Mechanical behavior of materials

Dr. Niraj Mohan Chawake

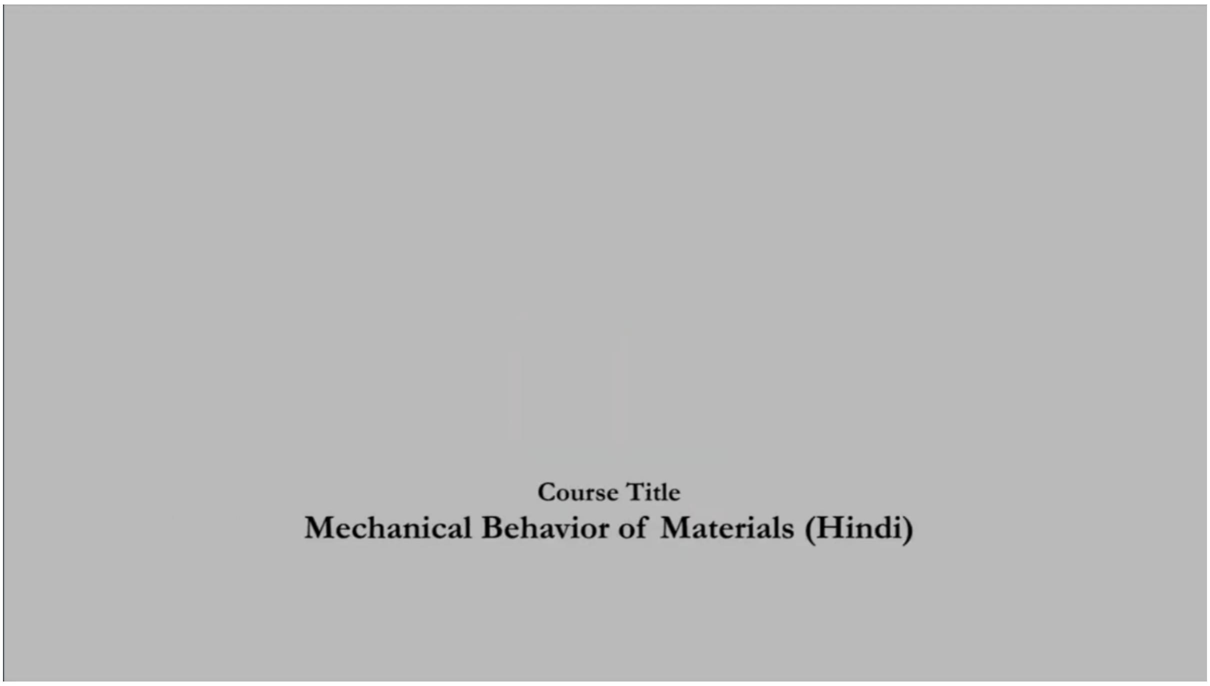
Department of Materials Science and Engineering

