Mechanical behavior of materials

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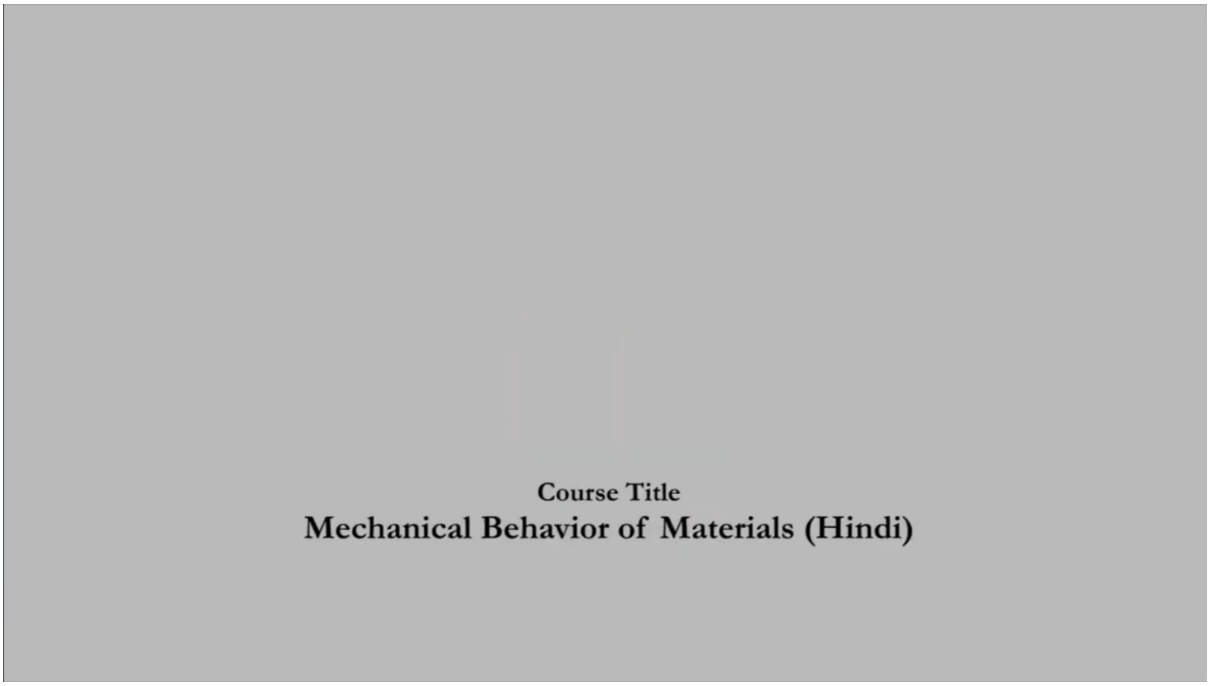
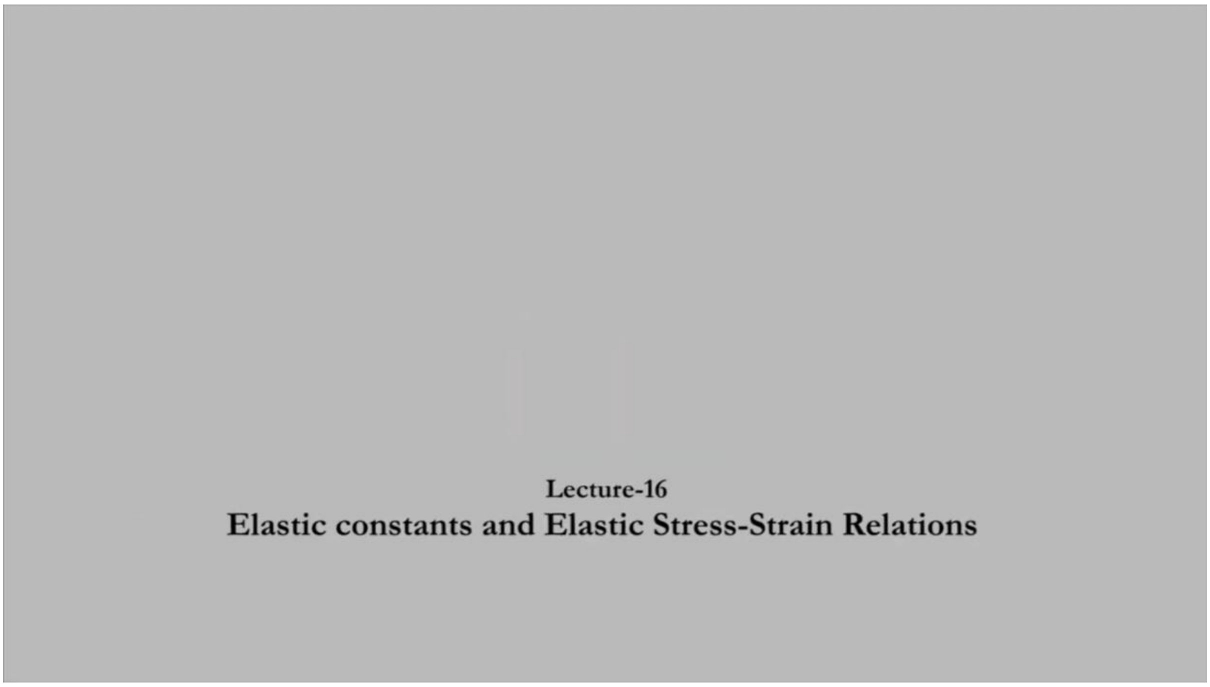
Week-2

