

Mechanical behavior of materials

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Week-2

Lecture-11

Principal stresses & Principal planes

Course Title

Mechanical Behavior of Materials (Hindi)

Lecture-11
Principal stresses & Principal planes

Namaskar! Toh humne last part mein dekha tha ki Mohr Circle (\odot) ko kaise plot karte hain. Abhi ek practical example dekhenge. Toh mere paas ek 2D Stress State hai jahan par maine coordinate axis mark kar diye hain, X aur Y. Toh hum jab plot karenge toh hum plot karenge σ (Sigma) jo hai Normal Stress, woh X-axis par plot karenge aur τ (Tau) jo hai hum woh Y-axis par plot karenge. Toh hum yahan par jo ek unit hai woh 10 MPa hai. Maine bola tha ki jo length hai woh stress ko represent karega. Toh yahan par hum dekhenge ki state of stress kaise hai. Toh ye jo A plane hai mera, toh A plane pe 80 MPa hai, par 80 MPa pe kya hai? Compressive Stress hai.

Kyunki against is plane ke act kar raha hai, against ye plane ka jo normal hai uske against act kar raha hai. Toh ye compressive stress hoga, toh yeh aa jayega mera negative X-axis pe. Aur yeh jo 25 MPa hai, yeh Shear Stress (τ) hai. Yeh shear stress kis direction mein aana chahiye? Ye jo mera shear stress hai, ye is element ko Clockwise (\curvearrowright) rotate kar raha hai, toh yeh positive Y-axis par aayega. Similarly, yeh jo point B aur plane B hai jo A se 90° apart hai, toh yeh jo point hai, ye stress hai, ye mera Tensile Stress hai, toh yeh positive X-axis pe aayega. Aur ye jo stress hai, ye stress mera isko rotate kar raha hai is element ko yaane ye τ_{yx} hoga. Yeh rotate kar raha hai mere element ko Counter-Clockwise (\curvearrowleft).

Toh ye negative Y-direction mein aayega. Toh hum isko plot karte hain. Point A aa jayega mera -80 jo negative side pe compressive stress hai isliye normal stress, aur jo shear stress hai is plane pe act kar raha hai woh is element ko rotate kar raha hai clockwise direction, toh isliye hum usko positive direction mein plot kiye. Aur mera Point B hai jo tensile stress hai, toh isliye positive X-axis pe aaya aur yeh jo stress hai 25 MPa, yeh rotate kar raha hai counter-clockwise toh isliye hum isko plot karenge negative Y-direction pe. Humne dekha tha hum is dono point ko join karenge kyunki ye 90° apart hain.

