakta hoon $\sin(2\pi x/b)$ isko likh sakta main simply $2\pi x/b$. To ye mere paas ek simple relation aa gaya. Abhi hum janenge ki ye jo position hai yahan se agar main kuch small displace karunga small displacement karunga to main yahan se kyunki ye jo small displacement aayega ye mere shear stress ke corresponding. Agar main small shear stress apply kar raha to ek strain develop hoga aur us ye distance mujhe pata hai a hai aur ye small displacement x main maan ke chal raha hoon. Jo strain shear stress ki jab main baat karunga to main τ ko is tarah se likh paunga agar mujhe shear modulus pata hai material ka to ye shear strain ki value main likh sakta hoon γ aur γ ki value main is tarah se likh sakta hoon x/a. Humne dekha hum hum shear strain jab calculate karte to ye γ likh sakte hain. To main likh sakta hoon ki $\tan\gamma \approx \gamma = x/a$ aur x bahut small hai to $\tan\gamma = x/a$ aayegi aur ye jo value hai shear stress ki τ ye main likh sakta hoon Gx/a. G mera shear modulus hai ye mera stress strain relation hai shear stress strain relation. Aur jab main isko is quantity se compare karta hoon to mere paas kuch aise relation aa jayega aur main kuch is tarah se likh paunga τ_{max} ko agar main τ_{max} ki value nikaal raha hoon ye jo maximum shear stress mujhe chahiye movement ke liye ye kuch is tarah se aayega:

$$\tau_{max} = \frac{G}{2\pi}\frac{b}{a}$$

Agar main a aur b ko equal maan leta hoon yahan pe agar simplify karna chahta hoon aur isko to a aur b agar main equal maan leta hoon to mere paas kuch τ_{max} ki value is tarah se aayegi $G/2\pi$. To ye ho gayi meri theoretical shear strength. Theoretical shear strength yani agar yahan tak main force apply kar paunga ya isko cross kar lunga to main material ko slip karunga yani atomic planes ko slip kar paunga aur yahan par mera plastic deformation hoga. To ye maximum theoretical shear strength aayi jiski value humne dekhi $G/2\pi$. G yahan par mera shear modulus hai material ka isko bhi likh sakte hain G ye mera shear modulus hai material ka. To ek simple relation aa gaya mera. To mujhe pata hai ki maximum shear stress kitna apply karna hai.



Abhi jaaniye dekhte hain ki maine different material yahan pe dekhe aur ye $G/2\pi$ yani mujhe agar shear modulus pata hai to main ek maximum shear strength ki value nikaal sakta hoon isko main likh leta hoon τ_m. To τ_m ki value jo hai $G/2\pi$ aur yahan par dekhenge ki$G/2\pi$ ki value aa rahi hai 4.6 GPa Silver ke liye aur aap dekhenge ki GPa mein hai saari values. 2π yani aap agar consider karenge to G kisi kisi books mein isko $G/10$ bhi likhte hain kyunki

2×3.14, 2π ki value agar dalenge to aapke paas $G/10$ lagbhag ye value aayegi. To aapke paas τ_{max} jo value hai theoretical shear strength ye values yahan par milegi.

[Table comparing theoretical and experimental shear strength values for different metals]

Aur laboratory mein hum shear strength ki value nikaal sakte hain experimentally aur ye jo values hai most of the metals aur materials ke liye ye MPa mein aa rahi hai aap dekhenge ki Silver ke liye yahan par theoretical shear strength 4.6 GPa Giga Pascal hai par yahan par experimental jo observed value hai 37 MPa yani ek bahut kam hai jo experimental observed value jo hai wo theoretical value se bahut kam hai. Agar hum inka ratio nikalenge to aap dekhenge ki jo ek theoretical jo shear strength hai wo experimental observed value se kam se kam chaar order higher hai Silver ke liye aur baaki material ke liye bhi aap dekhenge to ye kam se kam ek se chaar order higher hai. To ye values agar hum dekhenge to main conclusion likh sakta hoon ki meri jo maximum shear strength hai ya theoretical shear strength hai experimental observed value se ye 10^2 to 10^4 itni high hai yani meri experimental observed value hai bahut kam hai.

