Mechanical behavior of materials

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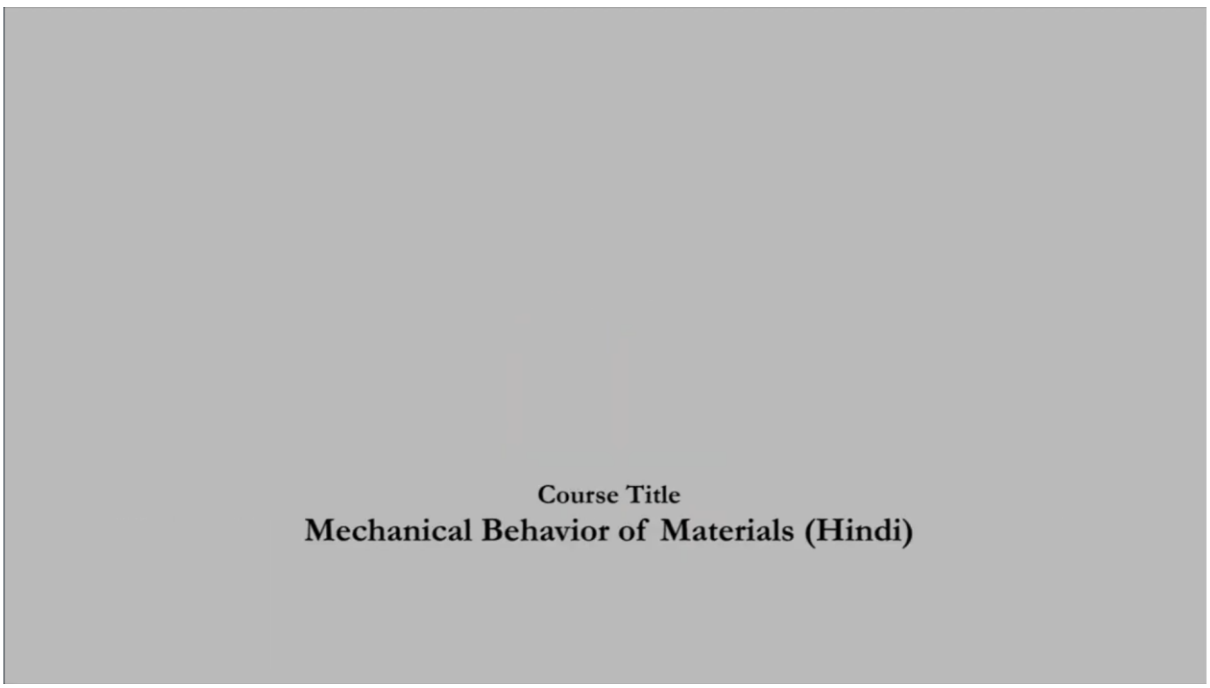
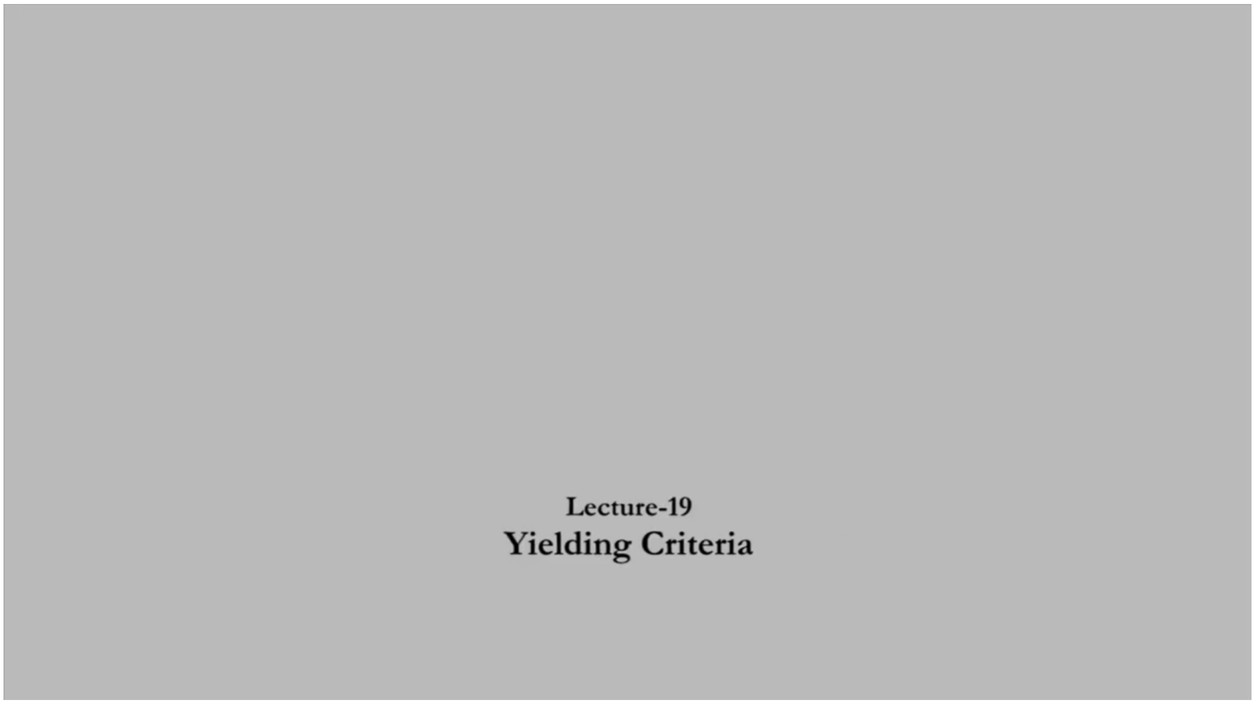
Week-3

Lecture-19

Yielding Criteria





Namaskar! Phir se swagat karta hoon is course mein jiska naam Mechanical Behavior of Materials hai jisko hum Hindi mein padhenge. Aaj ke is bhaag mein hum dekhenge ki Yielding Criteria kya hai. Last part tak humne dekha tha ki stress strain relations kya hain, stress tensor hai, strain tensor kya hai, unke relations kya hain. Is part mein hum dekhenge ki yielding kya hai. Yielding jab hum jaante hain tab yielding hum is tarah se samajhte hain jo elastic se plastic deformation behavior par jo change aata hai us point ko hum kis tarah se define kar payenge, yeh aaj hum is part mein dekhenge. Toh jab hum yielding criteria ki baat karte hain tab sabse pehle humare dimaag mein aata hai Uniaxial Tensile Test.

Toh humne ye stress strain plot dekha tha, ye engineering stress humne plot kiya tha Y-axis par aur engineering strain plot kiya tha X-axis par aur humein ek aisa curve mila tha jo hum tensile curve kehte hain. Toh jo pehla part tha humara woh linear part humne dekha tha jo elastic behavior denote karta hai humare material ka aur jo non-linear part hai woh plastic behavior dikhata hai. Chunki jo hum dekhenge yeh humne elastic relations ismein dekhe thae stress strain ke, abhi humein dekhna hai ki yeh jo elastic behavior se plastic behavior mein transformation hota hai aur transition hota hai woh kis point par hota hai, usko hum kisi stress ke dwaara jaan sakte hain.

Toh humne define kiya tha ek term Yield Stress. Humne yield stress dekha tha ki stress jiske upar mera material plastically deform hota hai. Plastically deform hota hai yaani iska matlab usmein permanent changes aate uske dimensions mein. Toh yahan par jo strain develop hoga woh recover nahi ho sakta. Is part mein, elastic part mein jo strain develop hoga woh recover ho jayega agar main stress relieve kar raha hoon toh. Toh humein jo bhi engineering applications hum dekhte hain wahan par humein chahiye ki material deform na ho ya uske upar koi permanent changes nahi uske aaye uske dimensions mein. Toh humein isliye yield stress nikaalna bahut important hai. Par yeh uniaxial tensile test hai ye simple test hai.

