Mechanical behavior of materials

Dr. Niraj Mohan Chawake

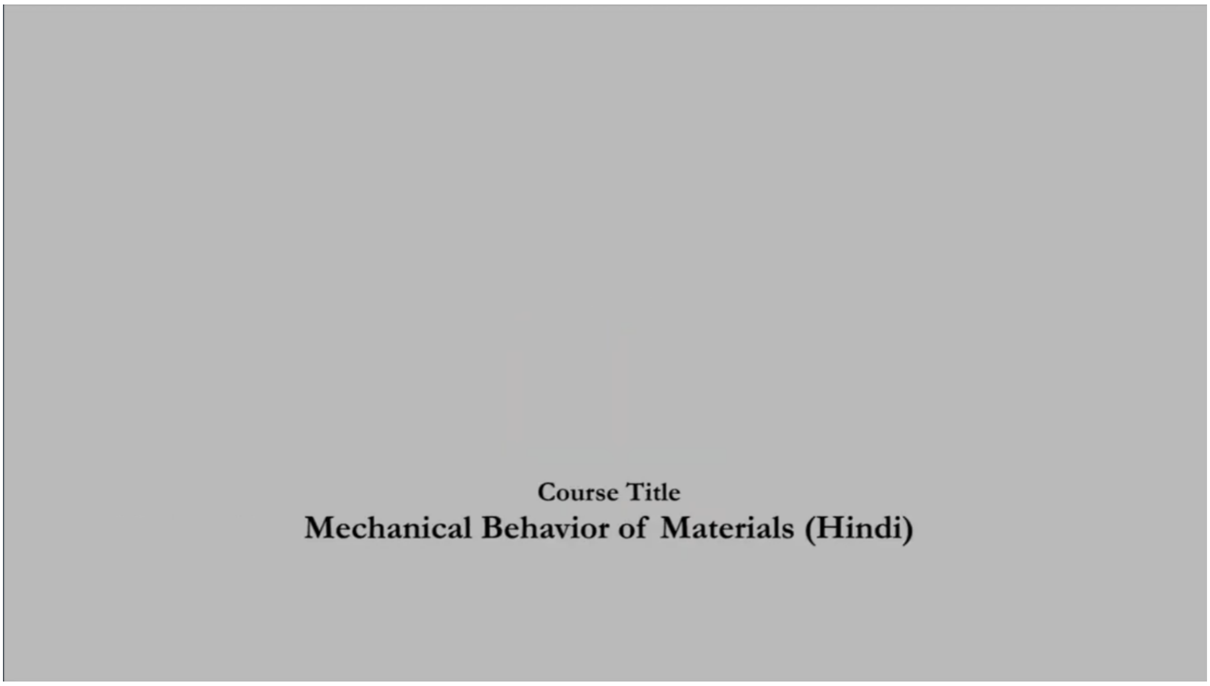
Department of Materials Science and Engineering

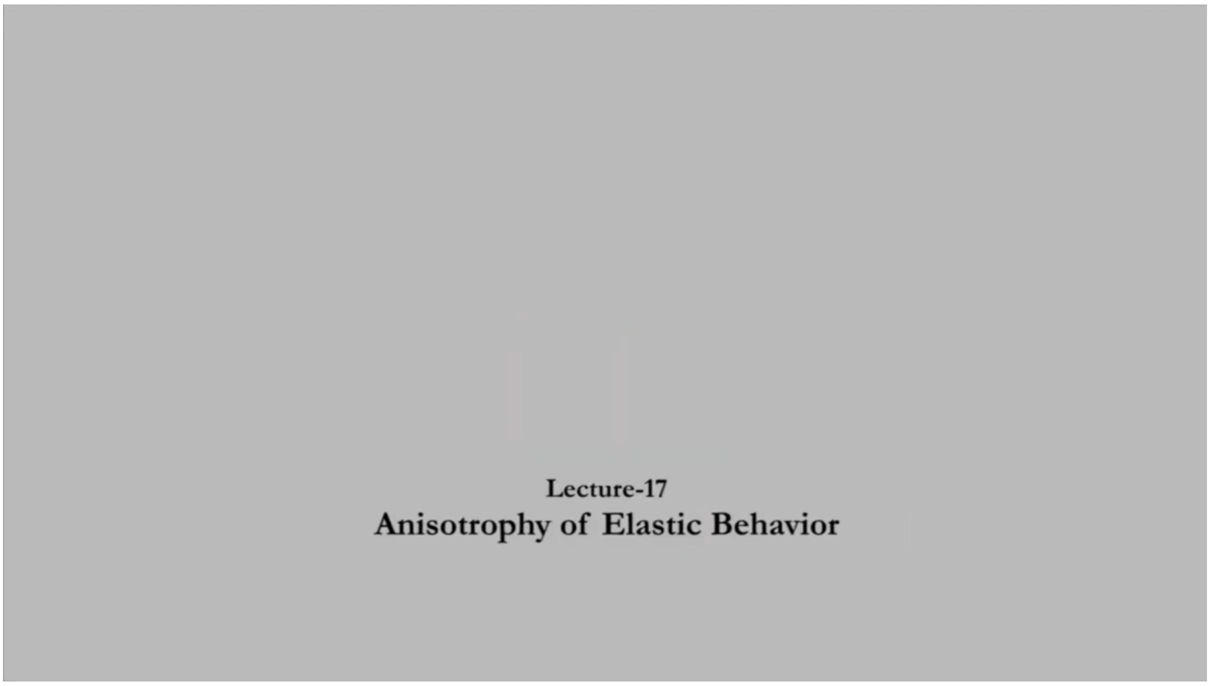
Indian Institute of Technology, Kanpur

Week-3

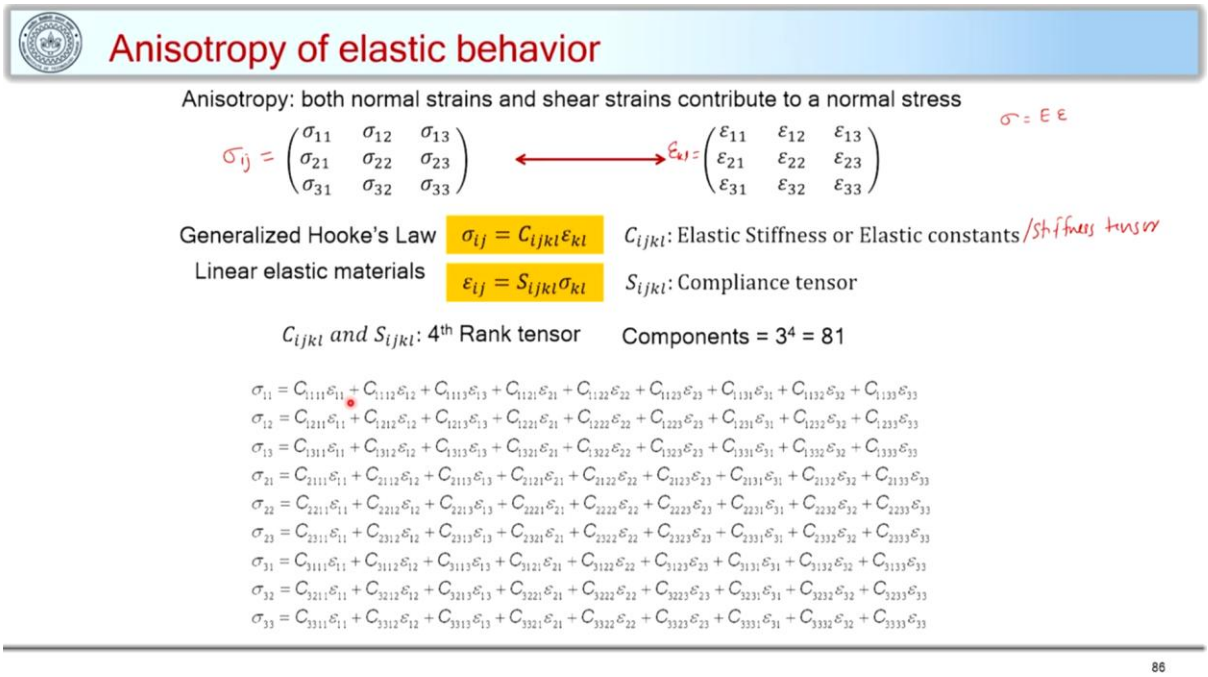
Lecture-17

Anisotropy of Elastic Behavior

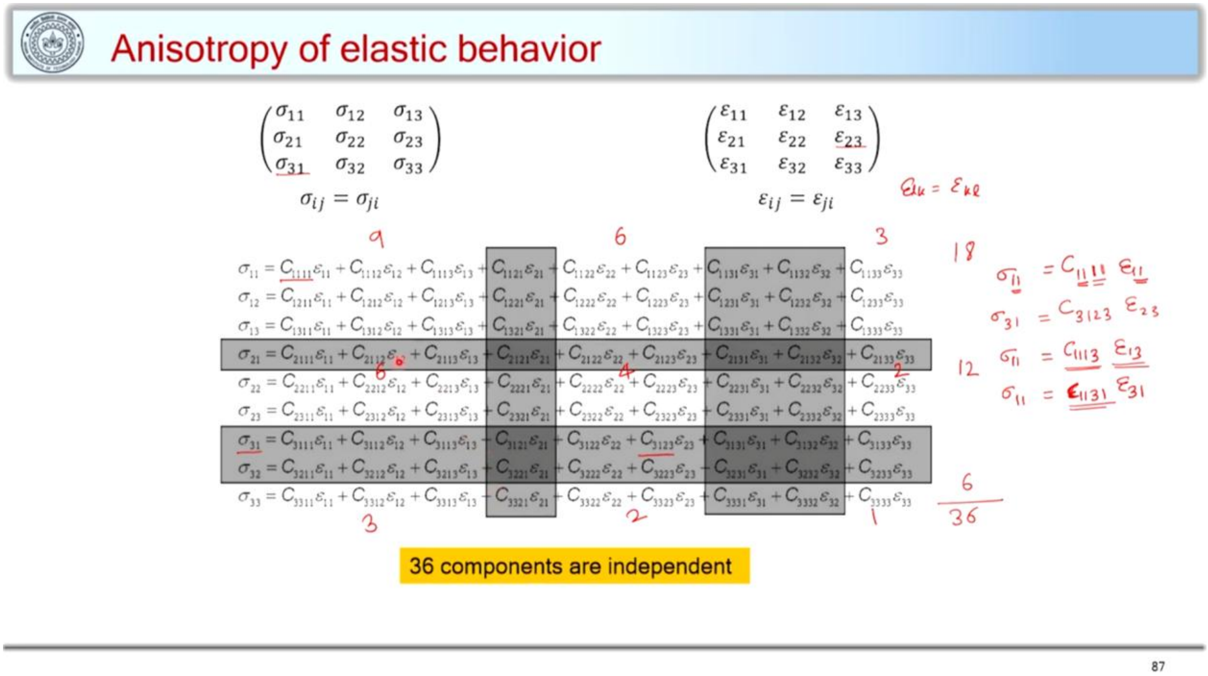




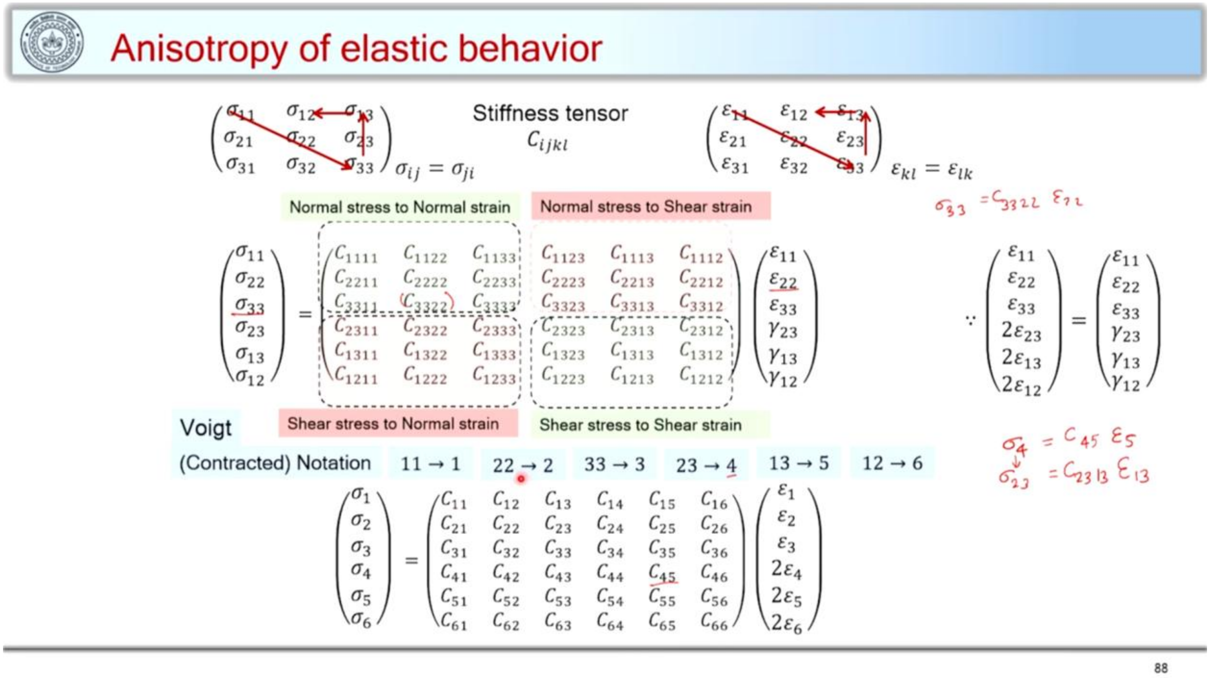
Namaskar aap phir se swagat karta hoon aapka last part mein humne dekha tha ki isotropic material ke liye stress strain relations kya hai is part mein hum thoda generalized view dekhenge anisotropy of elastic behaviour to hum jab anisotropy of elastic behaviour maan ke chalte hain to hum kehte hain ki jo stress strains hai wo orientation ya ya directions par depend karenge jab anisotropic hum baat karenge tab hum dekhenge ki normal strains aur jo shear strains hai yeh normal stress ko kis tarah se contribute karenge humne last time dekha tha ki principle of superposition ke baare mein dekha tha jo normal stresses. Hote hain normal strains ko hi develop karenge aur jo shear stresses hai wo shear strains ko hi develop karenge par ismein hum dekhenge ki dono kis tarah se ek doosre se dependent hai to maan ke chalte hain humare paas yeh general state of stress hai humne dekha tha ki yahan par nine components hai aur inmein se 6 components independent hai aur yeh jo stress tensor hai hum isko ya hum isko correlate karenge humare strain tensor se to strain tensor mein bhi humare paas nine components hai aur ismein bhi jo chhe hai wo independent hai. Isko hum jab correlate karne ki koshish karte hain hum usko correlate karte hain ek generalized Hooke’s law se kyunki stress strain ka relation jo hota hai hum usko Hooke’s law se define karte hain to yeh ho gaya humara generalized Hooke’s law yeh hum baat karenge abhi bhi linear elastic material ke liye linear elastic material yaani jo humara stress hai woh proportional hai strain se par woh linear change hoga to yeh jo stress strain relation hai agar main σ_{ij} likhoonga isko is tarah