Mechanical behavior of materials

Dr. Niraj Mohan Chawake

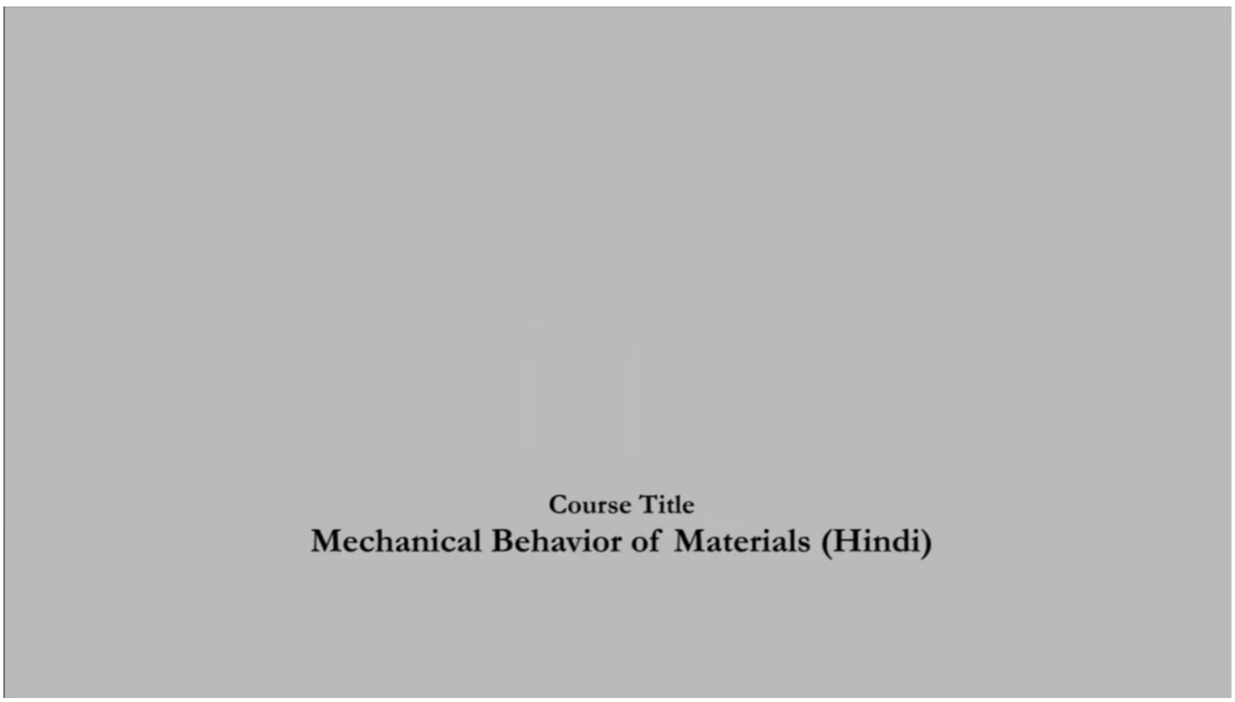
Department of Materials Science and Engineering

Indian Institute of Technology, Kanpur

Week-3

Lecture-18

Elastic Constants and Symmetry



Lecture-18

Elastic Constants and Symmetry

Namaskar! Main aapka swagat karta hoon Mechanical Behavior of Materials jo hum Hindi mein padhenge. Last part mein humne dekha tha ki Elastic Stiffness Tensor kya hai aur Elastic Constants kya hain. Is part mein hum dekhenge ki ye jo elastic constant hain ye Symmetry par kaise dependent hain. Toh last part mein humne dekha tha ki humare paas ye stress components hain aur ye strain components hain aur isko hum correlate karte hain ye stiffness components ke saath. Ye 36 stiffness components humne dekhe kyunki humare paas cheh (6) stress components hain aur cheh strain components hain. Isko Voigt contracted notation mein main likh leta hoon kyunki yahan par jo matrix likha hai humne woh Voigt contracted notation mein hai.

Toh humne dekha tha ki Voigt contracted notation ko kis tarah se likha jata hai. Toh 11 ko hum 1 likhenge, 22 ko hum 2 likhenge, 33 ko hum 3 likhenge, 23 ko 4 likhte hain, 13 ko 5 likhenge aur 12 ko 6 likhenge. Is tarah se humne likha tha aur yeh jo stiffness tensor hai humein yeh relation de raha hai, yeh stress tensor, stress component aur strain components ke beech. Aaiye dekhte hain inmein se jo 36 components hain usmein se humne dekha tha ki 21 independent hain kyunki humara jo stiffness tensor hai woh symmetric hai. Toh yeh bhi humne last part mein dekha tha. Isko likh lete hain ki humne dekha tha ki $C_{ij} = C_{ji}$ is tarah se humne dekha tha. Toh humare paas ye 21 independent components milte hain.

Abhi is 21 independent components symmetry pe kis tarah se dependent hain? Toh aaiye jaante hain ki symmetry kya hai. Pehle uske liye hum jaante hain ki Crystal Structure hum define karenge phir ek rotational symmetry dekhenge aur uske corresponding elastic constant dekhenge. Toh pehle hum dekhte hain Triclinic symmetry. Toh Triclinic symmetry aap kaise define karenge? Uh ye jo lattice parameters hain a, b aur c aur yeh angles hain α (alpha), β (beta) aur γ (gamma). Iske dwaara hum symmetry define karte hain. If $a \neq b \neq c$, $\alpha \neq \beta \neq \gamma$, isko hum kehte hain Triclinic crystal structure.

Triclinic crystal structure mein agar hum dekhenge toh koi ek aisa koi symmetric element hai nahi, iski symmetry 1 hai. Toh is keis mein humein saare jo components hain 21 component in sab ki zaroorat lagegi. Toh humare paas rotational symmetry yahan par None hai aur jo independent elastic constant hain humare paas woh humein 21 independent elastic constants lagenge stress ko correlate karne ke liye strains ke saath. Abhi hum dekhte hain Orthorhombic crystal structure. Abhi jaise-jaise hum doosri symmetry dekhenge aapko yeh idea ho jayega ki independent elastic constant symmetry pe kaise depend karte hain. Toh pehle abhi Orthorhombic crystal structure dekhte hain.

Orthorhombic mein kya relation aata hai lattice parameters aur angles ka? Toh hum dekhte hain ki $a \neq b \neq c$, jo angles hain $\alpha = \beta = \gamma = 90°$. Ye humare paas crystal structure ki yeh jo definition hai. Toh hum dekhenge ki humare paas ye maan lenge yahan par 90 degree hai. Toh humare paas Two perpendicular two-fold axis humare paas hain. Toh two perpendicular two-fold axis kahan par mere paas, agar hum dekhenge toh hum is tarah se mark kar sakte hain agar ye axis hai mere paas, ye ek hai aur yeh ek axis hai. Yeh two perpendicular two-fold axis humein mil jayegi.

Kyunki yahan par hum dekhenge symmetry is tarah se hum symmetry jab n-fold symmetry ki baat karenge toh is tarah se hum baat kar sakte hain jaise maine yahan par two-fold axis bola hai. Toh jaise mere paas cube hai, toh symmetry ko is tarah se jaante hain. Cube hai toh aur n-fold symmetry hai jaise cube ki chaar fold symmetry hoti hai toh humein 90 degree milega. Toh agar humein n-fold symmetry jab baat karta hoon toh cube ko aap dekhenge ki main 90 degree rotate karne ke baad woh apne initial position pe aa jayega. Jaise 1, 2, 3, 4 ye points agar main maan ke chaloon aur main isko 90 degree rotate kar doon toh 2, 1 ki position par chala jayega.