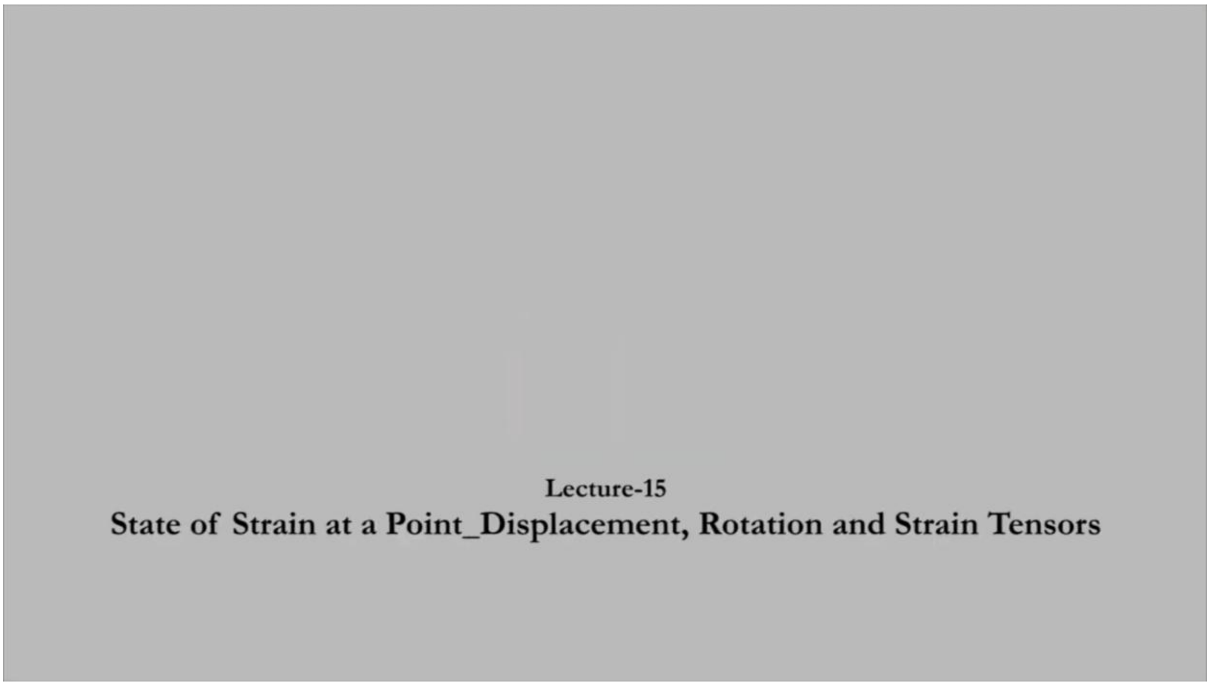
Indian Institute of Technology, Kanpur

Week-2

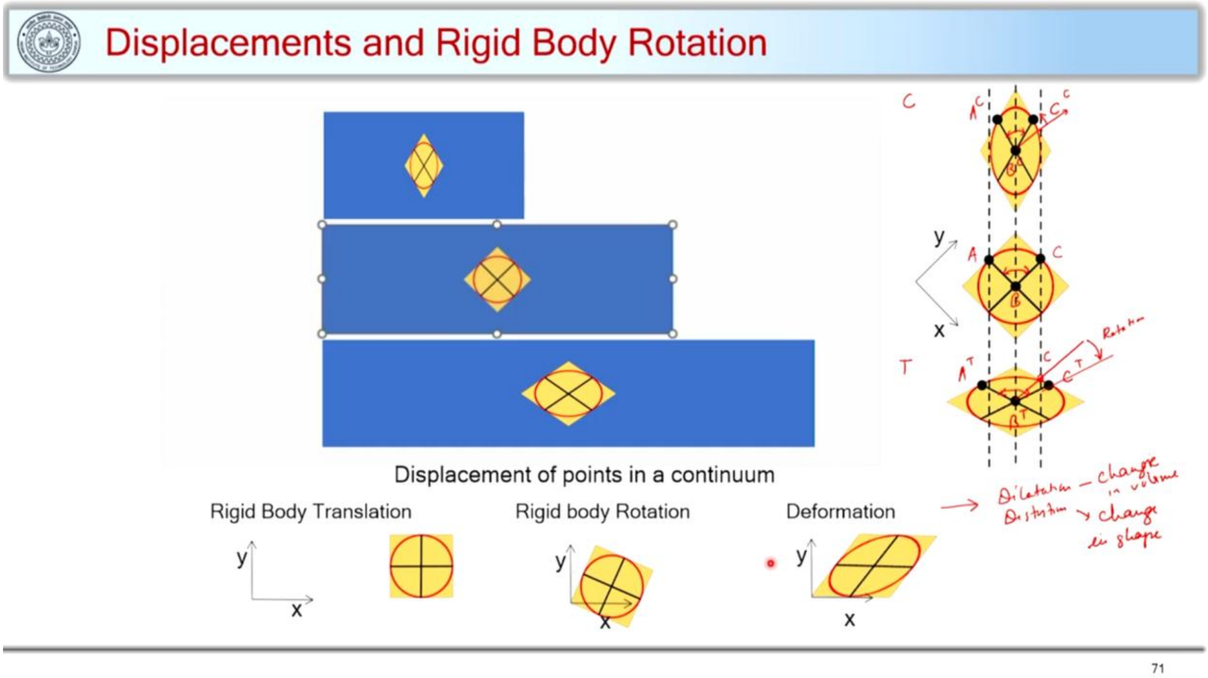
Lecture-15

State of Strain at a Point_ Displacement, Rotation and Strain Tensors



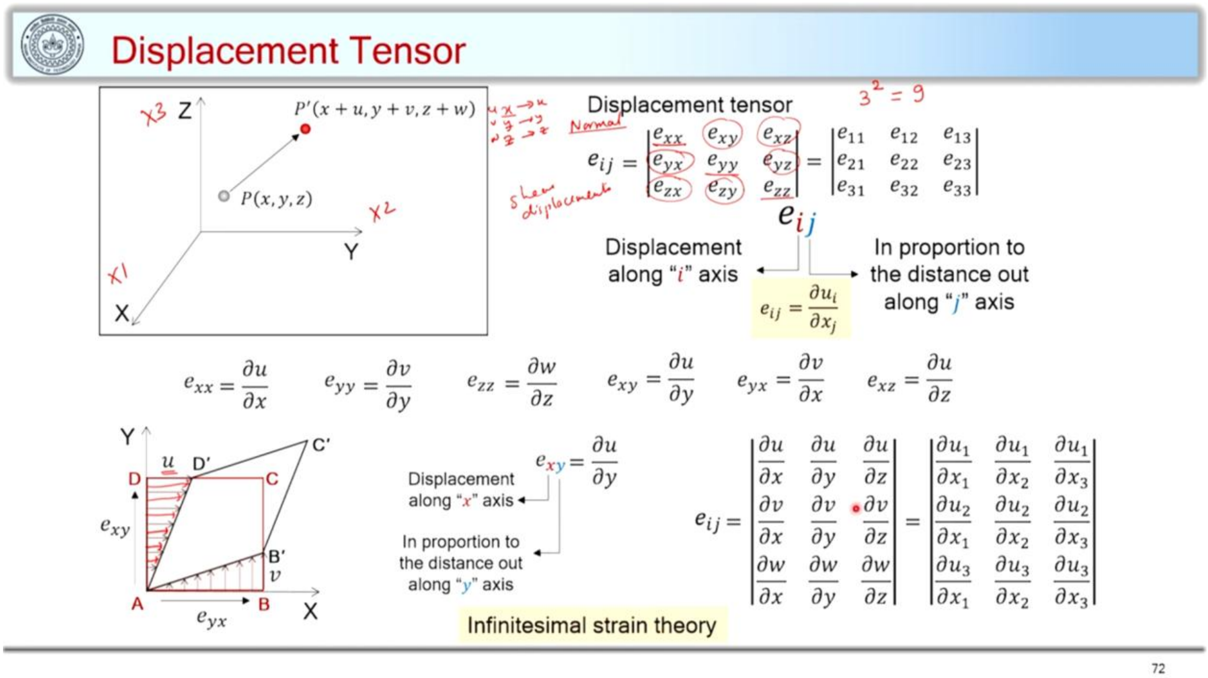


Namaskar, phir se swagat hai aapka. Last part mein humne dekha tha ki strain kya hai, yaani normal strain kya hai, shear strain kya hai. Is part mein hum jaanenge ki strain at point ya generalized strain ko kaise define karenge. To hum jaanenge ki strain tensor kya hai aur rotation tensor kya hai. To uske liye humein pehle jaanna padega ki displacement kya hai. Kisi bhi body jab deform hoti hai to displacements kya hote hain. To ek member maan lete hain aur humne kuch coordinate axes mark kar liye yahan par x aur y direction mein aur ek element mark kar liya is pe ye jo element hai. Bahut chhota element hai par yahan par maine bada dikhaya. To maan lijiye bahut chhota element hai kyunki hum bahut small deformations ki baat kar rahe hain. To isliye ye jo element hai ye bhi bahut chhota hoga aur ismein jo displacements honge ye bhi bahut kam honge. Par abhi hum isko ek samajhte hain ki ye deform agar main karunga tensile aur compressive deformation hota hai to displacement kaise hote hain. To maan lijiye agar isko main abhi tensile deform kar raha hoon yaani force apply kar raha hoon to ye long ho raha hai member. Abhi main usko compress kar raha hoon to ye compress ho raha hai yaani iska length kam ho raha hai. Usko main wapas se phir se normal stage par. Launga to ye mera element ho gaya. To is element ko maine mark kar liya aur hum do locations mark kar lete hain compression aur tension. Tension mein maan lijiye compression hai aur ye tension mein. To ye element jo ho raha hai ismein aap badlaav dekh sakte ho. In elements ko bhi hum mark kar lete hain is tarah se compression mein is tarah se aur tension mein is tarah se. Aur hum kuch points mark kar lete hain maan lijiye teen points main mark kar raha hoon A, B aur C. Aur ye jo lines hain ye tulna ke liye yaani comparison ke liye kaam mein aayengi compression mein kya ho raha hai aur tension mein kya ho raha hai. To in points ko main mark kar leta hoon maan lijiye mera point A hai yaani jab. Koi deformation nahi hai A aur B aur C mark kar leta hai. Ye mera tension mein hai aur ye mera compression mein hai. To inko main mark karunga A^C yaani superscript de raha hoon B^C aur C^C compression mein. Waise tension mein A^T, B^T aur C^T. Aur main maan leta hoon ki mera point B ek mera reference point hai. In teenon condition mein agar hum point A aur B dekhein, A aur C dekhein, to compression mein agar dekhenge to A aur C ye point B ke close