Mechanical behavior of materials

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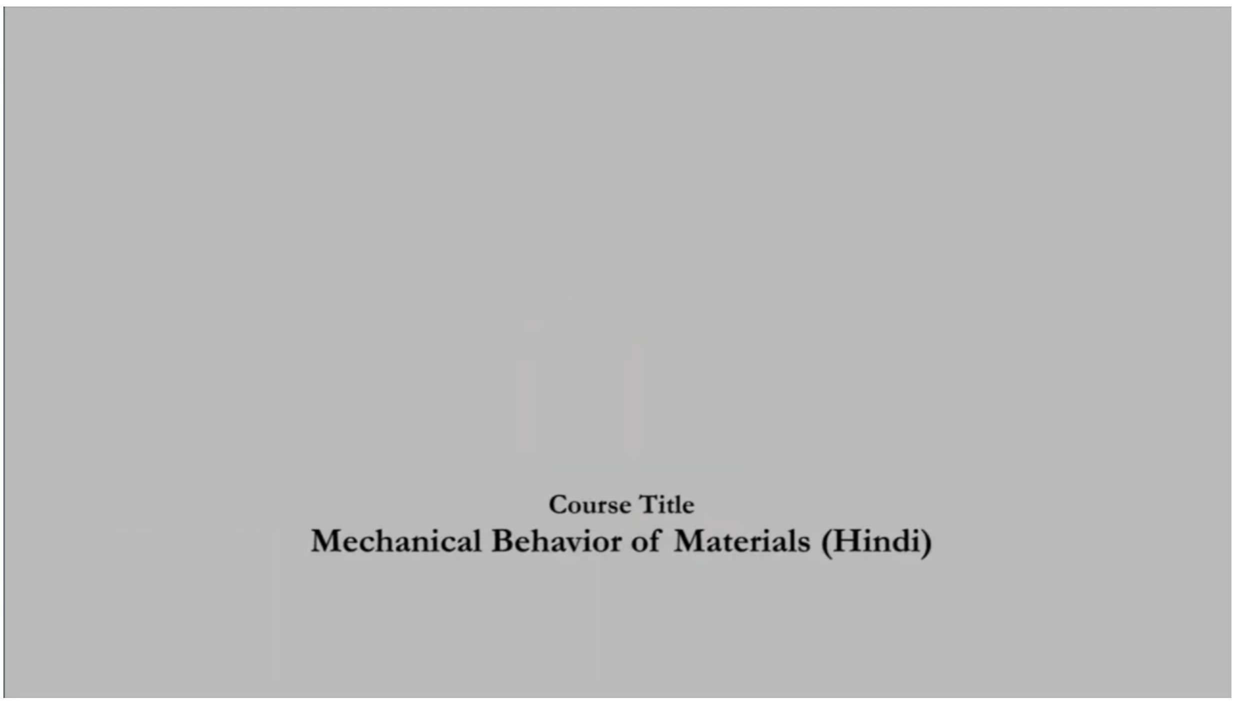
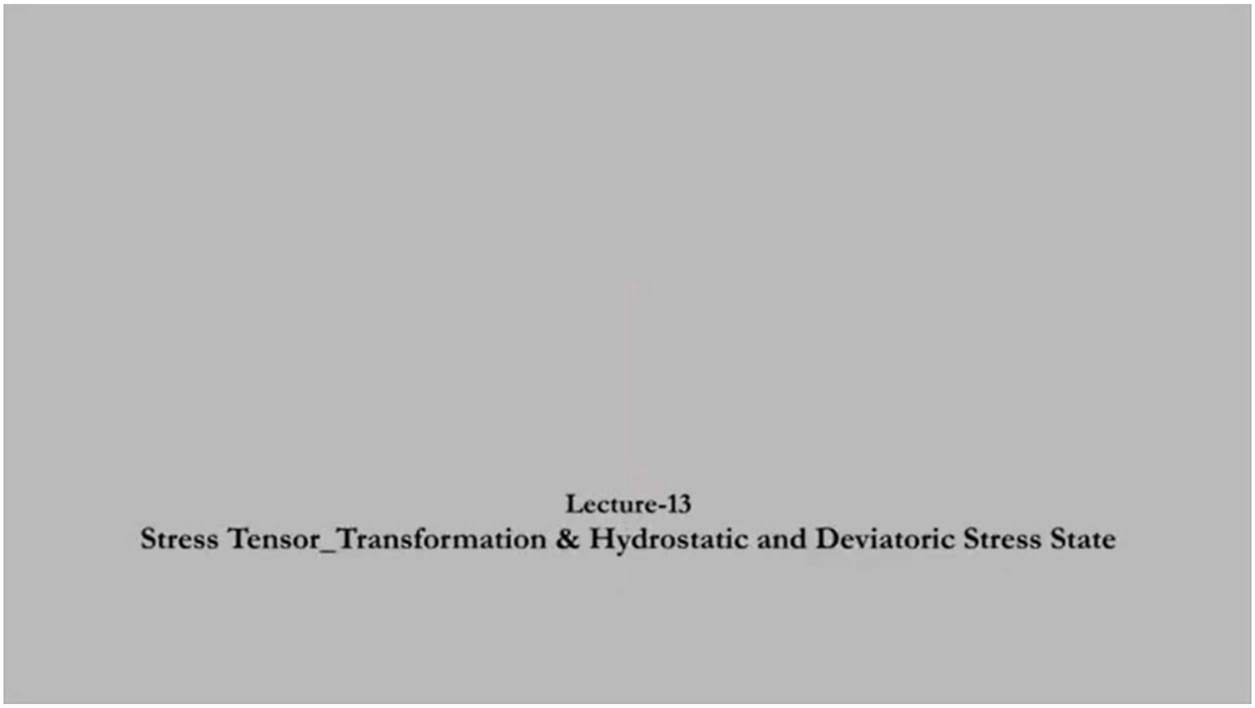
Week-3

Lecture-13

Stress Tensor_Transformation & Hydrostatic and Deviatoric Stress State





Namaskar! Main phir se swagat karta hoon is course mein jiska naam Mechanical Behavior of Materials hai. Last part mein humne dekha tha ki 2D stress state kya hai aur 3D stress state kya hai. Is part mein hum stress tensor aur ek general way jo transformation ka stress tensor ka, aur hydrostatic aur deviatoric stress state kya hai, yeh bhi is part mein hum dekhenge. Toh chaliye chaalu karte hain. Humne jab stress ko transform kiya tha ya kisi stress tensor hai mere paas usko transform karta hoon, toh mujhe kuch equations milenge doosre coordinate axis mein. Toh abhi hum case consider karenge 2D stress state ke liye.