Aur aap differentiate nahi kar paoge initial structure se final structure ke saath. Toh yahi humari n-fold symmetry hoti hai. Iske liye iske baare mein aap kisi bhi physical metallurgy books mein padh sakte hain ki n-fold symmetry hoti kya hai. Toh yahan par humare paas Orthorhombic crystal structure hai toh hum dekhenge ki yahan par yeh two perpendicular... yahan par hum dekhenge ek principle ko hum jaante hain isko main naam deta hoon Shear Decoupling. Yahan par do cheezein hain: Pehli cheez hai ki koi bhi normal stress, normal strains ko ya shear strains ko develop nahi karega (No shear coupling). Aur doosra hai... isko abhi is tarah se samajhte hain pehle.

Yeh part hum istemaal karte hain agar mere paas normal stress hai ek normal stress hai is direction mein, maan lete hain X_1 direction mein ek normal stress hai, let's say isko main maan leta hoon σ_{11}. Ek normal stress hai woh is plane par shear strain develop nahi karega. Koi bhi shear strain is plane par develop nahi hogi σ_{11} ke dwaara. Toh hum dekhenge jab humne yeh stiffness tensor dekha tha toh kuch parts jo thae yahan pe humne dekha tha ki normal stress ko normal strains ke saath correlate karte thae. Yahan par humne dekha tha ki normal stresses ko shear strains ke saath correlate karte hain components.

Ya shear stresses ko shear stresses ke saath correlate karte hain. Toh aap isko is tarah se jaan sakte hain jaise yeh jo nine (9) components hain is par main focus karunga: C_{14}, C_{15}, C_{16}, C_{24}, C_{25}, C_{26}, C_{34}, C_{35} aur C_{36}. Ye jo components hain ye kisko correlate kar rahe hain? Aap dekhenge main likh leta hoon isko jaise C_{14} hai, C_{14} kisko relate karega σ_1 ko ε_4 (epsilon 4) ke saath. Toh aap dekhenge ki ye normal stress hai, σ jo hai normal stress hai, ye kisko correlate kar raha hai shear strains ke saath. Toh agar mere paas yeh Orthorhombic crystal structure hai toh koi bhi normal stress jo main apply karunga woh is shear strains ke saath correlate nahi ho payega.

Toh yeh shear strains develop nahi kar payega. Toh iska matlab yeh jo terms, yeh jo nau component hain yeh saare shunya (0) ho jayenge. Toh mere paas n-fold rotations hain yeh maine explain kiya tha yaani 360/n will retain symmetry. Aur abhi hum dekhenge ki Orthorhombic mein yeh jo terms hain saare shunya ho jayenge. Isko mark kar lete hain kyunki mere koi bhi normal stresses shear strains ko develop nahi karenge yeh humara shear decoupling ka principle tha. Toh humare paas kitne components ho jayenge jo independent hain? Humare paas abhi 21 thae yahan par aur 21 se ghat ke kitne ho jayenge? Abhi 12 hain.

Abhi hum doosra principle dekhte hain ki koi bhi shear stress kisi doosre plane mein hai (Shear stress in one plane cannot cause shear strain in another plane). Isko is tarah se samajhte hain agar mere paas ek shear stress hai yahan par maan lijiye ye shear stress hai ek shear stress aise act kar raha hai yahan pe, let's say isko main plane 2 maan leta hoon aur yeh plane 1 maan leta hoon aur yeh plane 3 maan leta hoon. Toh ye jo shear stresses hain jo plane 2 mein act ho rahe hain woh plane 1 mein shear strain develop nahi karenge ya jo plane 3 hai ismein koi shear strain develop nahi karenge.

Toh shear stress ek plane mein jo act ho rahe woh shear strains develop karenge usi plane mein na ki doosre planes mein. Toh iske wajah se kya hoga hum dekhenge ki yeh jo constant hai yahan par, toh yahan par dekhenge ki yeh jo shear stress hai aur shear strains ke saath relate kar raha hai yaani C_{44} dekhenge toh C_{44} kis ke saath correlate kar raha hai isko bhi jaan lete hain. C_{44} yaani ye jo hai σ_4 woh ε_4 ke saath, ye shear strains ke saath. Toh mere paas ye shear stresses hain ye shear strains ke saath correlate karenge is components ke dwaara. Abhi hum dekhenge ki agar mere paas σ_4 hai ya σ_{23} hai woh sirf γ_{23} (gamma 23) ko generate karega na ki γ_{13} aur γ_{12}.

Similarly jo σ_{13} hai yeh shear stress hai yeh generate karega sirf γ_{13} na ki γ_{23} aur γ_{12}. Ya mere paas σ_{12} hai yeh generate karega sirf γ_{12} na ki γ_{23} aur γ_{13}. Toh aap dekhenge ki yeh jo component hain C_{45}, C_{46} aur C_{56} yeh shunya ho jayenge. Toh yeh bhi hum isko likh lete yeh bhi shunya ho jayenge mere paas abhi tak 12 thae aur 12 mein se ye teen aur chale gaye toh mere paas ho jayenge nau (9). Toh isko count kar lete hain pehle se, toh mere paas nine components ho jayenge. Toh Orthorhombic mein agar mujhe stress-strain ka relation nikaalna hai toh mujhe nau components ki zaroorat padegi.

Abhi jaante hain Cubic structure ke baare mein jiski highest symmetry hai. Toh Cubic hum jaante hain Cubic ko hum define karte hain $a = b = c$, $\alpha = \beta = \gamma = 90°$. Ye saare planes orthogonal rahenge aur identical rahenge. Toh cube mein kya fold symmetry hai? Cube mein four three-fold symmetry hai. Toh three-fold symmetry ko jaanenge hum is tarah kyunki highest symmetry hai toh agar main ek diagonal draw karta hoon ek diagonal agar yahan se draw karta hoon cube ka toh mere paas aise aise diagonal ho jayenge aur yeh jo diagonal hai iske around jo symmetry hai ye chaar diagonal main draw kar sakta hoon.

Aur ye jo diagonal hai iske around jo symmetry hai woh three-fold symmetry ye highest symmetry hai humare paas chaar diagonal hain aur chaar three-fold symmetry ho jayegi. Toh abhi hum dekhenge ki Cubic mein abhi humare paas yahan par toh nau (9) constant thae. Abhi cube ko stress-strain agar cubic crystal structure hai aur stress-strain ka relation nikaalna hai toh aapko kitne constant ki zaroorat padegi woh jaante hain. Toh agar hum dekhenge yahan par is tarah se jaante hain σ_{11} mera develop karega strain ε_{11} aur yeh rahega C_{11}. Aur σ_{22} develop karega ε_{22} aur yeh rahega C_{22}. Aur σ_{33} develop karega ε_{33} aur yeh rahega C_{33}.