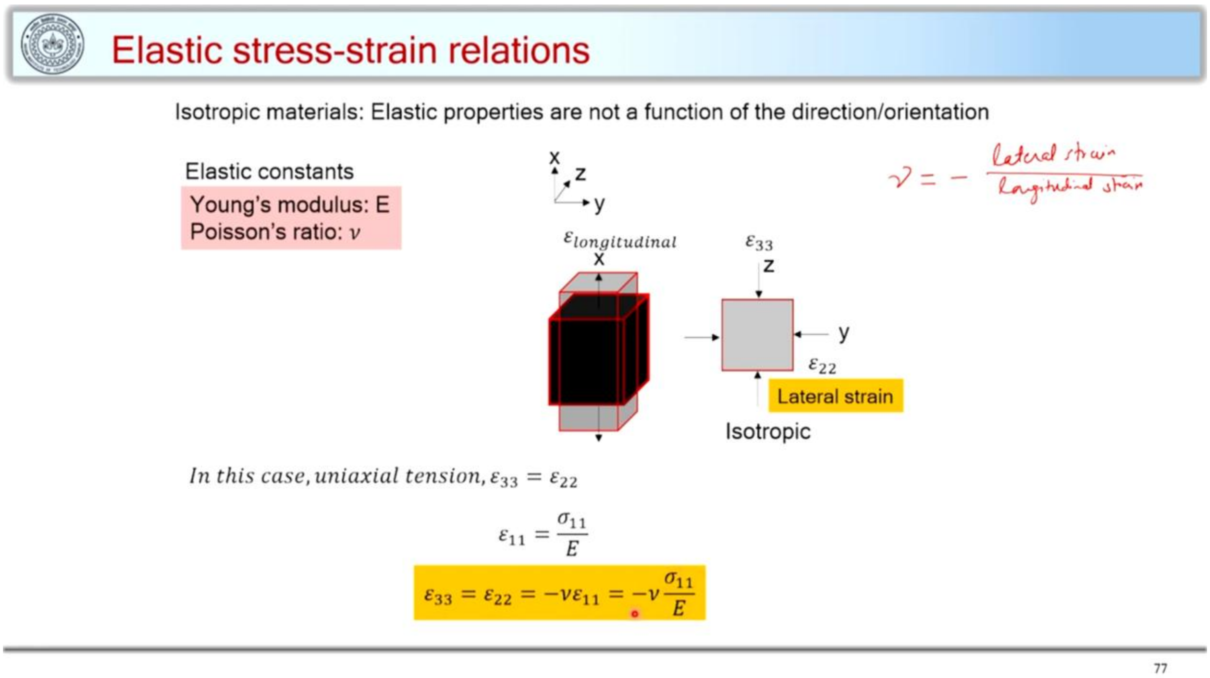
Lecture-16

Elastic Constants and Elastic Stress- Strain Relations



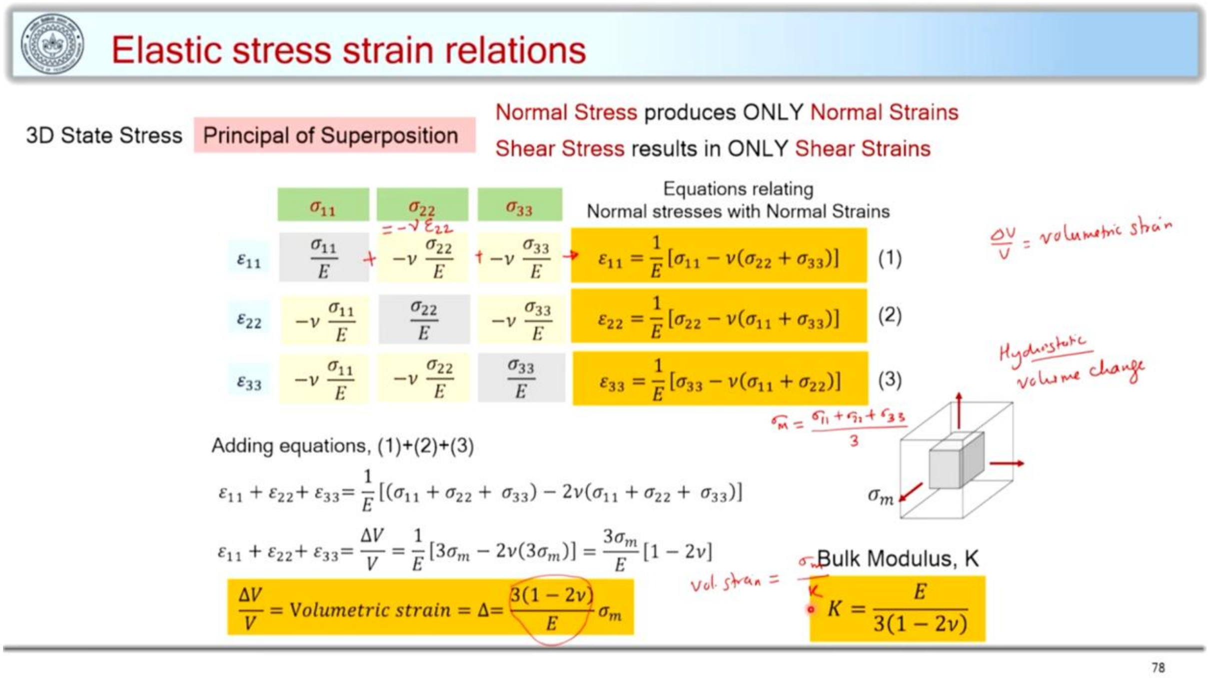


Aapka swagat karta hoon yeh jo course hai mechanical beha of material ka jo hum Hindi mein padhenge to humne last part mein dekha tha elastic jo stress hota hai stress tensor hota hai aur strain tensor hote hain inke components dekhe the kaun se kaun se components independent hai is part mein hum dekhenge ki kya elastic constant hai aur elastic stress starin jo relations hai inke baare mein padhenge to elastic constant woh hote hain jo elastic jo stress tensor hai aur strain tensor hai inka jo relation dikhate unko hum elastic constants kahenge aur hum is in constants ko jaanenge humare elastic stress relations ke dwara. To pehle to maante hain hum jo material hum study karenge woh isotropic material hai. Isotropic material yaani jo material jinki elastic properties kisi direction ya orientation pe depend nahi karti ya jo jinki elastic properties unke direction aur orientation ka function nahi hoti hai isi material ke baare mein hum dekhenge to maan lete hain ki humare paas ek member hai aur is member ko hum ek coordinate axis mein define kar lete hain yeh mera x direction hai y direction aur z direction hai is is yeh maan ke chalte aur hum force lagate x direction ke along jab hum yeh tensile force lagayenge x direction ke along to yeh material mein deformation aayegi ya material elongate hoga x direction ke along. Yeh humne dekha tha aur yeh jo abhi hum dikhayenge yeh cross section area hai is member ka yeh jo cross section area yeh humara zy plane hai to is zy plane mein hum dekhenge to pehle to hum dekhenge ki jab hum material ko deform kar rahe to ek elongation hoga to usko hum kehte hain longitudinal strain jo strain develop hoga woh longitudinal strain hoga aur agar humara material isotropic hai to yeh jo strains is zy plane mein develop honge woh constant rahenge ya same rahenge maan lete hain ki humare paas z direction mein strain develop hoga ε33 aur y direction mein develop hoga usko kahenge ε22. Yeh jo strains hai yeh humare saare normal strains hai kyunki hum yahan par ek normal stress apply kar rahe hain to yeh jo strains develop honge woh bhi normal strains hi develop honge woh longitudinal rahe ya humare lateral strain rahe to agar hum elastic constants agar dekhenge to humare paas kaun se elastic constants hai humare paas do elastic constants humne dekhe the ek Young’s modulus aur ek Poisson’s ratio to humne Poisson’s ratio ko is tarah se define kiya tha phir se hum likh lete uska definition Poisson’s ratio ko humne define kiya tha lateral strain ka ratio uske longitudinal strain se to yeh yeh humara definition hoga Poisson’s ratio ka to yeh definition abhi hum istemal karenge. Yahan pe to maan lete jab humare paas uniaxial tension hai aur yeh jo strains develop honge for isotropic material yeh same rahenge yaani ε33 aur ε22 yeh samaan hone chahiye to yeh condition hai to ε11 yaani jo longitudinal strain hai hum isko aise likh sakte hain ki jo stress hai 11 direction end x direction ke along σ11 / E yeh E Young’s modulus hai humara aur hum ε33 aur ε22 ko is ke dwara likh sakte hain to humare paas longitudinal strain kya hai longitudinal strain yahan pe humara ε11 hai aur lateral strains jo hai humare ε22 aur ε33 To ε33 aur ε22 ko hum is definition ke dwara aise likhenge − ν times ep.silon 11 aur phir ε11 ko hum replace karenge isko likhenge (σ11 / E) to humare paas aa jaayega ε33 = ε22 = − ν ×(σ11 / E).