se maan ke chalte hain yeh mera σ_{ij} hai aur yeh ε_{kl} hai isko main correlate karoonga. To yeh jo constant hai yahan par yeh ho jaayega Cijkl to yeh jo constant hai isko main kehta hoon elastic stiffness aur elastic constant agar aapse agar koi poochhe ki Hooke's law kya hai to hume abhi isko likhna hai na ki yeh relation likhna ($\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$) yeh generalized Hooke's law hai yeh ek special condition hai Hooke's law to isko hum elastic constant elastic stiffness kehte hain jo C jo tensor hai ya hum isko is tarah se bhi likh sakte hain agar hum strains ko likh rahe hain to hum isko is constant se likhenge Sijkl aur yeh jo Sijkl hai isko compliance tensor bhi kehte hain. To jo Cijkl isko hum stiffness tensor bhi keh sakte hain ab dekhenge ki jo C aur S hai yeh jo C hai yeh stiffness dikha raha hai or S hai compliance dikha rha hai to yeh is tarah se hum ulta likhte hain isko S agar S hai to hum isko compliance tensor kehte hain aur C hai to isko stiffness tensor kehte hain to yeh sirf yaad rakhne ke liye par yeh jo humara hai yeh generalized Hooke's law hai abhi hum isko dekhenge ki yeh jo components hai yeh iske Cijkl ke kitne components hai yeh humara fourth rank tensor hai. Kyunki humara second rank tensor hai aur yeh second rank tensor hai second rank tensor ko doosre second rank tensor se jab hum correlate karne ki koshish karte hain hum usko fourth rank tensor se correlate karte hain to humare paas yeh jo do tensors hai yeh fourth rank tensor hai to iske components kitne honge humne dekha tha ki yeh 3 D ke liye to three to the power four yaani yeh jo rank hai three to the power four to humare paas 81 components aane chahiye jab hum stress strain ko correlate karte hain. To dekhte hain ki isko hum kaise correlate kar sakte hain to yeh dekhiye humne σ_{11} ko saare jo strain components hai strain tensor ke components hai inke saath correlate kiya hai to humare paas yeh saare constants aa gaye yahan par ab dekhenge to mere paas 9 stresses hai aur 9 strain components hai to mere