

Mechanical behavior of materials

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Week-3

Lecture-12

Principal stresses in 3D & stress invariants

Course Title

Mechanical Behavior of Materials (Hindi)

Lecture-12
Principal stresses in 3D & stress invariants

Main phir se swagat karta hoon is course mein jo Mechanical Behavior of Materials hai jo usko hum Hindi mein padhenge. Toh last video mein humne dekha tha ki Principal Stresses kaise nikaalte hain 2D Stress State mein. Toh abhi hum is video mein dekhenge ki Principal Stresses 3D Stress State mein kya nikaalte hain kaise nikaalte hain. Toh humne hamari ye stress matrix hai ye Stress Tensor $[\sigma]$ hai jahan par stress components maine mark kiye. Toh pehle thoda revise kar lete hain ki humne dekha tha ki agar σ_{11} hi component hai aur baaki other components zero (0) hain toh isko hum kehte hain Uniaxial Stress State.

Aur hamare paas $\sigma_{11} = \sigma_{22}$ aur baaki components zero hain toh isko hum kehte hain Biaxial Stress State. Aur hamare paas σ_{11} , σ_{22} , σ_{33} hai toh isko hum Triaxial Stress State kehte hain. Zaroori nahi hai ki σ_{11} aur σ_{22} mathematically equal hon ya σ_{11} , σ_{22} , σ_{33} mathematically equal hon but yeh agar non-zero hain toh hum isko is tarah se dekhte hain. Toh ye humara general stress state hai aur humne dekha tha ki ismein nau (9) components hain usmein se hamare cheh (6) components independent hain. Toh is class mein hum dekhenge ki Principal Stresses 3D state mein kaise nikaalenge.

Toh uske liye mark kar lete hain hamari coordinate axis X1, X2 aur X3 aur kuch planes mark kar lete hain is tarah se. Toh hum dekhenge ki plane jaise yeh plane hai OBC yeh plane perpendicular

X1 direction ko hai toh yeh mera X1 plane ho jayega. Similarly yeh jo plane hai OCA yeh perpendicular hai X2 direction par toh yeh ho jayega X2 plane aur yeh jo plane hai AOB yeh perpendicular X3 direction pe ye mera X3 plane ho jayega. Agar isko main X, Y, Z agar aise likhoon isko agar main X, Y aur Z direction mein baat karunga toh yeh jo ho jayega mera yeh mera isko hum XY plane kahenge XY plane kahenge ya Z plane kahenge.

Toh dono cheezein sahi hain theek hai. Toh ismein humne dekha tha ki yeh jo stress components hain humne mark kiye thae ye teen orthogonal planes mein. Toh X1 plane mein hamare paas ek normal stress hai σ_{11} aur ye along ek direction hai yaani X1 direction hai. Yeh jo stress hai yeh mera shear stress (τ) hoga kyunki X1 plane mein hai aur yeh along X3 direction hai toh yeh mera σ_{13} ho jayega. Similarly ye mera σ_{12} ho jayega. Waise usi tarah se yeh do planes mein hum mark kar lenge do shear stresses aur ek normal stress aur yeh do shear stresses aur ek normal stress. Abhi ek stress mark karte hain σ jo ki is plane mein hum mark karenge is ko perpendicular hai.

Jo mera plane ABC jo plane hai isko perpendicular stress hai yeh stress σ hai yeh Normal Stress hai toh yaani yahan par koi bhi ek shear stress nahi hai. Toh yeh stress jo ho jayega σ is plane ke liye mera Principal Stress ho jayega aur ABC mera Principal Plane ho jayega. Principal plane ki humne vyakhya dekhi thi ki Principal plane par koi bhi shear stress (τ) nahi hoga. Toh jaise hum dekh paa rahe hain is plane mein jaise ye plane ABC hai ismein sirf σ plane hai jo ki perpendicular hai normal plane hai normal stress hai but yahan par koi bhi shear stress nahi hai.

Toh ye mera Principal Stress ho jayega σ . Toh agar hum dekhenge agar ye stress isko equilibrium mein rakh raha hai Static Equilibrium ($\sum F = 0$) mein rakh raha hai toh yeh condition satisfy honi chahiye yaani X1 plane par jo bhi forces ka summation (Σ) hoga woh zero hona chahiye, X2 plane pe jo bhi forces ka summation hoga woh zero hona chahiye aur X3 plane pe forces ka summation hoga woh zero hona chahiye. Agar hum yeh static equilibrium condition maan ke chalte hain toh hum yahan pe Principal Stress ki value nikaal sakte hain. Toh yeh approach humne 2D stress state mein dekha tha yeh summation of forces humne yeh zero kiye thae aur usse humne principal stress nikaalte thae.