yeh humare paas relation aa gaya abhi hum dekhte hain ki ek 3D stress state hai 3D stress state mein hum maanenge principle of superposition abhi isko hum thoda sa samajhte hain principle of superposition kya hota hai yaani humare jo normal stress hai woh humare normal strains ko hi develop kar sakte hain aur jo shear stresses hai woh shear strains ko hi develop kar sakte hain. Yaani normal stresses se humare paas normal strains milenge aur shear stresses se humko sirf shear strains hi milenge to yeh ho gaya humara principle of superposition agar yeh case hai to maan lete humare paas teen stresses hai agar humne uniaxial stress mein σ_{11} hi apply kiya tha abhi hum teen stress apply karenge teen normal stress apply karenge σ_{11} σ_{22} aur σ_{33} agar yeh teen normal stresses hai to yeh teen normal strains ko develop karenge humare definition ke hisaab se yaani principle of superposition ke hisaab se. To maan lete humare paas yeh teen strains develop honge ε_{11} ε_{22} aur ε_{33} humne abhi dekha ki agar main σ_{11} apply kar raha hoon to mere paas ε_{11} aayega to uska value jo rahega woh (σ_{11} / E) rahega agar main sirf σ_{22} apply karta hoon to mere paas ε_{22} ki value kya aani chahiye mere paas ε_{22} ki value aayegi (σ_{22} / E) agar main sirf σ_{33} apply karta hoon to mere paas ε_{33} ki value aayegi (σ_{33} / E) to yeh mere normal stresses se mere paas normal strains develop ho gaye. Par agar jab main σ_{11} apply kar raha hoon to mere paas lateral strains bhi develop honge to agar main sirf σ_{11} apply agar main σ_{22} apply karta hoon to mere paas lateral strain jo ε_{11} develop hoga woh is tarah se hoga $- \nu \times (\sigma_{22} / E)$ to isko samajh lete hain to yeh humare Poisson ratio Poisson ki definition se aise aayega yeh aaye ga yeh is tarah se aaye ga to main ε_{22} ko (σ_{22} / E) likh sakta hoon. To isi tarah se jab main σ_{33} apply karta hoon to lateral strain jo ε_{11} direction mein produce hoga woh is tarah se produce hoga jo $- \nu \times (\sigma_{33} / E)$ aaye ga similarly main agar σ_{11} apply kar raha hoon to mere paas ε_{22} jo lateral strain develop hona chahiye woh is tarah se aaye ga $- \nu (\sigma_{11} / E)$, $- \nu (\sigma_{33} / E)$ jab main σ_{33} apply kar raha hoon similarly agar main σ_{11} aur σ_{22} apply karunga to ε_{33} is tarah se develop hona chahiye. To agar hum dekhen agar hum teen ek saath apply karenge to humare paas stresses strains kya rahenge to strains is tarah se aaye ge agar main in teenon strains ko ε_{11} direction yaani 11 direction ke along add karunga summation karunga to main in teen terms ko add karunga to yeh strain mein agar add karunga to mujhe yeh ek identity milegi jo mujhe total strain milega ε_{11} agar main σ_{11} σ_{22} aur σ_{33} apply karunga to mere paas $\varepsilon_{11} = (1 / E) \times(\sigma_{11} - \nu \times (\sigma_{22} + \sigma_{33}))$ is tarah se aaye ga. Similarly agar main ε_{22} aur ε_{33} nikalne ki koshish karunga to mere paas yeh yeh identities aaye gi ε_{22} aur ε_{33}. abhi hum dekhenge ki yeh jo stresses hai yeh stresses kya hai stresses jo humne apply kiye hume normal strains mile to yeh jo equations hai yeh equation hume de rahe relation between normal stresses and normal strains yeh humare paas saare normal stresses hai aur yeh jo saare strains hai humare paas normal strains hai. To agar inko maan lete hum equation one two aur three to hum in teenon ko add karte hain add karne ke baad humare paas $(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})$ aaye ga agar hum in teenon ko add karenge to humare paas yeh term aaye gi one / E yaani one / E common nikal jaayega yeh aaye ga $(\sigma_{11} + \sigma_{22} + \sigma_{33}) -$ yahan par dekhenge aap σ_{11} do baar aa raha hai σ_{22}

do baar aa raha hai aur σ_{33} do baar aa raha hai to yeh aa jaayega $- 2 \times \nu \times (\sigma_{11} + \sigma_{22} + \sigma_{33})$. Abhi humne last time dekha tha maine ek term introduce ki thi σ_m jo mean stress humne dekha tha usko hum is tarah se likh sakte hain $(\sigma_{11} + \sigma_{22} + \sigma_{33}) / 3$ to agar main yeh σ_{11} σ_{22} aur σ_{33} ka addition jo hai usko main mean stress se agar replace karunga to main is tarah se kuch kar paunga yaani yeh mere paas mean stress hai in is state of stress ko main mean stress se replace karunga is equation ke dwara to mere paas yeh aa jaayega $(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})$ aur yahan pe aa jaayega $\sigma_{11} + \sigma_{22} + \sigma_{33}$ isko main likh sakta hoon 3 σ_m aur yahan pe yeh bhi addition ko main likh sakta hoon 3 σ_m abhi hum dekhte hain jab main normal stresses apply kar raha hoon to humne dekha tha yaani hydrostatic jo state of stress dekha tha humne dekha tha ismein volume change ho raha hai jo volume change ho raha hai isko hum is tarah se likhte hain. Agar hum teenon strains ko hum add karte hain normal strains ko to hum yeh likh sakte hain ki yeh volume volumetric strain hai aur volumetric strain ko is tarah se likh sakte hain volume change / original volume aur isko hum is tarah se relate karenge yeh yeh term hai to yeh jo addition of normal strains hai yeh mujhe deta hai volumetric strain isko hum likh lete hain $\Delta V / V$ ko hum kehte hain volumetric strain yahan par hum dekhenge ki jo hydrostatic state of stress tha woh volume change karta tha yeh humne dekha tha. To agar hum is tarah se likhenge isko to yeh 3 σ_m bahar nikalenge is term se to humare paas $(3 \sigma_m / E) \times (1 - 2 \nu)$. to yeh volumetric strain ko main Δ se represent kar raha hoon aur yeh term mere paas aa jaayegi aise to agar hum dekhenge yeh is identity ko is equation ko achhe se to mere paas kya hai strain hai volumetric strain aur ek term hai σ_m aur yeh term hai isko main poore is term ko main leta hoon $1 / K$ to hume mil raha hai ki ek relation mil raha hai strain aur stress ka relation aur in strain aur stress ke relation ko hum jab connect karte to hume constant milta hai. Us constant ko main naam deta hoon bulk modulus to yeh ho jaayega mera K bulk modulus aur K ki value kya aaye gi K ki value meri aaye gi $(E / 3) \times (1 - 2 \nu)$ to yeh ek other material constant ho gaya ek another material constant ho gaya material constant isliye keh raha hoon ki humare paas ek do constant hai E aur ν to isse main ek aur constant likh raha hoon jo constant kis ko relate kar raha hai volumetric strain to stress ko.



to yeh ek mere paas ek constant ho gaya to abhi hum dekhenge elastic stress strain relation jab isotropic material ke liye humne normal stresses ke liye dikhaya tha abhi shear stresses ke liye dekhte hain. Aur shear strains ke baare mein jo relation hai shear stresses aur shear strains ka iska straight forward relation hai humne dekha tha ki shear stress results only in shear strains yaani shear stress sirf shear strains ko produce kar sakte hain result kar sakte hain to humare paas shear stresses kaun se hain humare paas agar stress tensor dekhenge to humare paas yeh shear stresses hai yeh teen independent shear stresses hai σ_{12} σ_{23} σ_{13} jo produce karenge humare mere shear strains γ_{12} γ_{23} aur γ_{13} to γ_{12} ka simple relation hai $(1 / G \times \sigma_{12})$ jahan pe G jo hai woh main shear modulus hai. To yeh strain hai aur yeh stress hai iska jo agar constant define karenge woh shear modulus ke dwara define karenge yeh ek another elastic constant hai to γ_{23} ko main is tarah se likh sakta hoon $(1 / G)\ \sigma_{23}$ aur γ_{13} $(1 / G\ \sigma_{13})$ jahan pe mera shear modulus hai isko main constant maan ke chal raha hoon kyunki main isotropic material ke baare mein baat kar raha hoon to jo meri elastic properties hai woh change nahi hogi isliye main G constant maan ke chal raha hoon kisi bhi stress strain relations ko consider karta hoon jab to shear strain jab main strain tensor ke baare mein dekhta hoon jo yeh humne dekha tha ki yeh shear strain jab main baat kar raha hoon engineering shear strain ki baat kar raha hoon jab strain tensor jo component hai woh mera ε_{ij} rahega aur ε_{ij} aapko pata hai ki woh half of shear strain rehta hai yahan par hume pata hai ki i j not equal to j. To yeh agar main term use karta hoon to mere paas strain tensorial jo quantity hai $\varepsilon_{12} = (1 / 2G)\ \sigma_{12}$ isi tarah se main ε_{23} aur ε_{13} uske correspondingly stress jo shear stress hai uske sath ek relation likh sakta hu , aapko ek baat yaad rkhni padegi ye identity. to humare pass yaha pr 4 elastic constant humne dekhe

