Mechanical behavior of materials

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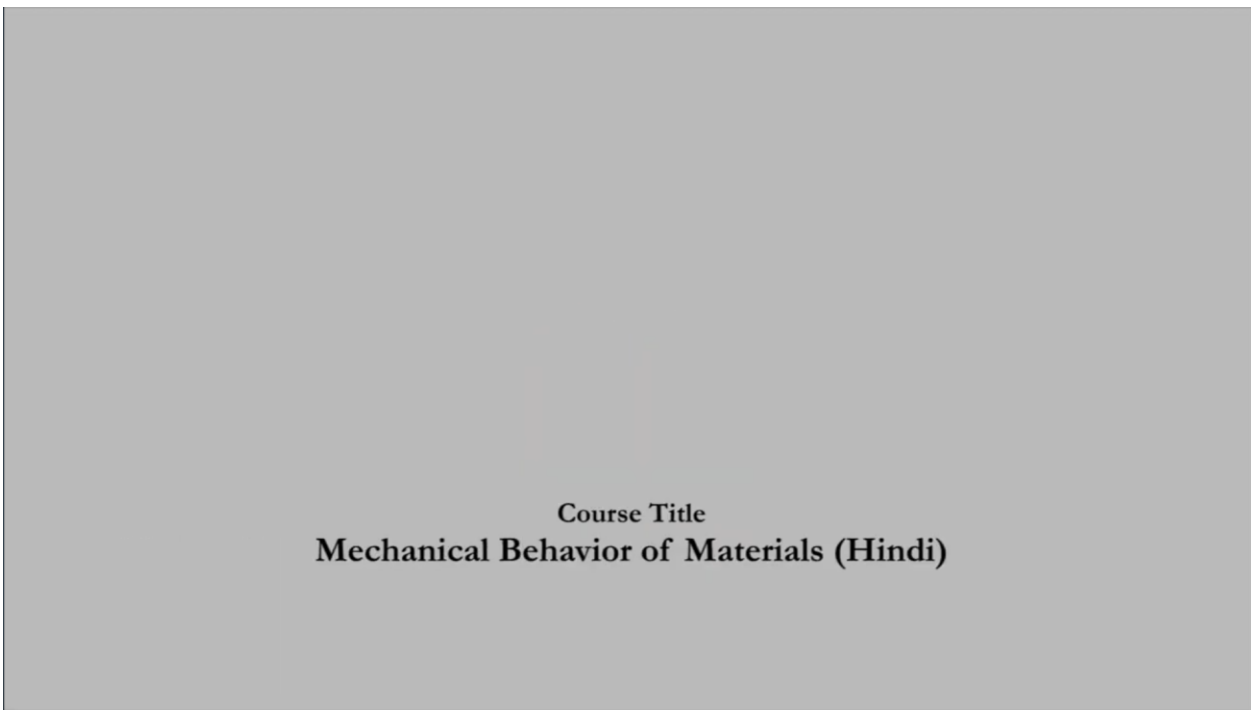
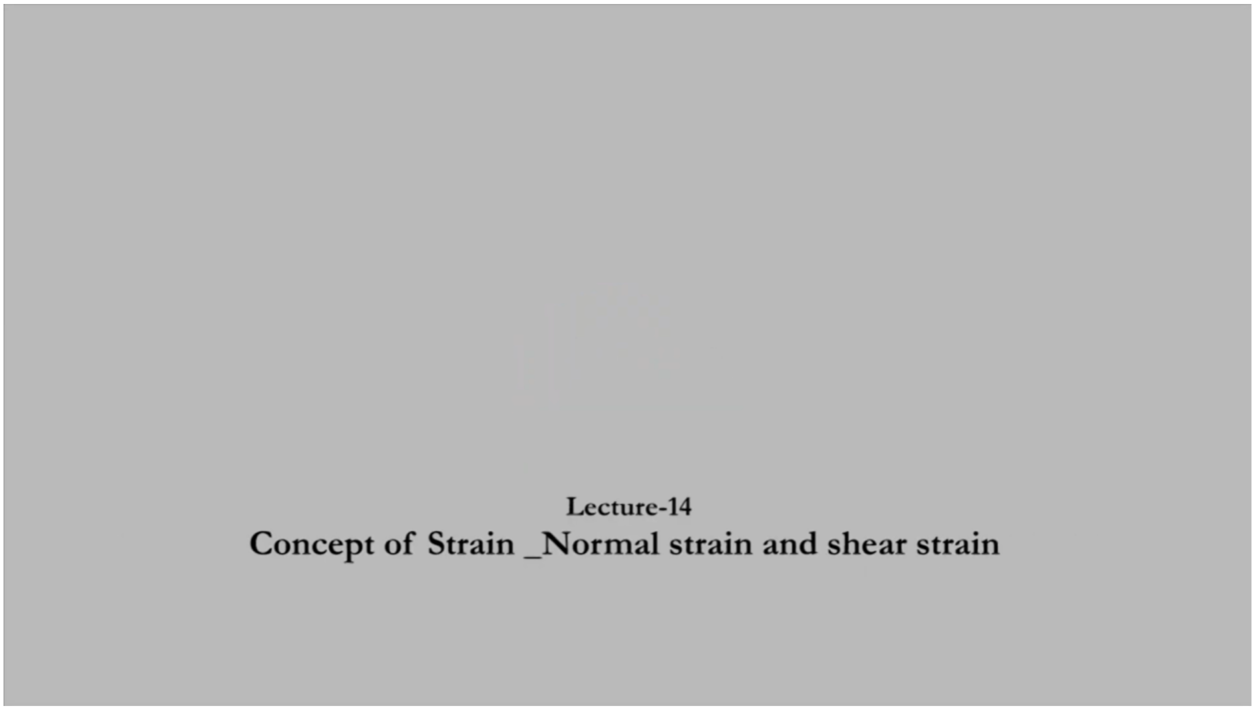
Week-2

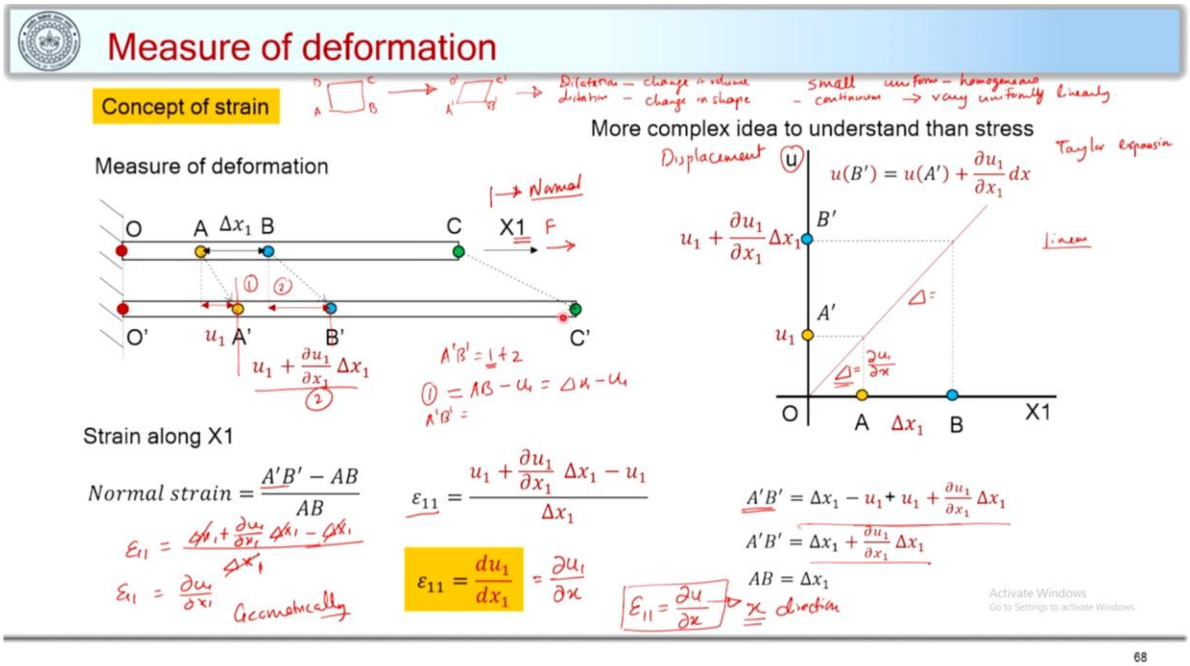
Lecture-14

Concept of Strain_Normal Strain and shear Strain

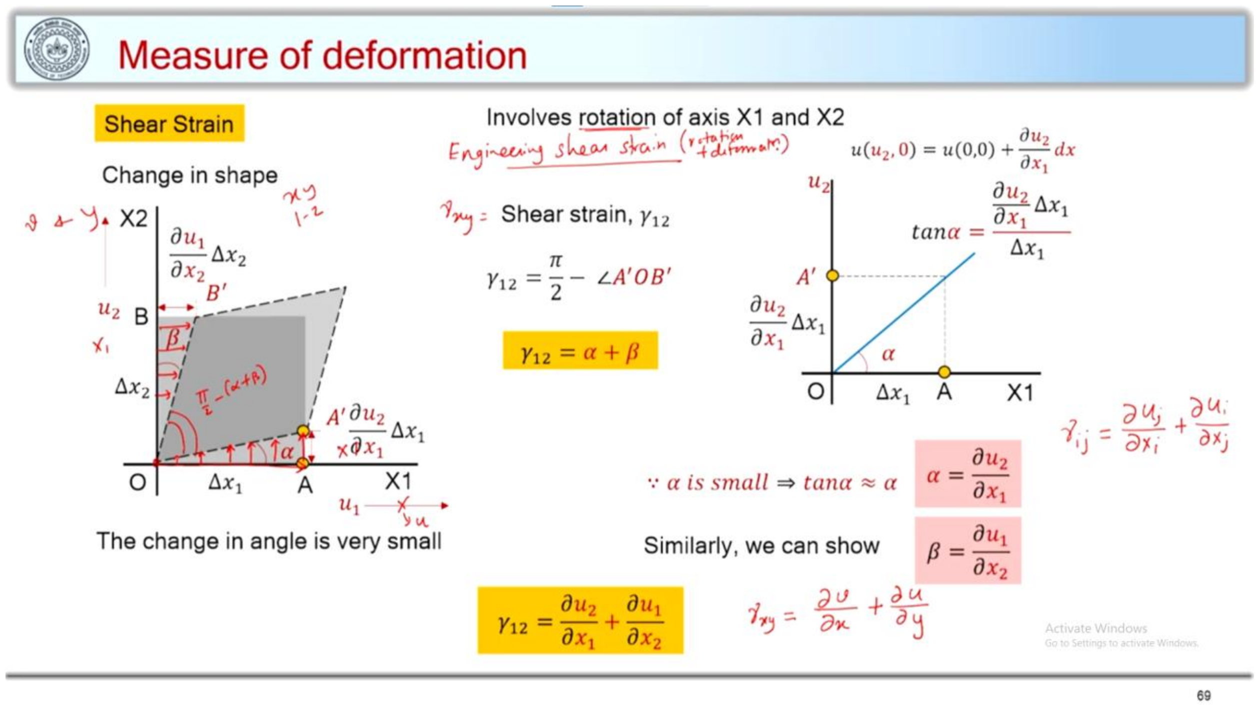




Namaskar, aapka swagat karta hoon is course mein jiska naam Mechanical Behaviour of Material hai, jisko hum Hindi mein padhenge. Is part mein hum measure of deformation, jo ki strain hai, uske baare mein jaanenge. To humne abhi tak stress ke baare mein jaana tha, ab strain kya hote hain iske baare mein jaanenge. To pehle concept of strain dekhte hain. To iske liye aise samjhiye ki mere paas ek element hai, isko main mark kar leta hoon A B C D, aur ye deform ho raha hai. Humne bola tha ki ye deform ho raha hai, deformation is tarah se ho raha hai A' B' C' aur D'. To ye jo deformation hai. Ye do tarah ke ho sakta hai. Humne last part mein bhi dekha tha ki ye dilatation ho sakta hai, yaani change in volume, ya ye is tarah se ho sakta hai, ya distortion ho sakta hai. Distortion ko hum bolenge change in shape. Iske baare mein hum jaanenge. Par aap maan ke chaliye ki yahan par agar main ye chhota sa ek element nikaal raha hoon aur ye deform ho raha hai, to ismein jo point A B C D hain, ye thoda alag tareeke se deform hote hain. To concept of strain samajhna thoda complex ho skta hai, pr is course me hum ye janege ki ye jo deformation hai yaani jo strain hai ye material ka bahut small hai. Isko hum phir se likh lete hain. Ye jo strains hain, ye small hain. Ye uniform aur homogeneous bhi hain. Aur hum jo material yahan par ya jo bhi element hum consider karenge ya body consider karenge, wo continuum body hai hamari. Aur ye jo strains hain, ye vary karenge, yaani badlenge uniformly, linearly also. Isko hum samjhenge is part mein. To jaise maine abhi concept of strain samajhne ki koshish karenge. Isko measure of deformation bhi kehte hain, kyunki jab humne stress–strain curve dekha tha, to ye jo