Iska derivation humne George Dieter mein diya hai toh hum isko is derivation yahan pe hum yeh derivation nahi dekhenge. Hum yahan par ek doosra approach istemaal karenge jo humne last class

mein bhi apply kiya tha jaise Matrix Equation ko apply karke. Toh humne yeh dekha tha ki agar mere paas ek koi stress matrix hai [A] aur uske Eigenvalues hain yeh jo λ (Lambda) hai woh eigenvalues hain is stress matrix ke toh ye eigenvalues jo hongy woh mere principal stresses dengy. Toh humne ek matrix equation $[\sigma]\{v\} = \lambda\{v\}$ likha tha aur yeh jo v hai yeh Eigenvector hai jo dikhayega ki principal planes kahan par hain toh uska location ye find out karne ke liye humein eigen vectors ki zaroorat hoti hai.

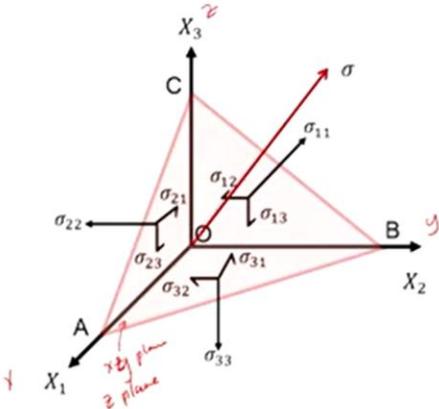
Toh abhi hum yahan par Principal Stresses nikalne hain toh hum yahan par hum iski eigenvalues nikaalenge. Toh yeh Determinant hum solve karenge $|A - \lambda I| = 0$ aur agar hum isko likhenge $(A - \lambda I)$ ko toh hum is tarah kuch likh paayenge. I mera Identity Matrix hai toh humein yeh equation mil jayega aur hum humne dekha tha ki yahan par yeh jo stress tensor hai yeh yahan par ye jo off-diagonal component hain equal hote hain. Toh jaise $\sigma_{12} = \sigma_{21}$ aur $\sigma_{13} = \sigma_{31}$ kyunki matrix symmetric hai toh yahan par yeh humne likha hai aur isko hum agar solve karenge toh humein kuch is tarah se equation milegi.



Principal stresses in 3D

$\sigma_{11} \neq 0$ uniaxial
 $\sigma_{11} = \sigma_{22} = \sigma_{33}$ biaxial
 $\sigma_{11} = \sigma_{22} = \sigma_{33}$ triaxial

$$A = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$



Assume plane ABC is a principal plane

σ is a Principal stress

$$\sum F_{X1} = 0, \sum F_{X2} = 0, \sum F_{X3} = 0$$

Similar approach as we dealt with 2D stress state

Derivation of this is given in Mechanical Metallurgy by G E Dieter

$$A \underline{v} = \lambda I \underline{v}$$

$$(A - \lambda I) \underline{v} = \underline{0}$$

$$\Rightarrow |A - \lambda I| = 0$$

$$\begin{bmatrix} \sigma_{11} - \lambda & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} - \lambda & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} - \lambda \end{bmatrix} = 0 \quad \begin{matrix} \because \sigma_{12} = \sigma_{21} \\ \sigma_{13} = \sigma_{31} \\ \sigma_{23} = \sigma_{32} \end{matrix}$$

6:36 - 7:30 Kuch is tarah se humein ek equation milegi jahan par mujhe milega λ^3 ek term, λ^2 ek term, λ ek term aur ek badi wali constant term = 0. Toh agar aap is equation ko solve karenge ya is tarah se hum dekhenge main is ye jo term hai $(\sigma_{11} + \sigma_{22} + \sigma_{33})$ isko main J1 likhta hoon aur

yeh jo term hai isko main likhunga J2 aur yeh jo badi term hai isko main likhunga J3. Agar is equation ko main is tarah se likhoon J1, J2 aur J3 ke form mein toh mujhe ye Cubic Equation is tarah se main likh paunga: $\lambda^3 - J1\lambda^2 + J2\lambda - J3 = 0$ Yeh mera cubic equation ho gaya.