E ν G aur K. to K ka relation humne dekha E / (3 × (1 − 2 ν)) to K independent elastic constant hai to iska answer hoga nahi kyunki K independent nahi hai kyunki hume pata hai ki yeh dependent hai E aur ν ke hisaab se agar mujhe E aur ν pata hai to main K nikal sakta hoon to K is not independent elastic constant. To humare paas ek question aata hai ki G independent elastic constant hai to humare paas chaar elastic constant hai jisme humne dekha tha K to independent nahi hai kyunki K ka relation main E aur ν ke hisaab se nikal sakta hoon to G independent constant hai ki nahi yeh hum dekhte hain to hume G aur E ka relation dekhna padega.

Elastic Stress-Strain relations

Shear stresses and shear strains (for isotropic materials)

Shear Stress results in ONLY Shear Strains

$\sigma_{12}, \sigma_{23}, \sigma_{13} \Rightarrow \gamma_{12}, \gamma_{23}, \gamma_{13}$

$$\gamma_{12} = \frac{1}{G}\sigma_{12} \qquad \gamma_{23} = \frac{1}{G}\sigma_{23} \qquad \gamma_{13} = \frac{1}{G}\sigma_{13}$$

where, G is the shear modulus

Another Elastic constant

Shear strain in terms of strain tensor

$$\varepsilon_{ij} = \frac{1}{2}\gamma_{ij} \quad i \neq j \qquad \varepsilon_{12} = \frac{1}{2G}\sigma_{12} \qquad \varepsilon_{23} = \frac{1}{2G}\sigma_{23} \qquad \varepsilon_{13} = \frac{1}{2G}\sigma_{13}$$

Four Elastic constants: E, ν, G and K

$$K = \frac{E}{3(1-2\nu)}$$

K is not independent
For Isotropic materials

Is G independent constant?