Ek general stress tensor hai mera 2D stress state aur usko transform karenge ek alag axis mein. Jaise yeh bhi ek equation humne dekhe thae jab humne 2D stress state ko transform kiya tha kisi aur coordinate axis mein. Toh main jo bol raha hoon, jaise mere paas ek coordinate axis hai usmein stress state hai aur isko main transform karta hoon ek another frame of reference ya kisi aur element par jo θ (theta) degree tilt hai is initial reference axis se, jaise ki yahan par x_1' aur x_2'. Toh yeh equation mujhe milenge, jaise main likh leta hoon yahan par. Mere paas agar main mere paas ek state of stress hai, yeh 2D stress state hai.

Stress state yaani σ_{11}, σ_{22}, σ_{12}. Isko main transform kar raha hoon kisi aur axis mein, toh mere paas σ_{11}', σ_{22}', σ_{12}' aur σ_{21}' milega. Humne dekha tha ki σ_{21} aur σ_{12} equal hote hain kyunki yeh symmetric matrix hoti hai. Toh isi equations ko humne transform kiya tha is equations mein. Toh yeh equations itni badi-badi hain, isko short form mein kaise likha jaaye? Iske liye ek elegant way hai. Jaise abhi isko humne dekha tha ki geometric form mein aasaani se kaise samjhein jo humne Mohr's circle ke dwaara dekha tha. Par agar computer mein humein yahi cheez calculations karni hai, toh ek simple tareeka hai iska naam hai Direction Cosines.

Toh Direction Cosines mein pehle toh hum define karte hain ki direction cosines hote kya hain. Toh direction cosine main define kar raha hoon ek hai a_{ij} jo ki cosine hai, kiska cosine hai? x_i' ka aur x_j ka. In dono ke beech ka jo angle hai uska cosine hai, yeh mera direction cosine rahega. Toh aaiye isko samajhte hain pehle toh. Jaise maine a_{11} maan liya, toh yahan pe i mera one hai aur j mera one hai. Toh yeh ho jayega x_1' aur x_1 ke beech ka angle ka cosine. Toh x_1 kahan pe hai? Yahan pe mere paas x_1 hai aur x_1' hai, iska beech ka angle hoga θ. Toh iska ho jayega jo cosine hoga yeh $\cos \theta$ hoga. Toh pehle isko ek represent kar lete hain is form mein.

Jaise isko maine ek matrix form mein likh liya kyunki yeh 2D stress state hai, toh main 2D matrix ke form mein isko likh raha hoon direction cosines ko. Jaise yeh a_{11}, a_{12}, a_{21} aur a_{22} hai. a_{12} kya hai? a_{12} represent karega cosine of x_1' aur x_2 ke beech mein. x_1' kahan pe hai? Yeh mera x_1' hai aur yeh x_2 ho jayega. Toh yeh angle hai mera $\pi/2 - \theta$, kyunki humein pata hai yeh x_1 aur x_2 ka angle hai $\pi/2$. Likh lete hain isko bhi. Toh yeh angle ho jayega $\pi/2$. Toh yeh angle hoga mera x_1' aur x_2 ka beech ka $\pi/2 - \theta$. a_{21} hai mera cosine kiske beech ka angle hai x_2' aur x_1 ke beech mein. Yeh mera x_2' aur x_1 ke beech mein, toh yeh total angle hoga.

Toh isko bhi mark kar lete hain, yeh poora angle jo hai yeh mera ho jayega $\pi/2 + \theta$. Aur a_{22} hai x_2' aur x_2 ke beech ka angle ka cosine. Toh yeh x_2' hai aur x_2 hai, iska beech ka angle ho jayega mera θ. Kyunki humein pata hai x_1' aur x_2 ka angle ho jayega... yeh x_1' aur x_2 ka angle hai 90 degree. Toh agar yeh theta degree tilted hai toh yeh bhi yahan se x_2 aur x_2' ka angle theta hona chahiye. Toh yeh angles humne mark kar liye yahan pe aur yeh humara direction cosine hai aur isko abhi hum is actual form mein likhte hain. Toh yeh a_{11} ho jayega $\cos \theta$, a_{12} ho jayega $\sin \theta$.

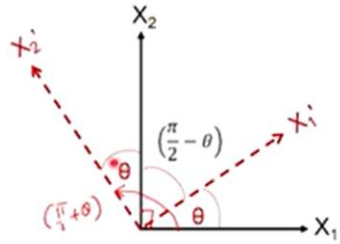
Transformation of the general stress tensor: 2D

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} cos2\theta + \tau_{xy} sin2\theta$$

$$\sigma_{y'y'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} cos2\theta - \tau_{xy} sin2\theta$$

$$\tau_{x'y'} = \frac{\sigma_{yy} - \sigma_{xx}}{2} sin2\theta + \tau_{yx} cos2\theta$$

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \xrightarrow{\theta} \begin{bmatrix} \sigma_{1'1'} & \sigma_{1'2'} \\ \sigma_{2'1'} & \sigma_{2'2'} \end{bmatrix}, \quad \sigma_{21} = \sigma_{12}$$



$$\sigma_{1'1'} = \sigma_{11} cos^2\theta + \sigma_{22} sin^2\theta + 2\tau_{12} \cdot sin\theta \cdot cos\theta$$

$$\sigma_{2'2'} = \sigma_{11} sin^2\theta + \sigma_{22} cos^2\theta - 2\tau_{12} \cdot sin\theta \cdot cos\theta$$

$$\tau_{1'2'} = \frac{\sigma_{22} - \sigma_{11}}{2} sin2\theta + \tau_{yx}(cos^2\theta - sin^2\theta)$$