deformation hai, ya elastic deformation ya plastic deformation, iska jo measure hai, iska jo quantification hai, hum strain ke dwara hi karte hain. To abhi hum isko aage dekhte hain. To ye thoda complex idea hai. Stress se samajhne mein. To ye complex kyu hai iska hum Chhota sa udaharan lete hain. Jaise mera ek member hai. Ye ek member hai. Ek side pe hinged hai is end pe, aur let's say ye point O pe hamara fix hai, move nahi ho sakta. Aur isko hum x1 direction ke along ek force lagayenge, tensile force lagate hain. To iska ek deformation hoga. Par pehle usse pehle hum mark kar lete hain kuch points is member pe. Uska naam rakh dete hain hum A, B aur C. Abhi hum dekhenge agar yahan par force lag raha hai x1 direction par. Isko mark kar lete hain. Let's say mere paas ek tensile force lag raha hai. To is tensile force ke wajah se ismein deformation aayegi, elastic deformation aayegi ya plastic deformation aayegi, jo bhi deformation aayegi. To wo hum dekhenge. Ye force agar mera lag raha hai to ye elongate hoga, yaani iska length badhega. To maan lete hain iska length is tarah se badh gaya. Abhi hum ye jo inka movement kaise hua is body par ye jaante hain. To maan lijiye mera point O ye to fix hai, to ye change nahi hoga. Mera point A yahan par move hua is tarah par. Mera point B aur zyada elongate hua, aur zyada displaced hua hai. Aur mera point C sabse zyada displaced hua hai. Agar aap dekhenge, agar ye saare points is member par hain, ye jo deformation hai ya iska displacement hai in points ka, ye point O pe sabse kam, yaani shoonya hai yahan par, aur point C par sabse zyada hai. Point B pe, point A ki tulna se zyada hai, par C ki tulna se kam hai. To mark kar lete hain kuch points inke. Ye jo displaced points hain, hum inko mark karenge O', A', B' aur C'. To agar main x1, ye jo points point ke agar location mein mark kar raha hoon, aur ye jo displacement hai, displacement yaani point jo move ho raha hai, to main isko displacement keh raha hoon ki mera displacement kya hai. To maan lete hain ye mera point