Humne dekha tha ki humein jo stress jo apply karte hain woh ek hi direction mein apply karte hain humare paas ek stress simple stress state rehta hai. Toh experimentally jab yield stress nikaalte hain woh laboratory mein nikaalna bahut aasaan hai par practically hum dekhenge ki jo bhi applications hain, engineering applications mein usmein simple uniaxial tensile loading nahi hoti hai usmein complex state of stress hota hai. Toh yielding jo yield stress hai yeh humein deta hai plastic deformation jahan par shuruat hoti hai. Toh humein isliye yeh stress nikaalne ki bahut aavashyakta hai aur humein yeh jaanna hoga ki ye yield stress jab hum nikaalenge uniaxial tensile test se nikaalenge but humare paas toh complex state of stress hai.

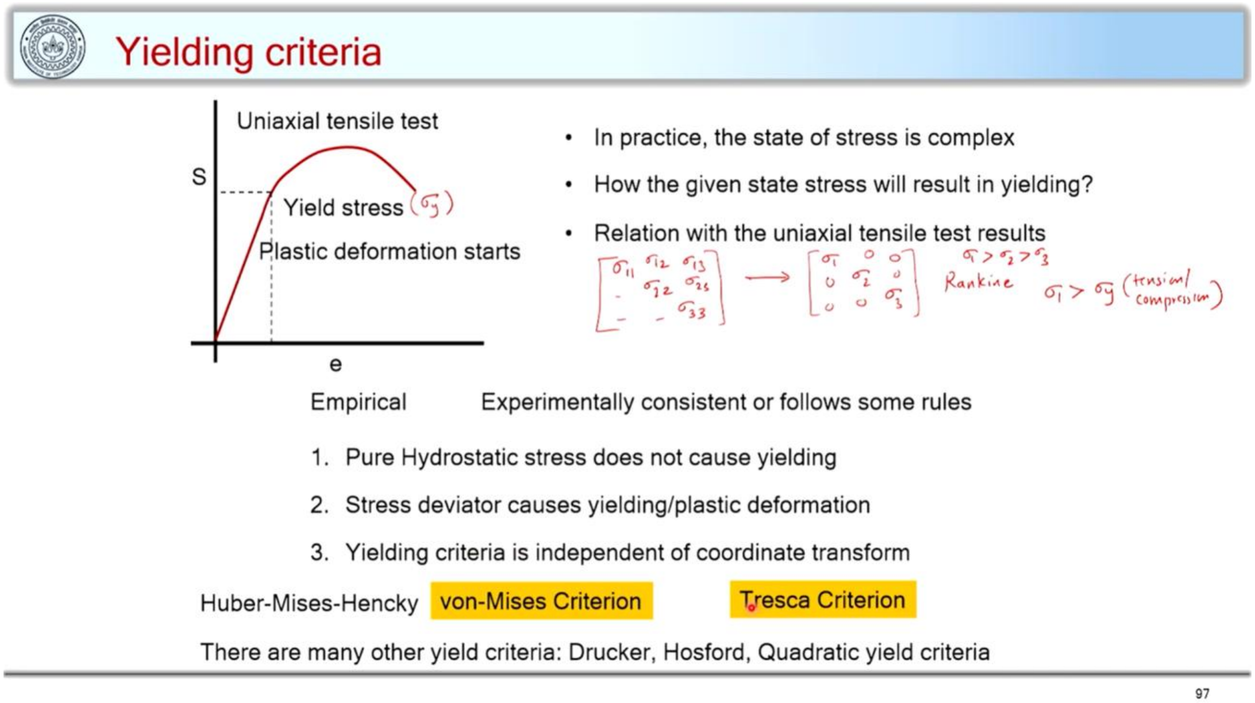
Toh agar hum complex state of stress hai toh ye iska yield stress pe effect hoga. Toh humein ek aisa criteria chahiye agar mere paas koi bhi stress state hai toh main bata paoon mera material yield hoga ya nahi hoga. Toh iske liye hum ek kar sakte ki jo bhi uniaxial tensile test mein jo bhi yield stress aata hai isko hum correlate kar sakte hain usi parameters ke saath jo humein complex state of stress mein milta hai. Toh complex state of stress mein hum isko correlate karne ki koshish karenge. Toh doosra point jo hai humein yielding criteria ke liye sochna hai ki yield stress jo bhi state of stress hai jo humein yielding degi toh yeh point humein dhoondhna hai.

Uniaxial tensile test mein isko nikaalna aasaan hai par ismein thoda difficult hai. Toh humein ek relation dena hai jo bhi maine baat ki ki uniaxial tensile test jo result kyunki hum laboratory mein isko aasaani se kar sakte hain kisi bhi material ke saath toh iska jo relation hai complex state of stress ke saath woh humein nikaalna padega. Toh ek simple relation hai hum dekh sakte hain agar mere paas mujhe yield stress pata hai aur mujhe mere paas koi bhi state of stress hai, state of stress ya main define karunga koi bhi state of stress hai jismein mere paas normal stresses hain aur shear stresses bhi hain. Ye humne dekha tha ki koi bhi yeh teeno stress same rahenge. Toh mere paas ek state of stress hai isko main convert kar sakta hoon kisi bhi Principal Stresses mein jahan par mere paas normal stresses rahenge.

Toh ek simple criteria aap dekh sakte hain ek Rankine criteria ki baat kar raha hoon main yahan par. Toh agar yahan par main maan leta hoon σ_1 yeh principal stresses hain σ_1, σ_2 aur σ_3 principal stress hain aur σ_1 highest principal stress hai toh Rankine ek criteria deta hai agar mera ye σ_1 greater ho jayega σy se, σy kya hoga mera yield stress, jo σy hai ye mera yield stress hai. Greater hoga tension ya compression mein tab mera material yield karega. Toh ye simple criteria hai par ye criteria itna simple hai ki yeh bahut saare materials par yeh valid nahi hai. Toh kuch aur empirical criteria hum dekhenge.

Empirical yaane jo experimentally consistent hai par kuch rules follow karte hain. Rankine criteria hum kabhi istemaal nahi karte ek simple criteria samajhne ke liye ki aap isko yaani hum ek state of stress ko hum principal stresses mein convert kar sakte hain aur principal stresses mein dekhenge ki jo maximum stress hai agar woh greater hai yield strength se toh mera material deform hoga par ye most of the material ke liye valid nahi hai. Toh abhi kuch empirical points dekhte hain yahan pe rules. Toh pehla pehla rule yeh hai yielding criteria ka ki jo Hydrostatic Stress hai woh kabhi bhi yielding nahi karega. Ye humne dekha tha ki hydrostatic stress aur deviatoric stress state ki baat ki thi tab humne dekha tha ki hydrostatic stress yielding nahi karta hai.

Aaj isko thoda aur detail mein dekhenge aur jo stress deviator hai yaani Deviatoric Stress jo hai wahi mera yielding ya plastic deformation karega yaani wahi reason rahega mere yielding aur plastic deformation ka. Third hai ki jo yielding criteria hai woh independent rahega mere coordinate transformation ke liye yaani agar main kisi bhi yeh jaise ek state of stress hai main doosre state of doosre reference mein jaunga coordinate transform karunga toh jo yielding criteria hai woh independent hona chahiye us coordinate transformation ke. Toh kuch kuch criteria hain toh aaj ke part mein hum dekhenge Von Mises criteria aur Tresca criteria. Von Mises criteria ko hum Huber-Mises-Hencky criteria bhi kehte hain. Bahut saare criteria hain jaise Drucker criteria hai, Hosford criteria hai, quadratic yield criteria hai.