Direction Cosines $a_{ij} = cos(\widehat{X_i' \; X_j})$

i.e.,

a_{11} : represents cosine X_1' & X_1
a_{12} : represents cosine X_1' & X_2
a_{21} : represents cosine X_2' & X_1
a_{22} : represents cosine X_2' & X_2

$$D = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$D = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

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Kyunki a_{11} yaani kya hai direction cosine x_1' aur x_1 ke beech mein toh yeh θ hai toh iska angle ho jayega cos θ. a_{12} ko samajhte hain, a_{12} yaani ho jayega cosine x_1' aur x_2 ke beech mein. x_1' aur x_2 ke beech mein yeh angle hai π/2 - θ, toh cos(π/2 - θ) ho jayega sin θ. Similarly agar hum a_{21} dekhenge toh yeh cosine hai x_2' aur x_1 ke beech mein. x_2' aur x_1 ke beech mein toh yeh total angle ho jayega mera π/2 + θ. Toh cos(π/2 + θ) ho jayega mera -sin θ. Aur a_{22} toh x_2' aur x_2 ke beech ka jo angle hai uska cosine hai. Toh yeh angle theta hai yahan pe toh yeh cos θ ho jayega. Toh yeh mera direction cosines ho gaye. Toh yeh maine define kiya hai.

Abhi abhi hum dekhte hain ki transformation kaise karenge. Toh yahi transformation humne equation ke dwaara dekha abhi yahi transformation hum direction cosines ke dwaara dekhenge. Toh yeh mera state of stress hai isko main prime (') ke term mein likhunga aur yeh transformation ho jayega mera theta jab main rotate kar raha hoon element ko ya mera reference axis rotate ho raha hai initial reference axis se theta degree se. Toh yeh matrix equation main likh sakta hoon. Yeh jo mera transformed state of stress hai ([σ']), yeh mera initial state of stress hai ([σ]), A aur yeh mera direction cosine hai aur yeh mera direction cosine ka transpose. Yeh equations maine dekh liye thae yahan par initial jo humne derive kiye thae equation.

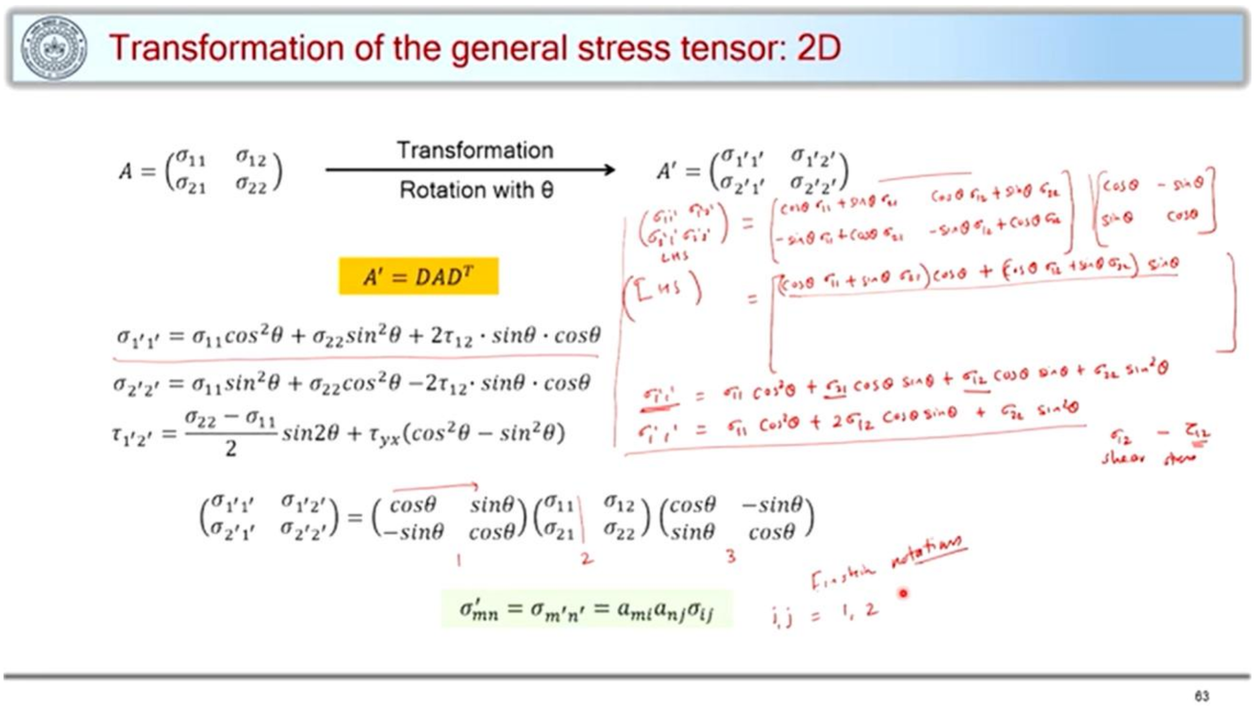
Toh abhi hum dekhte hain is matrix equation ko expand karte toh main yeh state of stress is tarah se nikaal paunga. Toh state of stress is tarah se nikaalna hai toh lagbhag abhi hum dekhte hain isko pehle solve karke dekhte hain. Toh agar matrix multiplication main karunga toh isko solve karke dekhte hain, jaise main isko aur isko pehle matrix ko ek, do aur teen ko multiply karna padega. Toh pehle hum aise multiply karte toh yahan par likh lete hain hum yeh σ_{11}', σ_{22}', σ_{12}' aur σ_{21}'. Yeh mera matrix ho gaya yeh wala. Toh left hand side agar main isko solve karunga toh pehle main aise multiply karunga.

Toh $\cos\theta\, \sigma_{11} + \sin\theta\, \sigma_{21}$ aur phir main multiply karunga is tarah se $-\sin\theta\, \sigma_{11} + \cos\theta\, \sigma_{21}$ aur isko multiply karunga aise $\cos\theta\, \sigma_{12} + \sin\theta\, \sigma_{22}$. Yeh close ho jayega yahan par $-\sin\theta\, \sigma_{12} + \cos\theta\, \sigma_{22}$. Aur mera third matrix $\cos\theta$, $-\sin\theta$, $\sin\theta$ aur $\cos\theta$.

