

Mechanical behavior of materials

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Week-4

Lecture-21

Resolved Shear Stress & Schmid's law



Mechanical Behavior of Materials (Hindi)

Resolved shear stress & Schmid's law

Namaskar phir se swagat karta hoon is course mein jiska naam mechanical behavior of material hai jisko hum hindi mein padhenge last part mein humne dekha tha ki plastic deformation ka ek mechanism jo slip hai par jo slip hota hai kis trace par hona chahiye aur uske liye kya law hai is part mein hum dekhenge to us usse pehle hum dekhte hain ki mere paas ek crystal hai ek undeformed crystal hai aur ye jo crystal hai ye jab main isko shear force apply karunga to ek kis ek direction mein deform hota hai ye humne dekha tha slip hota hai ek direction mein agar main ek shear stress apply kar raha hoon is undeformed crystal pe to ye is shear stress ke karan ye deform hoga wo is tarah se kuch deform hoga hum dekhenge ki ye upar wala jo part hai crystal ka wo slip hua hai niche wale part se to ye jo slip ho raha hai ye particular crystallographic plane par hota hai aur ek particular crystallographic direction hota hai yeh mera deformed part ho gaya to aap dekhenge ki mera shape jo yahan pe hai yahan par change ho gaya to agar hum dekhenge ki slip plane yahan par kahan hai to schematics mein hum dekhenge yeh mera slip plane ho gaya yeh jo plane ho raha hai aur yeh jo direction thi jis direction mein slip hua is direction ko hum kahenge Slip Direction ye mera Slip Plane ho gaya ye Slip Direction ho abhi hum jaanenge is part mein ki mujhe kitna stress apply karna padega is crystal ko deform karne ke liye to usse uske liye hum simple ek cheez karenge ki jo stress stress mujhe nikaalna hai slip ko occur hone ke liye slip ko

happen hone ke liye to iske liye main consider karunga ek Single Crystal to single crystal yaani yeh hota hai ki ismein koi bhi high angle grain boundary nahi hoti hai to abhi aap dekhenge yeh jo iske baare mein hum jaanenge high angle grain boundary ke baare mein iske liye aap pareshan nahi hue but jaaniye ki single crystal hota kya hai to single crystal hai mere paas aur yahan par koi interface nahi hai yahan par saare jo hai single crystal hai ek hi material ka bana hua hai to isko agar main deform karta hoon ya is tarah se agar mere paas tensile sample hai single crystal ka aur main usko is tarah se deform karta hoon to mujhe dhundhna hai ki ye jo stress jo apply kar raha hoon main tensile stretch jo apply kar raha hoon yeh kitna hona chahiye jab yeh material mera deform hona chalu hoga deform plastic deformation ki baat kar raha hoon to ismein jo shape change hona chalu hoga to iske liye hum jaanenge ki humne dekha tha ki maan lete ki iska ek jo cross section area pehle yeh A hai yeh jo cross section area hai yeh mera A hai aur ek plane maan lete jahan par mera slip ho raha hai yeh plane jo hoga mere material material ke liye yeh mera close packed plane hona chahiye humne dekha tha to maan lete koi bhi plane hai aur koi bhi plane yaane yeh close packed planes hai par ismein bhi kaun sa close packed plane hai yaani humne dekha tha ki mere paas 12 slip systems the FCC mein to 12 mein se kaun sa slip plane active hoga yaani jis par deformation chalu hoga yahi humein dekhna hai to maan lete hain mere paas ek slip plane hai aur uska cross section area main maan leta hoon A_s abhi main slip plane ko is tarah se define kar sakta hoon ek Slip Normal nikaal sakta hoon yaani is plane normal nikaala maine jisko maine n se represent kiya hai to ye n is plane ka normal hai abhi humne dekha tha ki jo slip hota hai ek particular direction pe hota hai aur abhi maan lete ki yeh jo n hai normal hai slip normal isko pehle location fix kar lete ye yeh jo normal hai ye ϕ angle bana raha hai is tensile axis ke agar hum dekhenge ki uniaxial tension stress hum tensile stress hum apply kar rahe hain to uska ek axis hoga us axis se yeh mera plane bana ek angle ϕ aur abhi hum mark kar lete hain meri slip direction slip direction meri slip plane mein hi honi chahiye humne dekha tha ki slip direction slip plane mein hoti hai to ek direction maine mark kar li d se aur iska jo angle hai tensile axis se slip direction ka isko main mark karta hoon λ se to λ ho gaya mera angle between tensile axis and slip direction.

abhi hum dekhte hain agar main stress apply karta hoon to slip kab hona chahiye to uske liye jaante hain pehle maine agar tensional force yahan par mujhe force nikaalna padega agar main force nikaalta hoon is area pe jo is area par act ho raha hai yeh force kaise aayega agar mere paas stress pata hai to main force nikaal sakta hoon to Force is equal to Stress into Area yeh aa jayega mera tensile force is single crystal ke cross section area par lag raha hai abhi hum jaanenge ki yeh jo component hai F humein kya nikaalna hai yeh jo component hai yeh is direction pe slip direction pe kaise lag raha hai yeh agar force ka component hum nikalenge to hamare paas aa jayega ki Shear Stress kya lagana chahiye ye slip ko happen hone ke liye occur hone ke liye to yeh jo force ka agar main yahan pe F nikaal raha hoon ye tensional force hai iska component is direction mein kya hoga d direction mein isko main mark kar raha hoon F_d se yeh ho jayega $F \cos \lambda$ kyunki angle iske aur iske beech ka λ hai to ye ho jayega $F \cos \lambda$ abhi main nikaalta hoon ki Resolved Shear Stress is plane pe kya hai yaani agar main shear stress nikaalne ki koshish kar raha hoon shear stress ki definition humne jaani thi ki shear stress ek plane pe act hota hai right to wo ek plane pe act hota hai to yeh humne shear stress is tarah se mark kiye the to yeh agar mere paas yeh stress hai hum kya kar rahe hain hamare paas ye stress hai aur hamare paas ek slip plane hai to ye stress ka component humein nikaal nikaalna hai yeh value humein nikaalni hai shear stress ki value to resolved shear stress isko hum kehte hain resolved shear stress to yeh slip plane pe jo act ho raha hai along slip direction yeh kya aana chahiye isko main define kar raha hoon τ_{RSS} humne dekha tha ki shear stress ko hum τ se represent karte hain aur isko hum kehte hain Resolved Shear Stress ye hoga mera F_d/A_s yaani jo component lag raha hai F_d shear component along d direction aur ye