Mostly metallic material ke liye main baat kar raha hoon toh hum aaj ke part mein ye Von Mises criteria aur Tresca criteria ke baare mein jaanenge iski concept jaanenge ki criteria hote kaise. Toh Von Mises criteria ke baare mein jaante hain abhi. Maan lijiye mere paas ek general state of stress hai jaise yeh nau components hain aur maine bataaya tha ki ismein se cheh independent components hain mere paas ye normal components hain aur yeh shear component hai. Is state of stress ko main transform kar sakta hoon ya isse main principal stresses nikaal sakta hoon σ_1, σ_2 aur σ_3. Yeh humne jaana tha ki eigenvalues jo hoti hain is stress matrix ki woh meri principal stresses ho gayi.

Toh mere paas aa gayi σ_1, σ_2, σ_3. Toh abhi aap jaan payenge ki humne yeh saari cheezein σ_1, σ_2, σ_3 ye jo principal stresses se kyun nikaali thi kyunki humein material ka yielding behavior jaanna hai toh humein ye principal stresses nikaalna aavashyak hai. Toh maan lete hain $\sigma_1 > \sigma_2$ aur $\sigma_2 > \sigma_3$. Abhi main is stress state ko aur doosre stress state mein convert kar sakta hoon jaise ye mere paas Hydrostatic Stress State maine convert kar li aur ye ho gayi mere paas Deviatoric Stress State. Hydrostatic stress state convert karne ke liye mere paas σ_1, σ_2, σ_3 hain toh main jab σ_m likhunga yahan pe.

Yeh yeh σm ho jayega in normal stresses ka, ye yahan par principal stresses hain in normal stresses ka average $(\sigma_1 + \sigma_2 + \sigma_3) / 3$, ye ho jayega mera mean stress. Agar ye teeno direction mein same hai, similar hai toh isko main kehta hoon hydrostatic stress state aur yeh jo doosra part hoga isko main kehta hoon deviatoric stress state. Is stress state mein aap dekhenge ki jo yeh part hai ismein shear stresses shunya hain. Toh isko bhi hum deviatoric stress state isliye kahenge kyunki ek special condition hai, special case hai humari deviatoric stress state ki jahan par shear stress shunya hai par yeh jo deviatoric stress state hai ye aapka plastic deformation karega.

Hydrostatic stress state ye plastic deformation nahi karta hai yeh sirf volume change karega. Toh aapko jaanna hai agar normal stresses equal hain teeno direction mein toh hi humara woh jo stress state hai woh hydrostatic stress state rahega aur hydrostatic stress state mein aapke shear stresses shunya rahengy yeh special condition hai deviatoric stress state ki. Par hum agar mere paas ek principal stress state ki condition hai usko main ek hydrostatic stress state aur deviatoric stress state mein convert kar sakta hoon aur humne jaana hai ki ek deviatoric stress state hai ye plastic deformation karta hai material ka.

Toh agar last part mein humne jab ye deviatoric stress state hydrostatic stress state nikaalne ki koshish ki thi tab maine aapko ek bola tha ki second invariant of deviatoric stress tensor is iski value nikaalne ke liye kaha tha toh yahan pe hum J_2 ki value is tarah se dekh payenge: $J_2 = \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2$ Ye meri J_2 ki value hai. J_2 kya hai mera second invariant hai mere stress tensor ka. Abhi main deviatoric stress tensor ka invariant nikaalunga toh uske liye abhi is keis mein agar ye stress state mein consider kar raha hoon toh mere paas σ'_{11} ki value kya hai $(\sigma_1 - \sigma m)$.

Von-Mises Yielding Criterion

General state stress

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \leftrightarrow \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \quad where, \sigma_1, \sigma_2 \; and \; \sigma_3 \text{ are principal stresses}$$

$$\Rightarrow \underbrace{\begin{pmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{pmatrix}}_{\text{Hydrostatic Stress state}} + \underbrace{\begin{pmatrix} \sigma_1 - \sigma_m & 0 & 0 \\ 0 & \sigma_2 - \sigma_m & 0 \\ 0 & 0 & \sigma_3 - \sigma_m \end{pmatrix}}_{\text{Deviatoric Stress state}}$$

Causes plastic deformation

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$\sigma_{11} = \sigma_1 - \sigma_m$$

$$\sigma_{22} = \sigma_2 - \sigma_m$$

$$\sigma_{33} = \sigma_3 - \sigma_m$$

$$\sigma_{12} = \sigma_{23} = \sigma_{13} = 0$$

Second Invariant of Deviatoric stress tensor

$$(\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2) = J_2$$

$$\frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = J_2$$

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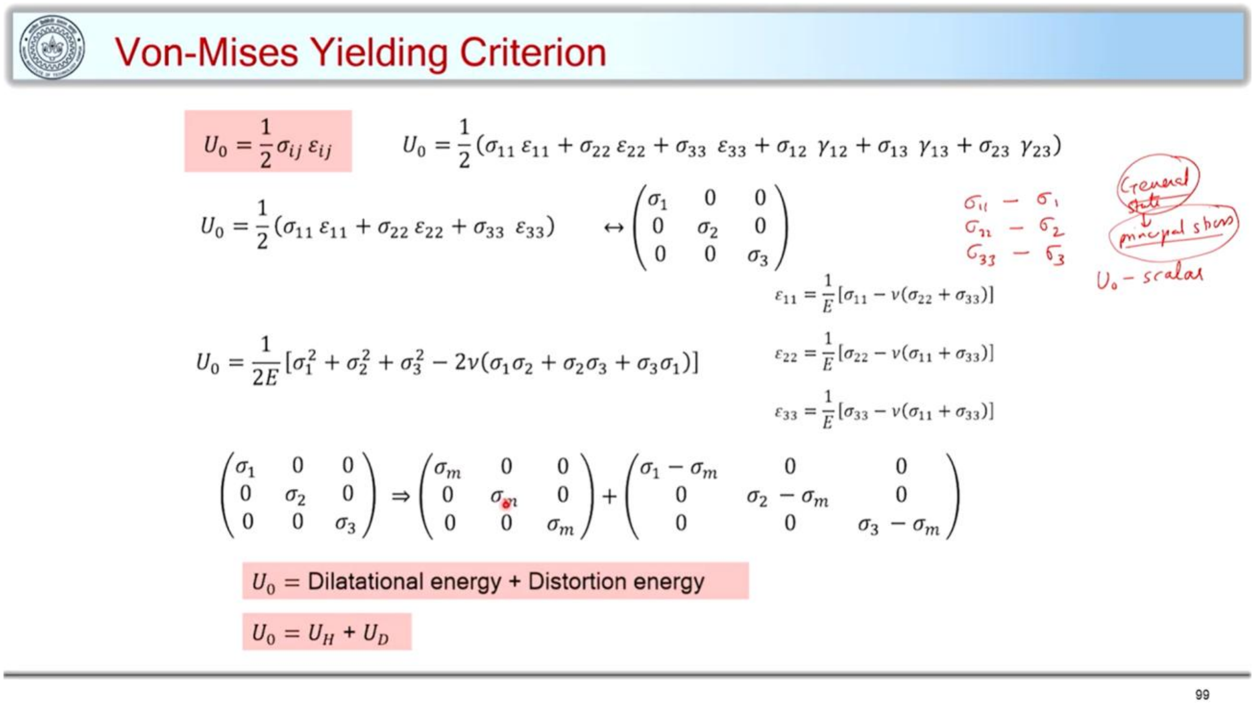
Usi tarah se σ'_{22} aur σ'_{33} ki value kya hai $(\sigma_2 - \sigma m)$, σ'_{33} ki value hai $(\sigma_3 - \sigma m)$ aur baaki jo stresses hain ye jo shear stresses hain is keis mein ye shunya hain. σ_{12}, σ_{23} aur σ_{13} ye shunya hain. Agar yeh values main is ismein put karta hoon is identity mein put karta hoon toh mere paas J_2 ki ek value aa jayegi ye aap solve karke dekhiyega agar main ye is yahan par daalunga aur simplify karke dekhoonga toh yeh mere paas ek kuch identity aayegi aur yahan pe σm ki value aapko pata hai ye average hai $(\sigma_1 + \sigma_2 + \sigma_3) / 3$ in stresses ka addition karke aapko mean lena hai toh ye mere paas ek J_2 ki value aayegi. Yeh aap thoda dhyaan rakhiye is isko hum istemaal karenge ek yielding criteria jo Von Mises yielding criteria hai iske dauran.