O hai, jiska displacement shoonya hoga. Ye x1, ye mera location hai is point par. Aur ye u jo hai Y axis par, ye mere displaced points ke location honge. To AB agar maine mark kar liya yahan par point, to let's say A B ke beech ka distance mera Δx1 hai, aur ye jo point A jo mera displaced hua hai, ye u1 displaced hua hai. Abhi hum dekh sakte hain ki ye jo displacement hai, point A, B, C ke jaise ki A', B', C' ye humare points jo displacement hain, ye linear vary ho rahe hain. Yaani iska matlab kya hai, jaise mera point yahan pe hai O ke kareeb, to iska displacement kam hoga. Ye point C jo hai, iska displacement sabse zyada hoga. To aap dekhenge ki iska jo variation hai displacement ka is member pe, wo uske length ke proportional hai. To wahi hum bolenge ki ye jo displacements hain, ye proportional hain, kyunki length ke anusaar ye jo point jahan ka jo location hai, uske anusaar ye vary honge. To agar mera point O hai, ya agar main yahan par maanoonga, to yahan par koi displacement nahi hai, yahan par shoonya displacement hai. Jaise jaise main aage badhoonga length ke along, waise-waise mera displacement badhta jaayega. To ye mera agar maan lete A ' jo displacement hai, u1 hai. Yaani ye mera reference point hai. Agar mera initial position tha A ka, to ye jo displacement hai wo A' ho jaayega. To yaha pr agar hum mark karenge to A ke corresponding displacement hai vo mera u1 hai. To ye humne u1 mark kr liya yah pe. Abhi hum nikaalenge ki point B ke corresponding kya displacement hai? Usko nikaalne ke liye hum kya kr sake hai. Hum mark kr lenge ki ye jo displacement hai mera, ye linear vary ho raha hai. To ye jo displacement hoga ye mera B', agar yahan par maine mark kar liya, to hum isko simple tareeke se is tarah se nikaal sakte hain. Jaise ye Taylor expansion ki tarah se hum likh sakte hain aasani se. To ye ho gaya mera displacement at point B'. Isko likhenge hum displacement at point A', yaani ye jo displacement hai, plus ye jo change hai, variation. Isko is tarah se hum samajh sakte hain. Agar ye slope