jo area hai slip plane ka isse main divide kar raha hoon to yeh mere paas aa jayega shear stress τ_{RSS} isko main kehta hoon agar hum dekhenge A_s ko main is initial jo cross section area ya cross section area hai mere single crystal ka iske hisaab se main likh sakta hoon to F_d ko pehle maine yahan par $F \cos \lambda$ likh diya aur A_s ko main is tarah se likh sakta hoon $A / \cos \phi$ aap dekhenge ki agar yeh yeh agar normal hai yeh angle ϕ bana raha hai to aap dekhenge ki yeh jo area hai initial cross section area aur yeh jo area hai A_s yeh bhi angle ϕ banayega ek dusre ke saath aur main A_s ko $A / \cos \phi$ likh sakta hoon is geometry ke hisaab se to main simply ye yahan par replace karunga denominator mein to mere paas ye kuch is tarah se term aa jayegi τ_{RSS} ko main is tarah se likh paunga $F \cos \lambda \cdot \cos \phi / A$ abhi aap jaanenge ki F/A ko main initial jo stress hai apply kar raha hoon σ is relation se isko main replace kar sakta hoon aur main τ_{RSS} ko $\sigma \cdot \cos \phi \cdot \cos \lambda$ is tarah se likh sakta hoon ya $\tau_{RSS} = \sigma \cos \lambda \cos \phi$ to mere paas ek equation aa gaya is equation ke hisaab se main likh sakta hoon agar mere paas σ pata hai mujhe aur mujhe mere paas pata hai ki mera plane kahan par located hai mujhe λ aur ϕ pata hai to main wahan par shear stress kitna lag raha hai yaani kitna zaroori hai deformation ke liye yeh nikaal sakta hoon kyunki yeh isliye nikaal rahe hain kyunki humne dekha tha ki slip jo hota hai wo shearing ke wajah se hota hai to abhi hum dekhte hain ki Critical Resolved Shear Stress kya hai ya τ_{CRSS} kya hai uske pehle jaante hain ki do special conditions kya hai ye mere paas ek relation aa gaya $\tau_{RSS} = \sigma \cos \lambda \cdot \cos \phi$ to ye jo term hai $\cos \phi \cos \lambda$ isko main replace kar raha hoon m se ek m number yeh isko main kehta hoon Schmid Factor yeh Schmid factor Professor Schmid ke naam par famous hai.

Resolved shear stress (τ_{RSS})

Undeformed

Deformed

Slip Plane
Slip Direction

What is the stress needed for slip to occur?

Tensile force acting on Area A
 $F = \sigma A$

Component of F in along slip direction
 $F_D = F \cos \lambda$

Resolved shear acting on slip plane along slip direction
 $\tau_{RSS} = \frac{F_D}{A_s} \Rightarrow \tau_{RSS} = \frac{F \cos \lambda}{A_s}$

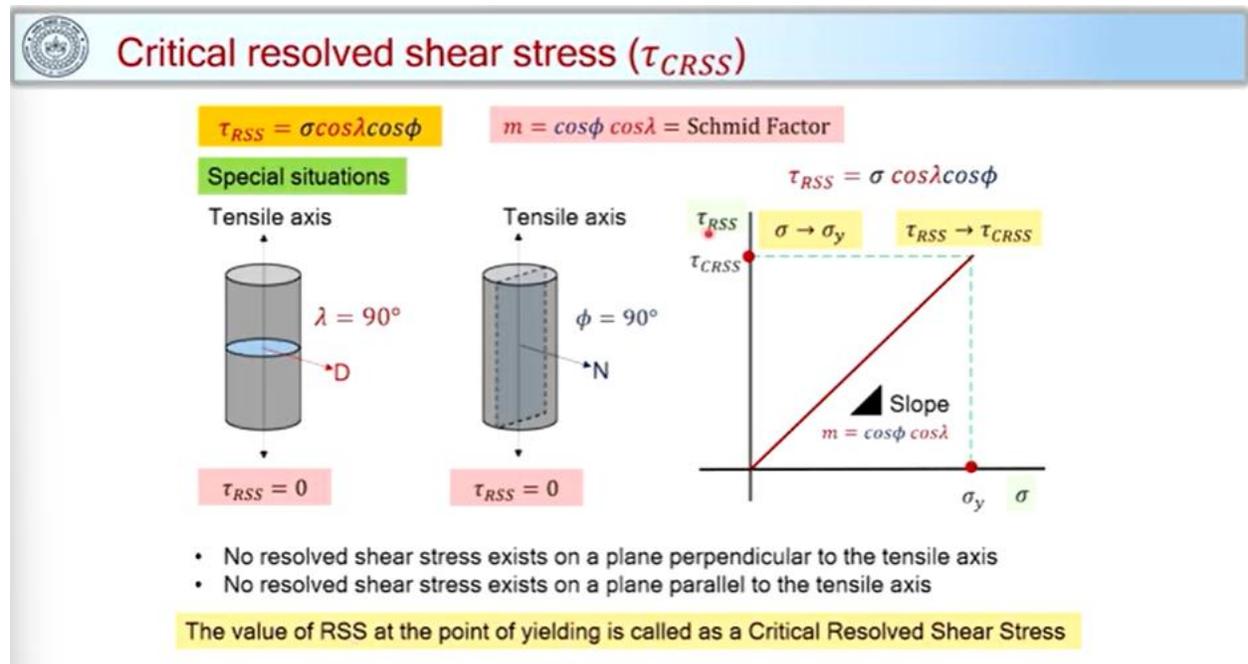
$\tau_{RSS} = \frac{F \cos \lambda \cos \phi}{A} \quad \because A_s = \frac{A}{\cos \phi}$

$\tau_{RSS} = \sigma \cos \lambda \cos \phi$

ϕ : angle between tensile axis and slip plane normal
 λ : angle between tensile axis and slip direction