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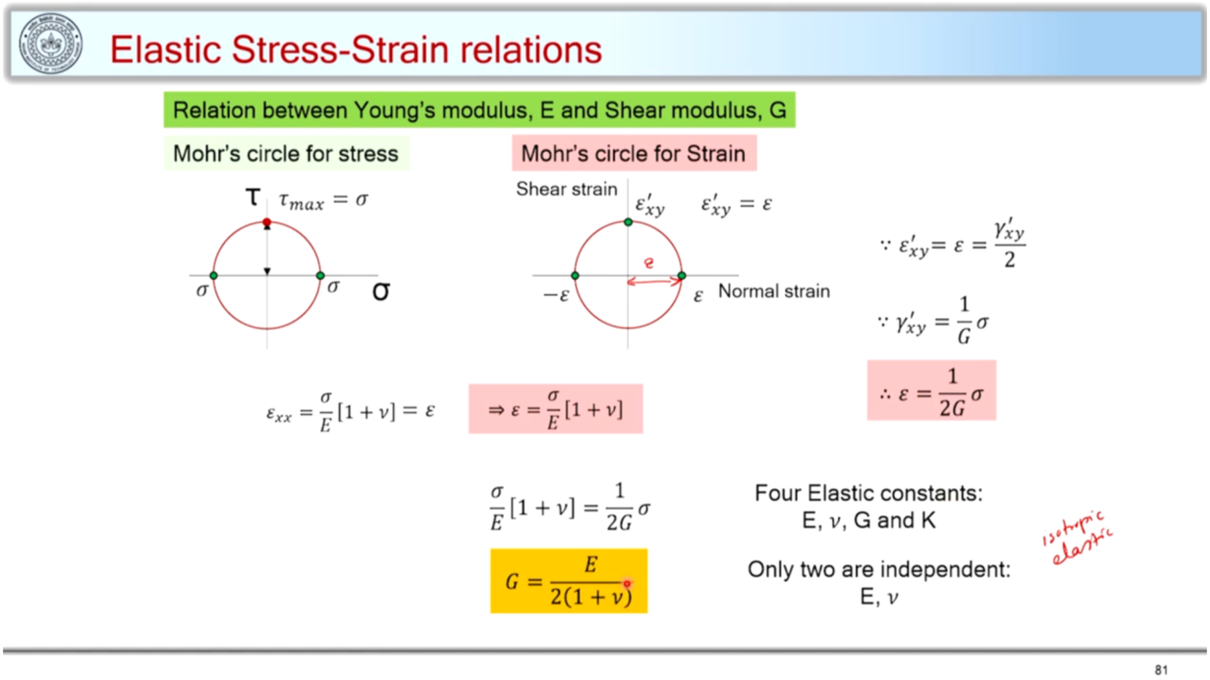
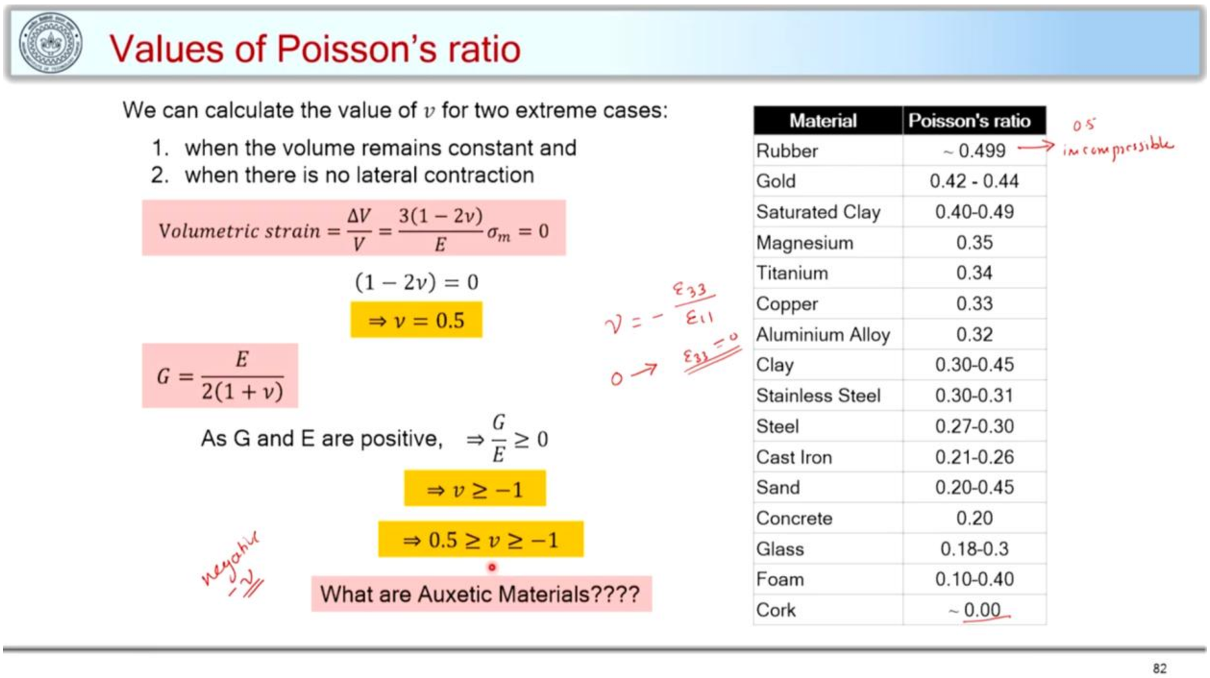
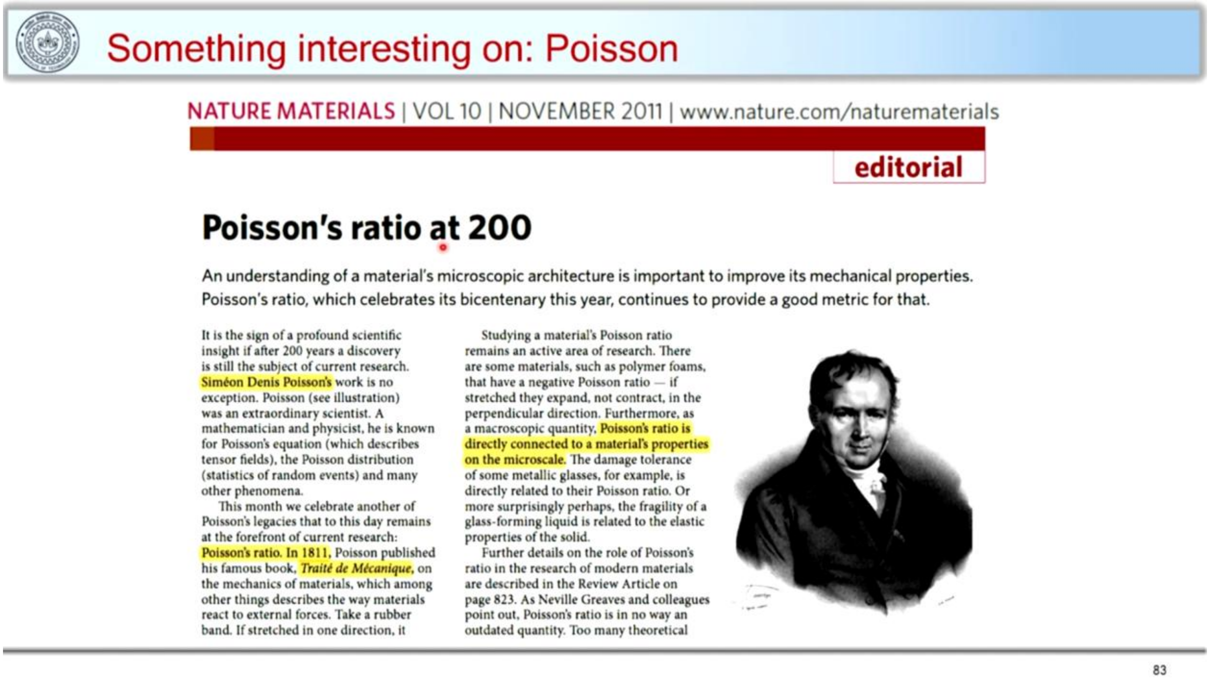
To dekhte hain abhi elastic stress strain relations mein E aur G ka relation kya hai yaani Young's modulus E aur shear modulus G ka relation kya hai yeh samajhne ke liye hume dekhna padega. Ek hum consider karte hain 2D stress state 2D stress state yaani ek x aur y ko net mein hum ek element mark kar lete hain aur ek simple stress state hum use karte jahan pe biaxial stress state use karte jahan par σxx aur σyy yeh normal stresses hai par yahan pe main is tarah se consider kar raha hoon jo σxx hai woh tension hai aur σyy yeh compression mein hai maan lete hain ki in dono ka magnitude same hai sirf nature different hai. ek tension hai ek compression hai to main – σ aur σxx ko likh raha hoon σ to humne state of stress dekha tha jab hum Mohr circle ke baare mein baat kar rahe the. To hum agar elastic stress strain relations likhenge to yeh normal strain hai εxx along x direction to isko hum is tarah se likh sakte hain 1 / E σxx − ν × yy yahan pe σzz zero hai to yeh term is tarah se hum likh paayenge agar main σxx ko σ se replace karunga aur σyy ko – σ se replace karunga to mere paas yeh identity aa jaayegi aur main is tarah se kuch likh paunga ε_{xx} ko $\sigma / E \times 1 + \nu$ abhi ε_{yy} ko hum likhte hain ε_{yy} kya hoga humara normal strain hoga jo σ_{yy} se develop hoga. To hum is tarah se likh sakte hain isko $1 / E \sigma_{yy} - \nu \sigma_{xx}$ yahan par σ_{zz} zero hoga to is tarah se agar aap dekh sakte σ_{xx} ko main is tarah se likh sakta hoon $1 / E \sigma_{xx}$ se develop ho raha hai aur yeh contribution aaye ga mera jab yeh σ_{xx} lateral strain rahega to ismein humara σ_{zz} zero hai kyunki hum 2D state state consider kar rahe hain to agar main yeh values agar rakhunga σ_{yy} ki $-\sigma$ aur σ_{xx} ki σ to mere paas ek quantity aa jaayegi $\varepsilon_{yy} - \sigma / E \times 1 + \nu$. Agar aap dekhen yeh do term ε_{xx} aur ε_{yy} to yeh inka magnitude jo hai woh same hai par sign jo hai woh opposite hai yeh mere paas positive hai aur yeh mere paas negative hai to main isko ek quantity se replace karunga εse aur yeh jab εhai to yeh ho jaayega $-\varepsilon$to ab agar hum Mohr circle draw karenge Mohr circle mein hum dekhenge ki x axis pe main σ normal stress plot karta hoon aur y axis pe shear stress plot karta hoon yeh humne dekha tha agar hum is state of stress ko yahan pe plot karte to hume kya milega ek positive hai σ yaani σ xx yahan pe aa jaayega σ aur ek negative hai yahan pe σ_{yy} jo hai yeh negative hai to yeh $-\sigma$ yahan pe aa jaayega aur main Mohr circle draw karta hoon to mujhe ek plane milega. Yeh plane jahan par mera σ normal stress zero hai yeh plane of maximum shear kehte hain aur yeh plane of maximum shear yaani tau max ki value kya rahegi yeh rahegi radius of Mohr circle to radius of Mohr circle yahan par rahega σ yeh jo radius hai yeh σ hai to tau max ki value bhi mujhe σ milegi to yeh coordinate ho jaayenge yahan pe normal stresses shoonya rahenge zero rahenge aur yeh plane hoga mera plane of maximum shear to plane of maximum shear agar main plot karunga mere coordinate axis ke hisaab se agar mera yeh initial coordinate axis hai to mera element is tarah se aaye ga jahan par sirf shear stresses present rahenge kyunki yahan pe normal stress shoonya hai. Aur yeh is tarah se orient rahenge mere original axis se 45 degree apart yaani humne dekha tha yahan pe 90 degree apart hai to yeh yahan pe actually reality mein 45 degree apart rahenge aur yeh stress jo rahega jo shear stress rahega maximum shear stress yeh rahega σ yahan pe yeh humne yahan pe dekha hai to isko kehte hain state of pure shear aur jo shear strain yahan pe develop hoga shear strain ke humne relation dekha tha last time to yeh main isko gamma' xy likh raha hoon kyunki main x dash y dash ke reference se baat karunga to yeh gamma' xy jo shear strain rahega yeh shear stress par depend karega shear modulus ke dwara. To yeh relation main likh sakta hoon aur isko main tau ko yahan pe replace karunga σ se to abhi yeh relation aapko yaad rakhna hai gamma' xy = $1 / G \times \sigma$ yeh hum aage chalenge aur isko use karenge to hume Young's modulus aur shear modulus ka relation nikalna hai tab hum yeh Mohr circle consider kiya tha Mohr circle of stress aur aur ek cheez hoti hai Mohr circle of strain bhi hota hai to Mohr circle of strain mein hum kya karte x axis par normal strains plot karte aur y axis par hum shear strains plot karte. To hume mila tha humare condition mein normal strain kya tha ek εtha jo ε_{xx} tha aur − εtha jo ε_{yy} tha to mere paas yeh do normal strain agar main Mohr circle plot karta hoon yeh centre pakad ke to mere paas yeh Mohr circle milega yeh Mohr circle of strain kehte hain aur yeh jo point rahega yahan pe normal strain shoonya rahega yaani yeh ho jaayega mera epsilon' xy aur epsilon' xy ki value yahan pe kya aaye gi yeh bhi aaye gi εkyunki yeh jo value hogi yeh Mohr circle ke radius ke barabar hogi. To yeh bhi value hume εmilegi to mere paas epsilon' xy aaye ga εaur hum epsilon' xy ko likh sakte hain gamma' xy / 2 kyunki yeh tensorial quantity hai isko hum shear strain mein jab convert karte to uska aadha rehta hai to hum likh sakte hain $\varepsilon = \gamma$x dash y dash / 2 humne last slide mein dekha tha ki $\gamma'_{xy} = \sigma / G \times \sigma$ hum jab yeh yahan pe replace karenge to hume milega $\varepsilon = 1 / 2G$ in sigma. To hum yeh do relation use karenge to hume yeh quantity milegi aur humne isse yeh nikala tha yeh Mohr circle se humne nikala tha ki $\varepsilon_{xx} = \sigma / E \times 1 + \nu$ is equal to humne likha tha ε to hum ε ko is tarah se likh sakte hain $\sigma / E \times 1 + \nu$ abhi mere paas εdo tarah se humne represent kiya hai ek normal stress aur yeh ho gaya Young’s modulus aur yahan pe εjo hai yeh maine represent kiya hai normal stress aur shear modulus. Agar hum dono ko comparison karte hain dono ko equate karte to mere paas yeh identity aa jaayegi aur main G ka aur E ka relation nikal paunga to mere paas aa jaayega $G = E / 2 \times 1 + \nu$ to yeh mere paas relation aa gaya shear modulus aur Young’s modulus ka to humare paas ek question tha ki humare paas chaar elastic constants the E ν G aur K humne dekha tha K to independent nahi hai to kya G independent hai to G bhi independent nahi hai kyunki main G ko E aur ν ke dwara present kar sakta hoon ya represent kar sakta hoon. To mere paas do hi independent elastic constants hai for isotropic material E aur ν yeh important hai aap yeh point hamesha isotropic elastic material ke liye hi yeh valid hai to mere paas do constants hai aur agar mujhe do constants pata hai to main baaki ke yeh chaar constants ko identify kar sakta hoon ya prapt kar sakta hoon abhi hum aage badhte hain aur dekhte hain ki yeh jo ν hai Poisson ratio hai iski values kya ho sakti hai