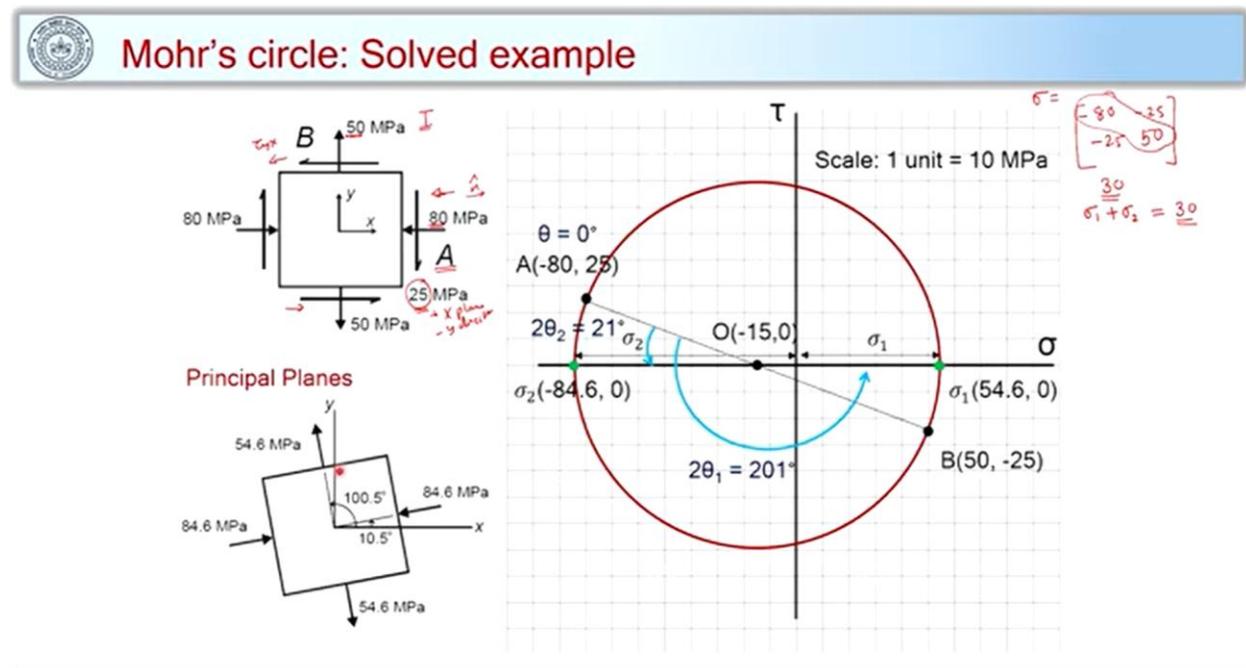
Toh Mohr Circle pe 180° apart karenge. Toh ye jahan par intersect karega, yeh mera ho jayega Center (C) of Mohr circle aur iski value aayegi (-15, 0). Yeh aap yahan se length measure kar sakte hain origin se. Hum jab measure karenge, saari lengths sab origin se hi measure karenge. Toh yeh aa jayega (-15, 0). Aur abhi humein center pata hai, hum yeh Radius (R) nikaal sakte hain aur radius se hum circle draw kar sakte hain. Yeh humare paas yeh circle aa jayega. Jab humare paas circle aayega toh ab hum dekhte hain ki Principal Planes kahan par hain. Toh mark karte hain ye do points. Principal planes yaane kya hota hai jahan pe shear stress (τ) ki value shunya yaane zero (0) hoti hai.

Humne dekha tha aur ye jo stress hoga, ye mera σ_1 ho jayega kyunki arithmetically yeh bada hai aur ye σ_2 ho jayega. Toh hum σ_1 ki value nikaal lete hain. σ_1 ki value hai 54.6. Agar measure karenge length yahan se origin se toh ye 54.6 aa jayega aur ye measure karenge toh ye aa jayega -84.6. Aap isko cross-verify bhi kar sakte hain. Agar main state of stress ko aise likhta hoon, jaise hum ye state of stress humne nikaala tha, 80 jo negative hai aur yeh agar hum notation dekhenge, yeh stress jo hai -25 hai, ye kis direction mein hai? Ye positive X-plane pe hai aur positive X-plane aur negative Y-direction ke taraf hai. Toh yeh state of stress ho jayega -25 yahan pe.

Aur yeh dono samaan hote hain toh isliye -25. Yeh mera aa jayega yeh mera state of stress hai ye element ke corresponding. Agar hum property dekhenge Mohr circle ki toh $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$ barabar hona chahiye. Toh hum $\sigma_1 + \sigma_2$ ko bhi add karke dekhenge toh iska bhi sum aana chahiye -30. Check kar sakte hain, -30 MPa aa raha hai. Toh hum yahan se bhi humara answer cross-check kar sakte hain ki Summation (Σ) of normal stresses remains invariant. Toh abhi hum inka location dekhte hain. Toh A ko main 0° maan raha hoon toh mera reference plane hai. Abhi σ_2 ke liye

mujhe kis direction mein move hona hai? σ_2 ke liye mujhe move hona hai counter-clockwise direction aur yeh jo angle hoga...

yeh hoga mera 2θ aur yeh jo angle maine measure kiya yeh 21° ho gaya. Toh yeh Mohr circle par toh 21° hai, actually yeh iska aadha ($1/2$) rahega jab hum physically isko plot karenge. Similarly, agar yeh angle dekhein doosre principal plane ka yahan par point A se ya plane A se toh yeh $2\theta_1$ aa raha hai mera 201° . Toh actually yeh rahega usse aadha. Toh hum jab principal planes plot karenge toh is par hum dekhenge yeh jo plane hai mera σ_2 pehle main plot kar raha hoon, ye kis direction mein move hai? Ye mera move hai counter-clockwise direction. Toh ye mera reference planes tha X aur Y aur main is direction mein move karunga counter-clockwise toh ye aa jayega 10.5° kyunki...