To yeh ho gaya mera Schmid factor yeh ho gaya mera special situation hum dekhte hain special situations kya ho sakti hai maan lijiye mere paas ek single crystal hai aur main usko deform kar raha hoon tensile force apply kar raha hoon to ye mere tensile axis ho gaye is axis ke around main force apply karunga ya stress apply karunga to mere paas do planes hote ho sakte slip planes maan lijiye mere paas yeh slip plane hai to aap jaanenge ki is plane jo hai wo perpendicular hai iska agar normal dekhenge to yeh along tensile axis hoga agar main normal draw karunga is plane ka to yeh along tensile axis hoga to yaani jo ϕ hai wo shunya hona chahiye aur jo λ hai yahan par yeh λ ho jayega agar d agar main mark karunga yahan par to ye slip plane mein hona chahiye to agar iska

normal parallel hai tensional axis se to jo direction hai ye perpendicular honi chahiye tensional axis se to mere paas λ ki value 90 aa jayegi to mere paas $\tau_{RSS} = 0$ aana chahiye to kyunki main $\cos\lambda$ agar 90 put karta hoon to ye value shunya aayegi aur mere paas τ_{RSS} ki value shunya aayegi similarly ek aur ek special plane agar mujhe mil sakta hai is case mein agar mere paas ye tensional axis hai is tarah se aur mere paas ye single crystal hai aur ye ek plane consider kar raha hoon ye jo plane hai to yaani iska matlab ye ho jayega ki ye jo iska normal hai plane ka normal ye ho jayega perpendicular to tensional axis yahan pe ϕ ki value 90 ho jayegi aur λ ki value yahan pe shunya ho jayegi to is special case mein bhi mujhe mere paas τ_{RSS} ki value shunya aayegi agar main is case mein ϕ 90 rakhta hoon to mere paas τ_{RSS} ki value shunya aayegi to ye mere paas do special planes hai jahan pe koi bhi resolved shear stress exist nahi karta hai do kaun se planes hai ek plane perpendicular to tensile axis ye perpendicular to tensile axis aur dusra plane hai jo parallel to tensile axis ye mere paas do conditions ho gayi jo do special planes hai jahan pe mere paas koi resolved shear stress exist nahi karta.



Abhi hum jaanenge ki agar main yeh equation ko dekhu carefully to aap dekhenge ya τ_{RSS} ko main likh raha hoon $\sigma \cos\lambda \cos\phi$ agar main σ yahan pe plot karta hoon x-axis par aur τ_{RSS} y-axis par plot karta hoon is tarah se to agar hum dekhenge yeh ek linear relation de raha hai aur yeh jo slope aayega yahan par $\cos\phi \cos\lambda$ yeh jo slope hai yeh mera Schmid factor rahega to agar aap dekhenge agar main is single crystal ko maan ke chal raha hoon main isko deform kar raha hoon tensile deformation de raha hoon to main σ ki value badha raha hoon aur ek limit aayegi jahan par mera ek single crystal deform hoga plastically to wo value hogi hamari Yield Strength hai na to wo agar yield strength hai to main wo yield strength yahan pe mark kar raha hoon yaani yahan pe aake mera material plastically deform hona chalu karega to ye jo normal stress hai ye σ_y jo value hai iske corresponding mujhe ek τ value milegi resolved shear stress ki value milegi isko main kehta hoon τ_{CRSS} yeh jo value hai yeh meri Critical Resolved Shear Stress hai yaani agar main σ_y cross karunga ya σ_y reach karunga to us plane pe ek shear stress ki value reach hogi wo rahegi meri τ_{CRSS} aur ya critical resolved shear stress ek critical value hai resolved shear stress ki isko

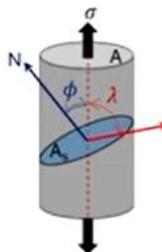
mathematically main kaise likh paunga agar mera $\sigma \rightarrow \sigma_y$ to τ_{RSS} yaani resolved shear stress $\rightarrow \tau_{CRSS}$ to aap dekhenge ki agar main σ_y tak ki taraf ja raha hoon to yeh jo τ ki value hai badh rahi hai aur yeh CRSS ki taraf pahunch rahi hai to mera single crystal kab deform hoga main is tarah se jaan sakta hoon jab mere us slip plane pe value jo hogi wo τ_{CRSS} tak reach hogi to yeh jo value hai ek kisi bhi plane ke liye ek ek critical value hai yeh critical value us plane par reach hogi tabhi wo jaake slip honge planes ek dusre ke upar to RSS jo hai the value of RSS at point of yielding is called Critical Resolved Shear Stress yahi humne yahan pe dekha hai ki jo RSS ki jo value hai τ_{RSS} ki jo value hai at yielding is τ_{CRSS} ye aapko yaad rakhna hai abhi hum ek example dekhte hain jaise Schmid's Law ko dekhte hain acchi tarah se to hum dekh sakte hain ki mere paas ek Schmid factor hai $m = \cos\lambda \cdot \cos\phi$ yeh geometry humne dekhi thi to hum dekh sakte agar meri yeh jo tensile axis hai agar isko main change kar raha hoon yaani yeh ϕ aur λ agar change kar raha hoon to mera Schmid factor change ho sakta hai hamare paas yeh relation hai yielding pe hum is tarah se likh sakte hain $\tau_{CRSS} = \sigma_y \cos\lambda \cdot \cos\phi$ agar hum yeh relation agar dekhenge to yeh jo factor hai yeh mera change ho raha hai to mere paas teen questions aa sakte hain to ye yielding pe bhi ye equation satisfy hona chahiye to agar ye yielding pe equation satisfy hoga to teen possibilities ho sakti hai ki mera σ_y bhi change ho sakta hai aur mera τ_{CRSS} constant reh sakta hai ya τ_{CRSS} change ho sakta hai aur σ_y constant reh sakta hai ya σ_y aur τ_{CRSS} dono change ho sakte hain isko thoda samajhiye agar main yeh value change kar raha hoon agar mera m change ho raha hai m kaise change hoga agar mera maine tensional axis dusri tarah pe liya hai ya dusre direction mein liya to yeh mera m ki value change hogi par yielding pe yeh equation satisfy hona chahiye agar ye m change ho raha hai to ek to σ_y change hoga ya τ_{CRSS} change hoga ya dono change honge kyunki yeh identity aapki valid honi chahiye yielding pe bhi to yeh jo puzzle hai yeh solve ki thi Professor Eric Schmid ne to unhone kuch careful experiments kiye the experiments kiye the unhone Magnesium single crystal pe yahan par maine result dikhaya unhone bahut saare single crystals pe experiments kiye the aur unhone dekha tha ki yahan par aap dekhenge ki $\cos\phi \cos\lambda$ yeh jo Schmid factor hai wo change ho raha hai yahan pe yahan par dekhenge ki shunya ki taraf se jaakar shunya ki taraf aa raha hai aur beech mein 0.5 hai to values yahan par change ho rahi hai aur aap dekhenge ki yahan par tensile stress unhone plot kiya hai to tensile stress unhone dekha hai ki tensile stress yahan par change ho raha hai mere Schmid factor ke saath to aap dekhenge ki σ_y change ho raha hai na ki τ_{CRSS} to unhone bahut saare careful experiments kiye aur inhone ek conclusion nikaala aur yeh law hai ki jo critical resolved shear stress hai wo independent hai orientation se yaani orientation change ho raha hai mere tensile axis ka mere slip system ke saath to uska koi critical resolved shear stress par koi asar nahi hota yaani τ_{CRSS} ek constant value hai kisi bhi given plane ke liye isko hum kehte Schmid Law.