Isko likh lete hain hum ye cubic equation ho gaya toh isko agar hum solve karenge equation ko solve karenge toh humein λ ke teen values milni chahiye kyunki yeh cubic equation hai toh iske teen roots hone chahiye. Toh yeh jo teen roots hain yeh humein yeh eigenvalues honggi aur yeh jo eigenvalue hai ye Principal Stresses ki values honggi. Toh yahan par humein teen eigenvalue milengi $\lambda_1, \lambda_2, \lambda_3$ aur phir ye eigenvalues unique honi chahiye kisi bhi given state of stress ke liye jo hamara state of stress kya tha yahan par state of stress hamara [A] tha jo humne dekha tha state of stress ye $\sigma_{11}, \sigma_{22}, \sigma_{33}$.

Ye hamara state of stress tha toh yeh hamara state of stress tha aur isko humne iska humne eigenvalues nikaalne ki koshish ki aur ye eigenvalue humein principal stresses degi aur iske jo eigenvalue hogi ye unique honi chahiye ki kisi bhi state of stress liye. Toh agar hum Coordinate Transform karte hain toh yeh state of stress same hona chahiye. Agar state of stress same hona chahiye toh yeh eigenvalues jo hain λ_1, λ_2 aur λ_3 change nahi honi chahiye yaani iska matlab yeh hai ki agar hume isko transform karta hoon transform yaane yaani maine rotation kar diya koi bhi ya maine coordinate axis change kar li yahan pe maine X1, X2, X3 aise kar liya tha.

Agar maine isko transform kar diya ya kisi bhi doosre coordinate axis mein maan lijiye kuch is tarah se X1', X2' aur X3' toh yeh agar transformation hua toh mujhe ek state of stress milega aur is state of stress ko main agar is tarah se likhoon [A'] likh leta hoon toh mujhe kuch is tarah value aayegi $\sigma_{11}', \sigma_{22}', \sigma_{33}', \sigma_{12}', \sigma_{23}', \sigma_{13}'$ aur iska agar main eigenvalue nikaalne ki koshish karunga jaise $|A' - \lambda I| = 0$ toh mujhe jo equation milega woh yahi equation hona chahiye woh equation change nahi hona chahiye.

Toh agar ye equation change nahi hoga toh iska matlab ye jo values hain J1, J2 aur J3 ye constant honi chahiye kisi bhi state of stress ke liye. Agar main yahan se equation nikaaloon aur yahan se ek equation nikaaloon toh yeh dono same hain iska matlab yeh jo J1, J2, J3 hain inki value same honi chahiye iska matlab kya hua yeh jo values hain ye Invariants hain yaani ye badlengi nahi kisi bhi state of stress ke liye. Toh ye Three Invariants mile humein. Humne last class mein dekha tha

ki hamare paas 2D stress state tha toh humein do invariant mile thae abhi hamare paas 3D stress state hai toh humein teen invariant milenge yeh humne last class mein bhi mention kiya tha.

Principal stresses in 3D

$$\lambda^3 - (\sigma_{11} + \sigma_{22} + \sigma_{33})\lambda^2 + (\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2)\lambda - (\sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{13} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2) = 0$$

$\sigma_{11} + \sigma_{22} + \sigma_{33} = J_1$
 $(\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2) = J_2$
 $\sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{13} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2 = J_3$

}

INVARIANT

Three invariants

$\lambda^3 - J_1\lambda^2 + J_2\lambda - J_3 = 0$

- The solution to this equation will yield three Eigen values: λ_1, λ_2 and λ_3
- The Eigen values are unique for a given state of stress
- Therefore, any coordinate transformation keeps the state of stress the same
- Eigen values: λ_1, λ_2 and λ_3 don't change, implies we will get the same Cubic equation

Toh hamare paas yeh teen invariants hain teen invariants matlab yeh jo values hain yeh change nahi hongy kisi bhi state of stress mein agar main σ_{11}' , σ_{22}' , σ_{33}' agar nikaaloon aur main is tarah se likho is stress state of stress ko is tarah se bhi main consider kar sakta hoon $\sigma_{11}' + \sigma_{22}' + \sigma_{33}'$ ye same honge change nahi honge. Us tarah se agar main ye equation σ_{11}' , σ_{22}' aur σ_{33}' mein likhoon ya is tarah se likhoon is is state of stress ka components lekar likhunga toh ye mujhe J_2 ki value milega woh same rahegi woh change nahi rahegi similarly J_3 ki value same rahegi agar main A' ke component istemaal kar raha hoon.