Toh yeh ho jayega mera product ho gaya one aur two ka. Abhi main phir se inko aise multiply karunga toh main agar aise multiply kar raha hoon toh mera pehla term aisa aayega $\cos\theta\, (\sigma_{11} \cos\theta + \sigma_{21} \sin\theta)$... pehli term likh raha yahan par... plus yeh term $(\sigma_{12} \cos\theta + \sigma_{22} \sin\theta) \sin\theta$. Yeh meri pehli term aayegi. Toh yeh pehli term hum dekhenge toh mera σ_{11}' yeh jo term hai, yeh agar LHS wala matrix aise likh raha toh σ_{11}' yeh term hogi meri. Agar isko main multiply karta hoon toh mere paas aayega:

$\sigma_{11} \cos^2\theta + \sigma_{21} \cos\theta \sin\theta + \sigma_{12} \cos\theta \sin\theta + \sigma_{22} \sin^2\theta$. Agar main isko solve karunga toh mere paas aayega $\sigma_{11} \cos^2\theta$ + yeh σ_{12} aur σ_{21} same hai toh yeh main isko is tarah se likh sakta hoon $2\sigma_{12} \cos\theta \sin\theta + \sigma_{22} \sin^2\theta$. Toh agar hum dekhenge σ_{11}' jo mujhe yahan par mila hai yeh multiplication se aur yeh jo term hai exactly same hai. Yaani maine bola bhi tha ki σ_{12} ko main τ_{12} bhi likh sakta hoon shear stress hai. Toh humein ek simple relation yaani yahi bada relation ek simple compact matrix multiplication se mil raha hai.

Toh yeh kiski utility hai? Toh hum yahi abhi mathematical identity istemaal karenge jab hum stress ko transform karne ki koshish karenge. Toh stress ko transform jab karenge toh yeh transformed stress hoga. Toh mujhe agar direction cosines pata hain toh main uska transpose nikaalunga aur mera initial state of stress ko main convert kar sakta hoon agar mujhe direction cosines mere mujhe pata hain toh. Toh in dono mein bhi agar main aise write karunga toh ismein mujhe pata nahi chal raha hai ki mera 2D stress state hai ya 3D stress state hai is mathematical form mein.



Yahan par humne toh general stress tensor likha tha 2D stress state liye, par hum aage bhi jayenge 3D stress state dekhenge toh yeh equation mujhe kuch bata nahi raha hai. Toh main usko is tarah se likhunga, ek elegant way hai likhne ka yaani kaafi accha way hai, isko hum kahenge... isko hum Einstein notation se likhte hain. Yeh jo notations dikh rahe hain mere toh yahan par main Einstein notations ke dwaara likhunga aur yeh yahan par agar 2D stress state hai toh mera i aur j ki value vary hogi one aur two ke beech mein. Toh aage badhte hain toh is course ke liye hum dekhte hain ki Einstein notations kya hain aur unki utility kya, ek brief introduction main aapko dena chahta hoon.

Toh dekhte hain ki tensor notation mein Einstein notations yaani isko hum Einstein summation notation aur convention bhi kehte hain. Iska ek brief introduction dekhte hain jo humare course ke kaam mein aayega. Toh ek iska rule ek simple rule hai, teen rule hain yahan pe. Toh ek rule one hai ki koi bhi index agar do baar repeat ho raha hai ek term mein, woh sum hoga uska sum hoga. Toh isko samajhte hain jaise main maan maine bola tha yahan pe index ek 1 to n woh kitna bhi vary ho sakta hai. Typically hum n yahan pe teen tak vary hoga yahan pe humare course mein kyunki hum 3D coordinate axis ko belong karte hain, 3D stress state tak hi jayenge toh n ki value teen tak hi aayegi humare course mein.

Maan lete hain humara simple relation humne likha tha $\sigma_{mn}' = a_{mi} a_{nj} \sigma_{ij}$. Toh a_{mi} aur a_{nj} humare direction cosines thae, 'a' humne denote kiya tha direction cosine ke liye. Toh yahan par dekhenge ki humare index jo do baar repeated hain, toh yahan pe kaun se index do baar repeated hain? Agar is term mein dekhenge right hand side pe, toh do baar repeat jo ho raha hai m ek baar hai, n bhi ek baar hai, i mera do baar aa raha hai, yeh do index aa raha hai yahan pe, a_{mi} mein bhi ek i aa raha hai aur σ_{ij} mein ek i aa raha hai, aur j bhi do baar repeat ho raha hai yahan pe. Toh yahan pe j aa raha hai is direction cosine ke term mein aur stress ki term mein bhi j do baar aa raha hai.

Toh yeh humare jo indexes hain woh sum up ho gaye. Toh maan lete hain humne 2D stress state consider kiya tha toh i ki value ek ya do ho sakti hai. Toh isko likhte hain, ek example lete hain ki j ki value maan lete hain ek se do vary hogi. Toh is term ko hum likhenge yahan pe pehle j ki value hum ek likhenge aur phir j ki value do hogi. Toh agar is term ko main expand karunga toh yahan par $a_{mi} a_{n1} \sigma_{i1}$ yahan par j ki value maine one li hai, aur yahan par j ki value one hai. Doosra term hoga $a_{mi} a_{n2} \sigma_{i2}$. Toh yahan par maine j ki value do likhi hai, yahan par one thi j ki value aur yahan par do ho gayi hai. Toh yeh mera summation ho gaya j ke liye.

j summed up ho gaya mera yahan pe. Abhi hum same value same cheez i ke liye karenge. Toh yahan par main pehle i ki value one likhunga is equation mein aur phir i ki value do likhunga aur iska summation karunga. Toh yahan pe yeh $a_{m1} a_{n1} \sigma_{11}$ (i ki value one rakhi hai), is term mein bhi i ki value one hogi $a_{m1} a_{n2} \sigma_{12}$ plus yahan pe i ki value main do rakhunga toh $a_{m2} a_{n1} \sigma_{21} + a_{m2} a_{n2} \sigma_{22}$. Toh yahan pe i aur j ki value humne 1 aur 2 se vary ki hai aur dono ka summation kiya hai. Toh yeh mera agar jab mera i aur j 1 to 2 vary ho raha hai, toh yeh term itna bada quantity represent karegi aur yeh summed up ho rahe hain yahan pe. Toh yeh humara...