Schmid law ye uska definition ho gaya ye mathematical equation ho gaya par ye uska definition ho gaya yahan par aap dekhenge ki yeh jo hai σ_y mera change ho raha hai ya mera stress change hota hai material ke orientation ke saath but τ_{CRSS} kisi bhi plane par wo constant rehta hai yeh hai mera Schmid's law abhi aap dekhenge acchi tarah se yahan par agar magnesium single crystal hai to aap dekhenge ki mera yield strength ye direction ke saath change ho raha hai isko hum kehte hain Anisotropy yahi meri anisotropy of single crystal hai aap dekhenge ki yeh value yahan par $\cos\lambda \cos\phi$ jo value hai Schmid factor ki ye ye point 5 hai aur yahan par aap dekhenge ki tensile jo stress mil raha hai mere crystal ka wo sabse kam hai isliye isko hum kehte hain Soft Orientation aur yeh jo orientation hai yahan par aap dekhenge yahan par ab tensile stress highest hai ya maximum hai yeh do part pe in dono ko hum kehte hain Hard Orientation to mera single crystal hard orientation ki kab kab jayega humne ek do special cases dekhe the jab ϕ ya λ yeh 90 degree

hoga mere tensile axis se ye jo ho gayi ye meri ye jo ho gayi ye meri hard orientation ho gayi aur meri soft orientation kya ho gayi meri soft orientation hogi jahan par mera Schmid factor maximum rehta hai jo ki jiski value hum 0.5 maan ke chalte hain ye 0.5 kab aayegi jab mera meri θ ki value ya θ_m yaane λ aur ϕ ki value 45 degree rahegi to aap dekhenge agar main λ ki jagah 45 rakh raha hoon aur ϕ ki jagah 45 rakh raha hoon to yeh jo product aayega iski value aayegi 0.5 to aap dekhenge ki jo plastic deformation hai usi slip system par chalu hogi jahan par yeh value maximum rahegi yaani jahan par Schmid factor maximum rahega isko hum thoda jaanenge kuch examples ke saath par aap Schmid's law ko acche se samajhiye τ_{CRSS} kisi bhi slip plane ke liye change nahi hota hai wo constant rehta hai yaani wo apne jo tensile axis hai uske orientation ke hisaab se wo independent rehta hai to ek solved example likhte hain to yahan pe



Schmid's Law



Prof. Erich Schmid
4 May 1896 - 22 Oct 1983



E. Schmid and W. Böse, "Kristallphysik" (Springer-Verlag 1935)

✓ We can change the Schmid factor i.e., $m = \cos\lambda\cos\phi$ by changing the orientation of a tensile axis

$\tau_{CRSS} = \sigma_y \cos\lambda\cos\phi$ At yielding, this equality must be satisfied

Then,

- σ_y changes and τ_{CRSS} remains constant?
- τ_{CRSS} changes and σ_y remains constant?
- Both σ_y and τ_{CRSS} changes?

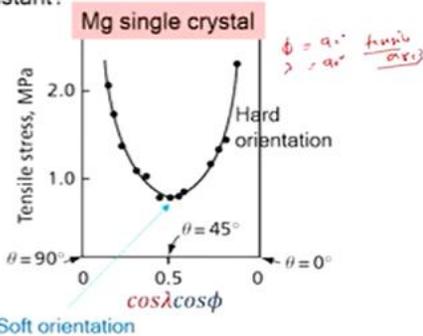
Schmid's Law

"Critical resolved shear stress is independent of the orientation of stress axis with respect to the slip system"

σ_y changes and τ_{CRSS} remains constant

Anisotropy of a single crystal

Plastic deformation start on a slip system with the highest Schmid Factor



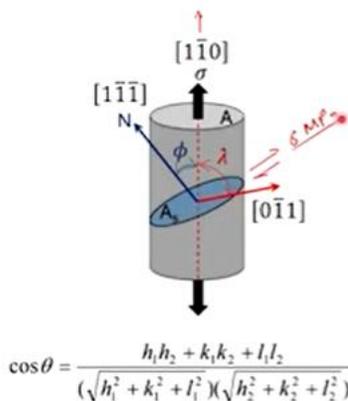
E. C. Burke et al. Trans. AIME, 194, 266 (1952)

Ek example hai determine the tensile stress that is applied along $[1\bar{1}0]$ axis of a silver crystal to cause slip on $(1\bar{1}\bar{1})$ plane and $[0\bar{1}1]$ direction system yeh meri slip system ho gayi aur Critical Resolved Shear Stress yahan par diya hai 6 MPa to yeh τ_{CRSS} aapko diya hai yahan par yeh meri tensile axis hai yeh meri tensile axis di hai aur mujhe diya hai silver crystal to silver crystal hota hai mera FCC crystal structure aur yahan par slip system bhi di yahan par ek particular slip system diya to yeh meri geometry ho gayi yahan par abhi mujhe ϕ aur λ ki value nikaalni hai to yeh tensile axis jo hai wo $[1\bar{1}0]$ direction par hai to yeh jo direction hai yahan par isko bhi mark kar lete yeh jo direction hai yeh meri $[1\bar{1}0]$ hai aur yeh jo plane diya hai ya yaani ye plane jo diya hai slip plane jo yahan par mark kiya hai yeh $(1\bar{1}\bar{1})$ hai aur cubic system mein aap dekhenge ki agar yeh mera plane hai to ye jo normal hai ye uske indexes bhi same rahenge aur jo slip direction di hai yeh $[0\bar{1}1]$ di hai to simple hai yeh jo direction hai $[0\bar{1}1]$ hai aur yeh jo direction hai humein nikaalni hai yeh cubic system ke liye $[1\bar{1}\bar{1}]$ hi hogi yaani yeh jo plane hai yahan par aap dekhenge ye plane indicate kar raha hai aur yeh jo indicate kar rahi hai ye direction hai abhi mere paas teen directions mere paas par pata chal gayi n pata chal gaya mere paas d hai aur mere paas σ ki direction hai to main unka unke beech ka angle nikaal sakta hoon to ye formula hum istemal karenge $\cos\theta = (h_1h_2 + k_1k_2 + l_1l_2)/(\sqrt{h_1^2 + k_1^2 + l_1^2} \cdot \sqrt{h_2^2 + k_2^2 + l_2^2})$ yeh hum nikalenge $\cos\theta$ ki value