aa raha hai, paas aa raha hai. Par aap tension mein dekhenge to point jo displace ho rha hai ye point jo displacement hai ye B se thoda door ho raha hai. Aur ye dotted line ke hisaab se aap dekh sakte hain. Is aap ye cheez dekh sakte hain ki yahan par jo angle hai, ek angle hum mark kar lete hain. Ye jo angle hai ABC, ye yahan par ghat raha hai, kam ho raha hai, aur yahan par badh raha hai. To agar hum dekhenge agar mera reference point yahan par tha to hum dekhenge ki yahan par rotation bhi aa raha hai. Yaani mera point C yahan par hona chahiye tha to C to C^T ek rotation aa raha hai yahan par. Waise yahan par agar aap dekhenge to ye point yahan par hona chahiye tha kahin to ye point is tarah se rotate bhi ho raha hai. Aur aap dekhenge ki jab hum displacement ki baat karte to displacement in continuum jab baat karte tab wahan par teen tarah ke displacement hote hain. Humne jaana ki pehla displacement kya hoga. Yahan par agar hum element mark kar rahe to agar dekhenge to ye jo element hai compression mein is tarah chala gaya aur tension mein ye positive x direction ki taraf aa raha hai. To isko hum kehte hain rigid body translation. To ye jo element hai translate ho raha hai. Translation yaane kya simple yahan par mera element hai ye translate hoga yaani move hoga xy direction par. Yahan par koi deformation nahi hai. Iska jo shape hai wo change nahi hua hai. Wo waise ka waisa hi hai. Waise ab humne dekha tha ki point C rotate ho raha hai compression aur tension mein to ek rigid body rotation bhi hota hai jab point displace hote hain. Rotation mein kya ho raha hai ye mera element hai aur ye simply rotate ho gaya. Agar z axis perpendicular maante hain is plane ko to ye z axis ke around sirf rotate hua hai. To is case mein bhi deformation nahi hai material mein. Ye sirf rotate ho raha hai. Yahan par translate ho raha hai ya phir rotate ho raha hai. Aur teesra hai rigid body deformation. Deformation mein kya hai ye jo mera element hai, ye jo consider kar raha hoon mera element, ye deform hoga yaani iska shape change hoga. Humne jaana tha deformation yaane kya hai. Deformation yaane humne jaana tha ki dilatation aur distortion ye do cheez humne dekhi thi last part mein bhi. Dilatation change in volume aur distortion mein dekha tha humne change in shape. To humein interested jab hum deformation ki baat karte to hum is part mein interested hain. Par jab hum displacement ki baat karte to ye teenon part hote hain displacement mein kisi bhi point mein. To humein dekhna hai ki humein ye jo rotation part hai wo humare displacement se nikaalna padega. Tabhi humein deformation ke baare mein pata chalega. To isko hum jaante hain aur achhi tarah se.