To ν ki jo maximum values hai kisi bhi material ke liye hum is tarah se nikal sakte hain to do extreme case hum consider karte hain jahan par volume remains constant yaani volume change nahi hoga jaise humne dekha tha ki hydrostatic state of stress apply kiya tha to wahan par volume volumetric strain hume mila tha agar volume change nahi ho to volumetric strain shoonya hona chahiye to yeh ek extreme condition hogi doosri condition hogi ki jahan pe hume koi lateral contraction na mile yaani Poisson ratio jo lateral strain hai woh shoonya rahe. To volumetric strain jab shoonya rahega to humne dekha tha ki volumetric strain hum is tarah se define kar sakte hain volume change V / original volume V = 3×1 − 2 ν / E×σ m yeh humne derive kiya tha agar hum isko shoonya maan ke chalte hain to hum dekhenge ki jo state of stress hai agar vo shoonya nahi hai aur humara elastic constant agar shoonya nahi hai to jo possible value hai yeh 1 − 2 ν shoonya honi chahiye. To agar main isko equate karta hoon 1 − 2 ν = zero to ν ki value mujhe milti hai point five to mere paas yeh ek maximum value hogi ν ki aur doosri condition mere paas doosra relation yeh tha G = E / 2 × 1 + ν hume pata hai agar koi material hai humare paas to shear modulus aur shear strains positive hone chahiye to agar G aur E positive hai to G / E greater than zero rahega hamesha agar yeh greater than zero hai to 1 − ν ki value kya honi chahiye. Hamesha ν should be greater than equal to − one yaani − one se woh bada hona chahiye hamesha to mere paas ν ki do values aa gayi yeh do extreme conditions ko maine consider kiya tab to ν ki value koi bhi material ke liye is domain mein lie karti hai yaani − one se badi aur point five se chhoti to yeh meri ν ki values ho gayi kisi bhi isotropic elastic material ke liye. To jaante abhi different material yahan par mark kiye aur uske Poisson ratio maine mark kiye humne dekha tha ki rubber ki value jo maximum value aa rahi hai point four nine nine ki aa rahi hai aur cork ki value maine yahan par mark ki hai jo zero hai to yeh extreme values hai to jab Poisson ratio ki value point five tak pahunchti hai to hum isko kehte hain incompressible material compressible nahi ho raha hai. Aur jab ν ki value shoonya rehti hai to iska matlab kya humne ν ko define kiya tha lateral strain yaani lateral starin ko main maan lunga ε33 yaha pe / ε11. to iska matlab kya hua ν ki value shoonya hai to lateral strain jo develop hoga ε33 woh shoonya hona chahiye yaani mera material koi lateral strain develop nahi karega agar ν ki value shoonya hai to yeh do extreme values humne dekhiye. Agar ν ki value negative hai us material ko hum kehte hain auxetic material to aap iske baare mein thoda padhiye ki auxetic material kya hote hain aur yeh negative Poisson ratio kyon dikhate hain main likh leta hoon ki auxetic material jo hote hain woh negative Poisson ratio dikhate hain to iske baare mein aap padhiye aur janiye ki iski value negative hoti hai.



To ek achha ek article hai Poisson ratio ke baare mein jo ki 2011 mein aaya tha Nature Materials mein Poisson ratio at two hundred kyunki yeh jo two thousand eleven saal tha woh is Poisson ratio ka bicentenary year tha yaani isko two hundred saal complete hue the aur yeh Poisson ratio jo diya tha woh Simon Denis Poisson ne diya tha aur yeh jo kaam publish hua tha woh Traite de Mechanique mein publish hua tha. Aap agar yeh article dekhenge to ismein bahut saari jo baatein batayi hai jo Poisson ratio jo main bata raha hoon yahan par woh microscopic deformation hai isko connect karta hai to jab hum microscopic deformation dekhte hain aur isko microscopic scale par kya mechanism chal rahe hain to Poisson ratio ek achha idea deta hai hume. To yeh jo article hai yeh baat karta hai amorphous ke baare mein liquid crystall ke bare me aur normal metallic material ke baare mein to aap isko padhiye thoda sa Poisson ratio ke baare mein


Something interesting on: Poisson

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editorial

Poisson's ratio at 200

An understanding of a material's microscopic architecture is important to improve its mechanical properties. Poisson's ratio, which celebrates its bicentenary this year, continues to provide a good metric for that.