nikaalne ke liye agar humein do vectors pata hai to un do vector ka angle isi formula se nikalta hai to aaiye nikalte hain ye angles ko to angle between abhi hum tensile axis nikalenge tensile axis yaani ye $[1\bar{1}0]$ hai aur yeh jo normal hai iska $(1\bar{1}\bar{1})$ plane ka is is tarah se hum nikalenge $\cos\phi$ ki tarah yahan par yahan par hum likh sakte hain jo h_1h_2 ye ye ho gaya mera $h_1 h_1k_1$ yeh ho gaya l_1 yeh ho gaya h_2k_2 aur l_2 to yeh jo product hai yeh nikaalunga aur unka square root nikaal ke main $\cos\phi$ ki value nikaal sakta hoon aur $\cos\phi$ ki value mere paas aati hai $2/\sqrt{6}$ usi tarah se $\cos\lambda$ ki value nikaalunga to $\cos\lambda$ ki value nikaalne ke liye mere paas yeh direction mujhe consider karni hai aur yeh direction yeh direction mujhe consider karni hai mere paas λ jab main λ nikaalunga to mere paas yeh direction honi chahiye aur yeh direction honi chahiye aur $\cos\phi$ jab nikaalunga to yeh direction honi chahiye jo normal hai aur ye tensional axis hai ye mere paas $\cos\lambda$ usi tarah se is formula se $1/2$ aayega humne dekha tha ki main σ ko is tarah se likh sakta hoon P/A aur P ko main likh sakta hoon $\tau_{RSS}/(\cos\phi\cos\lambda)$ aur yeh jo τ_{CRSS} ki value yahan pe mujhe di thi 6 aur $\cos\phi$ ki maine value nikaali $2/\sqrt{6}$ aur λ ki value aayegi $1/2$ to yeh value aayegi $6\sqrt{6}$ aur main isko solve karunga to mere paas aayegi 14.7 MPa to mujhe is is problem se example se mujhe yeh jo stress ki value aa rahi hai yaani mujhe kya diya tha jo slip ho rahi hai is plane par slip occur hone ke liye mujhe 6 MPa ki zarurat hai agar yeh mujhe pata hai to mujhe nikaalna tha ki yeh jo force yahan par lagaunga main is single crystal ko σ yeh mujhe nikaalni thi to mujhe yahan par 6 MPa ki zarurat thi to humne yahan par nikaala mujhe σ jo hona chahiye wo greater than 14.7 hona chahiye slip occur hone ke liye greater than or equal to 14.7 to is tarah se hum plastic deformation ko samajh sakte hain yeh ek example ho gaya.

 **Solved Example**

Example: Determine the tensile stress that is applied along the $[1\bar{1}0]$ axis of a silver crystal to cause slip on the $(1\bar{1}\bar{1})[0\bar{1}1]$ system. The critical resolved shear stress is 6 MPa.



The angle between tensile axis $[1\bar{1}0]$ and normal to $(1\bar{1}\bar{1})$ is

$$\cos\phi = \frac{(1)(1) + (-1)(-1) + (0)(-1)}{\sqrt{(1)^2 + (-1)^2 + (0)^2} \sqrt{(1)^2 + (-1)^2 + (-1)^2}} = \frac{2}{\sqrt{2}\sqrt{3}} = \frac{2}{\sqrt{6}}$$

The angle between tensile axis $[1\bar{1}0]$ and slip direction $[0\bar{1}1]$ is

$$\cos\lambda = \frac{(1)(0) + (-1)(-1) + (0)(-1)}{\sqrt{2}\sqrt{(0)^2 + (-1)^2 + (-1)^2}} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

From Eq. (4-13)

$$\sigma = \frac{P}{A} = \frac{\tau_R}{\cos\phi\cos\lambda} = \frac{6}{\frac{2}{\sqrt{6}} \times \frac{1}{2}} = 6\sqrt{6} = 14.7 \text{ MPa}$$

Abhi slip system dekhte hain ki kaun si slip system operative ho sakti hai to ek example aur ek lete hain ki mere paas ek single crystal hai copper ka copper bhi mera FCC structure hai aur isko main deform kar raha hoon tension par to loading axis jo hai meri yahan par di hai $[112]$ aur do sawal puche hain calculate the Schmid factors for the different slip systems dusra question hai if the critical resolved shear stress is 50 MPa what is the tensile stress at which the material will start to deform plastically to pehle to mujhe Schmid factors nikaalne hai different slip systems ke liye to yahan par humne dekha tha ki jo slip systems hai FCC mein to mere paas slip planes kaun se hai