Toh yeh yeh mere paas teen invariants hain abhi iska mahatva jaante hain ye mera state of stress hai toh mera J_1 main phir se likhunga isko $\sigma_{11} + \sigma_{22} + \sigma_{33}$ agar ye dekhenge toh yeh kya hai yeh summation of normal elements aur normal elements of this stress matrix. Toh yeh summation inka hai ka sum hai toh yeh sum of diagonal elements aur isko hum kehte hain Trace (tr) of a stress matrix jo diagonal elements ka summation hai usko hum kehte hain Trace of a stress matrix. Toh J_1 mera state of a stress matrix rahega. Similarly J_2 jo hai woh yeh term hai $\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2$. Isko main is tarah se likh sakta hoon.

Agar main isko solve karunga toh toh main is tarah se is determinant mein likh sakta hoon. Agar aap yeh dekhenge toh yeh hai mere ye jo hain Minors hain aur minors kiske hain mere diagonal elements ke. Toh minors kya hote hain agar aap matrix algebra padhenge toh jaise main σ_{11} ka minor nikaaloon toh σ_{11} ka minor hoga ye mera determinant theek hai. Jaise main σ_{22} ka minor nikaaloon toh main ye ye value chhod doonga yeh value chhod doonga yeh chhod doonga aur yeh bhi chhod doonga toh mere paas rahengi σ_{22} ka agar main minor nikaaloon toh yeh aa jayega $[\sigma_{11}, \sigma_{13}; \sigma_{31}, \sigma_{33}]$.



Principal stresses in 3D: Invariants

$$A = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \quad \text{Minor} = \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{13} & \sigma_{33} \end{vmatrix}$$

$$J_1 = (\sigma_{11} + \sigma_{22} + \sigma_{33}) \quad \text{Sum of the diagonal elements or the Trace of a stress matrix}$$

$$J_2 = (\sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{33}\sigma_{11} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{13}^2)$$

$$= \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{13} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{vmatrix} \quad \text{Sum of the minor of diagonal elements}$$

$$J_3 = \sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{13} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{13}^2 - \sigma_{33}\sigma_{12}^2 \quad \text{Det (A)}$$

Toh yahan par jo yeh isko main solve karunga toh mujhe yeh J_2 milega toh ye J_2 kya hota hai J_2 hota hai Sum of the Minors of Diagonal Elements. Diagonal elements ke agar main minor loonga toh mujhe ye J_2 milega. Similarly J_3 agar hum dekhenge toh yeh J_3 yeh jo stress matrix hai iska Determinant ($|A|$) hoga. Toh is tarah se hum J_1 , J_2 aur J_3 nikaal sakte hain agar mujhe koi bhi stress matrix pata hai toh. Toh inka ek importance hai toh abhi is class mein yeh mention karna chahta hoon ki jo J_2 hai iska istemaal karenge Yielding Criteria jab hum discuss karenge tab J_2 ka istemaal hota hai ye bas abhi ke liye aap yaad rakhen ki J_2 jo invariant hai iska hum istemaal aage jaakar karne wale hain.

Toh abhi yeh jo equation aaya hua tha mere paas ek cubic equation aaya tha is iske roots jo honge woh eigenvalue honge aur jo eigenvalue hain woh hamare Principal Stresses hain. Toh maan lijiye isko is equation ko hum normal way se bhi solve kar sakte hain yaane yeh cubic equation hai hamare paas toh isko normal way se iske roots nikaal sakte hain. Agar maan lijiye hamare paas ek root mil jata hai hum λ_1 milta hai toh yeh upar wala jo equation hai hum is tarah se likh paayenge $(\lambda - \lambda_1)$ aur ek quadratic equation aayega aur isko hum shunya maan ke challenge.

Toh agar iska hum solution nikaalne ki koshish karenge ek general solution ye cubic equation ka nikaalenge isse toh hum ek kaam kar sakte hain ki λ ki jagah hum $t + 1/3 J_1$ yeh value rakh ke dekh sakte hain theek hai. Toh yeh value rakhne ke baad humein yeh λ ki jagah humne yeh value rakh li toh humein yeh ek equation mil jayega J_1, J_2 aur J_3 ke form mein. Toh agar hum isko dekhenge is equation ko dekhenge toh yeh Depressed Cubic hai yaani depressed cubic yaani yahan par koi bhi square term nahi hai. Square term nahi hai na iska matlab yahan par dekhiye yahan par t^3 hai aur ek t hai aur yeh jo ye jo cheez hai yeh jo do terms hain yeh terms constant hain.