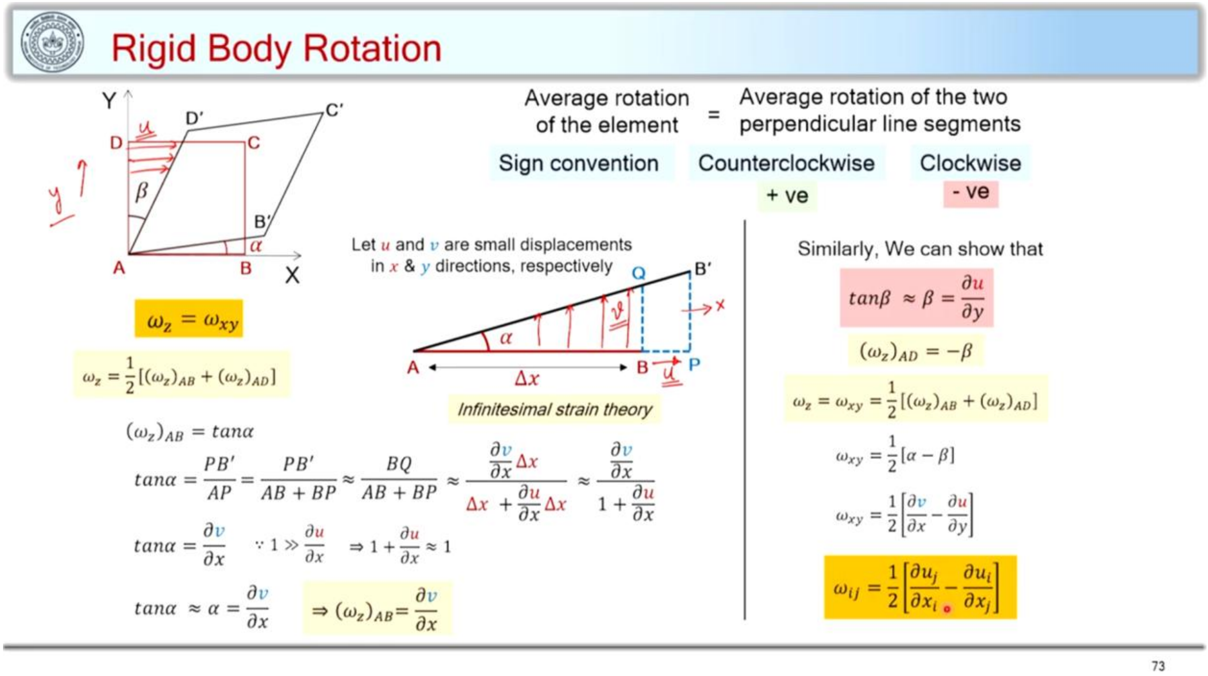
To maan lete hain mere paas ek coordinate axis hai x, y, z aur ek point hai wahan pe. Uske maine coordinates mark kar rakhe hain x, y aur z. Ab ye point displace ho raha hai 3D mein is tarah se. Is point ke coordinates hum mark karenge P' x + u, y + v, z + w. Yaane u, v aur w ye kya hai? Ye displacement hai along x, y aur z direction. To ye mere paas is point ka displacement aa gaya P'. to hum jab displacement tensor ki baat karenge kyunki humne jaana tha humne stress tensor ke baare mein jaana tha, abhi hum displacement tensor ki baat karenge. Ye jo coordinate axis hai mera 3D deformation hai aur ye bhi second order tensor hai. To is pe kitne components hone chahiye? Ismein nine components hone chahiye. To pehle hum displacement tensor ki jab baat karenge to hum isko is tarah se likhenge. eij Aur isko ye jo nine components hum likheng exx, exy, exz, eyx, eyy, eyz, ezx, ezy, ezz. Ye mere nine components honge. To isko likh lete hain. Three dimension tha aur second order tensor tha to ismein mere paas nine components hone chahiye humne dekha tha. To abhi hum displacement ki baat karenge. To isko pehle define karte hain ki displacement hote kya hain. To eij jab maine likha hai yahan pe to pehla suffix mera denote karta hai displacement along i axis. Aur jo doosra suffix hai, ye jo displacement hai, in proportion to the distance out along j axis. Yaane ye j axis ke along jo displacement badh rahe hain ye hum dekhenge. To isko hum samajhte hain eij kya hota hai. To eij ko hum likhenge is tarah se ∂ui / ∂xj. Yaane displacement along i axis in proportion to distance out along j axis. Maan lete hain exx agar hum dekhenge to isko is tarah se likhenge. x pehla suffix mera dikha raha hai displacement along i axis. To x axis ke along mera displacement kya hai? Yahan par mera displacement hai u. Aur ye kis direction mein badhna chahiye? x direction mein. To isko likhenge hum ∂u / ∂x. eyy kya hoga to mera phla suffix dikha rha hai displacement along i axis, yani displacement along yaha pe y axis hoga. y axis par displacement kya hai? v. Aur kis axis ke around displacement badhna chahiye? j axis. j yaha pe y hai To isko hum likhenge ∂v / ∂y. Similarly ezz ko z ke along displacement kya hai? w. Aur along z direction to isko hum likhenge ∂w / ∂z. To ye jo hai, ye jo displacement hai, isko hum normal displacement bhi kehte hain. To hum mark kar lete hain. Ye jo displacement hai exx, eyy, ezz ye x direction par displacement along x direction hai, y direction par displacement along y direction hai, z direction par displacement along z direction hai. To displacement jab baat karenge u, v, w ki baat karte hain to ye jo components hai ye mere normal displacements honge. Aur ye jo components hain off diagonal, off diagonal jo components hain ye jo 6 components hain off diagonal, inko hum kehte hain shear displacement. To humne normal displacements to dekh liye. Ab shear displacements ki baat karte hain. To uske liye ek example lete hain. Ek 2D element lete hain. x aur y coordinate mark kar liye aur ek element mark kar lete hain A, B, C aur D. Abhi hum iski displacement ki jab baat karenge to ye deform ho raha hai. Iska shape change ho raha hai jaise ki A B' C' D'. To hum dekhenge exy ko hum define karte hain. To exy kya hoga? Ye hoga displacement along x axis aur y hoga mera distance out along j axis. To displacement along x axis kahan pe? Yahan pe aap dekhenge to ye mera AD jo element ka jo part hai wo deform ho raha hai A'D' me . To yahan par mere jo displacement hain, ye agar aap dekhenge ye hai mere along x direction. Ye jo displacement hai ye mere along x direction hai. Isko main mark karunga u se. To aap samajhiye isko. Ye jo displacement hai agar dekhenge ye parallel hai x axis ke, yaani along x axis hai. To ye ho jaayega mera u. Aur ye displacement kaise badh rahe hain? Agar hum A se chalu karenge to ye displacement badh raha hai along y direction. To in proportion to distance out along y axis. Agar aap dekhenge ye displacement jo u jo badh raha hai, yahan se yahan pe shoonya tha, yahan pe badh raha hai aur yahan pe maximum mil raha hai humein displacement. Ye displacement along x axis hai, par ye badh kis tarah se rahe hain? Ye badh rahe hain y direction ke hisaab se. Yaani jaise jaise main y direction pe move ho raha hoon, waise waise mere x displacements badh rahe hain. Isko main. Kahoonga exy. Ye mere shear displacement se. Aur ye ho jaayenge mere exy. To exy ko main kaise define karunga? Displacement along x direction which are changing jo change ho rha hai y direction ke hisaab se. To ye ho jaayega ∂u / ∂y. Similarly main ye xy likh liya maine abhi eyx likhte hain. Agar hum dekhenge y direction par displacement hai, mere ye y direction pe change ho rhe hai . To isko main mark kar lunga v se kyunki y direction ke hisaab se change ho raha hai displacement. Par ye jo displacement badh rahe hain, agar dekhenge v kis direction mein badh raha hai? Ye badh raha hai mera x direction ke hisaab se. To is hisaab se isko main kahunga eyx. Isko hum likhenge kaise? eyx ko likhenge displacement along y direction, or y axis, which are v, jo change ho rahe hain mere x direction ke hisaab se. To ye ho jaayenge mere ∂v / ∂x. To similarly hum ezx ko likh sakte hain. Displacement along x direction kya hogye mere ? u ho gye to isko ∂u likhenge. Aur kis direction se badhna chahiye vo? z direction se. z axis se badhne chahiye to ∂z. To exz ho jaayega ∂u / ∂z. To eij ko main is tarah se likhunga. Ye saare components jo hain wo is tarah se main likhunga: ∂u/∂x, ∂u/∂y, ∂u/∂z. Yahan par mujhe. Normal displacement mil jaayenge aur jo off diagonal hain wo mere shear displacement mil jaayenge. To main isko e11, e12, e13 is tarah se bhi likh sakta hoon. To aap dekhenge agar main isko x1, x2, x3 likhunga to ye jo displacement hai main inko is tarah se represent karunga. Aur agar main is tarah se jab represent karunga to kisi textbook mein ye displacement is tarah se bhi likhe jaayenge. Confuse nahi hona hai aapko. Sirf ye definition yaad karni hai. Agar ye definition pata hai to aap kisi bhi coordinate axis, kisi bhi reference axis ho, jiska representation hai, wo aap is tarah se likh paayenge.