Toh humne last part mein dekha tha ki Elastic Strain Energy ko humne define kiya tha toh total elastic strain energy per unit volume ko main is tarah se define kiya tha ½ σ_{ij} ε_{ij} aur isko main expand karunga yeh mere paas normal stress hai aur corresponding normal strain hai aur yeh shear stresses hain aur corresponding shear strains hain aur main add karunga saare stress strain ka product toh mere paas total elastic strain energy aa jayegi ye humne dekha tha. Abhi main apna stress tensor jo hai principal stresses ke baare mein jo humne likha tha σ_1, σ_2 aur σ_3 uske liye hum ek elastic strain energy nikaal lene ki koshish karenge.

Toh is keis mein aap dekhengy yahan par mere paas shear stress shunya hai toh yeh saare terms shunya ho jayenge aur main U_0 agar nikaalunga toh yahan par yeh jo σ_{11} hai yahan par nothing but yeh sigma σ_1 hai aur uske corresponding strain ε_1 hai. Toh main total elastic strain energy corresponding to principal stresses nikaal paunga toh is keis mein ε_1, ε_2 aur ε_3 yeh principal strains hain. Toh main ε_1 ko is tarah se bhi likh sakta hoon $1/E\ [\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})]$. Yahan par main likh leta hoon σ_{11} jo hai woh mera σ_1, σ_{22} hai is state of stress ke liye σ_2 hai aur σ_{33} jo hai yeh σ_3 hai. Toh agar main yeh yahan pe strain ki jagah pe yeh values put karke nikaaloon toh mere paas ek total elastic strain energy aayegi.

Aur yeh aayegi $1/2E\ [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$. Ye mere paas ek identity aa jayegi yaani ye total strain energy corresponding to principal stresses hai us general state of stress se. Toh ye jo total strain energy rahegi yeh corresponding rahegi is principal stress tensor se. Kyunki humne ek general stress state liya tha aur usko convert kiya tha principal stress states mein toh yahan ki jo energy rahegi woh same honi chahiye is principal stress tensor ke liye bhi. Toh abhi hum aaiye chalte hain ye jo U_0 hai isko hum aur difference kar sakte hain.

Toh maine stress state ko do stress states mein convert kiya tha ek hydrostatic aur ek deviatoric. Toh ye jo U_0 hai iske liye corresponding jo energy rahegi isko main kahunga hydrostatic strain energy ya isko main U_v kahunga. U_v yaane main isliye kahunga ki ye jo stress state hai ye hydrostatic stress state hai ye sirf volume change karega aur yeh jo energy hai deviatoric stress state ke liye corresponding isko main Distortion Energy ya isko main elastic strain energy ko U likh raha hoon yahan par symbolically aur distortion energy ko likh raha hoon U_d. Toh agar main U_0 jaise nikaala yahan par is state ke liye agar aap dekhengy toh main bhi usi tarah se nikaal sakta hoon yahan par.



Yahan par main agar σ_1 ki jagah yahan par σm hoga, σ_2 ki jagah σm hoga, σ_3 ki jagah σm hoga. Toh mere paas U_v bhi main nikaal sakta hoon aur U_v mere paas is tarah se aayega aur isko simplify karunga toh U_v ki value aayegi 3/(2E) (1 - 2ν) σm². σm mera mean stress hai. Toh yeh mere paas U_v ki value aa gayi. Abhi main U_v ki value ko is tarah se likhunga σ_1, σ_2 aur σ_3 ke dauran. Toh humne dekha tha σm ye average value hai toh yeh σm ki jagah yeh value put kar di toh mere paas U_v ki jo value aayegi σ_1 aur σ_2 aur σ_3 ke ismein aa jayegi. Abhi mere paas U_0 ki value hai, U_v ki value hai, abhi mujhe U_d ki value nikaalni hai.

Toh U_d ki value main is tarah se nikaalunga agar main yeh jo U_0 ki value hai isko main isse main U_v ki value nikaal doonga toh mere paas U_d ki value rahegi. U_d = U_0 - U_v. Toh yeh U_d ki value main is tarah se nikaalunga. Yeh jo value thi mere paas ye U_0 ki value hai aap check kar sakte hain pichle slide mein humne dekha tha aur yeh U_v ki value jo hai yeh yahan par humne nikaali hai. Toh yeh agar main subtract karunga toh mere paas distortion energy aayegi. Ab yeh distortion energy ko main subtract karke simplify karunga toh mere paas yeh term aayegi.

Von-Mises Yielding Criterion

$$U_H = \frac{1}{2E}[\sigma_m^2 + \sigma_m^2 + \sigma_m^2 - 2\nu(\sigma_m\sigma_m + \sigma_m\sigma_m + \sigma_m\sigma_m)]$$

$$U_H = \frac{3}{2E}[1 - 2\nu]\sigma_m^2$$

$$U_H = \frac{3}{2E}[1 - 2\nu]\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)^2 \qquad \because \sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$U_H = \frac{[1-2\nu]}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right)$$

$$U_0 = U_H + U_D$$

$$U_D = U_0 - U_H$$

$$U_D = \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] - \frac{(1-2\nu)}{6E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$$U_D = \frac{(1+\nu)}{6E}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)$$

$$U_D = \frac{(1+\nu)}{6E}(6J_2) \qquad U_D = \frac{1}{2G}J_2 \qquad \because G = \frac{E}{2(1+\nu)}$$

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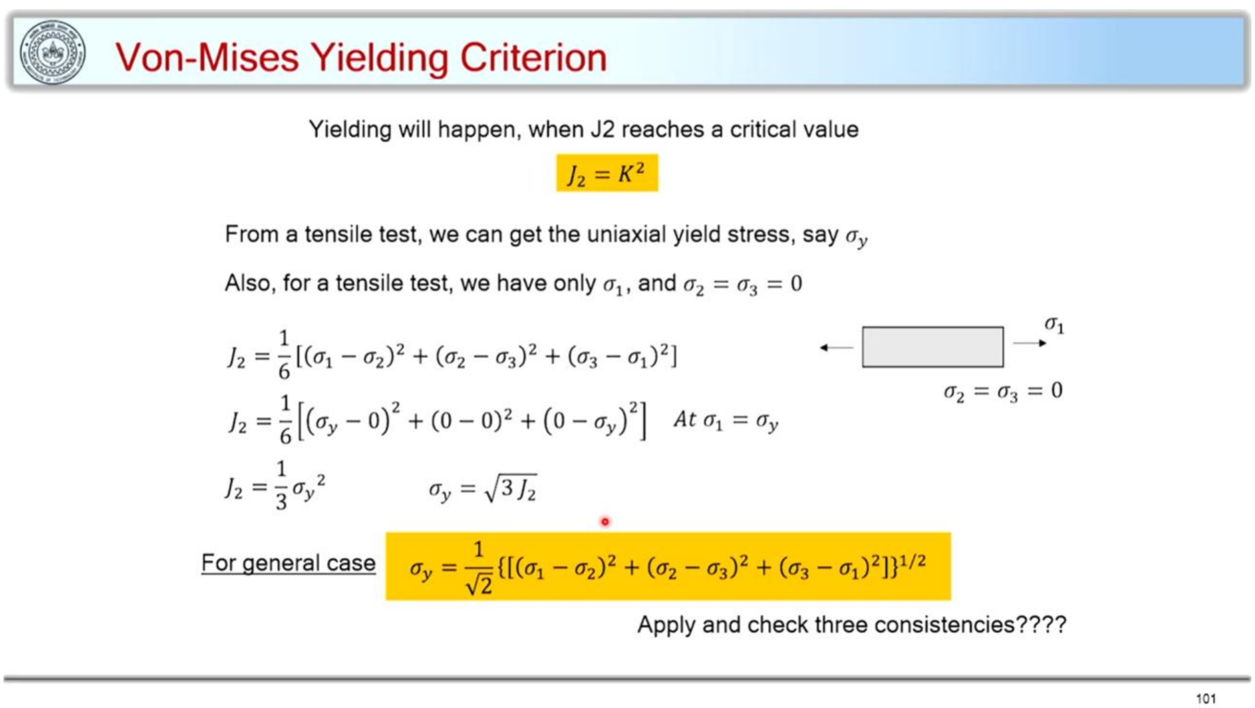
Abhi aap dekhengy yeh jo term hai $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$... yeh jo term hai yeh term meri isko main J_2 ke ismein likh sakta hoon. Ab humne J_2 ko derive kiya tha toh isko main $6J_2$ likh sakta hoon. Toh mere paas U_d ki value is tarah se aayegi $(1 + \nu)/(6E) * 6J_2$. Abhi yeh jo $(1 + \nu)/E$ hai isko main shear modulus ke ismein likh sakta hoon. Mere paas yeh relation hai shear modulus ka $G = E / [2(1 + \nu)]$. Toh main yeh jo term hai isko main G ke saath replace karunga toh mere paas simplified way se jo value aayegi U_d ki yeh aayegi $1/2G * J_2$. Toh ye distortion energy hai yeh main second invariant jo hai humare deviatoric tensor ka iske hisaab se likh sakta hoon.