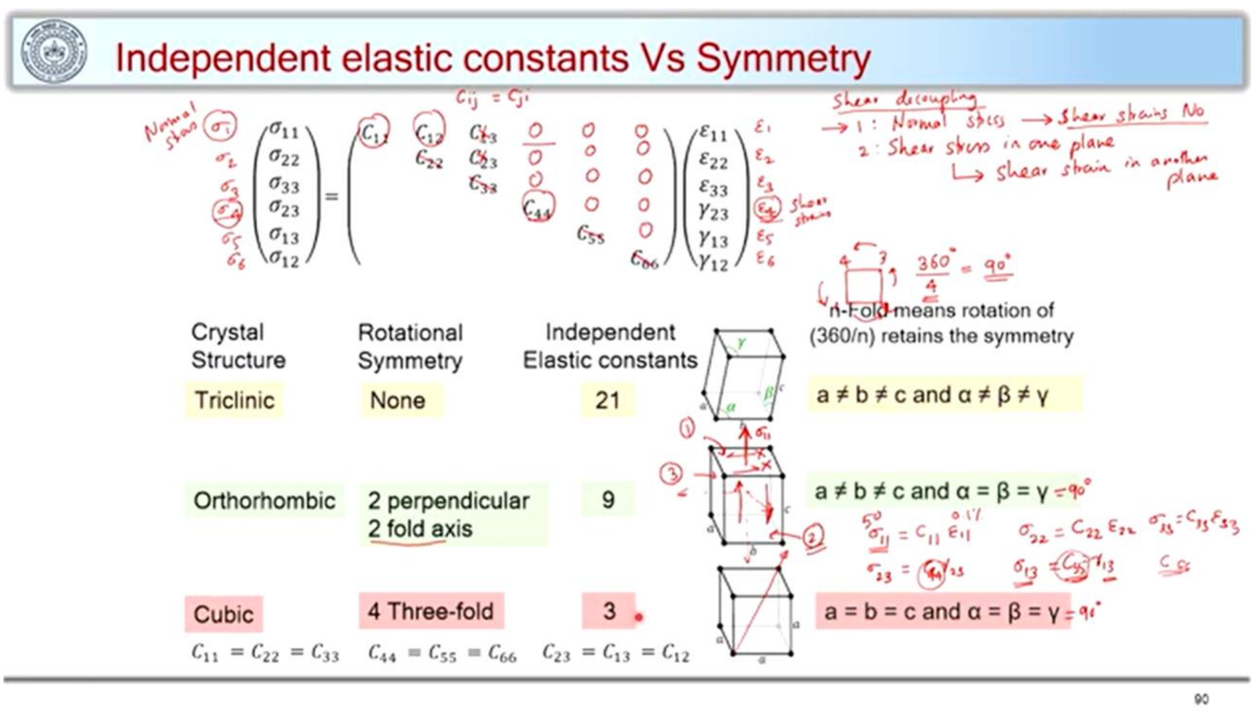
Toh ye mere paas normal stresses hain aur yeh normal strains develop ho rahe hain. Toh normal strains agar develop ho rahe hain toh agar hum dekhenge σ_{11} agar aap kisi bhi aur plane mein apply kariye jaise σ_{11} ki magnitude main maan leta hoon 50 MPa aur strain mera develop ho raha hai let's say 1%. Ye just main example dwaara de raha hoon. Toh agar main 50 MPa doosre plane pe apply karta hoon yaani plane 2 pe apply karta hoon toh mujhe strain utna hi milna chahiye 1% ya σ_{33} main 50 MPa third plane pe apply kar raha hoon toh mujhe strain 1% milta hai. Toh iska matlab yeh hoga agar mujhe ek relation pata hai is stress-strain ka σ_{11} pata hai aur ε_{11} pata hai.

Uske dwaara mujhe C_{11} pata hai toh mujhe C_{22} pata hona chahiye aur C_{33} bhi pata hona chahiye dono kyunki same hone chahiye. $C_{11} = C_{22} = C_{33}$ ye symmetry ki wajah se kyunki ye saare planes identical hain. Toh agar mujhe C_{11} pata hai toh is keis mein mujhe C_{22} ki zaroorat nahi hai aur C_{33} ki zaroorat nahi hai. Usi tarah se agar mujhe let maan lete hain isko bhi likh lete hain hum agar mujhe σ_{23} main agar nikaaloon toh $\sigma_{23} = G \cdot \gamma_{23}$ is tarah se likh paunga. Toh agar mujhe koi shear stress pata hai aur mujhe shear strain mil gaya toh mujhe shear modulus mil jayega.

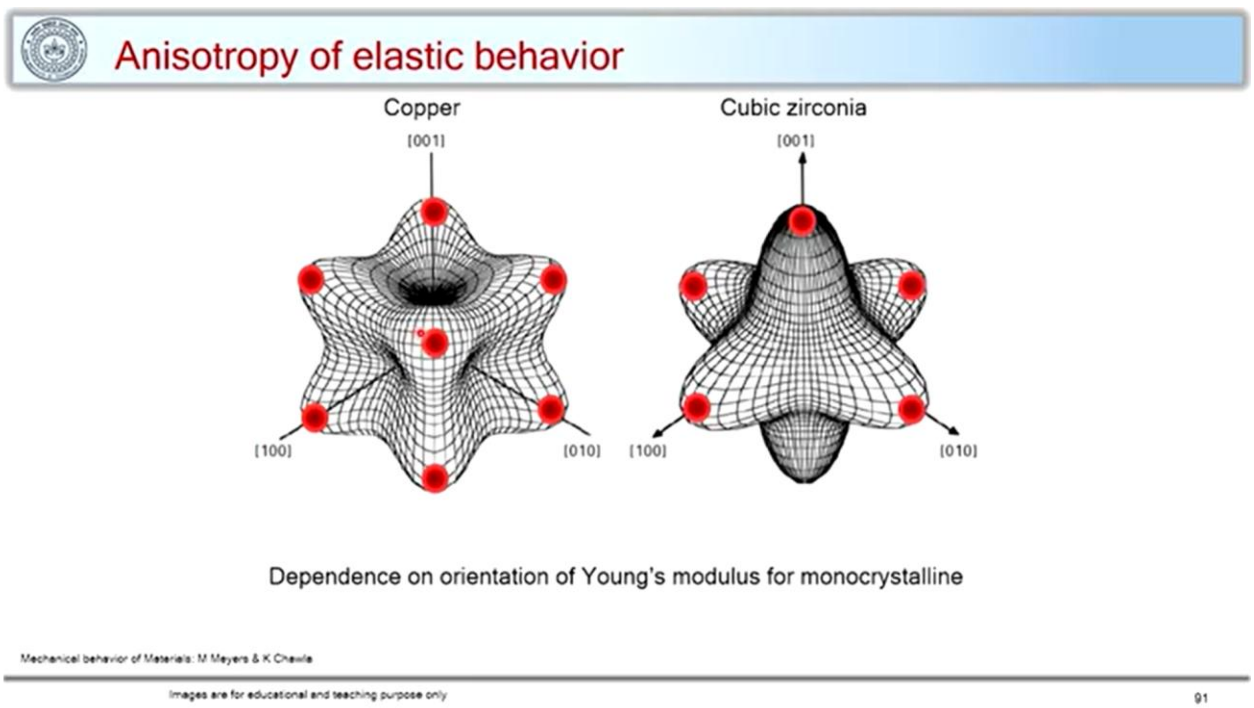
Shear modulus maine likha hai isko main G ki tarah likha tha par isko main C_{23} hai toh C_{23} yaani σ_4 hai aur ye ε_4 hai toh yeh C_{44} ke dwaara relate hoga. Toh ye C_{44} yaani shear modulus hoga par isko main C_{44} likh raha hoon yahan par. Toh yeh agar mujhe σ_{23} pata hai γ_{23} pata hai toh mujhe C_{44} mil jayega. Usi tarah agar main same stress jaise σ_{13} agar main apply kar raha hoon aur mujhe milega γ_{13} aur yeh jo constant rahega woh C_{55} rahega. Toh agar dono ka nature same hai plane kyunki saare plane same hain toh mujhe agar C_{44} pata hai toh mujhe C_{55} aaraam se pata chal jayega. Similarly mujhe C_{66} pata chal jayega.

Toh agar mujhe C_{44} pata hai toh mujhe C_{55} pata hai C_{66} bhi pata hai toh iski bhi zaroorat nahi rahegi agar mujhe C_{44} pata hai toh. Similarly main yeh bhi dikha sakta hoon ki yeh agar case yahan pe mujhe C_{12} pata hai toh main C_{13} ki zaroorat nahi rahegi aur C_{23} ki bhi zaroorat nahi rahegi. Toh C_{12} yaani kya hoga stress 1 yaani σ_{11} ko relate kar raha hai yeh relate kar raha hai mera ε_{22} ke saath. Toh mujhe C_{12} agar pata hai toh mujhe C_{13} aur C_{23} ki zaroorat nahi hai. Toh isko bhi likh lete hain agar mujhe pata hai toh mujhe kitne constants ki zaroorat hoti hai Cubic ko define karne mein. Mujhe sirf teen (3) constants ki zaroorat hoti hai.