To ye jo ho gaya mera displacement tensor, ye poora mera displacement tensor ho gaya. Koi bhi point ka agar main displacement kar raha hoon to mujhe jo displacement tensor milna chahiye wo main is tarah se likh sakunga. Ek cheez aap yaad rakhenge kyunki ye jo displacement hai, ye hum infinitesimal strain theory ke liye dekh rahe hain. Yaani strains jo develop ho rahe hain ya displacement jo hai wo bahut kam hai, chhote displacement hain. To isliye hum dekhenge ki ye jo definitions hain, ye saari infinitesimal strain theory ke liye hi available hain ya valid hain.

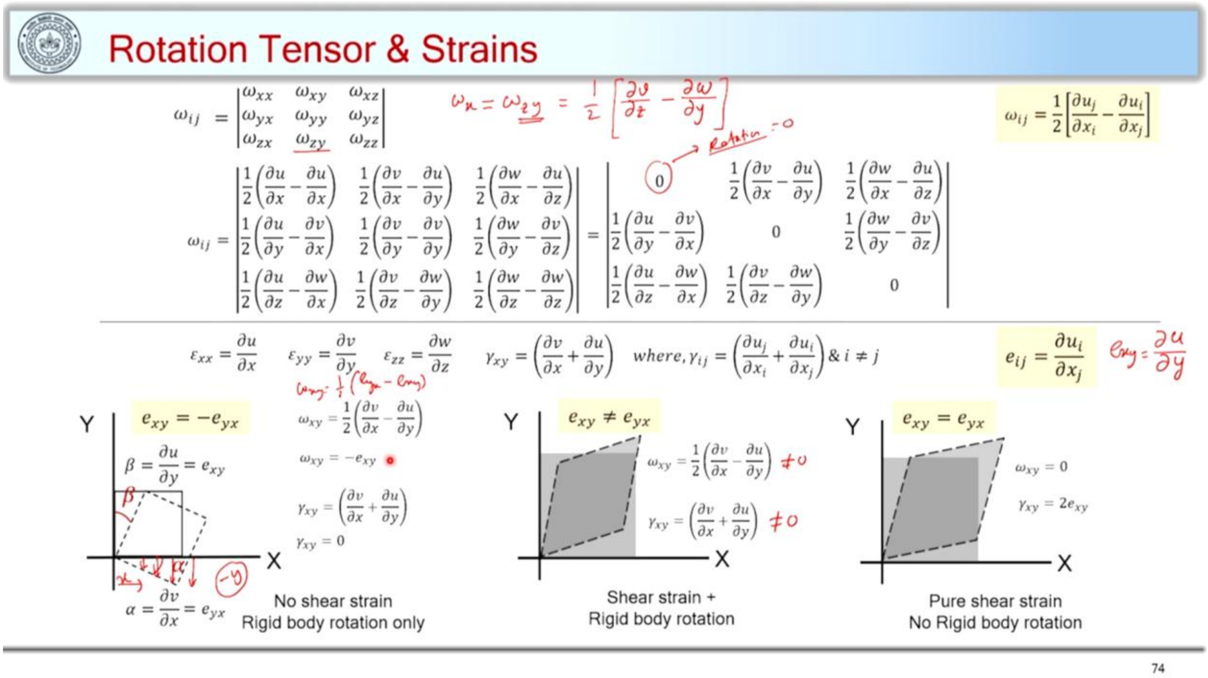


Abhi hum jaante hain ki rigid body rotation kya hota hai. To uske liye maan lete hain main element deta hoon x aur y. Direction mein ek 2D element hum consider karenge A B C D. Aur hum average rotation is element ka is tarah se define karte hain ki average rotation of the two perpendicular line segments. Iska matlab kya hai ki agar main is element ka rotation dekhunga A B C D ka, to mujhe agar do perpendicular line segments yahan par mark kar lein, maan lijiye A B aur A D. Ye perpendicular line segments hain, ek doosre ko perpendicular hain. To agar inka rotation ka average main nikaal loonga to ye ho jaayega is rotation ka average. To pehle sign convention jaan lete hain. Sign convention yaane agar mera rotation ho raha hai counter-clockwise to hum usko positive kahenge, aur ye jo line segment rotate ho raha hai clockwise to hum usko negative kahenge. Ye main sign convention yahan follow karunga jab rigid body rotation ki baat karunga tab. isko main is tarah se denote karta hoon ωz se. ωz isliye kyunki agar hum dekhenge ye x-y plane hai jo z direction ke perpendicular hai. To hum rotate krenge is element ko to ye rotate z axis ke along, yaani z axis ke reference se rotate hoga. To isliye hum isko ωz kehte hain. Ya hum is tarah se likh sakte hain isko ωxy. ωxy yaani jo rotation hai jo element ka hai wo x-y plane mein hai. To hum aage se is notation ko, ye two-suffix notation ko follow karenge. ωxy, kyunki humne stress tensor, strain tensor ya displacement tensor ye sab two-suffix notation mein denote kiye hain. To hum rotation tensor ko bhi ωxy yaani two-suffix notation se denote karenge. To ye jo ho gaya ωz ya ωxy, ye ho jaayega is element ka rotation with respect to z axis aur in x-y plane. to abhi dekhte hain isko hum is tarah se rotate ho gaya element main usko maan leta hoon A B' C' aur D' ke hisaab se abhi hum dekhenge ye definition ke hisaab se ωz hum is tarah se likh sakte hain ki mera A B rotate ho raha hai kaise rotate ho raha hai ye rotate ho raha hai A B' ke along yaani A B' ye rotation ke baad mil raha hai aur A D' mujhe rotation ke baad mil raha hai to ωz jo rotation hai is element ka main is tarah se likh sakta hoon pehle ye likhoonga ki A B ka rotation with respect to z axis aur A D ka rotation with respect to z axis in dono ko add karke unka average yaani ye do element hai to isko hum line segment hain to isko hum half kar denge to ye ho jaayega mera average rotation of A B C D Maan lijiye A B rotate ho raha hai α angle se aur A D rotate ho raha hai β angle se. To isko main likhunga. Agar isko mujhe jaanna hai to mujhe pehle α aur β nikaalne padenge. To main pehle nikaalta hoon ki A B ka rotation with respect to z axis. Isko hum maan lete hain $\tan\alpha$. To ye $\tan\alpha$ nikaalne ke liye mujhe ye geometry chahiye. Main isko thoda magnify kar leta hoon. Ye mera A hoga aur ye AB' ho jaayega rotation ke baad. Aur ye angle ho jaayega mera α. Agar main isko is tarah se extend kar loon, yaani B' par perpendicular bana loon AB ko aur isko extend kar loon, to mujhe kuch is tarah ka geometrical construction milega. Main ek line segment mark kar leta hoon BQ. Abhi $\tan\alpha$ mera ye ho jaayega P B' / A P. Ye $\tan\alpha$ ki definition ho jaayega. aur main is PB' / AP ko main is tarah se likh sakta hoon A B + BP aur phir agar main infinitesimal strain theory ki baat karoon to ye jo displacement hai bahut kam hai to main BP' ko consider karoonga equal to BQ yahan par main BQ maan ke chal raha hoon abhi ye jo distance hai A aur B ka ye mark kar leta hoon main Δx aur maanta hoon ki u aur v ye small displacement hai along x aur y direction to agar main yahan par dekhoonga to main BQ agar nikaalne ki koshish karoonga to aap dekhiye BQ ye jo yahan pe hai ye BQ is direction mein move ho raha hai yaani displacement agar main dekhoonga ye mere agar main BQ nikaalne ki koshish karta hoon to ye ho jaayega mera displacement along y direction to ye v ho jaayenge displacement aur ye kis kis direction ke saath change ho rahe hain ye change ho rahe hain mere x direction ke hisaab se to agar main nikaaloon to ye displacement main is tarah se nikaal sakta hoon $\partial v / \partial x$ aur ye total distance jo hai Δx inse multiply karoonga to mujhe B K ki length mil jaayegi $\partial v / \partial x$ into Δx usi tarah se AB ki value kya hai AB ki value to Δx hai aur agar hum ye nikaalne ki koshish karenge BP. BP agar nikaalne ki koshish karenge to agar ye dekhenge to ye mere displacement ki direction mein yahan se is direction mein hai ye u displacement hai ya along x direction to isliye u maan ke chal raha hoon aur agar mujhe change pata chalenge ki ∂u small x direction par kitna change ho raha hai aur ye total distance mujhe pata hai to main B p nikaal sakta hoon $\partial u / \partial x$ into Δx humne dekha bhi tha normal strains hum jab nikaal rahe the aur Δx mera positive hai kyunki distance hai to ye cancel ho jaayega yahan pe aur mere paas ye term bachegi $[(\partial v / \partial x) / \{1 + (\partial u / \partial x)\}]$ abhi hum infinitesimal strain theory ki baat kar rahe hain to hum jaanenge ki ye jo one hai ye bahut zyada hoga ye to fraction mein hoga bahut kam hoga to in dono ka summation agar karenge $(1 + \partial u / \partial x)$ wo one ke kareeb hi hoga to ye denominator wala term one ho jaayega to mere paas na α ki value aa jaayegi $\partial v / \partial x$ abhi hum small strains ki baat kar rahe hain to jo α hai jo angle hoga wo bhi bahut chhota hoga to main $\tan\alpha$ ko α likh sakta hoon to α is equal to $\partial v / \partial x$ aa jaayega to ωz ka jo ye term hai iski value aa jaayegi $\partial v / \partial x$ similarly main is tarah se dikha sakta hoon $\tan\beta$ $\tan\beta$ mera ye hai aur β is tarah se move jo displacement hai wo is tarah se hai to ye displacement se along x direction to mere u displacement ho gaye aur kis direction par change ho rahe hain ye along y direction par change ho raha hai yaani badh raha hai to main β ko $\partial u / \partial y$ likh sakta hoon aur ye jo angle hai agar hum ye humara sign convention follow karenge to ye jo change in angle hai line segment A D ka ye isko main $-\beta$ likhoonga kyunki ye change ho raha hai ye jo angle hai AD se A D' jab ja raha hai to mujhe clock wise move karna hai to ye negative value main consider karoonga A B se main AB' ja raha hoon to ye counter clock wise hai to main usko positive value consider karoonga to main ωz ya ωxy jo rotation hai component is element ka to main usko is tarah se likhoonga definition ke hisaab se to ye mere paas aa jaayega $\omega xy =$