Abhi main phir se Von Mises yielding criteria ko explain karna chahta hoon. Toh yeh jo criteria hai ye ek baat bataata hai kyunki ye empirical criteria hai toh jab yielding jab hogi kab hogi yielding jab J_2 ki value ek critical value ko reach karegi. Maan lete hain meri J_2 ki value k^2 ke barabar ho jaati hai tab hi yielding hogi. Yeh ek empirical relation hai toh ek humari yeh empirical formulation hai J_2 ka. Toh hum tensile test se agar hum dekhengy toh hum uniaxial stress state yield stress nikaal sakte hain. Isko main keh raha hoon σ_y. Maine pehle bataaya tha yielding criteria mein hum kya karte hain hum jo yield stress hai usko correlate kar rahe hain kisi parameter se.

Toh main tensile test mein mere paas σ_1 hai σ_2 aur σ_3 shunya rahengy kyunki main uniaxial tensile test kar raha hoon ek member hai usko main deform kar raha hoon yahan par main ek hi stress apply kar raha hoon σ_1. Toh J_2 agar main nikaalunga toh J_2 ki value yeh thi yahan par σ_2 aur σ_3 agar main shunya rakhta toh mere paas aur $\sigma_1 = \sigma y$ ho jayega tab hi yielding hogi. Tab main σ_1 ko σy consider karunga tab mere paas yeh identity aayegi J_2 ki ek value nikaal sakta hoon main yielding point par jab yielding hoga toh ye σ_1 ki value kya honi chahiye σy honi chahiye. Toh J_2 ki value main consider kar sakta hoon 1/3 σy^2. Yeh aap solve karenge toh aapko 1/3 σy^2 aayega.

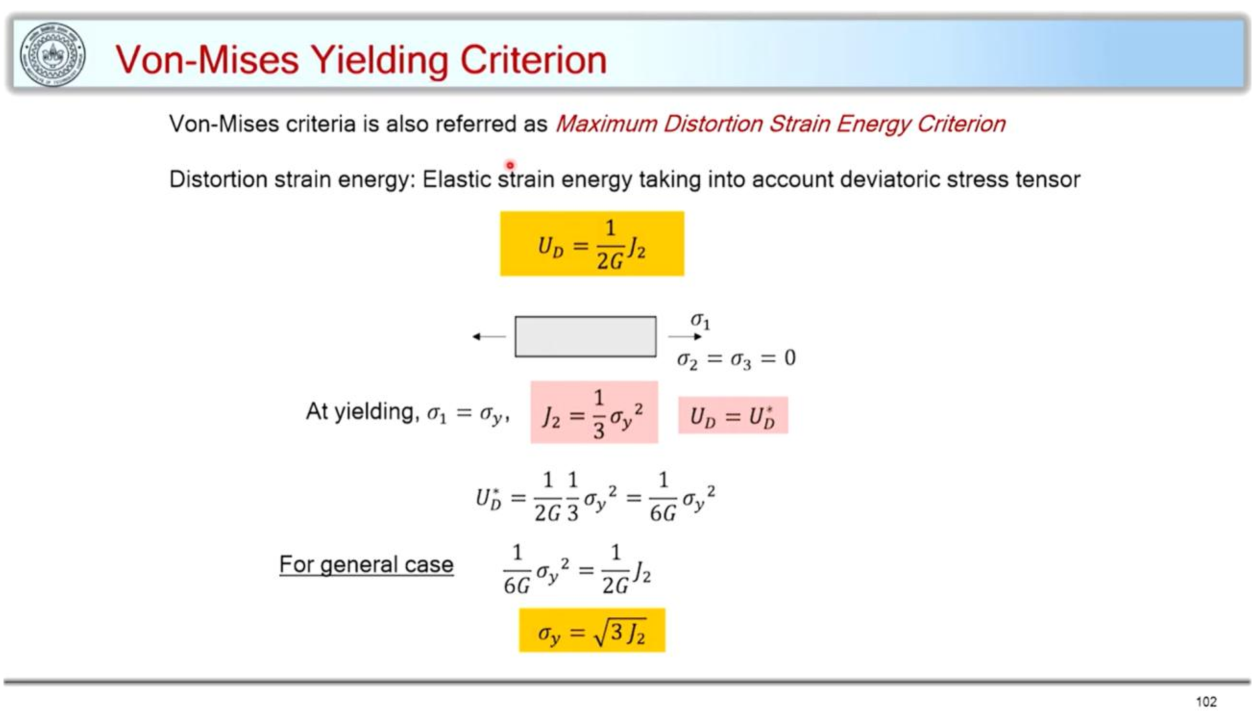
Toh J_2 ki value agar main dekhoonga aur σy ki value dekhoonga toh ye aayegi $\sqrt{3} * \sqrt{J_2}$. Toh aapke paas ye σy ki value aa gayi J_2 ke ismein. Toh ye jab σy ki value is tarah se pahunchegi tab tabhi meri yielding hogi material ki. General keis mein main is tarah se likh sakta hoon J_2 ko main expand karunga toh yeh aayegi. Agar J_2 main is tarah se expand karunga toh mere paas yeh general keis aayega aur jab J_2 ki value yahan pe pahunchegi is value tak pahunchegi $\sqrt{3} * \sqrt{J_2}$ tak pahunchegi tab hi meri yielding hogi ye meri general keis hai. Toh agar yahan pe hum humare teen consistencies hum check karenge teen consistencies kya thi yeh jo σy hai...



yeh humari pehla consistency thi ki yeh jo σy hai yeh coordinate transformation se independent honi chahiye. Toh agar aap dekhengy toh yeh σ_1, σ_2 aur J_2 jo hai woh stress invariant hai toh yeh jo term hai yeh invariant rahegi. Toh main koi bhi coordinate transform kar loonga toh J_2 invariant hi rahega. Toh agar aap do aur consistencies hain woh aap apply karke check kar sakte hain σy ke upar. Abhi main Von Mises yielding criteria ko is tarah se bhi dekh sakta hoon ye jo Von Mises yielding criteria hai isko hum kehte hain Maximum Distortion Strain Energy Criteria.