agar main nikaal loonga, ye slope agar main nikaal loonga yahan ka, to ye slope, ye jo variations hain displacement ke, ye linear variations hain. To isliye ye agar slope main nikaal leta hoon, to ye slope aur ye slope to same rahega. To ye slope kya aayega? Ye aayega slope mera small change u1 mein corresponding to x direction to ye aajaega mera, ye aajaega mera slope. To ye jo point hoga, is point ko main is tarah se likh sakta hoon. Ye u(B') mein u(A') hai mera u1, yaha pr maine u1 likh liya. Aur ye jo slope main is tarah se nikaaloon, ye aur ye slope to same rahega. To ye term mera same rahega. Par yahan se yahan tak jaane ka distance jo hai, ye mera $\Delta x1$ hai. To yahan par dx jo rahega, ye $\Delta x1$ rahega. To B' point ka jo displacement hai, wo main A' ke hisaab se likhoonga. To ye is tarah se main Taylor expansion ki tarah se likh sakta hoon. To ye linear response theory mein bhi humne dekha tha. Abhi ye jo displacement hai B ka maine mark kar liya. B' tak ka ye aayega $u1+\{(\partial u / \partial x1) \times \Delta x1\}$. Abhi humein agar strain nikaalna hai, to hum strain is direction mein nikaalenge, x1 direction mein, jisko hum kahenge normal strain. Normal strain isliye kyunki ye jo force lagaya hai, ye tensile force hai, ye normal force tha. To isliye hum isko normal strain kahenge. Aur strain ki simple vyakhya humne dekhi thi ki change in length / original length. To change in length kaise nikaalenge? Ye mera A' B' hai, ye mera after deformation hai, aur ye mera initial condition hai, A B ka length. To A' B' − A B mujhe dega change in length, / original length. Original length mera A B hai. To abhi dekhte hain ki A' B' hum kaise nikaalenge. To A'B' kya hoga? A'B' ka distance, yaani ye jo hai, ye jo corresponding displacement hai B' ka, ye hoga mera $[u1 +\{(\partial u1 / \partial x1) \times \Delta x1\} - u1]$. To ye term hoga mera change in length total, upon initial length. Initial length A B ka hai, $\Delta x1$. To ye jo dekhenge hum, ye initial length hamara $\Delta x1$ hai. To ye ho