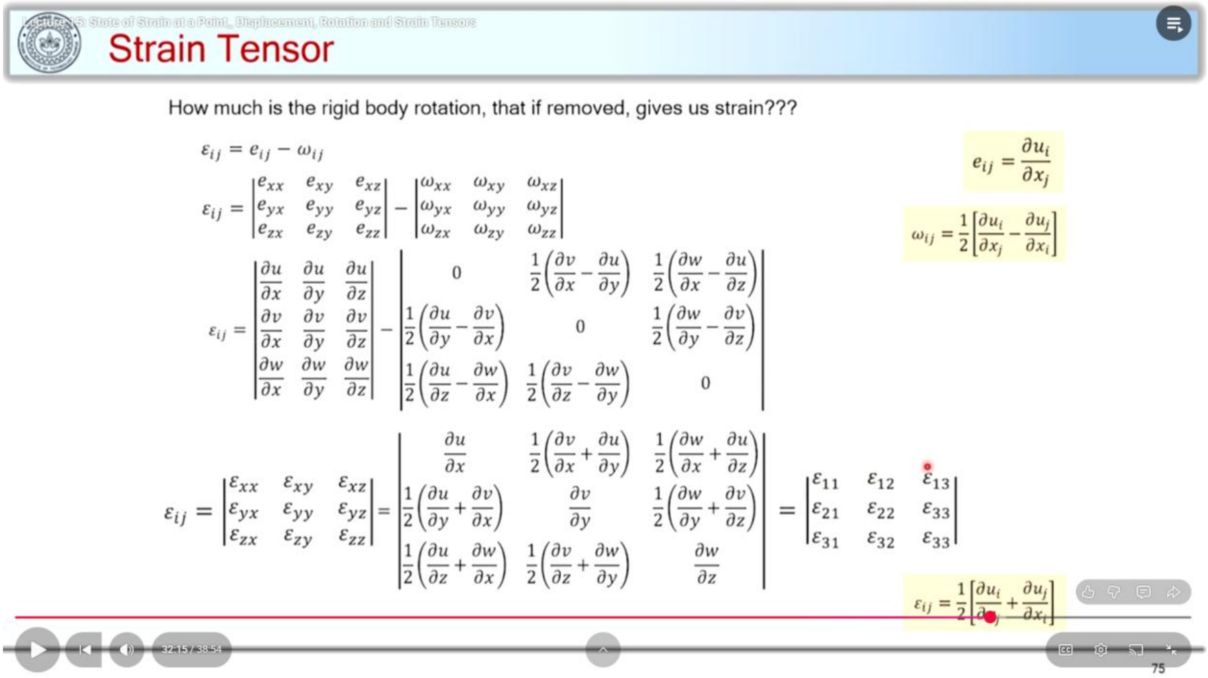
1/2 (α – β) ye mera rotation ho gaya is element ka in xy plane mein. To ω_{xy} ko jab main displacement ke hisaab se likhunga, to main isko likhunga 1/2 ($\partial v/\partial x - \partial u/\partial y$). Aur agar main einstein notation ke hisaab se likhoonga to ω_{ij} ko main likh sakta hoon yahan par aap dekhiye jab main ω_{xy} likh raha hoon to ye jo displacement hai pehla displacement hai humara y direction ke hisaab se aur change ho raha hai x direction ke hisaab se to ye ulta ho raha hai to aap dekhiye jo main ω_{ij} ko likh raha hoon $\partial u_j / \partial x_i - \partial u_i / \partial x_j$ ye mera jo notation hai einstein notation hai to ye humne dekha ki component hai rotation tensor ka