Toh mujhe mere paas C_{11}, C_{22} aur C_{33} equal ho gaye. C_{44}, C_{55}, C_{66} equal honge aur C_{23}, C_{13} aur C_{12} equal honge. Toh mere paas sirf teen constant ki zaroorat rahegi Cubic mein stress-strain relation obtain karne ke liye. Toh ye humne dekha tha ki humare paas 21 elastic constant hain jo humein require hain Triclinic mein jismein koi rotational symmetry nahi hai aur Cubic mein jahan par four three-fold symmetry hai mujhe wahan par sirf teen constants ki zaroorat hoti hai. Teen elastic constant ki zaroorat hoti hai stress-strain relation ko completely define karne ke liye. Toh abhi jaante hain ki Anisotropy kya hai ismein. Toh yahan par maine do example diye Copper aur Cubic Zirconia.



In dono ka structure cubic hai aur aap dekhenge ki yahan par maine Young's modulus plot kiya hai kisi bhi monocrystalline system ke liye jaise Copper monocrystalline, Cubic Zirconia monocrystalline. Toh aap dekhenge ki Young's modulus copper ke keis mein yeh jo <111> direction hai ismein highest hai ya main keh sakta hoon <100> direction par lowest hai. Aap dekhenge yahan par dip hai aur yahan par ubhra hua aaya hai toh yeh highest hai yahan par modulus copper ke keis mein. Cubic Zirconia ke keis mein aap dekhenge ki <100> direction pe Young's modulus highest hai yaani jo elastic constant hai uski value sabse highest hai par yeh jo keis hai yahan par <111> keis mein yahan pe lowest hai.



Cubic Zirconia... dono ka structure same hai toh yeh depend karta hai aapke material pe. Toh yahi anisotropy hai elastic behavior ki. Elastic behavior ki anisotropy ya iska matlab ki jo elastic properties hain woh aapke directions ke ya orientation ke hisaab se badalti hain aur uska function hoti hain. Toh abhi hum jaanenge anisotropy of elastic behavior. Toh hum abhi tak humne Stiffness Tensor jaana tha abhi hum Compliance Tensor dekhte hain. Toh jab hum compliance tensor ki baat karenge toh hum jaanenge ki jo C_{ijkl} isko main C_{mn} ye contracted notation mein likh sakta hoon. Toh ij ko main m likh raha hoon aur kl ko main n likh raha hoon yeh humne jaana tha jab hum contracted notation ki baat karte thae tab.

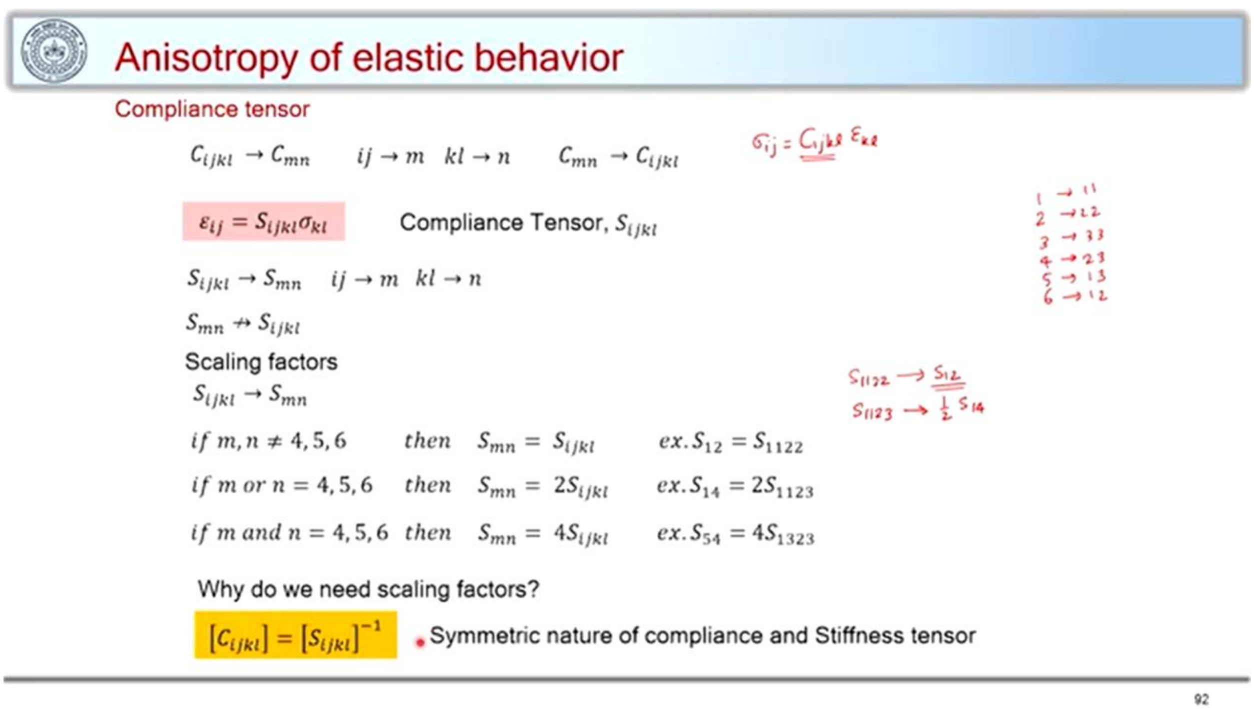
Ab ye jo C_{mn} hai isko main back C_{ijkl} likh sakta hoon par jab main strain ko stress ke saath correlate karta hoon yeh jo mera compliance tensor hai yeh mera stiffness tensor ka Inverse hai. Ye hum is tarah se likhte hain $\sigma_{ij} = C_{ijkl}\ \varepsilon_{kl}$. Toh ye mera stiffness tensor ho gaya aur yeh mera compliance tensor ho gaya par compliance tensor mein hum yeh cheez nahi kar sakte. Yeh mera compliance tensor ho gaya, compliance tensor mein bhi yahan par maine likh liya jab contracted notation ki baat karunga toh main ij ko m likhunga aur kl ko n likhunga. Par jab m, n se jaise contracted notation se waapas compliance tensor ke chaar indices ke form mein waapas karne ki koshish karunga toh yeh direct conversion nahi ho sakta.

Stiffness tensor mein ho sakta hai par compliance tensor mein nahi kar sakte uske liye kuch scaling rules hain toh Scaling Factors hain. Toh hum scaling factors dekhte hain. Hum jaanenge jab S_{ijkl} ko main S_{mn} likhta hoon toh agar m aur n {4, 5, 6} nahi hain toh hum main S_{mn} ko S_{ijkl} chaar form mein likh sakta hoon yaani do indexes ko chaar form mein directly convert kar sakta hoon. Jaise example lete hain jaise S_{12} hai mere paas, S_{12} ko main S_{1122} likh sakta hoon aur ulta bhi likh sakta hoon agar mere paas S_{1122} hai toh main usko S_{12} likh sakta hoon. Mujhe isliye koi scaling factor ki zaroorat nahi hai. Toh yeh kab hoga jab mera m aur n {4, 5, 6} nahi hain. Yaane mere paas {1, 2, 3} hai m aur n ki value.

Doosri condition dekhte hain m ya n {4, 5, 6} hai, dono mein se ek koi {4, 5, 6} hai toh mujhe S_{mn} jab main convert karunga chaar indexes ke keis mein toh mujhe ye do (2) se multiply karna padega. Jaise S_{14} hai toh ye mujhe likhna padega $2S_{1123}$. Toh yahan pe dekhenge n jo hai n ki value chaar (4) hai aur m ki value one (1) hai yahan pe. Toh dono mein se kisi ek ki value agar chaar, paanch ya cheh hai toh humein do se multiplication karna padta hai scaling factor interchange karne ke liye. Toh mujhe S_{1123} ko agar mujhe S_{14} likhna hai toh mujhe half (½) se multiply karna padega. Agar dono ki value {4, 5, 6} hai toh mujhe chaar (4) se multiplication karna hai.