rule one hai yahan par. Abhi hum dekhte hain i aur j jo hain inko hum dummy index kehte hain. Abhi dummy index kya hote hain? Toh dummy index woh hote hain jinko main replace kar sakta hoon jaise main chahoon waise. Toh yeh dummy index kyun hain? Toh kyunki yeh jo dummy index hai yeh main inko sum up kar sakta hoon isliye main inko dummy index kehta hoon. Aur doosri cheez hai ki yeh main unko replace karunga tab yeh wahi... jaise main agar main j aur i ko replace karunga kisi aur index se toh yeh already kisi is equation mein nahi hone chahiye tabhi main usko replace kar sakta hoon. Aur yeh jo doosra i aur j ko main agar...

replace kar raha hoon toh woh bhi vary honge mere one aur two se. Yahan par humne dekha tha i aur j vary ho rahe thae 1 to 2 se toh woh bhi index jo bhi main replace karunga woh one aur two hona chahiye. Ek example lekar dekhte hain. Toh maan lete hain ki main replace karta hoon i ko l se aur j ko k se. Toh yahi equation main is tarah se likh sakta hoon $\sigma_{mn}' = a_{ml} a_{nk} \sigma_{lk}$. Toh yahan pe maine replace kiya i ko l se aur j ko k se. Toh yeh l aur k ko maine replace kiya. Agar aap dekhenge toh is equation mein l aur k thae nahi toh isliye main i aur j ko l aur k se replace kar paaya hoon aur l aur k ki value...

vary hogi. Isko bhi likh sakte hain l aur k ki value yahan pe vary hogi one aur two tak hi kyunki humare i aur j ki value vary ho rahi hai one aur two se. Toh yeh dummy index hai aur dummy index ko main replace kar sakta hoon. Ab maante hain mera m aur n kya hai? Toh m aur n yeh ek hi baar aa rahe hain is equation mein aur ek hi term mein toh isko kehte hain free index. Isko main replace nahi kar sakta hoon yaani yeh index hamesha yahan par is equation mein rahengy. m aur n ko main replace nahi kar paunga aur jo indexes ko main replace nahi kar paata unko main kehta hoon free index. Rule two dekhte hain Einstein summation convention ka, yahan par no indexes occur three or more times.

Yaani kisi bhi term mein jaise yeh term maine maan li $a_{mi} a_{nj} \sigma_{ij}$... agar hum dekhoonga is term mein i teen baar aa raha hai toh yeh valid nahi hai. Par yeh term mein dekhta hoon yahan par yahan par maine do term likhi hain $a_{mi} a_{nj} \sigma_{ij} + b_{mi} a_{nj} \sigma_{ij}$. Toh yahan par main dekhoonga i toh teen baar aa raha hai par yeh term jab main kahunga toh meri ek term hogi toh is term mein i sirf do baar aa raha hai toh isliye yeh galat hai pehla wala kyunki i yahan par teen aa raha tha. Yeh ek hi term hai ismein i teen baar aa raha hai toh yeh allowed nahi hai, yeh allowed hai. Yeh mera ho gaya rule two. Abhi rule teesra dekhte hain Einstein notation mein.

Jo free indexes hain, free indices humne dekha tha jo hum kisi ko replace nahi kar sakte, yeh dono side same hone chahiye left hand side aur right hand side pe. Toh yeh dekhte hain agar i mera free index hai, i mera yahan pe free index hai aur j bhi yahan pe mera free index hai, match nahi ho raha hai yahan pe yaani do free indices yahan pe ek hi free index hai, match nahi ho raha toh yeh allowed nahi hai. Similarly $x_j a_{ik} + u_k$... agar main dekhoonga yahan pe mera k do baar aa raha hai, yeh mera dummy index ho gaya, i mera free index hai, j mera free index hai toh yahan pe is side j nahi hai aur is side i nahi hai toh yeh bhi allowed nahi hai...

Einstein summation convention ke anusar. Teesra dekhte hain ki mera k dummy index hai, i mera free index hai aur yahan pe i mera free index hai toh yahan pe dekhenge dono free index match ho raha hai left hand side aur right hand se so yeh allowed hai. Agar hum yeh bhi term dekhte hain $\sigma_{mn}' = a_{mi} a_{nj} \sigma_{ij}$ toh agar dekhengy mere free indexes m aur n hain toh m aur n is side mein match ho rahe hain yahan pe m ek baar hai yahan pe n ek baar hai, yahan pe m ek baar hai aur yahan pe n ek baar hai toh dono taraf yeh match ho raha hai toh Einstein summation ke anusar yeh aisa likhna uchit hai. Abhi aage badhte hain humare course ke liye iski utility dekhte hain Einstein notations ki.

Toh yeh humne simple transformation dekha tha stress ka direction cosines aur mera initial state of stress ke anusar. Toh maan lete hain mere paas 2D stress state hai toh i aur j ki value vary hogi one aur two tak. Toh isko humne expand kiya tha jab i aur j ki value one aur two hai toh isi compact form ko expand kar sakte hain isi form mein summation ke form mein. Maan lete hain hum m aur n ki value one aur one hain. Toh m aur n ki value jab 1 1 rahengy toh isko hi main σ_{11}' likh sakta hoon. Toh yeh m aur n ki value jab main one-one likhunga toh yeh ho jayega $a_{11} a_{11} \sigma_{11} + a_{11} a_{12} \sigma_{12} + a_{12} a_{11} \sigma_{21} + a_{12} a_{12} \sigma_{22}$. Toh yeh jo ho gaye mere direction cosines.