Kyunki ye constant isliye hai kyunki humne bola tha ki J_1, J_2 aur J_3 ye invariants hain agar ye invariants hain toh inse bhi jo bhi terms hum taiyaar karne wale hain J_1, J_2, J_3 ke combination se woh saari terms constant hi honi chahiye jaise $J_2 - 1/3 J_1^2$ yahan pe J_1 aur J_2 hai yahan pe mere paas J_1, J_2 aur J_3 teeno teeno hain. Toh ye jo terms hain ye constant ho gayi aur ye jo form hai yeh mere paas depressed cubic form hai toh main isko is tarah se likhunga yeh jo ye jo term hai usko main p maan ke chal raha hoon aur ye jo term hai isko main q maan ke chal raha hoon toh mere paas ye equation aa jayega $t^3 + pt + q = 0$.

Toh iske agar roots nikaalenge toh iska ye solution available hai toh yeh iske yeh jo equation ke roots hain yeh t ke equal to yeh term aa jayega jo p aur q ke form mein hai. Agar hum dekhenge ki yahan par k value hai yeh teen roots aa jayenge k ki value shuru hoti hai 0, 1 aur 2 yeh dhyaan rakhne wali baat hai. Toh yahan par jaise t_0, t_1 aur t_2 ye jo values aayengi t_0, t_1 aur t_2 aur mujhe pata hai λ isse main λ nikaal paunga kyunki yahan par main λ agar rakhunga toh mere paas aa jayegi value λ_0, λ_1 aur λ_2 kyunki humne t, λ ko is tarah se लिखा है. Toh yeh value agar main put karunga yahan par alag alag t_0, t_1 aur t_2 ki value.

Toh mujhe $\lambda_0, \lambda_1, \lambda_2$ ye mujhe mere paas teen eigen values aa jayengi aur yeh jo eigen values hain yeh mere ye meri Principal Stresses hain. Toh agar yeh real values milni hain ya real roots milne hain toh yeh condition satisfy honi chahiye jo ki hai $4p^3 + 27q^2 < 0$ yaani shunya se kam honi chahiye. Toh hamare paas yeh eigenvalue aa jayegi aur eigenvalue hi nothing but hamare principal stresses. Toh ek solved example main share karunga aapke saath jo ki aapka ek assignment hoga ek excel sheet ke form mein yeh hum ek 3D stress state lenge aur uske roots nikaalne ki koshish karenge is approach se.



Principal stresses in 3D: Solution to the cubic equation

$$\lambda^3 - J_1\lambda^2 + J_2\lambda - J_3 = 0$$

If we know one of the roots, say λ_1 , then the above equation reduces to

$$(\lambda - \lambda_1)(\text{Quadratic equation}) = 0$$

General solution to cubic equation

Substitute: $\lambda = t + \frac{1}{3}J_1$

$$t^3 + \left(J_2 - \frac{1}{3}J_1^2\right)t + \left(\frac{4}{27}J_1^3 + \frac{1}{3}J_1J_2 - J_3\right) = 0$$

Depressed cubic:
No square term

$$t^3 + pt + q = 0$$

$$\text{Roots: } t_k = 2\sqrt{\frac{-p}{3}} \cos \left[\frac{1}{3} \cos^{-1} \left(\frac{3q}{2p} \sqrt{\frac{-3}{p}} \right) - \frac{2\pi k}{3} \right] \quad k = 0, 1, 2$$

For real roots: $4p^3 + 27q^2 < 0$

Solved Example: Excel

J₁ J₂ J₃ invariants
t₀ t₁ t₂ → λ = t + 1/3 J₁
λ₀ λ₁ λ₂

Toh abhi dekhte hain ki hamare paas agar principal stresses aa gaye jaise $\lambda_1, \lambda_2, \lambda_3$ jo hamare principal stresses ki values ko darshaate hain toh ismein se humne dekha tha ki Major Principal Plane, Minor Principal Plane jo ki depend thi σ_1, σ_2 aur σ_3 ki values par. Toh by convention hum yeh maan ke chalte hain agar σ_1 bada hai toh σ_1 yaani arithmetically bada hona chahiye σ_2 se aur σ_2 arithmetically bada hona chahiye σ_3 se. Toh σ_1 humara algebraically greatest principal stress aur σ_3 algebraically smallest principal stress. Toh humne dekha tha ki Major Principal Plane, Minor Principal Plane...