(111) aur slip direction hogi meri $[1\bar{1}0]$ type is tarah se hogi meri slip planes aur slip directions aur yeh humne dekha tha ki mere paas 12 slip systems hai to ismein se kaun se slip system pe mere Schmid factor highest hoga ya mujhe calculate karna hai dusra hai ki mere paas ek critical resolved shear stress diya hai maine τ_{CRSS} diya hai 50 MPa to aapko nikaalna hai what is the tensile stress yeh sawal jo hai yeh hamara previous jo example hai usi ki tarah hai to aaiye pehle dekhte hain ki mera paas copper hai wo FCC structure hai aur slip systems mere paas ye $\{111\}$ type $\langle 1\bar{1}0 \rangle$ type hai to yeh ho gaya slip plane yeh ho gayi slip direction ye 12 slip systems hai mark kar lete mere paas maine bola tha ki chaar slip planes hote hain to chaar slip planes kaun se kaun se FCC mein ye mere paas $(111)(\bar{1}\bar{1}1)(1\bar{1}\bar{1})(11\bar{1})$ ye mere paas chaar planes ho gaye aur is chaar planes mein teen $\langle 1\bar{1}0 \rangle$ type ki slip directions hoti hai to mere paas ek plane pe agar yeh mera ek plane hai to ismein teen directions hai slip directions yahan par slip ho sakti hai to mere paas slip direction hogi to yahan par kul mila ke ya agar ye chaar plane aur ek plane pe teen directions to mere paas agar aap dekhenge iska product aata hai yeh 12 slip systems ho jayegi mere paas to yahan pe aap dekhenge mujhe $\cos\phi$ nikaalna hai to mere paas loading axis jo hai ye $[112]$ hai to $\cos\phi$ hoga main $\cos\phi$ agar nikaalunga agar aap formula ko yaad kareng to mere paas $\cos\phi$ ki jo value hai ya $\cos\theta$ ye $(h_1h_2 + k_1k_2 + l_1l_2)/(\sqrt{h_1^2 + k_1^2 + l_1^2} \cdot \sqrt{h_2^2 + k_2^2 + l_2^2})$ yeh formula aap istemal kar sakte hain yeh aa jayega mere paas ek formula is formula se main $\cos\phi$ aur $\cos\lambda$ nikaalunga aur $\cos\phi$ aur $\cos\lambda$ ka product jo hai yeh mujhe dega Schmid factor to aap dekhenge pehla case dekhiye yahan par yahan par $\cos\lambda$ ki value shunya aa rahi hai to agar ye shunya aa rahi hai to yeh material jo hai ya yeh jo plane hai ispe koi deformation nahi hoga similarly yahan par agar aap dekhenge yahan par jo Schmid factor hai in in planes par jaise yeh plane hai $(\bar{1}\bar{1}1)$ aur is direction jo hai $[\bar{1}\bar{1}0][101]$ ya $[0\bar{1}\bar{1}]$ is plane par aur is direction par jo Schmid factor hai wo shunya aa raha hai to yahan par koi deformation nahi hoga agar aap dekhenge closely yeh jo do planes hai yaani yeh plane aur yeh direction $(\bar{1}\bar{1}1)$ aur $[101]$ yahan par aap dekhenge ki jo Schmid factor ki value hai ye sabse highest hai $\sqrt{6}/6$ agar aap in saare Schmid factors mein dekhenge 12 slip systems mein yeh value hai $\sqrt{6}/6$ yeh highest hai is do slip systems ke liye yeh plane aur yeh direction yeh plane aur yeh direction iske liye meri slip Schmid factor sabse highest hai to aap dekhenge agar Schmid factor sabse highest hai to jo stress required hai woh kam rahega yahan par aap dekh sakte hain ki stress required for plastic deformation ye sabse kam hai in do slip systems ke liye to yeh jo slip systems hogi yeh do slip system yeh meri kehlati hai Primary Slip System aur primary slip system yaani kya hoga jo stress required to cause slip on the primary slip system is the yield stress of the single crystal ye mera definition ho jayega theek hai to mere paas do slip system hai yaani main is tarah se likhu mere paas kaun si do slip systems hai $(\bar{1}\bar{1}1)$ aur yeh direction hai $[101]$ yahan pe mera Schmid factor sabse highest hai $(1\bar{1}\bar{1})$ ya $[011]$ to yeh meri primary slip systems ho gayi ye dono slip system active ho sakti hai to probability in dono ke active hone ki saman hogi aur yeh meri sabse pehle active slip system hogi yaani sabse pehle jo plastic deformation hoga yeh slip system par chalu hoga abhi jaante hain ki yeh jo slip system se agar hum dekhenge yeh jo values hai 184 yahan par dekhenge ki yeh jo Schmid factor hai yeh yeh sabse highest tha yeh second highest hai to aap dekhenge ki yahan par jo stress lag raha hai σ lag raha hai yeh 122 ke baad yeh jo stress lag raha hai yeh sabse kam stress hai to yaani hum keh sakte ki yeh jo slip system se jo blue color se mark hai yahan par mera ye primary slip system active hone ke baad is slip system probability rahegi ki slip deformation chalu hogi to inko main kehta hoon Secondary Slip System yaani yahan par main jab load badhaunga to τ_{RSS} ki value yahan pe τ_{CRSS} ke hisaab se pahunch jayegi aur yahan par mere paas slip shuru ho sakti hai isko main kehta hoon secondary slip system to humne jaana hai ki primary slip system aur secondary slip system kya hoti hai to primary slip

system mein mera Schmid factor sabse highest rehta hai aur yahan par mera pehle sabse pehle plastic deformation ki shuruat hoti hai.



Slip system: Operative

A single crystal of copper is deformed in tension. The loading axis is $[112]$.

$\cos \theta = \frac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2} \sqrt{h_2^2 + k_2^2 + l_2^2}}$

- a) Calculate the Schmid factors for the different slip systems.
 b) If the critical resolved shear stress is 50 MPa, what is the tensile stress at which the material will start to deform plastically?

$[\ll (\bar{1}\bar{1}\bar{1}) < 1\bar{1}0 >$

Cu is fcc

Number of slip systems:
 $\{111\}\langle 1\bar{1}0 \rangle = 12$

Primary Slip system

$(\bar{1}\bar{1}\bar{1}) [101]$
 $(1\bar{1}\bar{1}) [011]$

The stress required to cause slip on the primary slip system is the yield stress of the single crystal

Slip plane (n)	Slip direction (s)	Cos ϕ	Cos λ	Schmid factor (cos ϕ cos λ)	σ (MPa)
(111)	$[\bar{1}10]$	$2\sqrt{2}/3$	0	0	Not deformed
	$[\bar{1}01]$	$2\sqrt{2}/3$	$\sqrt{3}/6$	$\sqrt{6}/9$	184
	$[0\bar{1}1]$	$2\sqrt{2}/3$	$\sqrt{3}/6$	$\sqrt{6}/9$	184
$(\bar{1}\bar{1}\bar{1})$	$[\bar{1}10]$	$\sqrt{2}/3$	$\sqrt{3}/3$	$\sqrt{6}/9$	184
	$[101]$	$\sqrt{2}/3$	$\sqrt{3}/2$	$\sqrt{6}/6$	122
	$[0\bar{1}1]$	$\sqrt{2}/3$	$\sqrt{3}/6$	$\sqrt{6}/18$	367
$(1\bar{1}\bar{1})$	$[\bar{1}10]$	$\sqrt{2}/3$	$\sqrt{3}/3$	$\sqrt{6}/9$	184
	$[101]$	$\sqrt{2}/3$	$\sqrt{3}/6$	$\sqrt{6}/18$	367
	$[011]$	$\sqrt{2}/3$	$\sqrt{3}/2$	$\sqrt{6}/6$	122
$(11\bar{1})$	$[\bar{1}\bar{1}0]$	0	0	0	Not deformed
	$[101]$	0	$\sqrt{3}/2$	0	Not deformed
	$[011]$	0	$\sqrt{3}/2$	0	Not deformed