Ab humne define kiya tha direction cosines agar aap recall karenge toh yeh mere direction cosines thae aur unki yeh values thae theta ke form mein cosines ke form mein: $\cos\theta$, $\sin\theta$, $-\sin\theta$, $\cos\theta$ 2D stress state ke liye. Agar yeh a_{11} aur a_{12} ki value main yahan par put karta hoon toh mujhe yeh relation mil jayega aur isko main solve karta hoon toh mere paas yeh yeh identity aayegi. Aur yeh identity aur initial identity jab humne dekhi thi toh yeh exactly same identity mujhe mil rahi hai. Toh yeh meri Einstein summation notations ki utility ho gayi. Toh agar main ek transformation likh raha hoon is tarah se toh main saare terms jaise σ_{11}' isi ek term se main denote kar sakta hoon.

Einstein notations: Utility

$$\sigma'_{mn} = a_{mi}a_{nj}\sigma_{ij}$$

Say, i, j = 1, 2

$$\sigma'_{mn} = a_{m1}a_{n1}\sigma_{11} + a_{m1}a_{n2}\sigma_{12} + a_{m2}a_{n1}\sigma_{21} + a_{m2}a_{n2}\sigma_{22} \qquad D = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Say, m=1 and n = 1

$$\sigma'_{11} = \sigma_{1'1'} = a_{11}a_{11}\sigma_{11} + a_{11}a_{12}\sigma_{12} + a_{12}a_{11}\sigma_{21} + a_{12}a_{12}\sigma_{22} \qquad D = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

$$\sigma_{1'1'} = cos\theta \cdot cos\theta \cdot \sigma_{11} + cos\theta \cdot sin\theta \cdot \sigma_{12} + sin\theta \cdot cos\theta \cdot \sigma_{21} + sin\theta \cdot sin\theta \cdot \sigma_{22}$$

$$\sigma_{1'1'} = cos^2\theta \cdot \sigma_{11} + 2cos\theta \cdot sin\theta \cdot \sigma_{12} + sin^2\theta \cdot \sigma_{22}$$

$$\sigma_{1'1'} = \sigma_{11}cos^2\theta + \sigma_{22}sin^2\theta + 2\tau_{12} \cdot sin\theta \cdot cos\theta$$

Verify this for $\sigma_{1'2'}\ or \sigma_{2'2'}$

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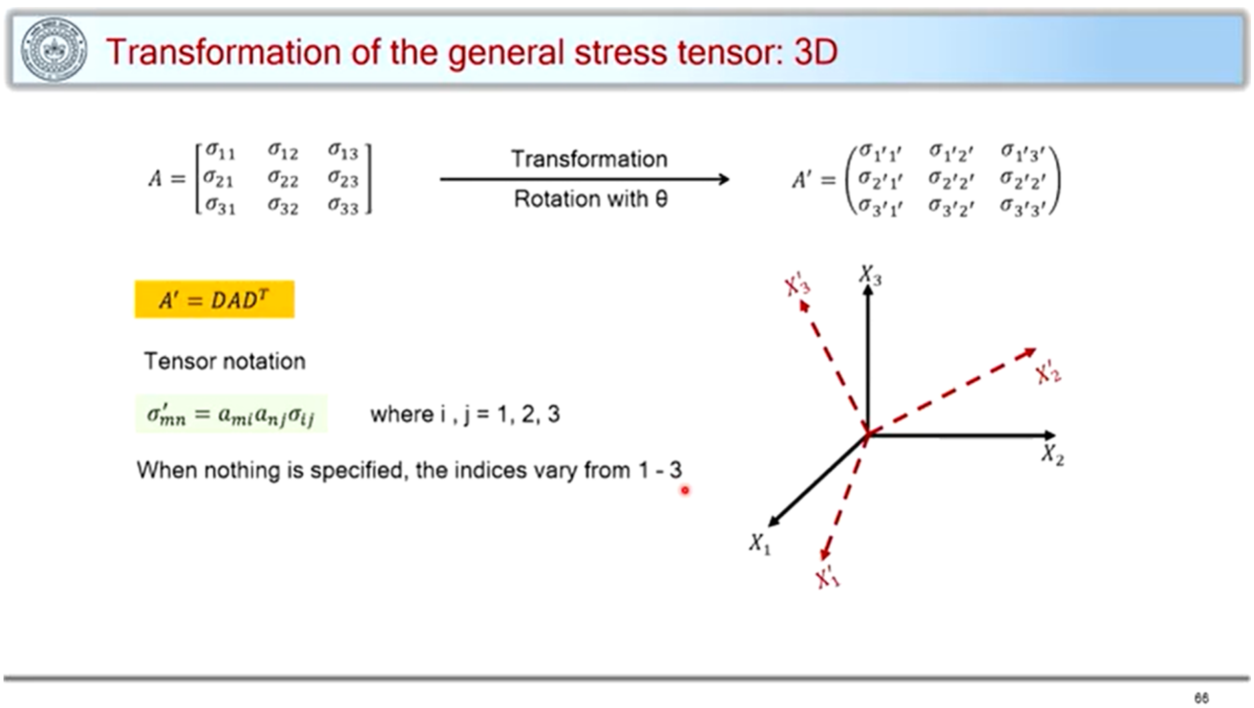
Toh yeh mera σ_{11}' ho gaya jo humne dekha ki initial jo relation hai usi ki tarah match ho raha hai. Aapke paas ab ek homework hai. Toh yeh jo homework hai aapko σ_{12}' aur σ_{22}' ke liye derive karna hai Einstein notations ke anusar. Toh aap yeh isko expand kariye yahan pe m aur n ki value kya hogi? Yahan pe m aur n ki value hogi m ki value hogi one yahan pe n ki value hogi two aur yahan pe m ki value hogi two aur n ki value hogi two. Toh agar aap isko expand karogy is term ko aur i aur j ki value one aur two tak vary hogi toh aapko ek identity milegi us identity mein aap yeh values put karke dekhein aur dekhiye aapko...

yeh initial values milti hain ki nahi. Toh yeh ho gayi humare Einstein notations ki utility. Abhi aate hain hum 3D stress state ke liye. Toh 3D stress state mein bhi similar approach hum istemaal karenge jaise mere paas ek stress tensor hai yahan pe mere paas nau components hain, usmein se maine mujhe pata hai ki cheh components independent hain humne dekha tha. Aur isko main transform karta hoon A' mein toh mere paas yeh transformed coordinate axis ya rotated element mein mere paas yeh nau component aayenge. Abhi hum isko is tarah se bhi samajh sakte hain mere paas coordinate axis hai aur maine yeh x_1', x_2' aur x_3' yeh rotated hain yeh mere initial frame of reference se.