paas 81 jo stiffness hai ya compliance tensor ke jo components honge wo eighty one rahenge to aap dekh sakte hain yahan par mere paas yeh nine components yahan par normal stress jo hai σ_{11} ko correlate kar rahe hain yeh saare strain components hai. Usi tarah se humare paas yeh 9×9 yaani 81 components hone chahiye aaye dekhte hain ki isko aur detail mein yeh jo saare components hai iske baare mein jaante hain



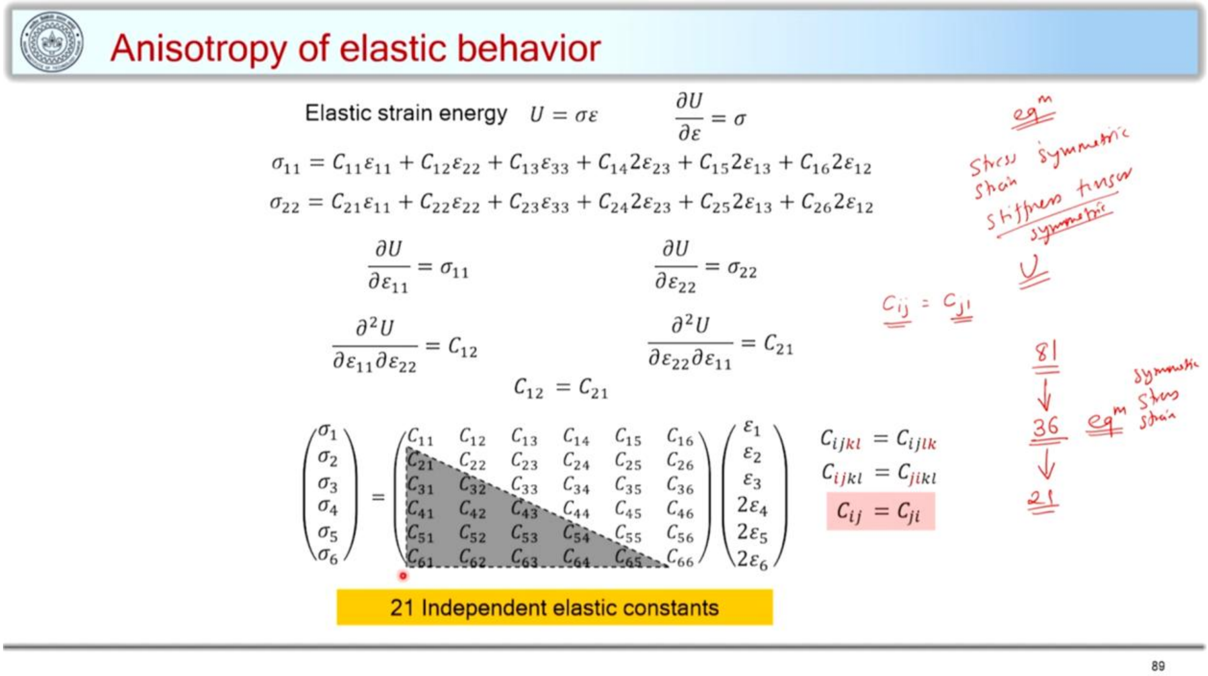
to mere paas stress tensor hai isko main correlate kar raha hoon strain tensor se to mere paas hai nine components to humne dekha tha ki yeh jo components hai jaise main σ11 ko correlate kar raha hoon ε11 se to mere paas yeh jo yeh jo one one hai theek hai aur yeh one one hai to main is tarah se likhoonga one one jo stress ka part hai stress component isko correlate karoonga main strain component se ε11. To mere paas yeh pehla constant ho gaya usi tarah se agar main maan ke chaliye mujhe σ31 ko correlate karna hai ε23 se to σ31 kahan par hai yahan par mera to main σ31 ko is tarah se correlate karoonga ε23 se to mere paas jo constant aayega yeh aayega C3123 to yeh 31 23 yeh component aayega usi tarah main saare stresses ko strains se correlate kar sakta hoon abhi mere paas pata hai ki yeh stress components hai aur strain components hai yeh symmetric hai. Yaani σij = σji aur εij = εji ya is tarah se bhi likh sakta hoon yahan par j likha hai to εlk = εkl to agar main isko correlate karna chahta hoon to aap dekhenge agar mere paas mujhe ε12 pata hai to main saare jo relations hai ε21 ka stress ke saath yeh mark kar sakta hoon agar mere paas ε13 pata hai to main ε31 ka jo relation hoga wo bhi same hona chahiye aur ε23 agar mujhe pata hai to ε32 ka relation mujhe pata hona chahiye. Isko is tarah se samajhte hain jaise agar mere paas yeh relation hai σ11 ka is tarah se ε13 se to yeh ho jaayega C1113 par hume pata hai ki ε13 aur ε31 same hai to agar main is tarah se correlate karoonga to yeh jo yeh jo do constants hai yeh jo C hai yeh same hone chahiye to agar yeh same hai kyunki yeh