Yaani aap dekhenge S_{54} toh m ki value 5 hai yahan pe aur n ki value 4 hai toh isko hum likhenge $4S_{1323}$. Toh humne dekha tha ki hum is tarah se likhte thae 1, 2, 3, 4, 5, 6. Toh 1 ko hum likhte thae 11, 2 ko likhte 22, 3 ko likhte thae 33, 4 ko likhte hain 23, 5 ko likhte hain 31 aur 6 ko likhte hain 12. Ye humne Voigt contracted notation dekhe thae toh yahi apne scaling factors hain. Toh ye scaling factors ki zaroorat hai toh iski zaroorat isliye hai kyunki jab hum convert karenge mere paas agar stiffness tensor hai usko main compliance tensor mein convert karunga yeh relation valid rahe isliye humein yeh scaling factor zaroorat hai aur isliye bhi zaroorat hai kyunki humara jo symmetric nature hai compliance aur stiffness tensor ka woh bana rahe.

Abhi aage chalte hain aur dekhte hain ki compliance tensor kya hai Isotropic Material ke liye. Toh mere paas maine dekha tha ki strains ko jab hum relate karte hain stresses ke saath toh humein zaroorat padti hai compliance tensor ki. Toh hum is tarah se likh sakte hain ye compliance tensor pehle main likh leta hoon jo main agar main ε_{ij} ko relate karunga σ_{kl} se toh mujhe S_{ijkl} jo hai yeh mera compliance tensor hai. Toh mere paas yeh strain components hain aur abhi main compliance tensor ko likhunga yahan par aur ye mere paas stress component hai. Toh humne isotropic material mein ye cheez dekhi thi ye ye jo term hai yeh normal stress ko relate karega normal strains se.



Toh is keis mein ye normal strains ko relate kar raha hai, karega normal stresses ke saath. Yaani normal strains kya hain yahan pe ε_{11}, ε_{22}, ε_{33}. Toh ye jo components hain, nau (9) components mujhe milenge ye relate karenge mere normal stresses ke saath σ_{11}, σ_{22} aur σ_{33}. Toh humne ye relationship dekha tha toh abhi isko hum is tarah se likh sakte hain, isko thoda sa expand karenge toh main ε_{11} ko is tarah se likh sakta hoon: $\sigma_{11}/E - \nu\ \sigma_{22}/E - \nu\ \sigma_{33}/E$. Toh yeh jo pehle teen components honge yahan par yeh mujhe ε_{11} jo main relation laaunga normal stresses ke saath toh mujhe pehle teen components is tarah se milenge.

Toh pehla component kya hoga mera? Mera pehla component hoga 1/E. Toh agar main 1/E rahega yaani S_{11} mera 1/E rahega. Ye jo doosra component rahega S_{12}, S_{12} rahega mera -ν/E (minus nu bata E) aur S_{13} aayega mera -ν/E. Usi tarah se hum baaki ke components bhi likh sakte hain. Toh for ε_{22} mera pehla component aayega -ν/E, doosra S_{22} aayega ya ε_{22} ka relation σ_{22} ke saath 1/E se aayega aur teesra component aayega S_{23} jo mera aayega -ν/E. Toh is tarah se mere paas nau components milenge jo normal strains ko normal stresses ke saath correlate karte hain woh ye equations humne dekhe thae pehli baar jab humne stress-strain relations dekhe thae.

Baaki jo components hain maine bola tha ki shear decoupling ka principle hai tab yeh jo components honge yeh shunya (0) honge kyunki yahan par agar hum dekhenge S_{14}, S_{14} yaani mera ε_{11} σ_{23} ke saath correlate ho raha hai. Toh σ_{23} ke saath correlate ho raha hai yaani normal stresses ko humne dekha tha ki normal strains ko hi develop karte hain par yeh jo shear stress hai yeh normal strain develop nahi karega. Toh yeh jo nau (9) components honge, yeh jo nau component honge jo normal stresses ko shear strains ke saath correlate karte hain ya shear strains ko normal stresses ke saath correlate karte hain, jo nau aur nau 18 component ye shunya ho jayenge. Abhi hum dekhenge ki shear stress ko humne is tarah se likha tha $\gamma = 1/G\ \sigma$.

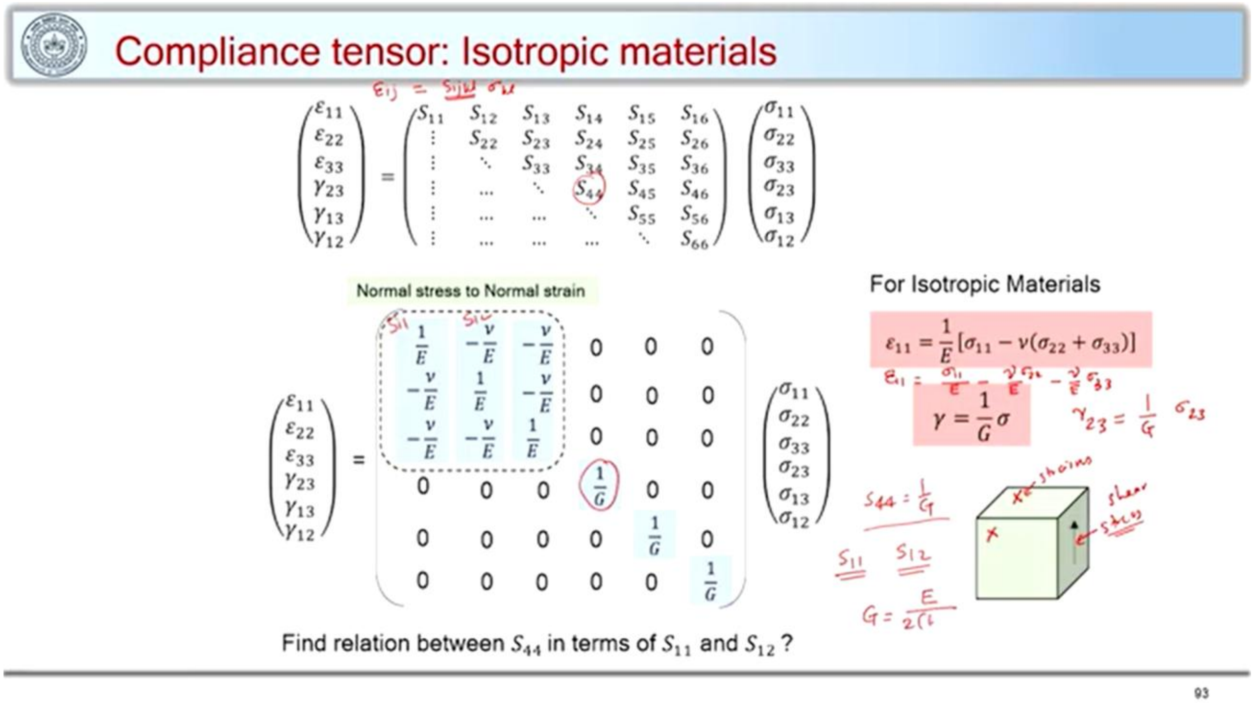
Toh agar hum dekhenge yahan par shear strains kaun se hain γ_{23}, γ_{13} aur γ_{12}. Toh γ_{23} ko main σ_{23} ke saath is tarah se likh sakta hoon. Toh is tarah se main samajh sakta hoon γ_{23} ko main σ_{23} ke saath likhunga toh yeh mera 1/G aana chahiye. Toh yeh jo term aayega yeh pehla term aayega ye 1/G ye mera shear modulus ka reciprocal rahega. Similarly main do terms is tarah se likh sakta hoon. Abhi hum dekhenge ki isotropic material ke liye yeh jo strains hain yeh kisi doosre stress ke dwaara nahi hone chahiye yaani maine bataaya tha ki humne shear decoupling principle, yaani agar mere paas ye shear stress hai is plane mein act ho raha hai toh yeh shear stress is planes par shear strains ko develop nahi karenge.