To abhi dekhte hain do material ke liye ki face centered cubic aur body centered cubic material ke liye kaun si slip system chalu ho sakti hai uske liye do tricks hai ye yahan main thoda sa mention karna chahta hoon to agar mere paas ek uvw direction hai jiske along meri tensile axis hai to in dono case mein kaun si slip system operate hogi iske liye kuch shortcuts hai aap dekhenge to aapko Schmid factor calculate karne ki zarurat nahi padegi aur yeh sirf face centered cubic aur body centered cubic material ke liye hi applicable hai aap dusre material ke liye isko apply nahi kar sakte to pehla hai OILS rule to yahan pe jo O jaisa dikh raha hai wo zero hai actually to ye jo OILS hai iska full form hai Zero Intermediate Lowest Sign to ye trick agar aap apply karenge to aap slip system yahan pe identify kar sakte hain to maan lete hain ye mere paas uvw tensile axis hai meri to humein dekhna hai ki inmein se sabse highest kaun si value hai highest value ye hai w isko main h mark kar raha hoon ye highest hogi uske baad sabse lowest hai u arithmetic mein u is less than v is less than w is tarah se hai to u meri lowest hogi aur v meri intermediate hogi to agar main in teeno ko mark kar liya highest intermediate aur lowest to main OILS rule se main nikaal sakta hoon to OILS rule kya kehta hai zero intermediate gives the slip direction to agar aap FCC mein dekhenge main FCC ki baat kar raha hoon FCC mein mere paas slip plane kaun sa hoga slip plane mera $\{111\}$ hoga $\{111\}$ type aur slip direction jo hogi woh meri $\langle 110 \rangle$ type hogi yeh fixed hai mere liye kyunki yeh close packed plane hai aur ek close packed direction hai ye FCC ke liye fix to zero intermediate kya kehta hai to mere paas zero jo intermediate sign hai yahan par agar main is tarah se likhunga to yahan par agar main zero rakh du isko samajhiye thoda sa sign agar main direction nikaal raha hoon to direction ke liye mujhe intermediate par zero rakhna hai aur baaki ki do values hogi one one yeh ho gaya mera direction aur lowest sign tells us that slip plane lowest sign yahan par kahan par hai yeh one hai lowest sign yahan par mujhe negative yaani bar one likhna hai agar mere paas 1 1 1 hai to yahan par is middle part pe main bar one likh dunga to mere paas slip system kaun si ho jayegi mere paas slip system hogi ye $[0\bar{1}1]$ aur $(1\bar{1}\bar{1})$ to yeh ho gayi mere paas slip system to aur ek example lete hain simple example lete hain jaise mere paas ek slip direction maine le li maan lete hain $[410]$ to yeh ho gayi meri sabse lowest yeh ho gayi mere sabse

highest aur ye ho gayi meri intermediate to agar mujhe slip direction nikaalna hai direction nikaalna hai slip direction to slip direction kehta hai zero intermediate to intermediate yahan pe one hai yahan pe to yahan pe main zero rakh dunga aur baaki jagah main one one rakh dunga ye ho gayi meri slip direction aur slip plane jo hoga plane jo hoga wo lowest sign lowest yahan pe shunya hai to yahan par main bar one likh dunga aur 1 1 to agar aap dekhenge to yeh meri slip direction hogi aur is mera slip plane hoga is tarah se main yahan par maine galti kar di hai yeh brackets is tarah se hone chahiye square brackets yaane direction dikhata hai yeh brackets yaane planes dikhate to yeh mere paas slip plane ho gaya aur ye slip direction ho to aapko same exercise BCC ke liye karna hai sirf BCC mein kya hoga slip plane yahan par kya hoga BCC mein slip plane humne baat ki thi $\{110\}$ ye mera slip plane tha aur slip direction ki baat karunga to direction mera $\langle 111 \rangle$ type tha to aapko sirf ulta karna hai yahan pe BCC mein to agar mere paas maan lijiye ye $[410]$ tensile axis hai aur mere paas BCC system hai ya alpha iron ki tarah main agar maan leta hoon alpha iron jo BCC system hai to yahan pe mujhe kaun sa slip system milna chahiye to plane ke liye mujhe yahan par sirf ulta karna hai to plane ke liye Lowest Sign tells us the slip plane lowest sign tells us the slip plane yahan pe to yahan pe sirf ulta karna hai to jo lowest sign hoga mujhe slip direction dega to slip plane ke liye mujhe a Zero Intermediate yeh wala karna padega to isko hum karte hain zero intermediate to yahan par hum dekhenge mark kar lete hain isko zero intermediate kahan hoga (011) yeh ho gaya mera slip plane aur slip direction jo hoga woh hoga mera $[1\bar{1}1]$ jahan par lowest sign jo hogi lowest value yeh hai shunya to yahan par meri ye direction hogi agar mere paas FCC hai to isko is tarah se mark kar lete confusion na rahe yeh mere paas FCC hai aur yeh mere paas BCC to mere paas agar FCC hai to to mere paas ye direction mil rahi hai theek hai is tarah se yeh mere paas FCC hai to yeh yeh jo axis hai agar aap dekhenge to $[101]$ direction hai yahan par (101) plane ho jayega aur yeh jo hai $[1\bar{1}1]$ yeh plane tha yeh yahan par BCC mein direction ho jayegi agar aap dekhenge ye slip system calculate karke is tensional axis ke liye Schmid factor calculate karke koshish karenge to aapko ye yahan par hi maximum Schmid factor milega to ye mera ek simple trick tha aur ek rule hai isko hum kehte hain Diehl's rule Diehl's rules ke liye aapko stereographic projection ke baare mein pata hona chahiye to aap is isko padh sakte hain agar yeh nahi pata hai to aapko pareshan hone ki zarurat nahi hai hum isko aur aage dekhenge stereographic projection ko to stereographic projection hum nikaal raha hoon to ye mere paas (001) stereographic projection to ye jo yahan par dikh rahe numbers yeh bata rahe poles hai mere planes jo hai crystal structure mein yeh poles uske mark kiye maan lete hain mere tensile axis yahan par hai kahin par yahan par ye jo red mark kiya hai yahan pe aap dekhenge ki yeh jo poles yahan par mark kiye (001) stereographic projection mein to aap dekhenge yeh jo poles hai ek to $\{111\}$ type hai ya $\{110\}$ type hai to hum hum dekhenge ki FCC aur BCC mein slip direct aur slip plane ye $\{110\}$ ya $\{111\}$ type hai to agar maan lete mere paas FCC structure hai FCC structure hai to yeh meri tensile axis hai to mujhe kya karna hai yahan par FCC mein humne dekha tha ki (111) plane active rehta hai to yeh tensile axis ke opposite humein jaana hai opposite yaani kahan jayenge yeh agar mirror hum yahan par yeh mirror maan ke chalte hain yahan par ye consider karenge agar ye tensile axis is side hai to (111) ke is side humein jaana hai yeh mera mirror ho gaya yahan par aur iska mirror image jo hoga yeh pole hoga to main is pole par aaunga maine yeh mirror kyun consider kiya kyunki aapko is triangle mein rehna hai yeh jo triangle hai yeh triangle mark kiya maine is triangle mein aapko rehna hai to yeh ho gaya mera slip plane ab slip direction ke liye bhi aapko kya karna hai yeh meri $\langle 110 \rangle$ type direction hoti hai slip direction FCC mein aur ye meri tensile axis hai to main iske opposite jaunga to iske liye jo mirror act hoga yeh mera mirror act hoga iske liye to main iska reflection idhar dekhunga to yeh mere paas direction aa gayi $[101]$ to agar mere paas yeh direction hai is triangle mein ye direction hai to mere