Toh yeh jo hai humne dekha tha ki J_2 jo hai woh main usko correlate kar sakta hoon distortion energy ke saath, strain energy ke saath tab yeh jo elastic strain energy jab deviatoric stress tensor ke around jo energy hai jo distortion strain energy hai ye critical value jab reach hoti hai tabhi meri yielding hogi isliye isko kehte hain Maximum Distortion Strain Energy Criteria. Maan leta hoon ye jo U_d hai humne nikaala tha 1/2G * J_2. Abhi main uniaxial tensile test agar kar raha hoon toh mere paas σ_1 rahega aur σ_2 aur σ_3 zero rahega aur yielding ke waqt yeh σ_1 rahega σ_y ke barabar. Toh main J_2 ko is tarah se likh sakta hoon 1/3 σ_y^2 yeh humne dekha tha abhi last slide mein.

Toh jab U_d yaane jo distortion energy ek critical energy hai, critical strain energy distortion energy main usko U_d* keh raha hoon ye iske barabar hogi tab mera yielding shuru hoga. Toh J_2 ki value maine nikaali yield stress ke isse aur U_d* jab nikaalunga tab yeh J_2 ki value main yahan par rakh raha hoon 1/3 σ_y^2 aur yeh jo value aayegi yeh aayegi 1/6G * σ_y^2. Yeh meri U_d* ho gayi. General keis ke liye kya hoga jab U_d aur U_d* equal hoga toh yeh jo do term hain 1/2G * J_2 yeh 1/6G * σ_y^2 hogi tab yielding hoga. Yahi Maximum Distortion Strain Energy ka criteria.



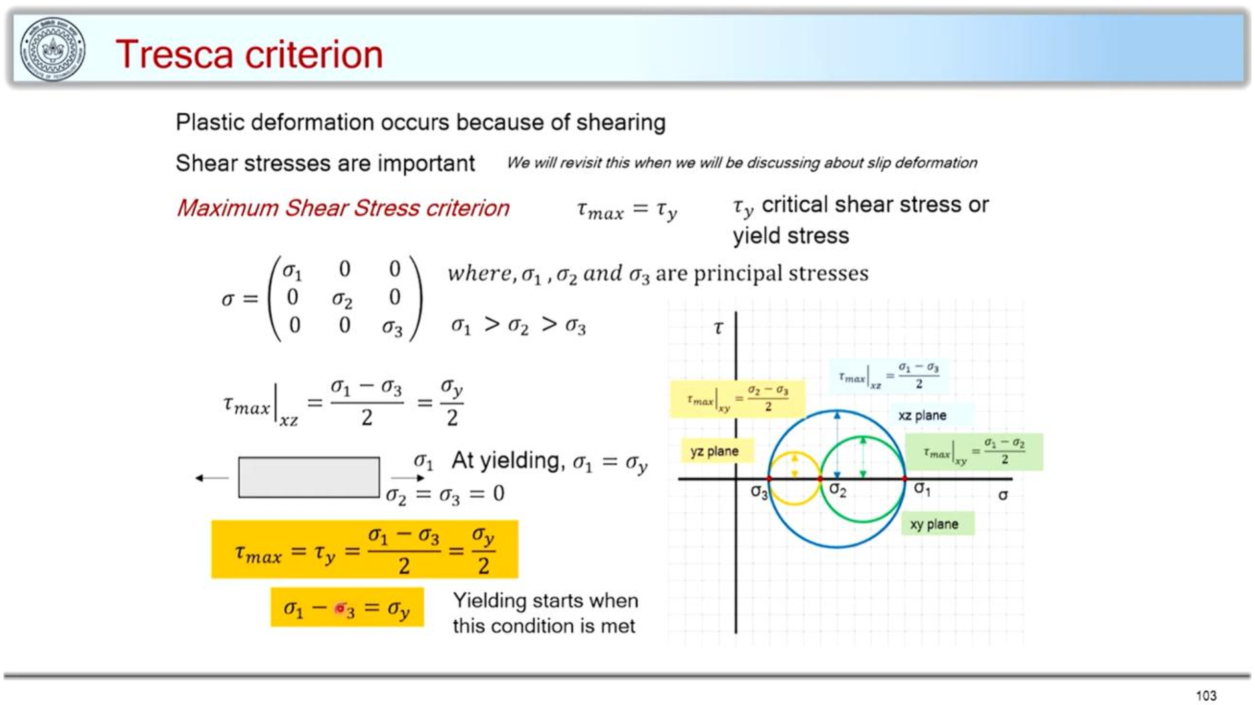
22:55 - 23:37 Toh isi se bhi humein ek stress aayega σy isko main nikaal sakta hoon $\sqrt{3} * \sqrt{J_2}$. Toh humne do cases se bhi dekha tha ki dono cases mein humein same result mil raha hai toh isliye is yielding criteria ko hum kehte hain Maximum Distortion Strain Energy Criteria. Abhi hum aage badhte hain aur ek criteria dekhte hain Tresca criteria. Toh Tresca criteria mein bola gaya hai ki jo plastic deformation hota hai shearing ki wajah se hota hai. Toh yahan pe shear stresses important hain. Hum dekhenge isko phir se dekhenge ki shear stresses kyun important hain kyunki shear stresses jo hain woh mere slip deformation karte hain jo hum micro mechanism dekhengy plasticity ke tab shear deformation shear stresses hi slip deformation laate hain.

Isliye shear stresses hi plastic deformation karte hain mere material mein. Toh ye Tresca ka criteria hai is criteria ko bhi kehte hain Maximum Shear Stress Criteria. Pichla wala criteria tha Von Mises criteria ye Maximum Distortion Strain Energy Criteria tha yeh Maximum Shear Stress Criteria hai. Toh maximum shear stress mein maan leta hoon $\tau max = \tau y$. τy mera ek critical shear stress ho gaya ya isko main keh sakta hoon yield stress simply. Maan lete hain mere paas ek stress state hai jisko maine convert kiya principal stresses ke dwaara σ_1, σ_2, σ_3 aur baaki ke shear stresses jo hain shunya hain.

Toh humne ek stress state dekha tha aur agar σ_1, σ_2, σ_3 principal stresses aur maan lete σ_1 algebraically bada hai σ_2 aur σ_3 se. Toh humne 3D stress state ke liye ek Mohr's circle nikaale thae. Toh hum humne dekha tha ki 3D stress state ko Mohr's circle ke dwaara kaise represent kiya jata hai tab us keis mein humein dikha tha ki maximum jo shear stress hai agar σ_1 aur σ_3 ye major principal stress hai aur yeh sabse lo smallest hai minor principal stress hai toh jo maximum shear stress woh σ_1 aur σ_3 jo stress act ho rahe usi usi plane par hoga.

Yahan par maine woh XZ plane mark kiya tha toh yeh jo shear stress hai yeh sabse maximum hai agar aap dekhengy teeno Mohr's circle toh yeh jo XZ plane par jo shear stress hai woh highest hai aur iski value hai $(\sigma_1 - \sigma_3) / 2$. Yeh humne dekha tha jab 3D stress state ka Mohr's circle humne draw kiya thae. Toh yeh value ho gayi mere paas τmax ki. Ab ye τmax ki value jo hai $(\sigma_1 - \sigma_3) / 2$ agar main uniaxial tensile test karta hoon toh ye jo σ_1 ki value hogi ye jab yield stress ki barabar hogi toh at yielding $\sigma_1 = \sigma y$ yield stress ke barabar hogi.

Toh yahan par main is tarah se consider kar sakta hoon ye jo $(\sigma_1 - \sigma_3) / 2$ agar σ_3 ki value yahan par shunya rakh do aap toh ye jo value aayegi aur σ_1 ki value σ_y ho jayegi tab ye $\sigma_y/2$ aa jayega τ_{max} ki value. Toh agar yeh identity mein solve karta hoon toh mere paas ye is tarah se aa jayega $\tau_{max} = (\sigma_1 - \sigma_3) / 2 = \sigma_y / 2$. Toh isko thoda samajhiye σ_1 ki σ_1 jab yielding yield point aayega toh σ_1 ki value kya ho jayegi σ_y. Aur yeh uniaxial stress state hai toh σ_3 yahan par shunya hai toh yeh τ_{max} ki value mere paas $\sigma_y/2$ aani chahiye.