Yeh mera stress hai aur yahan par koi strains nahi honge is stress ke wajah se is shear stress ki wajah se. Toh iska matlab yeh ho jayega ki yeh jo ek do teen yeh jo component hain aur ek do teen yeh component ye bhi shunya ho jayenge. Toh isko bhi mark kar lete hain toh yeh ho gaya mera compliance tensor aur compliance tensor ki actually values S_{11}, S_{12} aur S_{13} ki aur S_{44}, S_{55}, S_{66} ki. Toh abhi hum dekhenge jaise mere paas S_{44} hai toh main isko is tarah se likh sakta hoon S_{44} agar mere paas hai toh S_{44} nothing but mera 1/G hai. Yeh is tarah se main likh sakta hoon S_{44} ko main

1/G likh sakta hoon. Toh aapko nikaalna hai S_{12} aur S_{11}. S_{11} aur S_{12} yeh aapke S_{11} ho gaye yeh ho gaya S_{12}.

Aapko relation nikaalna hai S_{44} ka S_{11} aur S_{12} ke saath toh iske liye aap yeh istemaal kar sakte hain ye humne identity nikaali thi $G = E / (2(1 + \nu))$. Ye humne derive ki hai isotropic material ke liye. Abhi hum aage chalte hain aur compliance aur stiffness tensor ko aur acchi tarah se dekhte hain jaise for isotropic material. Toh mere paas strain hai isko main relate karta hoon compliance tensor ke saath stress ke saath toh yeh mera compliance tensor ho gaya yeh humne dekha tha abhi. Abhi hum dekhenge ki stress ko jab main strains ke saath compare karta hoon toh mujhe stiffness tensor ki zaroorat hoti hai toh yeh mera stiffness tensor ho gaya. Toh yahan par λ (lambda) jo hai yeh Lame's constant hai humne dekha tha.

Jaise relation humne jab dekha tha σ_{11} ko ε_{11} ke saath agar correlate kiya tha toh mere paas yeh equation aaya tha jahan par humne lambda ko define kiya tha ye Lame's constant. Toh agar aap dekhenge yeh jo Δ (delta) hai delta ki value hum yahan se phir se likh sakte hain, delta ki value hai $\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ ye Summation of normal strains ho jayega. Toh yahan pe delta ki yeh hai agar main isko is tarah se multiply karunga jaise σ_{11} ki value nikaalne ki koshish karunga toh main matrix multiplication karunga toh yeh yeh jo terms hain isko main is tarah se multiply karunga toh mujhe yeh identity milegi. Toh is tarah se main stiffness tensor ko nikaal raha hoon.

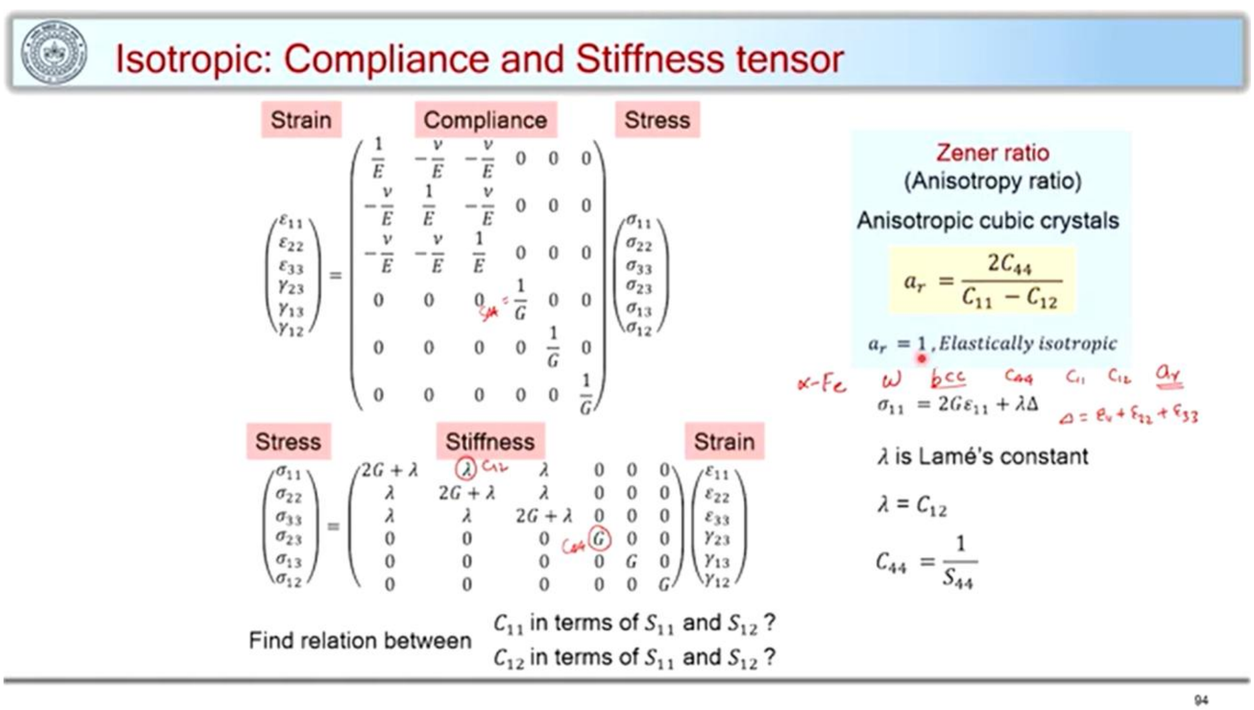


Toh yahan par aap dekhenge ki jo lambda hai ye C_{12} hai mera yeh agar dekhenge yahan par lambda ki value C_{12} hai aur C_{44} jo hai yahan pe C_{44} hum dekhenge uh iski value hai ye G hai maan lete hain ye mera C_{44} hai ye mera C_{44} hai aur ye mera S_{44} hai. Toh agar aap dekhenge C_{44} aur S_{44} ka relation is tarah se milega $1/S_{44}$. Yahan pe aap dekh sakte hain S_{44} 1/G hai toh C_{44} G hai toh ye ek relation humein yahan se mil gaya aur lambda C_{12} hai jo humne Lame's constant define kiya tha toh C_{12} mera is tarah se main likh leta hoon ye mera C_{12} ho gaya. Abhi aapko ek relation nikaalna hai jo C_{11} ka relation nikaalna hai S_{11} ke ismein ye mera C_{11} hai S_{11} ke ismein aur S_{22} ke hisaab se aapko nikaalna hai.

Aur C_{12} ka relation nikaalna hai S_{11} aur S_{22} keis se toh aapko S_{11} ki zaroorat hogi S_{12} ki zaroorat hogi C_{11} ki zaroorat hogi aapko yeh relation pata hai aapko lambda pata hai aur isse aap yeh cheezein aap nikaal sakte hain. Abhi ek main parameter define karunga cubic material ke liye toh yeh parameter hai Zener ratio isko hum kehte hain Anisotropy Ratio. Ye sirf cubic materials ke liye hi define hota hai toh ye anisotropy cubic crystals ke liye. Toh ye parameter is tarah se define hota hai: $Ar = 2C_{44} / (C_{11} - C_{12})$. Toh yeh jo ratio hai yeh mujhe dega Zener ratio Ar. Yeh jo ratio bataayenge ki mera material kitna isotropic hai. Toh main ek exercise aapko dena chahta hoon.

Toh aap iska Iron aur Tungsten, alpha-iron yaane dono BCC hain inke stiffness tensors nikaaliye components nikaaliye jaise C_{44} find out kariye C_{11} find out karein aur C_{12} find out karein aur uske baad ye anisotropy ratio nikaalne ki koshish kariye. Toh yeh jo anisotropic ratio hai yeh 1 ke around hai toh utna hi mera material elastically isotropic hai. Toh aap dekhenge ki Tungsten ke liye yeh ratio exact 1 aata hai aur Iron ke liye yeh 1 se deviate hota hai. Toh aap dekhenge ki Tungsten mera elastically isotropic hai par Iron elastically isotropic nahi hai. Toh abhi hum Elastic Stored Energy ke baare mein dekhenge.