paas slip system kaun si honi chahiye ($1\bar{1}1$) aur $[101]$ ye meri direct agar main crystal structure change karta hoon BCC to sirf mujhe yeh bracket change karne hai kyunki yeh meri direction ho jayegi aur yeh ho jayega mera plane to yeh do rule se aap aasaani se slip systems nikaal sakte hain face centered cubic aur body centered cubic ke liye to aapko yaad rakhna hai sirf yeh do crystal structure ke liye hi applicable hai.

Which slip system will operate?

FCC
 $(111)[110]$

Face-centred Cubic (fcc) or Body-Centred Cubic (bcc) metals

If given $[u\ v\ w]$ tensile axis, which slip systems will operate?

The OILS rule

OILS:

zer0 Intermediate, Lowest Sign

Ex. If the tensile axis is along $[2\ 1\ 3]$ the indices are identified as **[L H]**

Zer0 Intermediate gives the slip direction as **[011]**

Lowest Sign tells us that the slip plane is **$(1\bar{1}1)$**

So, the slip system in fcc: **$[011] (1\bar{1}1)$**

Do the same exercise for bcc?

α -Fe

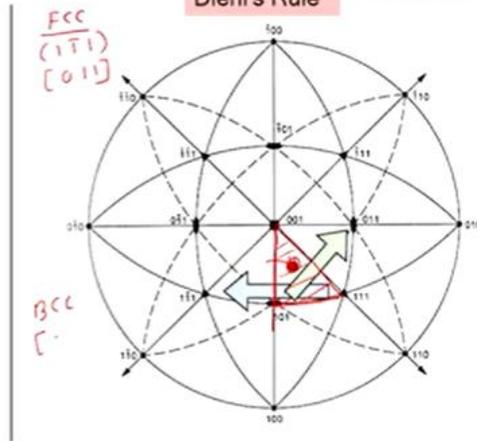
BCC plane = (101)
direction = $[1\bar{1}\bar{1}]$

FCC
H I L
E L H
 $[4\ 1\ 0]$
D = $[101]$
P = $[1\bar{1}\bar{1}]$

slip plane (110)
direction $[111]$

Diehl's Rule

Stereographic



abhi hum dekhte hain jab main deform karta hoon material ko agar mere paas single crystal hai aur main yahan par tensional deformation kar raha hoon to yeh slip planes maine mark kiye agar yeh free move hote hai agar yeh jo edges hai ya jahan par maine hinge kiya hai mere sample ko yeh free to move hai yaani main keh sakta hoon ki end constraint nahi hai to mujhe slip is tarah se milni chahiye is planes par mera Schmid factor highest ho jayega aur yeh slip ye plane slip hona chalu ho gaya theek hai is tarah se slip honge to yeh sample is tarah se deform hoga par tensile axis mein humne dekha hai ki ye jo ends hai wo constant rehte hai ye change nahi hote inki position to aap dekhenge ki jo slip planes hai yeh rotate ho rahe hain ya kuch slip planes hai jo yahan par aap dekhenge ki ye bend ho rahe hain to iske baare mein hum jaanenge jab strain hardening ke baare mein baat karenge FCC ke par aapko yeh yaad rakhna hai ki agar mera material constrained hai to wahan par mere slip planes bend honge ya rotate honge to kis tarah se rotate honge agar aap dekhenge yeh mera slip plane tha ya slip direction tha agar main is tarah se direction aap dekhenge ki ye jo slip ho raha hai wo is direction pe ho raha aur yeh jo plane hai ye jo plane mark hai yahan par yeh mera slip plane hai aur yeh direction hai ye jo direction hai aap dekhenge yeh is tarah se tensile axis ki taraf rotate ho rahi hai jab hum tensile deformation karte likh leta hoon jab hum tensile deformation karte to slip direction meri tensile axis ki taraf rotate hoti hai slip direction tensile axis ki taraf rotate hoti hai yaani aap dekhenge ki in dono ka angle kam ho raha hai tensile axis aur slip direction ka jab hum compression karte hain to aap dekhenge ki ye jo slip plane hai slip normal hai yeh compression axis ke parallel ho jayega compression axis ki taraf rotate karega hum baad mein jaanenge iske baare mein jab hum strain hardening ki baat karenge aap is jo deformation hai yeh rotation hai ya bending hai aap isko bhi stereographic projection ke hisaab se samajh sakte hain to hum dekhenge ki kaun sa slip system operative hoga during tensional

deformation to aap dekhenge ki ye jo slip plane hai jahan par Schmid factor highest rehta hai wo deform hoga agar constant hai to aap dekhenge ki jo slip direction hai wo rotate hogi tensile axis ki taraf to abhi ke liye main yahan par rukta hoon is part mein humne dekha ki slip jo hai humne dekha tha ki particular plane pe hoti hai crystallographic plane pe aur crystallographic direction par hoti hai jo ki generally close packed rehte hai dono aur kis planes par honi chahiye agar mere paas slip system hai to jahan par Schmid factor highest hoga to yeh humne jaana abhi hum aage badhenge ki is yeh jo humne dekha tha ki slip planes par aise deformation hota hai hum jaanenge ki ek theoretical value kya ho sakti hai slip ki yaani ki agar mere paas different material hai to stress ki value kya honi chahiye ye hum jaanenge next part mein abhi ke liye rukta hoon dhanyavad