dono same hai aur yeh same hai to yeh dono bhi same hone chahiye to agar mujhe yeh pata hai to yeh jo yeh jo components hai wo independent nahi hai wo dependent ho jaayenge. Is tarah se agar mujhe σ12 ka relation pata hai in saare strain components ke dwara to main σ21 ka nikal sakta hoon similarly mujhe σ13 pata hai to main σ31 ka nikal sakta hoon aur agar mujhe σ_{23} ka pata hai to main σ_{32} ka nikal sakta hoon to mere paas independent jo components hai wo kitne components hai yahan pe agar count kar lete hain hum yahan pe mere paas 9 components hai yahan pe 6 components hai aur yahan pe 3 components hai yahan pe 6 hai yahan pe 4 hai yahan pe do hai aur yahan pe teen hai yahan pe do hai aur yahan pe ek hai. To yeh ho gaye mere 18 components yahan pe 9 aur 6 aur teen 18 6 4 plus 2 to yeh ho jaayenge 12, 3,2 aur 1 yeh ho jaayenge chhe to mere paas in total 36 independent components hi hai to humne dekha ki mere paas 81 components the par usmein se independent kitne the usmein se independent 36 components the. To abhi hum dekhte hain in general ki in 36 components mein se hum inko kaise represent kar sakte hain



To mere paas stress state hai aur yeh strain state hai aur isko humne likha tha ki $\sigma_{ij} = \sigma_{ji}$ aur $\varepsilon_{kl} = \varepsilon_{lk}$ abhi hume pata hai ki in nau components mein se humare paas chhe components hi independent hai aur humare paas in nau components mein bhi chhe components independent hai. To humare paas jo stiffness tensor aayega usmein bhi 36 components hi independent hone chahiye 6×6 to jo stiffness tensor hai C_{ijkl} isko aur aasani se hum likh sakte hain to dekhte hain ki usko kaise likh sakte hain to hum abhi jo correlation likhenge C_{ijkl} ka wo is tarah se likhenge hum is tarah se move honge σ_{11} σ_{22} σ_{33} σ_{23} σ_{13} aur σ_{12}. Aaiye hum usko is tarah se likh lete hain ab main is tarah se move hua tha diagonal phir upar phir aise to σ_{11} σ_{22} σ_{33} σ_{23} σ_{13} aur σ_{12} to yeh pehle jo teen components hai stress ke yeh normal stresses hai aur yeh jo teen