Toh yahan par 3D state mein yeh ho jayega mera Major Principal Plane aur yeh ho jayega Minor Principal Plane. Ab 2D Stress State dekhte hain ki hum isko kaise likh sakte hain. Toh maan lijiye

mere paas ek 2D stress state hai yahan par 2D stress state yaane mere paas σ_1 ya σ_2 aur ek third direction mein jo stress hoga woh shunya hai yahan par woh zero hai. Toh pehle abhi inke toh hum signs assign kar lete hain kyunki normal stresses saare in teen case mein yeh saare stresses jo hain normal stresses hain normal yaane perpendicular (perp) to yeh plane hai isko perpendicular hai. Toh yeh maan lete yeh hamara Positive Stress hai kyunki tensile stress hai.

Yeh bhi hamara positive stress ho jayega kyunki ye tensile stress hai. Yeh hamara positive stress hai kyunki ye tensile stress hai yeh Negative Stress hai kyunki ye compressive stress hai yeh negative stress ho jayega aur yeh bhi negative stress ho jayega kyunki dono compressive stress hain. Toh yahan pe σ_1 kaun si σ_1 hoga mera 80 MPa aur ye jo ho jayega mera σ_2 kyunki 80 MPa ye arithmetically bada hai 50 MPa se. Is keis mein σ_1 mera 80 hoga aur yeh σ_2 ho jayega aur par yahan par aap dekhiyega ki -80 negative hai toh yeh arithmetically small hai toh yahan par σ_1 mera 50 hoga aur σ_2 ho jayega -80 MPa. Toh similar approach hum 3D state liye lete hain jaise maan lete mere paas 150, 100 aur -200 hai.

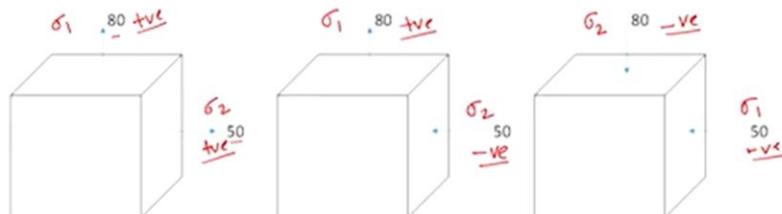


Three principal stresses: $\sigma_1 > \sigma_2 > \sigma_3$???

By convention:
 $\sigma_1 > \sigma_2 > \sigma_3$

σ_1 is the algebraically greatest principal stress
 σ_3 is the algebraically smallest stress

major principal
minor principal



Similar, approach is taken in the case of a 3D stress state

150, 100, -200
↓ ↓ ↓
 σ_1 σ_2 σ_3

20:46 - 21:47 Toh yahan par hum dekhenge ki arithmetically sabse bada mere paas 150 hai toh yeh σ_1 ho jayega aur arithmetically sabse chhota mere paas -200 hai toh yeh σ_3 ho jayega aur σ_2 mera 100 MPa ho jayega. Toh abhi hum dekhte hain ki isko yeh jo principal stresses hain inko

geometrically kaise represent karte hain. Toh humne dekha tha ki 2D stress state mein humne stresses ko represent kiya tha geometrically Mohr Circle se. Toh 3D stress state mein bhi hum dekhte hain ki Mohr circle pe jo 3D stress state hai kaise represent karte hain. Toh maan lete hain hamare paas ek stress cube hai jispe hamare paas different stresses lag rahe different planes pe.

Toh yeh saare stresses yahan par normal stresses diye hain toh maan lete hain hum apne pehle coordinate axis mark kar lete hain X_1, X_2, X_3 . Toh maine kaha tha ye isko hum X plane bhi keh sakte X direction bhi keh sakte Y direction bhi keh sakte aur Z direction bhi keh sakte hain. Toh yahan pe mere paas teen values aa jayengi jaise $\sigma_1 = 100$ MPa ho jayega, $\sigma_2 = 50$ MPa aur $\sigma_3 = 20$ MPa. Toh agar hume isko Mohr circle pe represent karna hai toh yahan pe humein pata hai ki hum circle ko represent karte hain is axis pe hum likhte hain Normal Stress (σ) aur is axis pe hum mark karte hain Shear Stress (τ). Toh yahan pe is keis mein toh mere paas shear stress toh shunya (0) hai sirf mere paas normal stresses hain.

Aur normal stresses agar hum dekhenge toh saare stresses positive hain yaani tensile stresses hain toh yeh saari mark karenge hum positive direction pe positive X direction pe. Toh mark karte hain teeno stress toh yeh mere mera ho jayega σ_1 yeh ho jayega mera σ_2 aur yeh ho jayega σ_3 . Toh humne dekha tha ki circle par jab hum mark karte hain stress toh jo length hoti hai yeh hamare stress ke barabar hoti hai. Toh isko jahan mark karenge $\sigma_1, \sigma_2, \sigma_3$ toh abhi hum Mohr circle nikaalenge. Toh Mohr circle kaise nikaalte hain? Toh Mohr circle pehle hum nikaalenge σ_1 aur σ_2 ke beech mein.