It is the sign of a profound scientific insight if after 200 years a discovery is still the subject of current research. Siméon Denis Poisson's work is no exception. Poisson (see illustration) was an extraordinary scientist. A mathematician and physicist, he is known for Poisson's equation (which describes tensor fields), the Poisson distribution (statistics of random events) and many other phenomena.

This month we celebrate another of Poisson's legacies that to this day remains at the forefront of current research: Poisson's ratio. In 1811, Poisson published his famous book, *Traité de Mécanique*, on the mechanics of materials, which among other things describes the way materials react to external forces. Take a rubber band. If stretched in one direction, it

Studying a material's Poisson ratio remains an active area of research. There are some materials, such as polymer foams, that have a negative Poisson ratio — if stretched they expand, not contract, in the perpendicular direction. Furthermore, as a macroscopic quantity, Poisson's ratio is directly connected to a material's properties on the microscale. The damage tolerance of some metallic glasses, for example, is directly related to their Poisson ratio. Or more surprisingly perhaps, the fragility of a glass-forming liquid is related to the elastic properties of the solid.

Further details on the role of Poisson's ratio in the research of modern materials are described in the Review Article on page 823. As Neville Greaves and colleagues point out, Poisson's ratio is in no way an outdated quantity. Too many theoretical

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Abhi hum aage chalte hain aur dekhte hain stress strain relations kya hai humne to abhi elastic constants ke baare mein jana abhi hum stress strain relations dekhte hain to yeh stress strain relation humne dekha tha. Normal stresses aur normal strains ke baare mein to main ε_{11} ko is tarah se likh sakta hoon isko equation mein one maan ke chal raha hoon yeh constant humne likha tha jo shear modulus tha main usko elastic modulus yaani Young's modulus ke saath relate kiya tha $E / 2 \times 1 + \nu$ yeh mere paas isko main doosra equation maan ke chal raha hoon aur volumetric strain ko main delta define kiya tha change in volume / original volume aur volumetric strain ko main is tarah se bhi likh sakta hoon $\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \Delta$. aur yeh relation hume mila tha abhi hum kya karenge abhi hum yeh σ_{22} p σ_{33} ki value nikalenge agar main σ_{22} p σ nika loonga is relation se to mere paas yeh kuch identity aa jaayegi $E / 1 - 2\nu \times \Delta - \sigma_{11}$ to yeh agar main e ko yahan pe multiply karoonga yahan yeh $1 - 2\nu$ denominator mein aa jaayega aur σ_{11} ko main subtract karoonga to mere paas ek third equation main maan ke chal raha hoon abhi main kya karoonga yeh jo value hai $\sigma_{22} + \sigma_{33}$ idhar rakhoonga yahan pe rakhoonga to mere paas aa jaayega yeh yeh identity ε_{11} to maine sirf $\sigma_{22} + \sigma_{33}$ mein yeh value yeh term likh liye abhi main m se sub in saare jo bracket ke andar jo terms hai isko multiply karoonga yahan par mere paas σ_{11} hai aur yahan par σ_{11} hai to main mere paas aa jaayega $-\nu \times \sigma_{11}$ to mere paas ek $+ \sigma_{11} \times \nu$ aur isko main common nikaal loon. To mere paas yeh aa jaayega $(1 + \nu) \times \sigma_{11} - E\nu / (1 - 2\nu) \times \Delta$ yeh mere paas ek identity aa jaayegi ε_{11} main likh raha hoon σ_{11} ke dwara abhi main σ_{11} ko likhunga agar main isko solve karunga aur σ_{11} ki value nikalne ki koshish karunga to mere paas yeh identity aaye gi to aap rearrange karenge terms σ_{11} ko is side le aayenge to aapke paas yeh identity milegi. Agar aap yeh term dekhen $E / (1 + \nu)$ to main isko shear modulus ke hisaab se likh sakta hoon to mere paas yeh aa jaayega $E /(1 + \nu)$ ko main likh sakta hoon 2G to yeh isko main replace karunga 2G se aur yeh jo term hai isko main replace kar raha hoon lambda se aur yeh lambda jo hai isko main kehta hoon Lamé constant yeh jo term hai $E \nu / ((1 + \nu) \times (1 - 2 \nu))$. To aap dekhenge yaha pe 2 constant hai elastic constant elastic modulus yani youngs modulus and poisson ratio iske dwaara main likh rha hu, to ye constant ho jaega isko main kahta hu Lamé constant jo lambda se hum represent karte hain. To abhi aap dekhenge to agar main σ_{11} ko is tarah se likh sakta hoon to main similarly σ_{22} aur σ_{33} ko ε_{22} aur ε_{33} ke dwara likh sakta hoon similar treatment agar doonga to yahan pe aap yaad rakhiye ga yeh hai $\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ summation of normal strains hai yeh to yahan pe hum dekhenge ki yeh jo elastic stress strain relation hai yeh jo pehla relation hai yeh mujhe agar mere paas normal stresses available hai to main normal strains nikal sakta hoon. Agar mere paas normal strains mujhe pata hai aur yeh material constants mujhe pata hai to main normal stresses nikal sakta hoon to main yahan par yeh jo exercise humne ki yeh interchange karne ke liye yaane stress ko strain mein represent karne ke liye aur jo pehle humne nikala tha jo strains ko hum stresses mein represent kar rahe the to yeh mere paas ho jaayenge normal stresses ke relation normal strains ke dwara aur agar hum dekhen shear stresses yeh jo shear stresses hai inke relation shear strains ke dwara yeh simple relation hai. Yeh σ_{12} ko main likh sakta hoon $G \times \gamma_{12}$ to aap dekhenge ki yeh mera shear strain hai isko main agar is tarah se bhi likhunga to agar aapke paas tensorial quantity hai to aap is tarah se likh sakte ho $2G\, \varepsilon_{12}$ baaki ke strains bhi aap is tarah se likh sakte ho

Stress-Strain relations

$$\varepsilon_{11} = \frac{1}{E}[\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})] \quad (1)$$

$$G = \frac{E}{2(1+\nu)} \quad (2)$$

$$\frac{\Delta V}{V} = Volumetric\ strain = \Delta$$

$$(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) = \Delta = \frac{(1-2\nu)}{E}(\sigma_{11} + \sigma_{22} + \sigma_{33})$$

$$\Rightarrow (\sigma_{22} + \sigma_{33}) = \frac{E}{(1-2\nu)}\Delta - \sigma_{11} \quad (3)$$