components hai yeh shear stresses hai aur aap dekhenge ki yeh jo chhe components maine likhe hai yeh chhe hi sirf independent hai baaki ke teen shear stresses to dependent components hai. Usi tarah se main strain components mein move karoonga ε_{11} ε_{22} ε_{33} phir ε_{23} ε_{13} aur ε_{12} aur is tarah se main likh loonga yahan pe maine shear strains likha hai kyunki kuch books mein yeh bhi relation hum likhte hain to par aapko yeh relation hamesha yaad rakhna hai ki main shear strains ko tensorial quantity mein is tarah se convert karta hoon to confuse nahi hona hai bas aapko yeh interchangeably dekhna hai ki agar γ hai to main usko tensorial quantity mein is relation se convert kar sakta hoon. To mere paas yeh 36 stiffness tensor ke jo components hai yeh mujhe mil jaayenge jaise aap dekhenge ki normal stresses hai aur shear stresses hai yeh normal strains hai aur yeh shear strains hai to aap dekhenge ki yeh pehle nau components yahan par yeh stiffness tensor ke yeh represent karenge normal stress to normal strain ka relation to dekh lete hain ek example dekh lete hain jaise main σ_{33} ko normal strain yahan par lets say ε_{22} ke saath relate kar raha hoon. To σ_{33} ko main ε_{22} ke saath relate kar raha hoon to mere paas stiffness jo component aayega wo C_{3322} aayega to yeh jo component hai C_{3322} yeh relate karega mera σ_{33} to ε_{22} usi tarah se hum agar dekhenge to yeh jo components hai jo nau components hai yeh wale yeh aapke relate kar rahe shear stress to shear strain yeh jo shear stresses hai inko is shear strains ke dwara jo relation hai yeh is component ke dwara milega. Aur yeh jo do hai yeh milega mujhe normal stress to shear strain aur yeh jo component hai mujhe relation denge shear stress to normal strain abhi hum dekhenge ki yeh representation thoda hum aur aasaan kar sakte hain to hum is tarah se kuch kar sakte hain jaise Voigt (contracted) notation ke dwara hum likh sakte hain agar main 11 ko 1 likhta hoon 22 ko 2 likhta hoon 33 ko 3 likhta hoon 23 ko 4 likhta hoon 13 ko 5 likhta hoon aur 12 ko 