Isko main transformation matrix ke equation mein aise likh sakta hoon $A' = A \cdot A^T$. Yeh mera matrix equation ho gaya transformation ke liye aur tensor notation mein main simple yahi relation istemaal kar sakta hoon par yahan par previous case mein i aur j ki value kitni thi 2D stress state mein one aur two thi, is case mein i, j vary honge 1, 2 aur 3 tak. Yeh ho gaya mera 3D stress state ka transformation. Toh yeh relation abhi hum yahi istemaal karenge humare course mein yeh Einstein summation notation ke dwaara. Toh jab kabhi bhi agar yeh i aur j ki value mention nahi ki hogi course mein, toh i aur j vary honge hamesha 1 to 3 tak kyunki hum hamesha ek coordinate axis...



jo hum teen coordinate axis maan ke chal rahe hain aur Cartesian coordinate maan ke chal rahe hain aur 3D stress state hum consider karenge toh i aur j ki value 1 aur 3 tak vary hogi. Abhi yeh ho gaya humara general transformation stress state ka 3D tak humne dekh liya hai. Abhi hum dekhte hain ki hydrostatic aur deviatoric stress state kya hai. Toh yeh mera stress tensor hai. Stress tensor aap kabhi dekhenge toh kisi-kisi books mein yeh sigma ke niche do underline hoti hain, is tarah se bhi denote kiya jaata hai tensor ko. Toh mere paas yeh nau component hain aur unmein se kitne component mere independent hain? Unmein se mere six independent component hain kyunki hum dekh rahe hain yeh...

jo cheh kaun se independent hain? Yeh, yeh, yeh aur diagonal kyunki humne dekha tha $\sigma_{ij} = \sigma_{ji}$. Aur yeh kyun independent hain kyunki hum static equilibrium consider kar rahe hain isliye, stress matrix humara symmetric hona chahiye isliye. Toh is stress tensor ko main is tarah se bhi likh sakta hoon... main is tarah se likh sakta hoon agar main stress matrix ko is tarah se matrix algebra agar main apply karunga. Toh yahan par dekhenge ki mere paas sirf yahan par diagonal elements hain aur off-diagonal element jo bhi hain woh saare zero hain.

Aur in dono ka summation agar main dekhoonga toh mujhe yeh initial stress matrix mujhe milni chahiye ya initial state of stress mujhe milni chahiye. Toh main matrix algebra apply karke yeh isko is tarah se likh sakta hoon. Toh agar aap dekhenge main in dono ko add karunga yeh term aur yeh term, toh mujhe σ_{11} milna chahiye. Toh is state of stress mein jahan par off-diagonal terms zero hain... toh off-diagonal terms agar main maanunga toh yeh off-diagonal terms kya hongi? Yeh meri shear stresses ho gaye. Humne dekha tha ki yeh jo values hain, yeh jo diagonal terms hain yeh represent karti hain meri normal stresses ko aur yeh jo off-diagonal terms hain yeh...

represent karti hain shear stresses ko. Toh is case mein hum dekhenge ki sirf mere paas diagonal term yaani ki sirf normal stresses hain toh isko hum kahengye hydrostatic stress state. Is stress state ko hum kahengye jahan par mere paas shear component hai is stress state ko kahengye hum deviatoric stress state. Toh iski utility kya hai abhi hum dekhenge. Yahan par hum pehle σ_m define kar lete hain. Toh σ_m main define kar raha hoon isko main mean normal stress ya hydrostatic pressure aur stress kehta hoon. Toh σ_m ko main is tarah se likh sakta hoon $(\sigma_{11} + \sigma_{22} + \sigma_{33}) / 3$. Yeh in iska mean stress ho gaya teeno...

stresses ka, teeno normal stresses ka mean stress ho gaya. Yahan par normal stresses kaun se kaun se σ_{11}, σ_{22}, σ_{33} ka average lunga toh yeh mere paas mean stress hoga. Kisi-kisi books mein yeh is tarah se bhi likha jaata hai σ_m ya σ_h bhi likha jaata hai. Toh yeh hydrostatic stress state aur yeh hydrostatic pressure ko denote karta hai. Isko main σ_1, σ_2 aur σ_3 ke dwaara bhi likh sakta hoon. Yeh σ_1, σ_2, σ_3 mere principal stresses hain. Yeh humne dekha tha 3D stress state mein. Agar main principal stresses nikaaloon toh main is tarah se bhi likh sakta hoon kyunki humne dekha tha ki inka addition $\sigma_{11} + \sigma_{22} + \sigma_{33}$...