Toh humne abhi tak yahan par stress-strain ke relations dekhe thae yahan par ek concept humne dekhi thi jab humne stiffness tensor ko derive kiya tha ki woh symmetric hai tab humne elastic stored energy ka concept evoke kiya tha. Toh elastic stored energy kya hoti hai? Toh simply hum isko likh sakte hain ki work done in elastically deformed body. Elastic behavior kya hota hai material jaise maine stress apply kiya usmein koi strain develop hoga par jab maine stress remove kar diya toh strain chala jayega toh yeh ho jayega mera elastic behavior. Toh jo work done hota elastic deformation mein usko main elastic stored energy kahunga ya isko Potential Energy bhi keh sakta hoon main.



Kyunki jaise isko is tarah se samajhte kyunki ek material hai isko maine force lagaya toh force lagaya toh usmein kuch elongation hoga agar ye force remove kar diya toh yeh elongation waapas ho jayega. Toh yeh jo waapas jaane ki prakriya hai yeh kyunki yahan par ek stored energy hai yeh isko push back karegi toh isko hi hum elastic stored energy kehte hain. Isko thoda sa aur samajhte hain ki jaise maine load remove kiya yahi maine yahan par explain kiya toh yeh potential energy recover ho jayegi. Toh hum ek linear elasticity ke liye assume karte hain kyunki yahan par hum linear elasticity dekh rahe thae toh ek member consider karte jo linearly elastically deform hota hai uska area maan lete hain A aur length maan lete hain L.

Aur isko force apply kar raha hoon X direction par yeh mera X direction hai let's say mera ek force apply kar raha hoon F_x. Toh is force ke under iska length change hoga aur length change hoga isko main maan leta hoon small length change jo ho raha hai woh dx hai. Ab maine force apply kiya bada wala force F aur zyada toh iska total length change hoga x. Toh main ek elastic stored energy is tarah se samajh sakta hoon jaise main ek force apply kar raha hoon yahan par aur mujhe yahan par elongation milega X direction pe. Toh yeh linear elastically deform ho raha hai toh yaani agar mujhe yahan par deformation chahiye is tarah se mark kar lete hain.

Agar mujhe X direction pe deformation chahiye elongation chahiye toh mujhe us tarah se yeh force jo apply karna hai woh linearly apply karna hai. Toh main agar mujhe x deformation chahiye toh mujhe F jo hai force apply karna padega toh main usko Work Done is tarah se likh sakta hoon: Force into displacement yaani force required and this is a change in length dx. Is tarah se main ek small work done likh sakta hoon. Agar mujhe total work done nikaalna hai poora from zero se x length tak deform karne ke liye toh main is tarah se likh sakta hoon is part ko main integrate kar sakta hoon shunya (0) se x tak. Toh mere paas ye total work done milega.

Toh ye total work done kya hoga yeh total work done nothing but mera potential energy rahega ya total work done rahega W. Toh yeh mera U rahega ya potential energy yahan par main baat kar raha hoon toh U ko main is tarah se samajh sakta hoon yeh U rahega area under this curve. Toh isko main likh sakta hoon 1/2 into force into displacement, $1/2 \cdot F \cdot x$. Area is triangle ka jo aayega yeh rahega $1/2 \cdot F \cdot x$. Yeh area ki main baat kar raha hoon. Isko main likh sakta force ko main is tarah se likh sakta hoon $\sigma \cdot A$ ye simplistic way se main is tarah se likh sakta hoon. Aur yeh jo dx hai toh humne dekha tha ki strain ki definition hum is tarah se likh sakte hain.

Strain ki agar main definition le raha hoon toh small change in length upon original length toh yeh aa jayega mera yahan par small change yahan par dx era original length agar main L pakad raha hoon toh ye dx ko main is tarah se likh sakta hoon $\varepsilon \cdot L$. Similarly stress force ko main stress into area ya stress ko humne define kiya tha Force / Area toh main force yahan par replace kar raha hoon. Toh yeh jo main consider kar raha hoon yeh bahut hi small displacement aur linear elastic material ke liye toh ye jo formulation hai aapko yaad rakhna hai ye small displacement aur linear elastic material ke liye hi hai. Toh maine jab is tarah se likha toh main kuch rearrangement is tarah se kar sakta hoon.

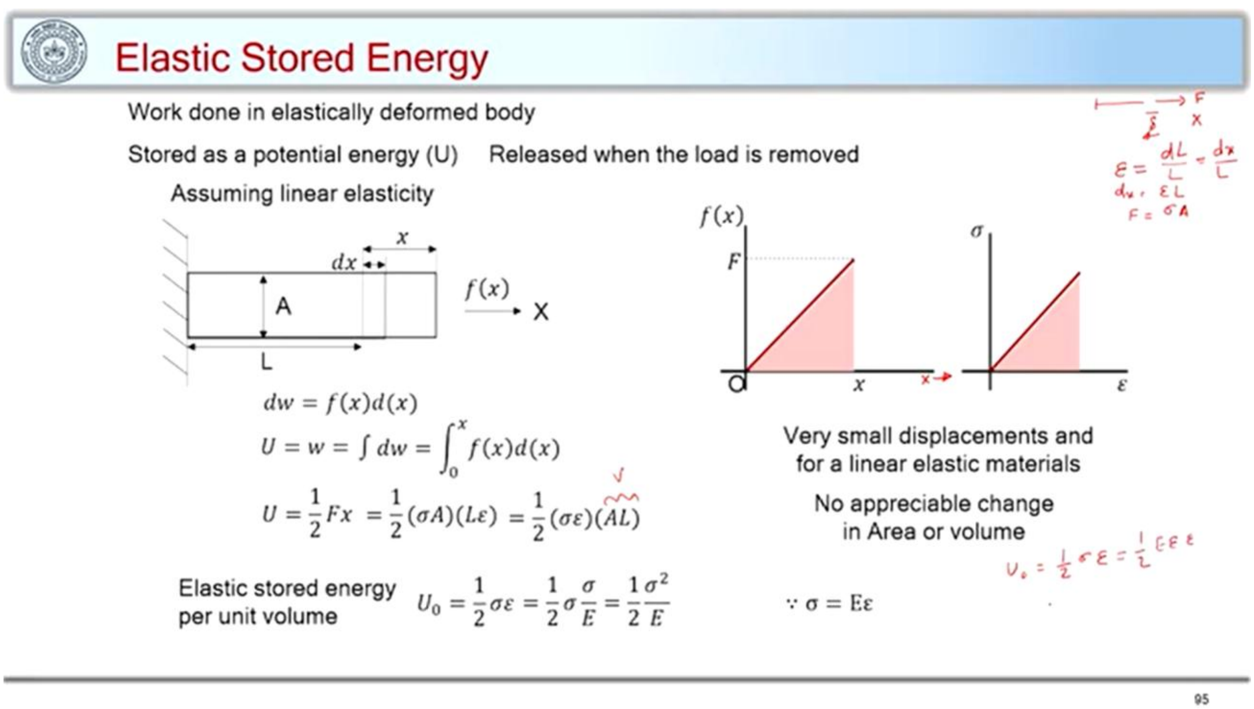
Aur yahan par ek cheez yahan par appreciable area change ho raha hai aur volume change ho raha hai woh nahi ho raha hai yaani large volume change nahi hai small volume change hi hai. Toh yeh main apply karta hoon ye main thode terms rearrange kar raha hoon toh main epsilon ko is tarah se laa raha hoon stress ke paas aur area ko length ke paas laa raha hoon. Toh agar aap yeh term dekhenge $\sigma \cdot \varepsilon$ ye ek term hogi jo maine deliberately yahan par dono ko combine kiya. Ye jo term hai yeh $A \cdot L$ yeh kya hoga yeh mere paas area hai aur length hai toh yeh ho jayega Volume of this member. Toh yeh volume hai aur yeh volume main is tarah laata hoon is left hand side ke taraf laata hoon.