jaayega mera normal strain. To isko main $\varepsilon 11$ likh raha hoon. Isko $\varepsilon 11$ isliye likh raha hoon kyunki ye one dir ection mein hai, aur ye one plane jo hai perpendicular plane hai. To is direction mein normal strain hai ye. To isliye hum isko $\varepsilon 11$ likhenge. Iska bhi hum dekhenge ki kaise represent karte hain. Ye sirf strain ka concept samjhaane ke liye main abhi yahan par bol raha hoon. To isko aur ek tarah se hum likh sakte hain. To $\varepsilon 11$ kya aayega? $\varepsilon 11$ aayega mera $(\partial u1 / \partial x1)$. Aur isko is tarah se likh sakte hain, partial derivative mein. Hum isko is tarah se bhi likh sakte hain. $u1 / \Delta x1$. Main isko yaha pr $dx1$ hi likh raha hoon. To $\varepsilon 11$ ko main $\partial u1 / \partial x1$ likh raha hoon. To ye jo displacement hai, u jo displacement hai, ye jo displacement hai along x direction hai. Ab abhi isko geometry se bhi dekh sakte hain. Agar main A B' nikaaloon, to AB' agar main nikaalne ki koshish karoonga, to AB' mera ye distance hai. To maan lete hai ye mera one hai. To AB' kya hoga? One plus two hoga. Aur one kya hoga? To one agar main nikaalne ki koshish karoon, to one hoga mera $AB - u1$. A B mera $\Delta x1$ hai, $- u1$. To ye ho jaayega mera one. Aur two jo hai, wo two mera part hoga. Aur in dono ko main add karoonga. one aur two ko add karoonga to mere paas aa jaayega AB', jo ki mera ye length hai. Ye length. To isko agar main normal strain ko likhoonga, $\varepsilon 11$, to $\{\Delta x1 + (\partial u1 / \partial x1) \times \Delta x1\} / \Delta x1$, aur AB hai mera $- \Delta x1$. To ye ho jaayega mera poora strain. To $\Delta x1$ to positive hai aur shoonya nahi ho sakta, kyunki ye distance hai dono ke beech ka. To ye cancel out ho jaayega. To mere paas bachega $\partial u1 / \partial x1$. Ye humne dekha ki geometry kaise likh sakte hain strain ko. To isko hum normal strain kahenge. Aur normal strain humein milega $\partial u1 / \partial x1$. u jo hai, ye displacement hai koi bhi point ka along x direction.