Theoretical vs Experimental Yield Strength

Material	$\tau_m = G/2\pi = \frac{G}{10}$		Experimental Yield Strength		
	GPa	10^6 psi	MPa	psi	τ_m/τ_{exp}
Silver	4.6	0.67	0.37	55	$\sim 1\times10^4$
Aluminum	4.2	0.61	0.78	115	$\sim 5\times10^3$
Copper	7.2	1.05	0.49	70	$\sim 1\times10^4$
Nickel	12	1.78	3.2-7.35	465-1,065	$\sim 4\times10^3$
Iron	13	1.91	27.5	3,990	$\sim 5\times10^2$
Molybdenum	19	2.76	71.6	10,385	$\sim 3\times10^2$
Niobium	5.8	0.84	33.3	4,830	$\sim 2\times10^2$
Cadmium	3.8	0.56	0.57	85	$\sim 7\times10^3$
Magnesium (basal slip)	2.8	0.4	0.39	55	$\sim 7\times10^3$
Magnesium (prism slip)	2.8	0.4	39.2	5,685	$\sim 7\times10^1$
Titanium (prism slip)	6.3	0.92	13.7	1,985	$\sim 5\times10^2$
Beryllium (basal slip)	23.4	3.39	1.37	200	$\sim 2\times10^4$
Beryllium (prism slip)	23.4	3.39	52	7,540	$\sim 5\times10^2$

$$\frac{\tau_m}{\tau_{exp}} = 10^2 - 10^4$$

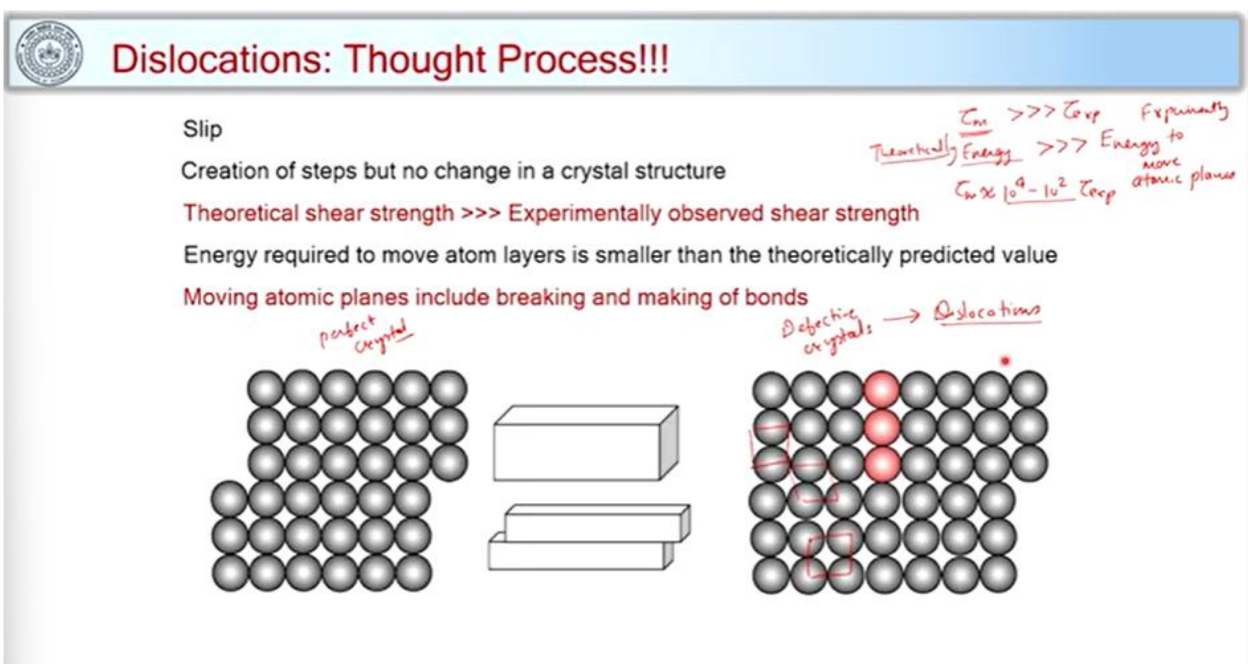
Why the theoretical strength is much higher than the experimentally observed strength???????

To iska reason kya ho sakta hai ye abhi humein dekhna hai. To ek question humein aayega ki theoretical shear strength ye experimental observed value se kyun zyada hai ye humein dekhna hai. To iske liye concept Dislocation ka concept scientists ne nikala hai aur ye concept pehli baar teen scientist ne propose kiya tha Polanyi, Taylor aur Orowan ne. So in teeno ne dislocation ka ek concept nikala jo ye jo discrepancy hai theoretical shear strength aur experimentally observed shear strength ki discrepancy ko mitayega. To ye dislocation ka ne 1934 mein propose kiya tha aur jo theoretical shear strength evaluate kiya tha material ke liye Frenkel ne 1926 mein yani kam se kam 8 saal ke baad ye dislocation jo concept hai ye proposed hua aur ye jo dislocation concept hai ye explain karega ki mera theoretical shear strength experimental observed shear strength se kyun zyada hai.