Toh σ_1 aur σ_2 ke beech mein agar Mohr circle nikaalna hai toh mujhe iski radius pata honi chahiye. Toh radius hogi ya σ_2 aur σ_3 ke beech mein nikaalenge toh is tarah se circle aayega. Pehle dekh lete hain ki radius kaise nikaalte hain toh humein radius pata hai ki radius hogi yahan par σ_1 aur σ_2 ke beech mein isko main R1 mark kar raha hoon $(\sigma_1 - \sigma_2) / 2$. Par humein radius bhi pata hai par iska Center (C) bhi humein pata hona chahiye toh hamare paas iska center hoga H1 isko main keh raha hoon ye aayega $(\sigma_1 + \sigma_2) / 2$. Humne dekha tha ki radius aur center kaise nikaalte hain 2D stress state ke liye similar approach hum yahan par apply karenge.

Toh hum pehle σ_1 aur σ_2 consider kar rahe toh yeh mere paas ek circle ho gaya uska radius yeh ho jayega $(\sigma_1 - \sigma_2) / 2$ aur center ho jayega iska $(\sigma_1 + \sigma_2) / 2$. Similarly hum baaki cheezein bhi

mark kar lete hain jaise radius do (R_2) agar mujhe mark karna hai toh yeh main mark kar raha hoon σ_2 aur σ_3 ke beech mein toh ye radius iski aa jayega $(\sigma_2 - \sigma_3) / 2$ aur iska center aa jayega ye ye jo yellow circle hai iska center aayega $(\sigma_2 + \sigma_3) / 2$. Similarly agar hum σ_1 aur σ_3 ke beech mein mark karenge circle toh woh is tarah se aayega yeh blue circle aayega.

Toh iska radius hoga iska radius hoga R_3 aur is R_3 ko hum mark karenge $(\sigma_1 - \sigma_3) / 2$ aur iska center aayega H_3 jo $(\sigma_1 + \sigma_3) / 2$. Toh is tarah se hum teen circles mark kar sakte hain jo green circle hai σ_1 aur σ_2 ke beech mein hai, jo yellow circle hai σ_2 aur σ_3 ke beech mein hai aur jo blue circle hai yahan par woh σ_1 aur σ_3 ke beech mein. Toh yeh kya darshaata hai? Jaise main σ_1 aur σ_2 ke beech mein jo green circle hai yeh mera represent karega koi bhi stress XY plane par. Toh XY plane kahan par yahan par toh XY plane mera yeh hai yeh mera XY plane hai X_1 aur X_2 ke beech mein.