Toh main usko keh sakta hoon Elastic Stored Energy per unit volume. Toh elastic stored energy per unit volume ko main is tarah se bhi likh sakta hoon sigma aur epsilon ke against plot karunga jo area yahan par aayega mera stress-strain ka ye ho jayega mera elastic stored energy per unit volume aur isko main mark karta hoon U_0 aur isko likh sakta hoon $1/2 \cdot \sigma \cdot \varepsilon$. Ye term mere paas rahegi kyunki maine yeh jo L hai isko mark kar lete hain ye jo L term hai yeh meri volume rahegi is volume ko main left hand side le aaunga toh mere paas elastic stored energy per unit volume mil jayega.

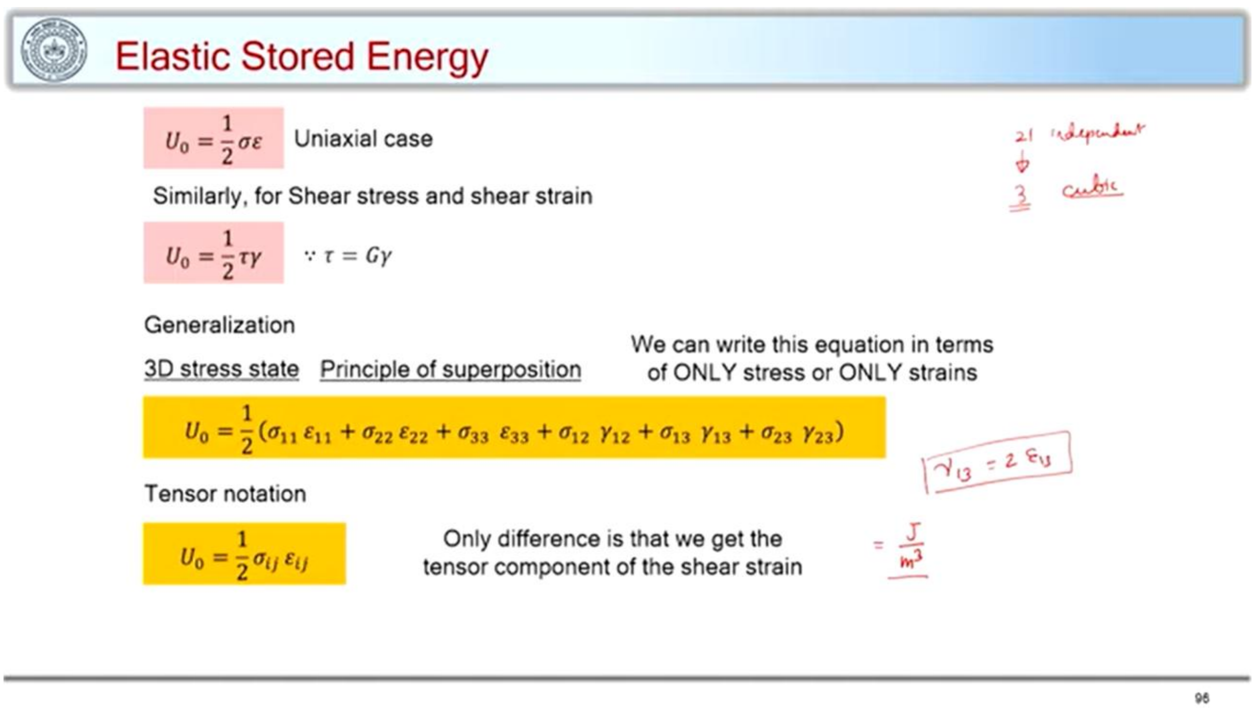
Toh mere paas $1/2 \cdot \sigma \cdot \varepsilon$ hai. Agar main epsilon ko stress-strain relations ke dwaara likhunga jaise yeh uniaxial tension mein humne deform kiya toh main epsilon ko σ/E likh sakta hoon toh yeh mere paas ek relation aa jayega $\frac{1}{2} \sigma^2/E$. Yeh yeh identity maine yahan par istemaal ki thi toh main elastic stored energy ko sirf stress ke dwaara represent kar sakta hoon ya strains ke dwaara represent kar sakta hoon. Agar main isko is tarah se bhi likh sakta hoon U_0 agar mere paas $\frac{1}{2} \sigma \varepsilon$ hai toh main sigma ko replace karunga $\varepsilon \cdot E$. Toh mere paas U_0 half $E \varepsilon^2$ bhi aayega.

Toh main koi bhi elastic stored energy per unit volume ko strains ya stress ke dwaara bhi represent kar sakta hoon. Yeh jo humne kiya yeh yahi pe maine dikhaya yahan par. Abhi hum dekhte hain elastic stored energy ka mahatva kya hai yaani agar humne uniaxial keis consider kiya toh mere paas U_0 aaya ½ σ ε. Aur jab hum shear stresses ki baat karenge toh hum shear strains ko develop karenge shear stresses ke dwaara. Toh U_0 jo aayega sirf shear stresses agar main apply kar raha hoon toh isko main ½ τ γ. Yahan par yeh jo shear stress hai aur yeh shear strain hai aur tau ka relation kya hoga shear strains ke saath?

Toh ye shear stress hai yeh shear strain hai ye mera shear modulus hai toh main shear stress ko shear modulus ke saath correlate kar raha hoon. Toh jab main generalized way se likhunga jab mere paas toh normal stresses hain aur shear stresses bhi hain toh main generalized way se jab likhunga aur 3D stress state ke liye likhunga toh main Principle of Superposition istemaal karunga yaani shear stresses shear strains ko develop karenge aur jo normal stresses normal strains ko develop karenge. Toh mere paas jo total energy aayegi energy per unit volume toh main isko is tarah se likh sakta hoon: addition likh sakta hoon saare stresses ka unke corresponding strains ke saath.



Agar normal stresses hain toh mere paas normal strains ka product main unka consider karunga ye shear stresses hain toh main shear strains ka product consider karunga. Toh tensorial notation mein main is tarah se likh paunga ya Einstein notation mein is tarah se likh paunga $U_0 = \frac{1}{2} \sigma_{ij} \varepsilon_{ij}$. Aur yeh jab main likhunga toh mujhe pata hai humne abhi dekha tha ki main in terms ko sirf stresses aur sirf strains ke saath likh paunga agar mujhe elastic constants pata hain yaani E pata hai aur G pata hai toh main unko is tarah se likh paunga. Toh ye jab likhenge tab difference only kya hai sirf difference hai ki jo tensor component hai of shear strain woh mujhe consider karna hai.



Yaani jab main γ_{ij} hai jaise γ_{13} aur ε_{13} hai ye inka relation humne dekha tha is tarah se. Toh aapko yeh hamesha yaad rakhna hai jab main tensorial notations mein elastic energy bhi nikaal raha hoon toh. Toh yeh ho gaya mera elastic stored energy per unit volume, iska unit hona chahiye Joules per meter cube (J/m^3). Toh humne abhi tak dekha ki stress-strain relations kya hain humne aaj ke part mein dekha ki jo humare paas usko bhi likh lete hain humare paas 21 independent constants thae usmein se teen constants mujhe chahiye cubic system ko define karne ke liye stress-strain relation define karne ke liye.

Toh yeh aaj ke part mein humne dekha toh yaane ye jo constants hain aapki symmetry par depend karte hain aur aaj ke part mein humne yeh bhi dekha ki elastic stored energy ka mahatva kya hai aur usko hum kis tarah se mathematically likh sakte hain. Toh abhi ke liye hum yahan rukte hain. Next part mein hum dekhenge ki Yielding Criteria ya ye humne abhi tak toh elastic deformation dekha tha agle part se hum chaalu karenge plastic deformation kaise hota hai aur uska criteria kya hai. Dhanyavaad