to hum rotation tensor ko phir se likhenge to rotation tensor ko main ω_{ij} se abhi represent kar raha hoon aur ye mere nine components mujhe milenge aur ye nine components main is definition ke hisaab se yahan par main likhunga to maan lete hain mere paas main maan leta hoon main ω_{zy} nikaal raha hoon to ω_{zy} kya hai ω_{zy} hai mera element ka rotation in zy plane to isko main ω_x bhi likh sakta hoon aur ye ho jaayega mera average to ye ho jaayega pehle ∂v ye displacement along y direction with respect to displacement along z direction − ∂w ye displacement along z direction upon displacement along y direction to hum dekh paayenge ki ye is tarah se hum likhenge saare components ko to ye ye mera ω_{zy} ho jaayega to is tarah se hum saare elements likhenge aur hum solve karenge to hum dekhenge ki ye jo normal aur diagonal components se saare zero aayenge to ye hum dekhenge jab yahan pe normal displacement the to rotation yahan pe zero hona chahiye aur jo non diagonal elements hain wo non zero rehta hai rotation jab normal displacement hai tab mere paas rotation zero hota hai to abhi hum dekhte hain ki strain components hum likhte hain Humne ε_{xx} likha tha $\partial u/\partial x$, $\varepsilon_{yy} = \partial v/\partial y$ aur $\varepsilon_{zz} = \partial w/\partial z$. to ye humne last part mein dekha tha ki normal strain kaise define karte hain. Aur shear strain, jo engineering shear strain hai, humne define kiya tha jaise $\gamma xy = \partial v/\partial x + \partial u/\partial y$. Generalized form mein hum isko likh sakte hain γij, jahan $i \neq j$. Ye definition humne engineering shear strain ke liye dekhi thi. Abhi hum dekhte hain ki ek element lete hain aur dekhte hain ki rotation aur strain kya develop ho raha hai material mein ya element mein. Maan lijiye mere paas ek element hai aur ye element ko main deform ho raha hai is tarah se deform ho raha hai. To ye angles main mark kar loonga α aur β. Jab main angles mark karunga α aur β, to ye β maine dekha tha $\partial u/\partial y$ ho jaayega. Kyunki ye jab β ho raha hai to ye x direction par displacement hai aur jo increase ho raha hai y direction ke hisaab se. To ye $\partial u/\partial y$ ho jaayega. A ho jaayega mera $\partial v/\partial x$. To main eij ko is tarah se likhunga $\partial ui / \partial xj$. Ye mera displacement definition rahega. To ye agar main dekhoonga is definition ke hisaab se to ye $\partial u/\partial y$ kya hoga? Mera exy ho jaayega. Agar main yahan par exy ko likhne ki koshish karunga is definition ke hisaab se, to ye displacement hoga along x direction which are increasing with respect to y direction. To $\partial u/\partial y$ ho jaayega, jo ki mera exy hoga. Similarly ye agar main dekhoonga to ye ho jaayega eyx. Aur abhi hum aur ek scenario dekhte hain. Ye element is tarah se deform ho raha hai aur ye element is tarah se deform ho raha hai. Aur dekhte hain ki is condition mein gar hum dekhenge isko hum e − e y x likh sakte hain − e y x isliye kyunki agar aap dekhenge ye jo displacement hai yahan par ye is direction hai negative y direction par ho raha hai aur jo badh rahe hain x direction ke hisaab se to isliye jo displacement hain negative honge to isliye hum isko likh sakte hain exy = − eyx. aur is condition mein hum dekhenge exy is not equal to eyx. Aur is condition me likhenge exy is equal to eyx. Ye saare mere displacement hain. Abhi hum rotation dekhenge is element ka. To rotation ki definition ke hisaab se, ye agar hum dekhenge, ye x–y plane mein dekh rahe hain. To isko hum likhenge $\omega xy = 1/2\ (\partial v/\partial x - \partial u/\partial y)$. To agar hum solve karne ki koshish karunga. To mere paas ωxy ko main is tarah se likh sakta hoon. 1/2 $\partial v/\partial x$ aayega mera eyx, aur ye jo aayega $\partial u/\partial y$, ye aa jaayega exy. To agar main dono ko ek hi notation mein convert karoon, to main ωxy ko −exy likh sakta hoon. Kyunki mera eyx ye jo hai, wo eyx jo hai −exy hai. To ωjo rahega wo −exy aa jaayega. Agar main shear strain ki baat karoon is definition se, to shear strain mera kya aayega? Ye mera aayega zero. To hum dekh pa rahe hain yahan par material ka aur jo element ka hai, wahan par shear strain develop nahi ho raha hai, sirf rigid body rotation ho raha hai. Kyuni α aur β yahan par same rahenge aur ek hi direction mein ho rahe hain. To hum dekhenge ki ye rotation ho raha hai aur yahan par koi shear strain develop nahi ho raha. Agar hum dekhenge ωxy yahan par, to ye mera definition hai. exy ≠ eyx, to ye non-zero aayega. Aur ye jo dono terms hain, ye dono non-zero aayenge. To inhein mark krlete hai ki ye non zero aenge. mein mere paas is condition me mere pass strain bhi develop ho raha hai aur rigid body rotation bhi ho raha hai. Is condition mein hum dekhenge ω_{xy} agar hum dekhenge $e_{xy} = e_{yx}$, to ye term ω_{xy} zero ho jaayegi. Aur is condition mein γ_{xy} ho jaayegi 2 e_{xy}. To yahan par humare paas sirf pure shear strain milega aur yahan par koi rotation nahi hoga. Kyunki ω_{xy} shoonya hai yahan par. To isliye ω_{xy} shoonya hai yahan par koi rigid body rotation nahi hoga, sirf shear strain rahega. Agar hum dekhenge is case mein bhi ye rotation jo ho raha hai, wo mera ye rotation ho rha mera clockwise direction mein. To displacement ke sign ke hisaab se hum dekhenge to minus sign aana chahiye. To yahan par minus sign dikh raha hai. To humara sign convention yahan par bhi follow ho raha hai. To ye ho gaya mera rotation aur shear strain. Jab main deformation condition ki baat karta hoon,