Abhi hum aage badhte hain. Ye to humare normal strain hue. Abhi hum dekhenge ki shear strain kya hota hai. To shear strain jo hota hai, jo shear stress se develop hoga, wo shear strain hoga hamara. To isko hum jaanenge ki jo shear strain hai, ye important hai, kyunki ye hamara jo element hai, uska shape change karta hai. To isliye deformation mein shear strain bahut important hai. To agar hum dekhenge x1 aur x2 hum plot karenge. Ek element lete hain, aur is element mein mark kar lete hain do point A aur B. ye orthogonal hai points A aur B ye jo element hai is tarah se ek to cube consider kr skte hai ya rectangle consider kr skte hai. Hum general case rectangle consider krte hai aur maan lete hai ki ye jo original se hai A ka distance hai $\Delta x1$ aur B ka distance hai $\Delta x2$. Abhi hum shear force agar apply krenge is element ko to ye deform hoga. To maan lete hai ye is tarah se deform hoga. To ye jo deformation hai aap dekhenge ki main isko is tarah se bhi bol sakta hoon ki ye jo shape change ho raha hai mera ye do angles mark kar leta hoon, is x1 aur x2 direction ke hisaab se, α aur β. To ye jo angles mark kiye maine, ye thode zyada exaggerated way se dikhaye, bahut bade dikhaye yahan pe. Par actually hum small deformation consider karenge. To angle jo hoga wo bahut small rahega, jo angle change hoga wo bahut hi kam hoga. Aur jab hum angle change ki baat karte hain, tab yahan par hum ek aur cheez dekh sakte hain ki ye jo element hai, ye rotate ho raha hai x1 aur x2 ke hisaab se. Isko bhi hum jaanenge. Par abhi ke liye samajh lete hain ki shear strain ka hum definition jaan lenge. To ye jo deformation hai, yaani shape change hai, ismein ek rotation term bhi involved hai. To abhi hum shear strain define karte hain. γ_{12}. γ_{12} isliye kyunki main yahan pe dekh raha hoon ki ye mera x1 direction hai aur x2 direction hai. Aur ye jo element hai, is plane mein deform ho raha hai, ya one-two plane mein deform ho raha hai. Isko main is tarah se bhi dekh sakta hoon. Ye x1 nothing but hum isko x keh sakte hain, aur isko y keh sakte hain. To ye plane jo hai, wo x-y plane hai y 1-2 plane hai. To isko γ_{12} ko main γ_{xy} bhi likh sakta hoon agar main x aur y direction consider kar raha hoon. To isliye shear strain main γ_{12} define karoonga. Aur ye jo deformation hai, agar hum dekhenge, ye O A yahan pe deform ho raha hai OA', aur OB deform ho raha hai OB'. To yahan pe displacements hain. Aur ye jo shear strain abhi main define karoonga, ye change in angle ke hisaab se define karoonga. Aur change in angle yahan pe kya hai? $\pi/2 - $ angle A'OB'. Ye angle. Ye mera angle hoga , Ye angle kya hoga? Ye angle mera hoga $\pi / 2 - \alpha+\beta$. To agar is definition se main agar dekhoonga, to ye jo shear strain aayega mera, ye shear strain aayega γ_{12} equal to $\alpha+\beta$`. Abhi hum ye α aur β displacement ke hisaab se nikaalne ki koshish karenge. To maan lete hain ye mera x1 direction maine plot kiya, aur ye jo displacement hai, ye u1 aur u2 rahenge. To maine pehle part mein bataya tha ki x1 direction par displacement mein yahan par main u1 consider karoonga, aur x2 direction mein displacement karoonga u2. To agar ye x direction agar consider karoonga kisi books mein, to yahan par displacement likhenge u displacement. Aur y agar hai, to main v displacement ke dwara mark karoonga. To agar hum dekhenge, ye point A mark kar lete hain. Point A ye point hai $\Delta x1$ apart from point O. Aur ye jo displacement hai, wo linear vary karenge. Linear vary karenge. To yahan ke yahan par hum dekh sakte hain. Ye jo displacement hai, displacement yaani main ye baat karoonga. Main O se chalu karoonga aur A tak ja raha hoon. To main kis direction mein move kar raha hoon? Main x1 direction mein move kar raha hoon. Par agar main displacement dekhoonga, to ye saare displacements hain, ye is direction mein hain, yaani ye x2 