Toh main general keis is tarah se likh sakta hoon $\sigma_1 - \sigma_3 = \sigma_y$. Ek special condition li thi humne yahan par aur yeh general keis ho gaya. Toh yahi mera Tresca criteria hai toh yielding starts when this condition is met. Toh mera σ_1 yaane major principal stress minus minor principal stress jo value hai unka difference hai jab woh equal ho jayega yield stress ke barabar tab yielding chaalu hogi ye mera Tresca criteria hai. Ab abhi hum dekhte ki Yield Surface kya hai. Toh ye do criteria humne dekhe Maximum Distortion Energy Criteria aur Maximum Shear Stress Criteria. Abhi hum yield surface kya hai yeh dekhte hain.

Maan lijiye mere paas ek stress state hai aur iske coordinates maine mark kar liye X_1, X_2, X_3 aur stresses mark kar liye σ_1, σ_2 aur σ_3. Maan lete hain 2D stress state hai aur σ_3 yahan par shunya hai is tarah se maan leta hoon main. Toh σ_3 ko main zero consider kar raha hoon aur abhi main mark karta hoon kuch stresses mark karta hoon. Toh main σ_1 yahan par mark kar raha hoon σ_2 yahan par mark kar raha hoon. Toh positive stress hai σ_1 toh yeh positive side par aayega aur positive σ_2 hai toh ye positive side par aayega. Toh agar yielding kab hogi yahan par yielding Tresca criteria ke anusar yielding tabhi hogi...

jab $\sigma_1 - \sigma_2$ should be less than equal to σy, ye criteria hai. Aur agar main 3D stress state consider karunga toh mujhe ye do aur aur conditions consider karne padengy $\sigma_2 - \sigma_3 \leq \sigma y$, $\sigma_1 - \sigma_3 \leq \sigma y$. Toh agar main yeh condition satisfy karunga is isse main ek yield surface ek yield surface create kar sakta hoon. Toh jaante hain ki kaise yield space create kar sakte hain maan lete hain mere paas ek 2D stress state hai aur jab hum dekhengy is tarah se likhte hain yahan par σ_1 is quadrant mein, quadrant jo one hai ismein σ_1 aur σ_2 dono greater than zero hain, positive hain, positive likh leta hoon dono positive hain.

Toh maan lete hain ki σ_1 ki value σy se zyada hai tab yielding hogi. Toh kahan par hogi jab main σ_1 ki value σy se cross kar loonga. Agar main is side hoonga toh meri yielding shuru hogi. Similarly agar σ_2 hai σ_2 ki value yeh bhi maan lete hain ki agar main compression mein hoon, compression mein hoon toh bhi main σy ki jab value cross karunga compression side mein toh meri yielding shuru hogi kyun hogi kyunki hum isotropic material ke liye consider kar rahe hain yaane agar tension mein agar yielding σy pe ho rahi hai toh compression pe bhi σy par hi hogi.

Similarly σ_2 ki dekhte hain agar σ_2 ki value σy se badi hai toh yahan par ek yielding shuru hogi. Yahan par aap jaan sakte hain ki yahan par σ_1 ki value shunya hai agar dekhengy agar main is axis pe badh raha hoon toh σ_2 ki value jab σy se cross hogi toh meri yielding shuru hogi. Agar main X-axis par hoon is side par hoon agar main σ_1 ki value σy se cross ho jayegi tab meri yielding shuru hogi. Abhi hum dekhengy agar main is side hoon ya idhar is state mein move ho raha hoon toh main dekhoonga jaise σ_2 ki value compression mein bhi yahan par yielding meri shuru ho gayi humne dekha tha ki isotropic material ke liye.

Abhi maan lete hain ki agar mere paas dono stresses hain σ_1 aur σ_2 toh hum dekhengy ki σ_1 aur σ_2 se kaun sa stress bada hai waise hi yielding hogi jaise σ_1 agar algebraically bada hai toh yahan par dekhengy mod liya hai maine toh agar yeh dono ka subtraction agar σy se bada hai tab meri yielding shuru hogi. Toh criteria satisfy karne ke liye main yeh line kheechoonga yahan par. Toh yahan par yeh line aa jayegi aur yahan par ek line aa jayegi toh yeh ek relation agar hum dekhengy toh yahan par agar dekhengy yeh jo value hai yeh agar mere principal stress bada hai toh yeh condition yahan pe satisfy honi chahiye ya isko main equal to maan leta hoon.

Isko main is equal to maan leta hoon aur yeh jo yeh jo line hai yeh aa jayegi meri $\sigma_2 - \sigma_3 = \sigma y$. Ye jo condition yahan pe meet hogi tab meri yielding shuru hogi. Isko is tarah se jaan sakte hain agar mera σ_2 principal stress hai major principal stress hai aur σ_3 minor hai toh yeh value dominate karegi. Agar mera σ_1 principal stress hai major principal stress aur σ_3 minor principal stress hai toh ye value dominate karegi. Abhi isi tarah se is third quadrant mein kyunki ye dono value agar hum dekhengy toh yahan par σ_1 negative hai compressive hai aur σ_2 bhi compressive hai.

Abhi hum dekhengy is keis mein jahan par sigma yaane second quadrant mein hum dekhengy toh σ_1 meri negative hai aur σ_2 positive hai. σ_1 negative hai aur σ_2 positive hai toh agar hum dekhengy toh σ_1 meri negative hai aur yeh bhi yahan pe negative sign hai toh yeh dono stresses add ho jayenge toh mere paas ek line aa jayegi yaani agar aap dekhengy toh yeh jo equation hoga is line ka is tarah se hoga $\sigma_1 - \sigma_2 = \sigma y$. Similarly agar aap dekhengy yeh quadrant mein fourth quadrant mein phir se isko likh leta hoon fourth quadrant mein mere paas σ_1 positive hai aur σ_2 negative hai.

Toh agar hum yeh equation dekhengy $\sigma_1 - \sigma_2$ toh yeh positive hai yeh negative hai aur yeh σ_2 negative hai toh negative negative positive ho jayenge toh in dono ka addition ho jayega toh isi tarah se mere paas ek ye ek line mil jayegi aur iska equation bhi yahi rahega. Toh yeh jo yield surface maine create kiya hai yeh mere Tresca criteria se generate kiya hai. Toh yield surface ka matlab kya hai agar main koi bhi state of stress agar consider kar raha hoon aur yeh yield surface ke andar ho toh yahan par material yield nahi hoga. Agar is surface se baahar ho toh yahan par yielding shuru hogi. Toh aapko dekhna hai ki mera state of stress agar andar hai toh yielding nahi hogi aur baahar hai toh yielding hogi.

Abhi Von Mises criteria dekhte hain aur us uska bhi yield surface nikaalne ki koshish karte hain. Toh humne dekha tha ki σy ki value hai $1/\sqrt{2} * \sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$. Toh yeh ho gayi meri Von Mises criteria ki. Abhi humein 2D state ke liye consider karunga toh yahan par σ_3 ki value shunya hai aur σ_1, σ_2 ki yahan par rakhunga toh mere paas ek simplified relation aayega: $\sigma y^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$. Yeh jo equation dekhengy yeh mere equation of ellipse hai, ellipse ka equation hai. Toh agar main yahan par plot karunga equation yeh toh mere paas kuch aisa yield surface aayega.

Yeh jo yield surface hai blue wala yeh mera Von Mises criteria ka yield surface hai. Toh humein Tresca criteria se ek simplified yield surface mila tha aur Von Mises se ek another yield surface criteria mila. Abhi hum dekhengy ki yeh jo values hain $\sigma_1 - \sigma_2$, $\sigma_2 - \sigma_3$, $\sigma_3 - \sigma_1$ yeh jo values hain aap dekhengy yeh jo $\sigma_1 - \sigma_2$ kya tha yeh diameter tha diameter kiske aur kiske beech ka agar main Mohr's circle nikaalunga σ_1 aur σ_2 ke beech mein. Toh yeh jo $\sigma_1 - \sigma_2$ hai ye mera diameter rahega Mohr's circle between σ_1 and σ_2.