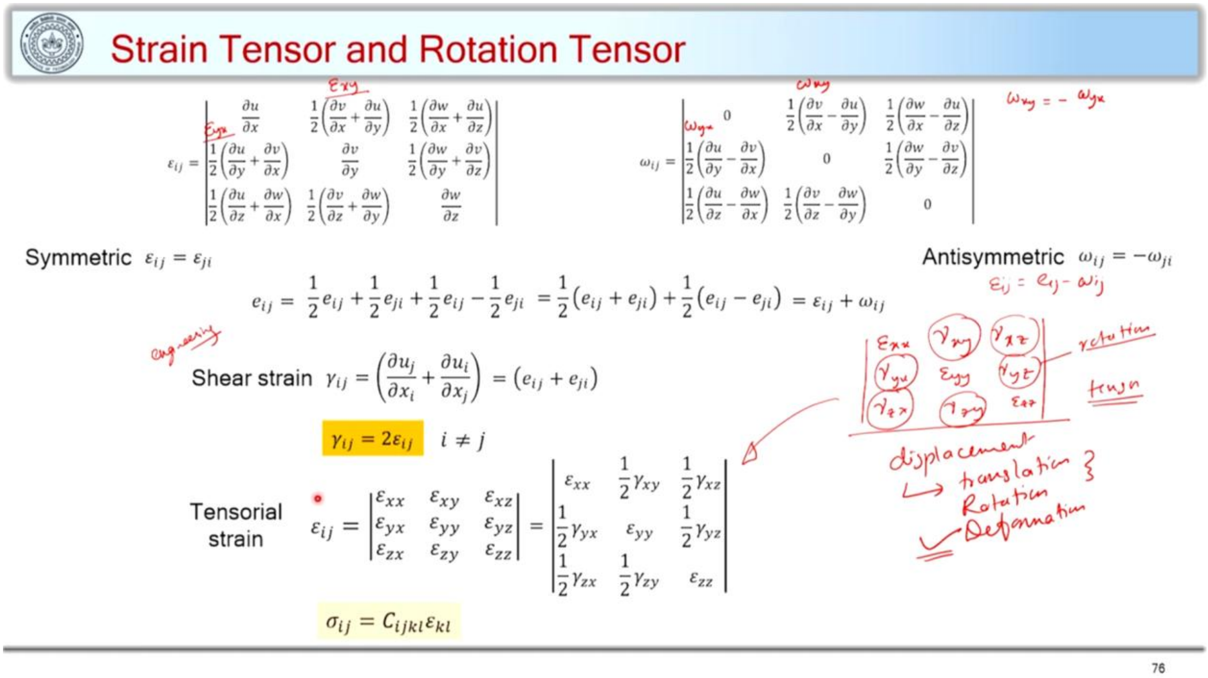


to abhi humein nikaalna hai ki strain tensor jo bhi strain values hain, wo kaise nikaalte hain. Ye dekhenge ki jab hum deformation ki baat karte hain, to jitna hum rotation remove karenge displacement mein se, to humein strain milega. To isko main ε_{ij} ke hisaab se mark karoonga. Ye main define krta hu e_{ij}, ye mera displacement tensor hai aur ye mera rotation tensor hai. To jo main rotation tensor displacement tensor se subtract karoonga, to mere paas strain tensor rahega. To isko hum likhte hain. e_{ij} ke jo components hain, wo displacement components hain. Aur ye rotation components hain. main subtract karoonga, to mere paas definition se e_{ij} aur ω_{ij} hai, is hisab se main sare components likh lunga to ye sare components maine 9 components displacement tensor ke likh liye, 9 components maine rotation tensor ke likh liye. Jb main substract krunga to mere pass jo bachega mera wo bachega strain tensor. To ye strain tensor mere paas hai Iske nine components hote hain. Ye jo normal strain aur shear strain hote hain. To kabhi agar main isko subtract bhi krunga general form mein agar Einstein notation mein likhoon, to mere paas ε_{ij} ki definition aa jaayegi: $1/2\ \{(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)\}$. To ye ho gaya mera strain component, strain tensor. To abhi humne dekha tha ki displacement jo hai usme se mein agar rotation nikaalunga, to jo bachta hai wo mera strain rhega, strain part rhega . Books mein is tarah se bhi likha jaata hai. Jab main x_1, x_2, x_3 coordinate axis likhoonga, to mere paas strain components main is tarah se likhunga.



Abhi hum aage badhte hain aur strain tensor aur rotation tensor ko dekhte hain. To mere paas ye strain tensor hai jo humne nikaala hai aur ye rotation tensor hai jo humne nikala. Agar aap dekhenge yahan par to ye mera hojaega ε_{xy} aur ye ho jaayega mera ε_{yx}. agar hum dono ka, dono ko dekhenge to ye dono same hain. Par is case mein agar hum dekhenge, agar main phir se likhoonga ω_{xy} aur ω_{yx}, to mujhe milta hai $\omega_{xy} = -\omega_{yx}$. To agar hum dekhenge ye dono tensors, to ye jo tensor hai, strain tensor hai, ye symmetric tensor hai. Agar aap dekhenge jo off-diagonal components hai ε_{ij} is equal to ε_{ji} pr is case me ye aisa nhi hai. Yaha aap dekhenge rotation me ω_{ih} is equal to $-\ \omega_{ji}$. To isko hum antisymmetric tensor kahunga ya skew-symmetric tensor kehte hain. To main jab displacement tensor likhta hoon, to ek jo tensor ka postulate hai do tarah se divide kar sakta hoon. ek symmetric tensor aur ek antisymmetric tensor ke hisaab se to main isko is tarah se likhoonga $1/2\ e_{ij} + 1/2\ e_{ji} + 1/2\ e_{ij} - 1/2\ e_{ji}$ agar hum dekhenge to main in dono ko combine karke is tarah se likh sakta hoon aur in dono ko combine karke is tarah se likh sakta hoon to agar aap dekhenge ye jo definition hai ye definition mera ε_{ij} ka hai yaani mera strain tensor ho jaayega aur ye jo definition hai mera

ω_{ij} ka hai to humne last slide par dekha tha ki $\varepsilon_{ij} = e_{ij} - \omega_{ij}$ to ye summation hum is tarah se bhi likh sakte hain jagah agar humare paas displacement tensor hai to ek baat hum dekhenge ki jo shear strain hai jo shear strain humne define kiya tha γ_{ij} ye is tarah se define kiya tha aur ye agar hum dekhenge to ye ye part mera e_{ij} ho jaayega aur ye jo part hai mera e_{ji} ho jaayega to aap dekhenge ye agar jab main shear strain likh raha hoon ye shear strain humne is tarah se bhi define kiya tha engineering shear strain define kiya tha to ek humein identity milti hai ki ye jo γ_{ij} hai ye is equal to two ε_{ij} ye jo mera hai ye mera tensorial part hai strain ka aur ye jo part hai mera shear strain hai jo isko main engineering shear strain bhi kehta hoon to ye jab ye rahega jab i is not equal to j to ye ek identity hai ye hum hamesha use karte hain to jo tensorial strain hai to uske components main is tarah se likh raha hoon nine components hain mere paas aur isko main agar engineering shear strain ya shear strain ke part mein likhoonga to mere paas ye jo off diagonal hai ε_{yx} ye ho jaayega mera $1/2\ \gamma_{yx}$ to jo non diagonal terms hain jo shear agar engineering shear strain ke hisaab se agar main ye tensor likhoonga to mujhe is tarah se likhna padega to aapko dhyaan rakhna hai ye jo books mein ye interchange karke likhte hain to ye kyun chahiye humein kyunki jab main stress ko relate karoonga strain se to mujhe ek constant milega jo C_{ijkl} ye jo fourth order tensor hai ye ye mera elastic modulus ko define karega to ye mera generalized Hooke's law hai to abhi aapko ek baat hamesha dhyaan rakhni hai ki abhi main ek strain tensor likhne ja raha hoon kisi books mein ye aise bhi likha rehta hai ε_{yy} γ_{yz} aur γ_{zx} γ_{zy} ε_{zz} ye agar likha rahega to isko main directly ye jo strain tensor hai isko main directly stress tensor se co relate nahi kar sakta hoon mujhe stress tensor ko is is is quantity ko ismein ya strain tensor mein hi convert karna padega is part mein convert karna padega ya is part mein convert karna padega to ye mera tensor part nahi hai ye mera tensor nahi hai kyunki yahan par ye jo term hai γ_{xy} γ_{xz} non diagonal jo terms hain jo shear strains hain mere ye ye ismein rotation involve hai mujhe jab main rotation subtract karoonga tab mujhe ye term milegi aur tab jaakar main ye jo strain tensor hai isko main stress tensor se compare karoonga is fourth order tensor se jo hum baad mein dekhenge jab stress strain relation dekhenge tab is part mein humne dekha ki mere paas displacement hai jab main deform karta hoon material ko mere paas displacement milte hain points ke us displacement mein mere paas teen components hote hain ek rigid translation yaha pr likh lete hai isko usmein mere paas teen components milte hain ek translation doosra rotation aur teesra deformation to humein interested hain is part mein deformation mein kyunki ye dono deformation nahi karte to jab hum displacement ki baat karenge to humein ye dono isse subtract karne padenge tab humein deformation milega to translation to humne as such dekha tha par abhi rotation ko subtract kiya displacement se to humein deformation mila to ye jo deformation jab milega to humein strain tensor ke hisaab se milega aur hum ye strain tensor is stress tensor ke saath compare kar paayenge.



To abhi ke liye main yahan par rukta hoon. Next part mein hum dekhenge ki stress aur strain ke relations kya hain. Dhanyavaad.