From (1) and (3)

$$\Rightarrow \varepsilon_{11} = \frac{1}{E}\left\{\sigma_{11} - \nu\left[\frac{E}{(1-2\nu)}\Delta - \sigma_{11}\right]\right\}$$

$$\Rightarrow \varepsilon_{11} = \frac{1}{E}\left\{(1+\nu)\sigma_{11} - \frac{E\nu}{(1-2\nu)}\Delta\right\}$$

$$\sigma_{11} = \frac{E}{(1+\nu)}\varepsilon_{11} + \frac{E\nu}{(1+\nu)(1-2\nu)}\Delta$$

$$\sigma_{11} = 2G\varepsilon_{11} + \lambda\Delta \quad \text{From (2)}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

where, λ is Lamé's constant

$$\sigma_{11} = 2G\varepsilon_{11} + \lambda\Delta$$

$$\sigma_{22} = 2G\varepsilon_{22} + \lambda\Delta$$

$$\sigma_{33} = 2G\varepsilon_{33} + \lambda\Delta$$

where, $\Delta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$

$$\sigma_{12} = G\gamma_{12}$$

$$\sigma_{13} = G\gamma_{13}$$

$$\sigma_{23} = G\gamma_{23}$$

$\sigma_{12} = 2G\varepsilon_{12}$

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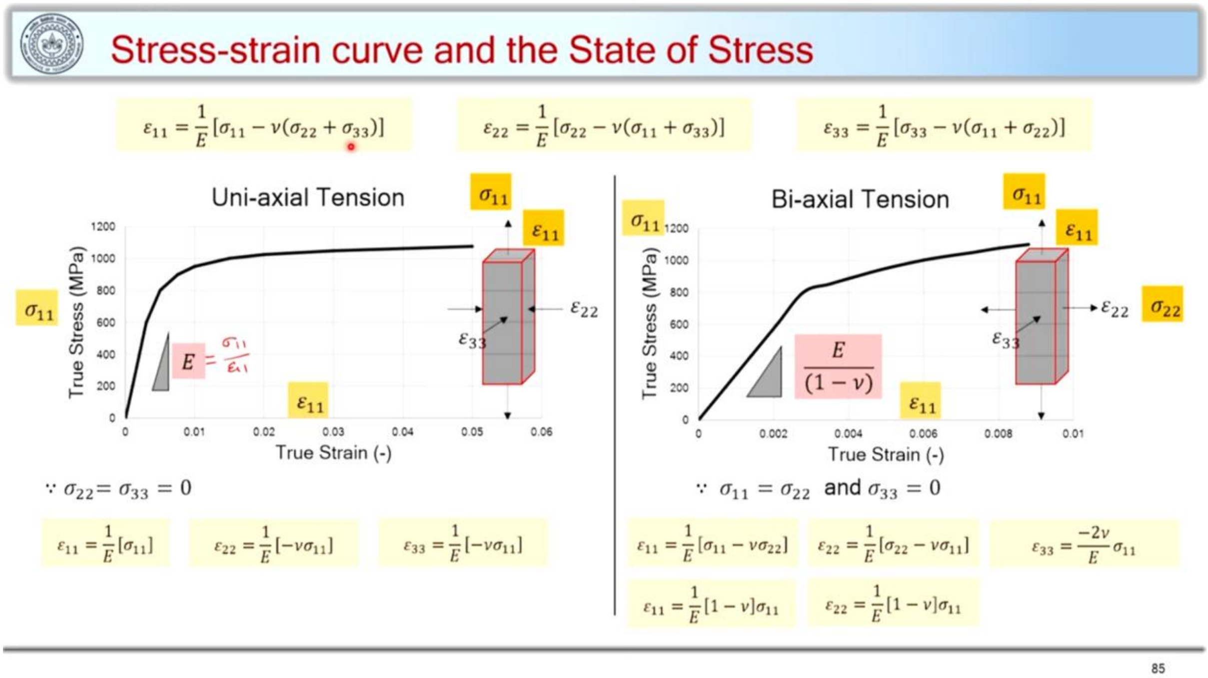
to yeh ho gaye mere stress strain relations abhi hum dekhte hain ki kuch simple cases aur stress strain curve pe is ka kya asar hota hai state of stress ka. To mere paas yeh relation hai normal strains ke normal stresses ke dwara yeh mere paas teen relations hai humne derive kiye the lets say ek mere paas member hai isko main stress kar raha hoon isko main ek normal stress apply kar raha hoon $\sigma 11$ to kya hoga ismein ek normal strain develop hoga $\varepsilon 11$ aur lateral strains bhi develop honge $\varepsilon 22$ aur $\varepsilon 33$ yeh humne dekha tha. Agar hum uniaxial tension ki baat kar rahe to mere paas yeh hum plot karte y axis pe true stress aur true strain to y axis pe aaye ga mera $\sigma 11$ aur $\varepsilon 11$ agar hum yeh equations dekhen aur yeh uniaxial tension ki baat karenge to mere paas $\sigma 22 = \sigma 33 =$ zero aaye ga. To agar hum yeh relations mein rakhte hain yeh values to mere paas milega $\varepsilon 11 = 1 / E\ \sigma 11$ aur $\varepsilon 22$ ko milega $(1 / E)(-\nu\times\sigma 11)$ aur $\varepsilon 33$ ki value aaye gi $1 / E - \nu\times\sigma 11$ aap yahan pe $\sigma 33$ aur $\sigma 22$ ki value shoonya rakhenge to hume yeh teen identity milengi normal strains ki. To agar hum yahan pe dekhen uniaxial tension test mein agar yeh jo slope hai initial slope hai yeh iski value rahe gi E kyunki hum slope ki value kya aaye gi humare paas isko likh lete hain yeh aaye gi $\sigma 11 / \varepsilon 11$ jab hum plot karte hain normal stress versus normal strain to hume jo slope milega woh elastic modulus milega. Abhi consider karte another case jab humne $\sigma 11$ apply kiya to hume mila $\varepsilon 11$ aur jab humne $\sigma 22$ apply kiya ek biaxial stress apply kiya $\sigma 22$ direction mein to hume mila $\varepsilon 22$ aur ek lateral strain develop hoga jo $\varepsilon 33$ hai to mere paas biaxial tension ek test main kar raha hoon. Jab hum plot karenge y axis pe to yahan par bhi main $\sigma 11$ plot karunga aur x axis pe plot karunga $\varepsilon 11$ jo ki true strain hai mera agar hum ismein maan lete $\sigma 11 = \sigma 22$ yeh special condition maan lete hain aur $\sigma 33$ zero hai to agar hum yahan pe $\sigma 11$ aur $\sigma 22$ rakhte hain aur $\sigma 33$ shoonya rakhte to mere paas ek relation mil jaayega ε_{11} ka aur ε_{11} ka. Agar $\sigma_{11} = \sigma_{22}$ hai to main isko σ_{22} ko σ_{11} se replace kar sakta hoon aur isko common nikaal ke bahar nikaal sakta hoon is bracket ke bahar to mere paas $\varepsilon_{11} = 1 / E \times 1 - \nu \times \sigma_{11}$ aise milega waise hi ε_{22} milega mujhe to ε_{22} ko bhi main is tarah se likh sakta hoon $(1 / E)\times((1 - \nu) \times \sigma_{11})$. Aur ε_{33} yeh is tarah se milega $- 2 \nu / E \times \sigma_{11}$ yahan pe main value rakhunga σ_{33} ki value shoonya $\varepsilon_{11} = \varepsilon_{22}$ to mere paas yeh identity aa jaayegi agar main dekhunga to mere paas yeh jo slope hai $\sigma_{11} / \varepsilon_{11}$ agar main yahan se nikalne ki koshish karunga to jo slope aaye ga yeh aaye ga mera $E / 1 - \nu$. To humne dekha tha ki uniaxial tension mein mujhe slope milega E aur biaxial tension mein agar main kar raha hoon testing to mujhe slope milega $E / 1 - \nu$ to yahan par humne dekha tha ki uniaxial tension aur biaxial tension mein slopes alag alag aa rahe hain to yeh slopes alag alag aa rahe yeh hum is elastic jo stress strain relations hai iske dwara hum samajh sakte hain. To humne is part mein jaana ki elastic constants kya hote hain aur elastic stress strain relations kya hote hain humne strains ko stresses ke dwara likha tha aur stresses ko strains ke dwara bhi likha tha. to abhi hum yahan pe rukte hain aur next time hum dekhenge ki elastic constants ko general tareeke se kis tarah se correlate karenge.



Abhi ke liye rukta hoon dhanyavaad