To abhi hum dekhte hain ki ye thought process kaise hui. Humne jaana ki jab plastic deformation hota hai to uska ek mechanism hai slip aur yahan par hum dekhenge ki jo slip hota hai humne dekha tha ki slip steps taiyar hoti hai but wahan par koi crystal structure change nahi ho raha. Aur abhi humne dekha ki meri jo theoretical shear strength hai wo meri experimental observed shear strength se kaafi zyada hai. To agar ye zyada hai to agar hum dekhenge ye mere paas ek crystal hai aur ye mera deformed humne dekha tha ki slip plane aur slip direction hai to humne dekha tha ki humein agar atoms atomic planes ko move karna hai atomic layers ko move karna hai to humein energy chahiye. Agar energy chahiye agar hum dekhenge hum is tarah se kuch samajh sakte hain agar meri theoretical value jo maximum shear stress value bahut zyada hai experimental observed value se to main bol sakta hoon ki yahan par mujhe jo energy chahiye maximum energy ye bhi zyada honi chahiye mujhe experimental energy yani atomic planes move karne ke liye to move atomic planes yani atomic planes ko ek dusre ke upar move karne ke liye experimentally mujhe kam energy chahiye jo main estimate kiya tha theoretically wo zyada energy dikha raha hai. Iska matlab kya ho sakta hai?

Iska matlab aap is tarah se samajh sakte hain ki jab main atoms atomic planes ko move karta hoon wahan par bonds banenge aur tutenge yani agar atom main ek dusre ke upar move kar raha hoon to wahan par bonds kabhi banenge kabhi tutenge aur ye jo making aur breaking of bonds hai iske liye humein energy chahiye aur theoretically jo energy hum predict kar rahe hain wo zyada hogi meri experimentally observed value se. To kuch is tarah se samjhte hain ye mere paas atomic planes hai aur main isko deform karna chahta hoon agar main shear stress apply karta hoon agar ye mera crystal hai aur isko main deform kar raha hoon ye mera perfect crystal hai aur is tarah se main usko deform karunga to atom atomic level pe kuch is tarah se aayega humne dekha tha ki yahan par kuch bonds bane kuch bonds toote yahan par aap dekhenge ki ye agar iska dekhunga ye atom isse bhi bonded hai isse bhi bonded hai par ye ye agar structure main consider karunga to yahan par mera crystal structure change ho raha hai. To main phir se isko deform karunga aur mujhe yahan par surface step milegi. To mere paas yahan par do baar mujhe atomic bonds break karne pade aur make karne pade maine do steps involve thi agar mujhe ye outcome chahiye to.

Agar agar aisa sochiye ki mere paas material mein microscopic yani atomic level pe hi jo bonds hai wo already toote hue hain yani wahan pe ek local defects hai. Agar main is tarah se dekhunga ye mera crystal structure hai humne dekha tha ye mera crystal structure hai aur yahan par mera crystal structure hai dono crystal structure same hai par yahan par agar dekhunga mere paas to ye yahan par local defect taiyar ho rahe hain to ye already local defects material mein present hote hain to ye jo local defects hai ye dislocation concept ko rise karenge. Hum janenge dislocation ke baare mein kya hai par aap yahan par hum vichar dekh rahe ki ye dislocation ki concept kaise sochi gayi thi. To hum dekhenge ki yahan par already bonds jo hai locally yahan par agar is planes par agar aap dekhenge ye locally already broken hai to agar main yahan par shear stress apply karta hoon is tarah se to main yahan par surface step taiyar ho jayegi aur yahan par mere paas ek perfect crystal mil jayega. To mujhe sirf ek step mein hi surface step mil gayi ya atomic planes yahan par ek hi step mein mujhe wo move ho gaye. To humne dekha yahan par agar dekhenge to yahan par mere paas zyada energy lag rahi hai aur is case mein mere paas kam energy lag rahi hai. Hum dono case mein agar ye meri theoretical value thi ye mere paas perfect crystal tha ye maine perfect crystal consider kiya tha aur yahan par maine defective crystal consider kiya tha. Aur ye jo defects hote hain isko hi hum dislocations ke naam par janenge aur iske baare mein hum next class mein dekhenge ye dislocations hote kya hai.



Is part mein humne dekha ki jo theoretical shear strength hai material ka wo kaafi zyada hota hai kitna order zyada hota hai wo 10^4 to 10^2 order zyada hota hai mere experimentally observed value se aur ye zyada kyun hota hai kyunki main consider karta hoon jab theoretical shear strength calculate karta hoon to main perfect crystal consider kar raha hoon aur jab experiment karte hain to hum defective crystal pe karte hain kyunki generally hamare paas jo material hote hain usme inherent defect hote hain yani pehle se hi defect present hote hain aur wahi defects mere dislocations kehlayenge. Wo is ye jo defect hai iske baare mein aur janenge dislocation ke baare mein ye hum janenge next part mein. Abhi yahan par rukta hoon dhanyavad.