Usi tarah se σ_2 aur σ_3 ka jo Mohr's circle rahega uska diameter $\sigma_2 - \sigma_3$ rahega aur σ_3 aur σ_1 ke beech ka jo Mohr's circle rahega uska diameter $\sigma_3 - \sigma_1$ rahega aur agar aap dekhengy toh yeh yield stress ko main is tarah se bhi likh sakta hoon ki root mean square diameter of three Mohr's circles. Toh is tarah se main likh sakta hoon. Abhi aur ek exercise main aapko deta hoon agar aap dekhengy ye major axis aur minor axis is ellipse ka yaani agar ye axis le rahe ho aur minor axis le rahe ho inka agar ratio loge inke length ka agar ratio loge toh aapke paas $\sqrt{3}:1$ aana chahiye.

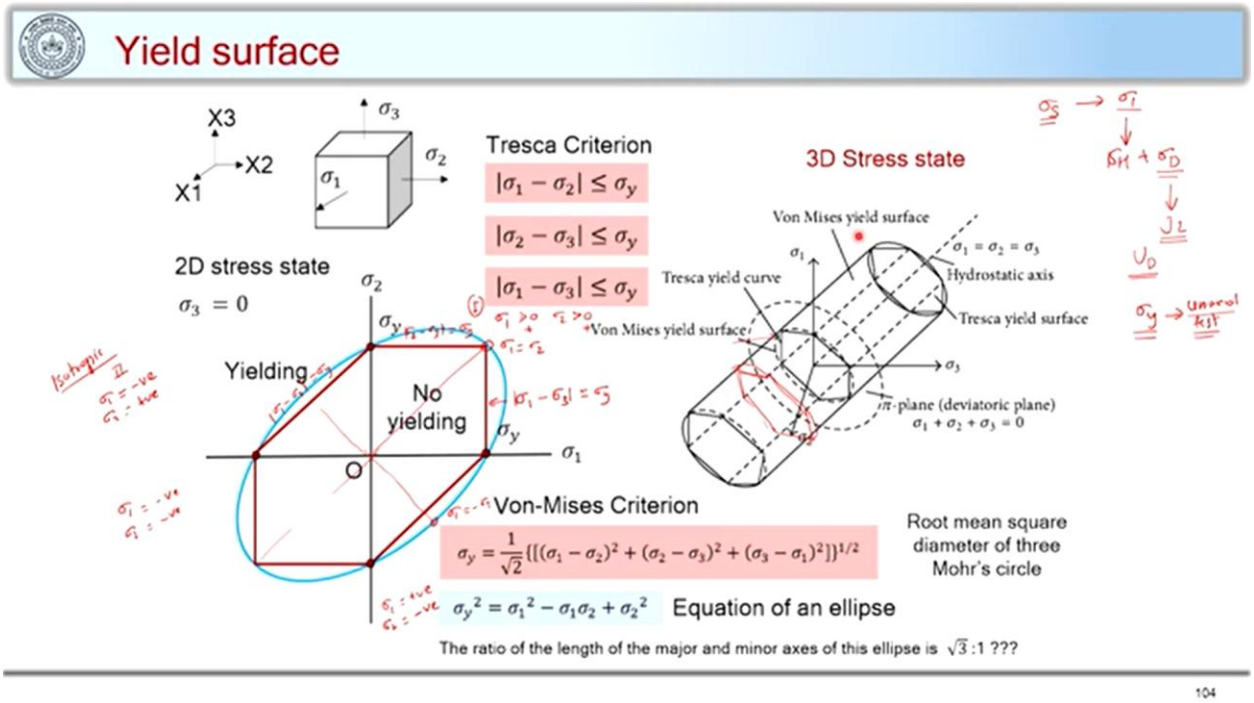
Toh aap ye exercise karke dekhiyega agar main yeh yeh length nikaal raha hoon yeh meri major axis ho gayi is ellipse ki aur ek minor axis ho gayi. Is keis mein aap dekhengy ki mera $\sigma_1 = \sigma_2$ hai aur is keis mein aap dekhengy ki mera $\sigma_1 = -\sigma_2$ toh agar aap yeh value yahan pe put karenge length nikaalne ke liye toh aap aap yeh prove kar sakte hain toh aap exercise karke dekhiyega abhi. Toh maine simple keis consider kiya tha 2D stress state ke liye abhi main 3D state consider karunga toh σ_3 shunya nahi rahega σ_3 ki kuch value rahegi par aap is tarah se kuch samajh sakte hain.

Agar mere paas yeh stress state hai 3D stress state agar main inka har stress state ke liye loci nikaaloon toh mere paas yaani main is tarah se kuch yeh mere paas ek ek stress state thi 2D stress state agar main doosra nikaaloon yahan par toh mere paas kuch aise aayega doosra yeh aa jayega mera Tresca criteria ke liye aur Von Mises criteria ke liye mere paas kuch aise aayega. Toh agar main in saare sabka loci nikaaloon toh mere paas ek surface milega. Toh agar surface main Von Mises yield criteria ke liye nikaaloon agar ellipse ko saare ellipse ko main 3D direction mein dekh raha hoon toh mere paas ek ek cylinder type aayega yaani elliptical cylinder aayega.

Aur yeh jo aayega Tresca yield surface jo aayega mera 3D stress state mein yeh aayega mera ek prism aayega jo distorted hexagon ki tarah dikh raha hai yahan pe yeh ek yield surface aayega. Toh aap dekhengy ki yahan par ye jo axis hai is cylinder ki yeh yahan par $\sigma_1 = \sigma_2 = \sigma_3$ rahegi aur yeh jo plane rahega yahan par yeh ek plane hai yeh koi bhi plane main consider karta hoon isko main pi plane consider karunga. Yeh mera rahega deviatoric plane yahan par $\sigma_1 + \sigma_2 + \sigma_3$ ki value shunya rahegi. Toh humne dekha ki ye 3D stress state ke liye main sirf in sab ko agar stack karunga third direction mein toh mere paas yeh yield surface taiyaar honge Von Mises ke liye aur Tresca surface.

Toh abhi is part mein humne dekha ki koi bhi stress state agar mere paas hai koi bhi stress state general stress state mere paas hai uskon main convert karta hoon principal stresses mein aur principal stress state ko main consider kiya tha hydrostatic stress state aur deviatoric stress state. Is deviatoric stress se maine J_2 ki value nikaali thi aur J_2 ki value maine isliye nikaali thi ki mujhe distortion energy nikaalni thi aur is distortion energy se main ek yielding criteria diya tha usko maine kaha tha Von Mises criteria aur jab hum shear stresses ki baat karengy tab ek criteria humne diya tha uska naam hai Tresca yield criteria.

Toh yeh Tresca criteria humne dekha tha toh aur yahan se hum yield surface taiyaar kar sakte hain isi tarah se hum dekh sakte hain koi bhi stress state mere paas hai aur main bata sakta hoon ki yielding hogi ya nahi hogi agar mujhe yeh aapko pata hona chahiye aapko sirf yield strength nikaalni hai laboratory uniaxial tensile test se. Agar yeh mujhe pata hai aur koi bhi stress state pata hai toh agar main do criteria apply karta hoon toh main material ka yield hoga ki nahi hoga. Yield kya hai? Yield mera elastic to plastic behavior ka transformation bataata hai.



Toh abhi ke liye hum yahan par rukte hain aur next part se hum jaanengy ki plasticity kya hai yaani micro mechanism of plasticity kya hai ye toh microscopic criteria tha aur actually atomistic level par jaakar plasticity kis tarah se define hoti hai. Dhanyavaad.