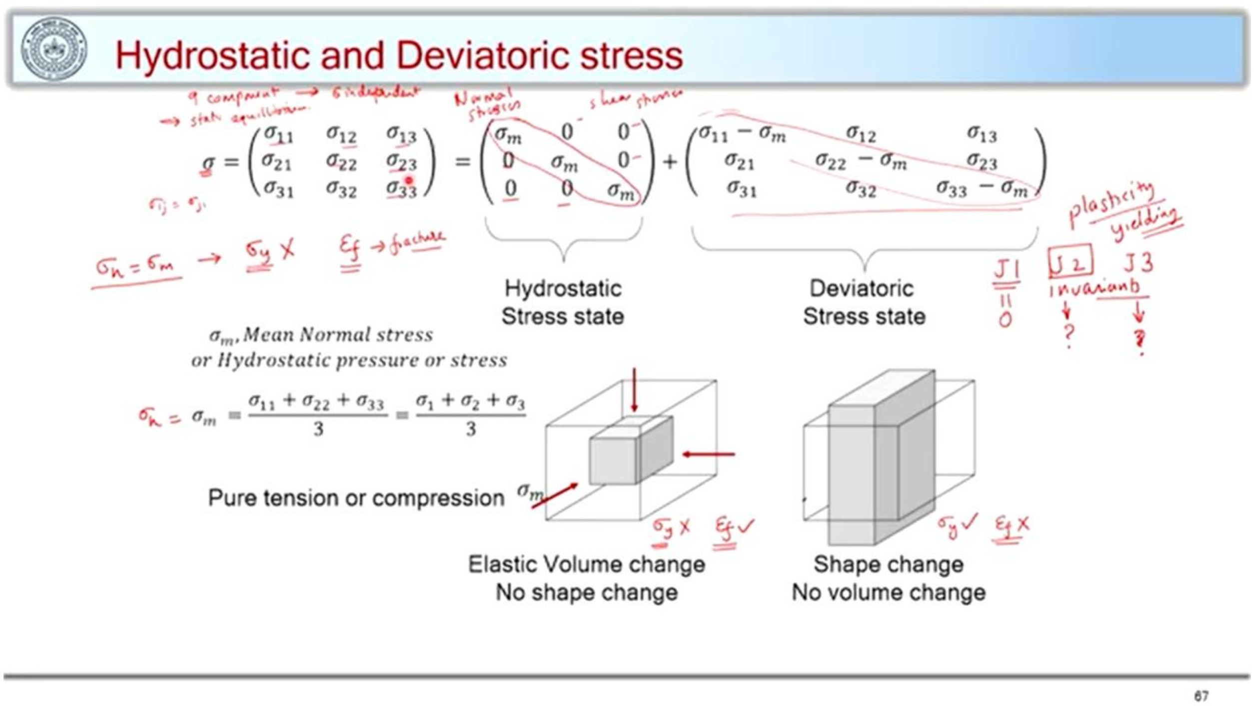
barabar hai $\sigma_1 + \sigma_2 + \sigma_3$. Yeh ek invariant hai toh main isko in principal stresses ke dwaara bhi likh sakta hoon. Toh abhi dekhte hain yeh hydrostatic stress state aur deviatoric stress state karta kya hai. Toh maan lijiye mere paas initial cube hai ek aur isko main ek pehle case mein agar hum isko represent karenge, ek jo open cube hai isko main teeno tarah se compress kar raha hoon. Yahan par maine maan liya ki yeh sigma jo m hai yeh ek toh tension hoga nahi toh compression hoga. Toh yeh ek toh pure tension ya pure compression ho gaya. Maan lijiye is case mein main pure compression ko dikha raha hoon, yeh sigma mean stress jo hai. Toh ab dekhenge ki yeh normal stresses hain teen direction pe aur yeh isko...

compress kar rahe hain. Toh yeh cube jo hai woh iska volume change ho gaya aur iska shape change nahi hua. Agar dekhenge toh yeh cube ka cube hi rahega, initial cube tha aur final mujhe cube mila. Toh yahan pe hum dekhenge yahan par sirf mera elastic volume change hoga yahan pe shape change nahi ho raha hai. Toh mera hydrostatic stress state kya karta hai? Hydrostatic stress state mera volume change karega par shape change nahi karega. Toh agar hum dekhenge is case mein deviatoric stress state mein, yeh mera yahan pe kyunki yahan pe mere paas normal stresses hain aur shear stresses bhi hain toh yeh shape change karega. Par yahan pe dekhengy ki mera initial cube hai aur mera final...

jab deform ho gaya because of this stress state, yeh stress state ki wajah se toh yeh shape change yahan pe mujhe milra hai par volume change nahi milra hai. Toh deviatoric stress state mera volume change nahi karega sirf shape change karega. Yeh hum dekhenge deformation hum jaise-jaise aage badhenge jis plasticity mein yeh deviatoric stress state ki bahut utility hai. Toh yahan par main likhna chahta hoon jab yeh jo hydrostatic stress state hai sigma m hai, yeh mera elastic volume change karega toh isliye yeh yield strength... jo material hum...

deform hota hai jahan par elastic to plastic transition hai iske upar koi affect nahi karega. Par yeh fracture strain par affect karega. Yeh hum dekhenge jab hum fracture ki baat karenge ya plasticity mein dekhenge. Toh yeh jo stress state hai sigma m, yeh sigma y ko affect nahi karega, yeh fracture strain ko affect karega. Yeh jo deviatoric state hai yeh sigma y ko affect karega, yeh generally fracture strain ko affect nahi karta hai. Toh abhi hum dekhenge yahan par abhi main ek aapko exercise dena chahta hoon. Aap J_2, J_3...

yeh teen invariants thae humne dekhe thae 3D stress state mein humein teen invariants milte hain. Toh yahan par aap dekhenge ki J_1 aap calculate karke dekhenge is case mein deviatoric stress state mein. Toh J_1 kya hota hai? Summation of diagonal elements. Yeh jo hai summation agar aap dekhenge toh yeh J_1 agar aap quickly karenge toh yeh shunya aata hai yahan pe. Abhi main aapko example diya tha J_1 ka, ab aap J_2 nikaal ke dekhiye aur J_3 nikaal ke rakhiye. Toh yeh jo J_2 hai yeh plasticity mein ya hum dekhenge yielding criteria mein kaam mein aayega, isliye aap is J_2 ke upar aap isko calculate karke dekhein ki is deviatoric...



stress state liye J_2 kya aata hai. Toh humne do cheezein dekhi yahan par: koi bhi stress state mera main usko do tarah se divide kar sakta hoon - hydrostatic stress state aur deviatoric stress state. Hydrostatic stress state mera elasticity part ko govern karta hai majorly aur deviatoric stress state mera shape change yaani plastic deformation mein yeh deviatoric stress state kaam mein aata hai. Toh koi bhi stress ke components main is tarah se divide kar sakta hoon yeh humne iski utility jaani. Abhi next class mein hum dekhenge ki strain kya hai. Abhi tak yahan par humne dekha ki stress kya thae, different type of stresses ya stress matrix...

humne dekha tha, stress tensor humne dekha tha, phir humne dekha ki stress ke jo components hain woh independent kitne hain aur stress ka transformation kaise hoga kisi bhi coordinate axis mein. Abhi humne dekha ki ek stress state ko kis tarah se hum divide kar sakte hain hydrostatic aur deviatoric stress state mein. Next part mein hum jaanenge ki strain kya hai. Abhi yahan par main rukta hoon. Dhanyavaad!