6 likhta hoon. To main yeh jo stress components hai inko is tarah se likh paoonga σ_1 σ_2 σ_3 σ_4 σ_5 σ_6 is tarah se usi tarah se main jo strain components hai is tarah se likh paoonga ε_1 ε_2 ε_3 ε_4 ε_5 ε_6 aur yeh jo stiffness hai isko main kuch is tarah se correlate kar paoonga to aap dekhenge ki yeh jo chaar notation wala tha isko humne do notations mein convert kar liya to yeh aasani ho jaayegi humare analysis mein. To jaante hain lets say main σ_4 ko ε_5 se relate kar raha hoon to likhte hain agar main is tarah se likhoonga σ_4 ko ε_5 se correlate kar raha hoon to yeh jo mera stiffness aayega yeh C_{45} aayega C_{45} yeh aayega to σ_4 yaani kya hai yahan pe agar main four ko consider kar raha hoon to four ko maine represent kiya tha to yeh aa jaayega σ_{23} back agar main calculate kar raha hoon aur ε_5 jo aayega wo aa jaayega ε_{13} aur yeh jo constant aayega yeh aayega $C_{23\,13}$. To agar aap dekhenge yeh jo C_{2313} hai yeh mera nothing but C_{45} hai isi tarah se aap saare components is Voigt notation se nikaal sakte hain



to abhi dekhte hain ki ek ek strain energy ko main define karna chahta hoon isko hum aage aur dekhenge par isko main yahan par is tarah se define karta hoon product of stress into strain to isko main kahoonga elastic strain energy. Aapne padha bhi hoga koi bhi strain energy agar hum nikaal rahe hain jaise humne toughness nikaala tha ya stiffness nikala tha to hum isko is tarah se, resilience nikala to is tarah se defend kiya tha, product of stress into strain. to yeh main ek elastic strain energy term introduce karta hoon agar main iska partial differncial leta hu to mujhko is tarah se kuchh relation milega, ($\partial U / \partial \varepsilon = \sigma$) agar main iska differentiation leta hu to ye term nahi chahiye simple mathematics hai agar main Voigt notation agar use karta hoon jaise $\sigma 11$ ke liye likh raha hoon aur $\sigma 22$ ke liye likh raha hoon. Saare saare six strain components jo ki independent strain components hai agar main kuch is tarah se likhta hoon ($\partial U / \partial \varepsilon 11$) ko likhoonga to mujhe $\sigma 11$ milna chahiye agar main isko phir se differentiate karta hoon differentiate kiss se krunga, main $\varepsilon 22$ se differentiate kr rha hu yaha pe agar aap dekhenge ye jo $\sigma 11$ ye term hai jb main isko differentiate kruga is pure term ko kis se $\varepsilon 22$ se agar main baaki ke term ko constant le kr chl rha hu to mere paas answer aayega C12 usi tarah se agar main isko yeh elastic energy hai isko main pehle $\varepsilon 22$ se differentiate krunga to mere pas aajaega $\sigma 22$ definition ke hisab se agar main ise phir se differentiate krunga derivative partial derivative loonga to $\varepsilon 11$ ke respect se to mujhe mere paas C21 milna chahiye. Jaise yeh term hai isko maine differentiate kiya mere pass $\sigma 22$ is constant ke dwara likha hai maine hai elastic strain ka relation. To agar main differentiate krunga to main sirf C21 ko yani ye term hai , yhi differentiate ho paegi baaki to constant hogi, inka differentiation zero aaega to mere pass answer aa jaega C21. Agar aap dekhenge differentiation humne kiya to ye jo term hai aur ye jo term hai vo mathematically equal hai agar yeh mathematical equal hai to main yeh is tarah se likh sakta hoon $C_{12} = C_{21}$ to hume ek relation aur ek milega ki yeh jo stress strain relations humne nikaale the yeh 36 constants the humare paas yaani humare paas six independent stresses the aur 6 independent strain components the unko relate kiya tha 36 stiffness component ke dwara. To agar $C_{12} = C_{21}$ hai to isi tarah main is tarah se bhi dikha sakta hoon baaki ke components ke liye jaise C_{ij} hai to main isko C_{ji} dikha sakta hoon ya yeh kar sakta hoon main yeh elastic strain energy ke dwara to agar aap dekhenge to main kuch is tarah se likh paoonga $C_{ijkl} = C_{ijlk}$ aur ya $C_{ijkl} = C_{jikl}$ ya simply is tarah se likh paoonga $C_{ij} = C_{ji}$. To agar main is tarah se likh paoonga to aap dekhenge ki yeh jo shaded components hai wo independent nahi rahenge yaani agar mujhe C_{12} pata hai to main