direction pe hain. To isliye main yahan pe u2 displacement mark kar raha hoon. To agar main point A consider karta hoon, to mujhe ye displacement yahan par nikaalne hain. To ye yahan par main A' karke mark kar raha hoon simple way se. Aur ek Taylor series expansion ke hisaab se main yahan par phir se displacement nikaalne ki koshish karoonga. Yahan par hum dekhenge ki ye jo u2 displacement hai, yaani is point par displacement hai, wo main mark kar raha hoon u2. Aur ye jo displacement hai point A pe, ye shoonya rahega, kyunki yahan par koi displacement nahi hai. Yaani zero-zero ke hisaab se agar main displacement mark karoonga, to ye mere displacement yahan par rahenge. Aur iska slope agar main nikaalne ki koshish karoonga, to yahan par slope kya hai? Mere displacements vary ho rahe hain x2 direction pe. To ye small change in displacement in x2 direction with respect to x1 direction, kyunki main is direction pe move kar raha hoon x1 direction pe, aur ye displacement badh rahe hain is direction pe. To isliye $\partial u2 / \partial x1$ aayega × $\partial x1$. $\partial x1$ mera ye jo bhi length hoga $\Delta x1$ us hisaab se rahega. To ye jo displacement aayega A' pe, ye aayega $(\partial u2 / \partial x1) \times \Delta x1$. Ye u zero-zero main shoonya maan ke chal raha hoon. To yahan pe dx ki value aegi $\Delta x1$. To ye displacement mera ho jaayega $(\partial u2 / \partial x1) \times \Delta x1$. Ye jo angle hai, ye aap dekh sakte hain, ye α angle hai. To α angle main definition nikaaloon. Mujhe ye displacements pata hain ye wale, aur ye ye distance pata hai. To main $\tan\alpha$ define karoonga, opposite side / adjacent side. To ye opposite side ka jo length hoga, $\partial u2 / \partial x1 \times \Delta x1 / \Delta x1$. To ye mera $\tan\alpha$ ho jaayega. To humne dekha tha ki ye jo displacement hai bahut small hai. Maine already mention kiya tha aapko ki displacement ya angle change bahut small hai. To hum likh sakte hain $\tan\alpha$ is equal to α for small angles. Aur jab bhi α hum nikaalne ki koshish karenge, to ye aa jaayega $\partial u2 / \partial x1$. To similarly hum β nikaalne ki jab koshish karenge, tab humein dekhna hai ki ye jo displacement hai B aur B', ye jo displacements hain, main phir se mark kar leta hoon. Ye jo displacement hai, ye kis direction mein badh rahe hain? Ye jo displacement hain, mere x direction mein, x1 direction mein. To agar hum dekhenge, β nikaalne ki koshish karenge, to β mera $\partial u1 / \partial x2 \times \Delta x2$. To ye jo displacement hai, ye badh rahe hain mere x2 direction pe. To isliye $u1 / \partial x2$ aayega, $\times \Delta x2$. Aur hum $\tan\beta$ bhi bahut small hai. To β ki value aayegi $\partial u1 / \partial x2$. Agar hum shear strain nikaalne ki koshish karenge, to shear strain aayega summation aayega α aur β ka. Humne α aur β nikaale hain. To $\partial u2 / \partial x1 + \partial u1 / \partial x2$. To hum is tarah se likh sakte hain. To agar main x aur y consider karoonga, jaise maine abhi bola tha. x1 agar main consider karoonga, to ye γxy mera aisa aayega. u2 ke side displacement mera v rahega along x direction. Aur u1 jo displacement hai mera, wo u rahega along y direction. To main γxy is tarah se define karoonga. To humne Einstein summation notation dekhe. To main γij is tarah se define karoonga. Ye aa jaayega mera $\partial ui / \partial xj + \partial uj / \partial xj$. Is tarah se main Einstein summation se is general notation shear strain ke liye likh sakta hoon. Jo shear strain hai, isko main already ek aur terminology use karna chahta hoon. Ye mera engineering shear strain hai. Engineering shear strain mein rotation bhi involved hota hai. To rotation rahega, rotation aur deformation dono cheez yahan par rahegi. To ye sirf dhyaan mein rakhna hai. Isko hum agle class mein clarify karenge ki rotation kya hota hai aur deformation kya hota hai.