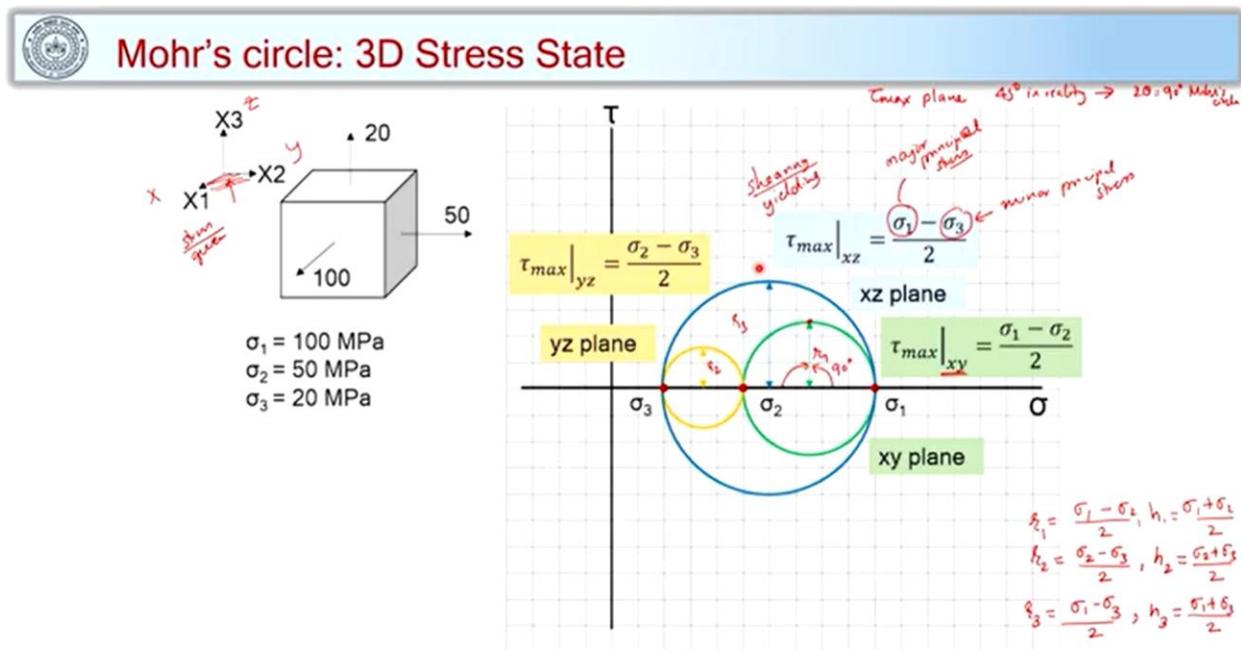
Toh yeh mera XY plane hoga yahan par jo bhi stress hoga toh woh green circle par mark kiya jayega mark hoga. Similarly yeh jo stress yahan pe jo stress hoga σ_2 aur σ_3 yeh mera YZ plane hoga YZ plane yaani yeh plane hoga X_2 aur X_3 ke beech wala. Toh jo bhi state of stress hoga is plane mein woh yeh yellow circle par mark hoga. Similarly jo bhi blue circle par hai woh mera XZ plane ka jo bhi stress hoga woh is circle par mark kiya jayega. Toh abhi hum nikaalenge ki Maximum Shear (τ_{max}) kya hoga. Jaise hum agar is plane ki baat karen XY plane ki σ_1 , σ_2 ke beech mein toh maximum shear mera mujhe pata hai ki ye equal to radius of circle hoga.

Toh yeh jo hoga radius of circle hoga toh is plane pe maximum shear ki value yeh aani chahiye XY plane par jo maximum shear ki value aayegi woh $(\sigma_1 - \sigma_2) / 2$ jo ki radius of this circle. Toh isko specifically mark kar lete hain ye R_1 hoga similarly ye jo hoga ye R_2 hona chahiye aur yahan par jo aayega woh R_3 hona chahiye blue circle ke liye. Toh ye R_2 hoga jo τ_{max} jo yahan par milega YZ plane pe woh aayega $(\sigma_2 - \sigma_3) / 2$ aur XZ plane par jo aayega woh R_3 aayega woh aayega $(\sigma_1 - \sigma_3) / 2$.

Toh agar aap dekhengy jaise R_1 hai ya is plane par jo shear hai maximum shear hai ye aa raha hai woh humne dekha tha ki Mohr circle par Principal Plane se 45 degree rahega toh Mohr circle par woh 90 degree rahega. Toh hum dekhenge ki dono jaise hum dekhenge maximum shear plane jo rahega woh 45 degree in reality rahega but Mohr circle par woh rahega 2θ yaani woh 90 degree

rahega Mohr circle par iska location. Toh agar hum dekhenge toh yahan par in teeno mein se maximum jo shear ki value hai woh mujhe XZ plane par mil rahi hai jo $(\sigma_1 - \sigma_3) / 2$ hai.

Toh agar hum dekhenge yeh jo σ_1 hai yeh mera Major Principal Plane hai aur major principal stress hoga aur ye jo σ_3 hai ye mera minor hoga principal stress. Toh yeh mere paas do major aur minor principal stresses aayenge aur inka jo difference hoga wahi mujhe maximum shear ki value dega. Toh yeh jo value hai woh bahut important hai hum isko bhi dekhenge agli classes mein jiska mahatva hum dekhenge ki shearing mein kya hota hai. Shearing mein as well as Yielding mein iska kya importance hai. Toh ye hum agle kuch classes mein dekhenge.



Toh humne yahan par dekha ki ek state of stress agar hamare paas hai 3D stress sstate hai usse humne eigenvalues kaise nikaalni hain. Eigenvalues jo hain woh principal stresses ki value humein degi. Jo principal stresses hain hum usko geometrically kahan par represent karenge Mohr circle se ye humne yahan par dekha aur Mohr circle se humne maximum shear stresses ki values nikaalne ki koshish ki. Toh abhi hum yahan rukte hain agle class mein hum dekhenge ki general way se stress ko kaise transform kiya jayega. Dhanyavaad.