C_{21} nikaal sakta hoon kyunki yeh same same hi ho jaayenge to yeh jo stiffness tensor hai yeh bhi symmetric ho jaayega aur symmetric ho jaayega aur ismein se kitne components independent rahenge to independent humare paas 21 independent elastic constants hume chahiye. Agar humare paas 21 elastic constants agar mujhe pata hai to main saare stress strain ka relation nikaal sakta hoon to aap dekhenge count karenge to yeh 6 diagonal components ho gaye aur yeh 15 off diagonal components ho gaye to humne dekha tha ki stress tensor symmetric hai stress tensor aur strain tensor symmetric hai yeh humari equilibrium conditions thi. Par humara jo yeh stiffness tensor hai yeh isliye symmetric nahi hai kyunki equilibrium condition satisfy honi chahiye yeh isliye symmetric hai kyunki elastic strain energy jo hum dekhenge woh constant hoti hai agar main uska derivative le raha hoon ε_{11} aur ε_{22} ke dwara agar main same strain component use kar raha hoon to woh jo elastic strain energy milegi woh same honi chahiye aur isliye yeh jo stiffness tensor hai yeh symmetric ho jaata hai. To humare paas kitne the humare paas pehle humare paas 81 elastic constants chahiye the mujhe aur 81 se hum aaye 36 elastic constants ke liye yeh kyon aaye kyunki humare paas equilibrium yaani humare paas equilibrium condition ke wajah se jo stress aur strain tensor tha woh symmetric tha is wajah se hum 36 elastic constants tak aaye. Aur 36 se hum 21 elastic constants par aaye to abhi main yeh kehna chahta hoon agar mere paas yeh 21 elastic constants hai aur mere paas ek linear elastic material hai to main jo saare stress ka relation hai aur strain ka relation hai main find out kar sakta hoon yahin par hum rukenge aur next part mein hum dekhenge ki elastic behaviour ka relation jo yeh relations hai yeh elastic energy ke saath kaise correlate kar paayenge. Aur hum aur dekhenge ki yeh jo constants hai 21 elastic constants hai yeh material ke par kaise dependent rehte hain aur isko hum aur kam kaise kar sakte hain



yeh bhi hum jaanenge next part mein dhanyavaad