To abhi hum sign convention dekhte hain. Humne stresses ke liye sign convention likhe the. Humne normal strain dekha hai aur shear strain dekha hai. To normal strain ke liye to simple hai. Agar tensile hai to extension hoga mere member ka. To isko hum positive likhenge. Aur compressive hai to length uska reduce hoga. To isko hum compressive strain ko negative likhenge. Shear strain ke liye thoda humein samajhna padega, jaanna padega. To yahan par thoda dhyaan dijiye. To ye mera element hai is tarah se, cubic element hai. Aur ye deform ho gaya is tarah se deform ho gaya. Ya kya ho gaya? Humne bola ki yahan par angle change ho raha hai. Ye jo angle hai, ye batayega ki aapka shear strain kitna hai. To agar ye main stresses mark karoon. Ye jo angle yahan pe aise milna hai, yahan pe $\pi / 2$ tha. Ye reduce hoke α ho gaya hai. To ye agar milna hai, to mujhe jo shear stresses hain, wo is tarah se apply hone chahiye. To agar hum dekhenge, ye jo shear stress hai, agar main coordinate axis mark kar loon, ye 2D state ke liye x aur y. To agar hum dekhenge, to ye mera x plane hai, positive x plane hai. Aur ye jo shear stress ki direction hai, ye positive y ki taraf hai. To ye jo shear stress hoga, ye mera positive shear stress hai. Aur ye bhi shear stress hai, ye mera positive shear stress hai. Kyunki aap ye dekhenge ki ye positive y plane pe act ho raha hai along positive x direction. Ye agar shear stress dekhenge, to ye negative x plane pe act ho raha hai along negative y direction. To ye bhi shear stress jo hoga, wo positive hoga. To ye jo shear stress hai, ye deformation laayega. Jo deformation laayega, isko hum ye jo angle decrease hoga, to is shear strain ko kahenge positive shear strain. Agar angle badh raha hai, α badh raha hai, to is strain ko hum kahenge negative shear strain. To agar ye deformation laana hai mujhe, to shear kis direction mein hone chahiye? Mere shear stresses is direction mein. Ya ye negative shear stresses hain. Agar aap dekhenge, to ye positive shears positive plane hai, positive x plane hai, aur negative direction pe ja raha hai ye shear stress. To ye jo shear strain develop karega, wo negative shear strain develop karega. To hum is tarah se bhi dekh sakte hain. Decrease in angle agar decrease in angle hai, to shear strain positive hai. Aur increase in angle hai, to shear strain negative hai. To normal strain ka to simple hai. Tensile hai to positive rahega, compressive hai to negative rahega. Shear strain mein humein dekhna padega ki positive shear stresses will cause positive shear strain, and negative shear stresses will cause negative shear strain. Hum isko dekh sakte hain ki element kaise deform ho raha hai. Agar angle ghat raha hai to wo shear strain positive hai, aur angle badh raha hai to shear strain negative hai. Strains ke baare mein abhi aur hum achhe se jaanenge agle part mein. Abhi main yahan par hi rukta hoon. Dhanyavaad.