yahan pe 21° hai iska aadha aa jayega 10.5° . Aur yahan pe jo stress lag rahi hai woh hai 84.6 jo ki compressive hai, toh main is direction se usko yaane plane ke against usko mark karunga. Agar hum dekhenge ye jo σ_1 plane hai woh kitna degree apart hai? Ye yahan pe Mohr circle pe 201° apart hai toh ye iska aadha ho jayega, toh yeh aa jayega 100.5° aur yeh jo stress yahan pe lag rahi hai yeh tensile stress hai jo 54.6 hai. Toh agar hum dekhenge yahan pe koi shear stress nahi hai

toh ye mera element ho gaya jahan par sirf principal yaane principal element hai jo yahan par maine...

principal plane dikhaye jahan par shear stress shunya (0) hai. Toh ye ho gaya mera principal planes. Abhi hum dekhte hain ki shear stress plane of Maximum Shear (τ_{max}) yeh bhi dekhte hain. Toh yeh maine principal planes plot kiye aur jab main yahan par ja raha hoon toh maximum shear mujhe yeh milega. Toh aur ya is side pe mil sakta hai. Toh magnitude dono ka same hoga jo radius (R) rahegi meri. Toh radius maine dekha tha, radius humne dekhi thi radius kya honi chahiye? Isko likh lete hain, radius aayegi meri: $Radius = (\sigma_1 - \sigma_2) / 2$. Yeh ho jayega mera maximum shear stress. Toh agar hum isko calculate karenge...

toh ye aa jayega 69.6. Yeh mera τ_{max} hoga. Isko bhi likh lete hain τ_{max} mera radius ke barabar rahega aur yahan pe normal stress bhi rahega. Normal stress kya aayega? Normal stress ki value aayegi meri -15 aani chahiye. Toh normal stress ki value kyunki center ke barabar rahegi toh yeh normal stress ki value bhi -15 aani chahiye. Toh agar hum iska location dekhte hain plane of maximum shear point A se nikaalte hain toh ye mera yeh reference point hai toh main isko kis direction mein ja raha hoon? Main ja raha hoon Clockwise (\curvearrowright) direction mein. Ye aa raha hai mera 69° . Yeh Mohr circle par aa raha hai toh physically aayega iske...

aadha. Ya ab jab σ , τ_{max} humne ki baat karenge toh yeh aa jayega mera $(\sigma_{xx} + \sigma_{yy}) / 2$. Toh yeh yeh bhi value aapko yaad rakhni hai ki jo center hai mera Mohr circle ka aur τ_{max} plane of maximum shear hai uska jo normal stress hai uski value yeh rahegi hamesha. Toh yeh aa jayegi meri (-15, 69.6). Aur main agar isko plot karunga is tarah se toh yeh jo mera X aur Y references hain toh main kis direction mein move ho raha hoon? Main move ho raha hoon clockwise direction mein toh main is tarah se clockwise direction mein move hoonga. Toh ye yahan pe 69° hai toh iska aadha ho jayega 34.5° . Aur yahan pe yeh plane maine draw kiya hai toh yahan pe...

normal stress kitna hai 15 jo ki negative value hai toh yeh compressive stress hoga. Toh maine is tarah se mark kiya isko -15 yaane against this plane. Aur shear stress jo hum dekhenge yeh hai 69.6. Toh yeh shear stress main mark karunga is direction par aur jab hum dekhenge yeh plane bhi hum nikaalenge yeh plane agar hum nikaalenge toh humein milega yeh plane 55.5° apart. Toh yeh

agar hum calculate karenge toh yeh iska double ho jayega aur yahan par bhi state of stress... peeche jata hoon... yahan par bhi state of stress aa jayega. Main likh leta hoon yahan...

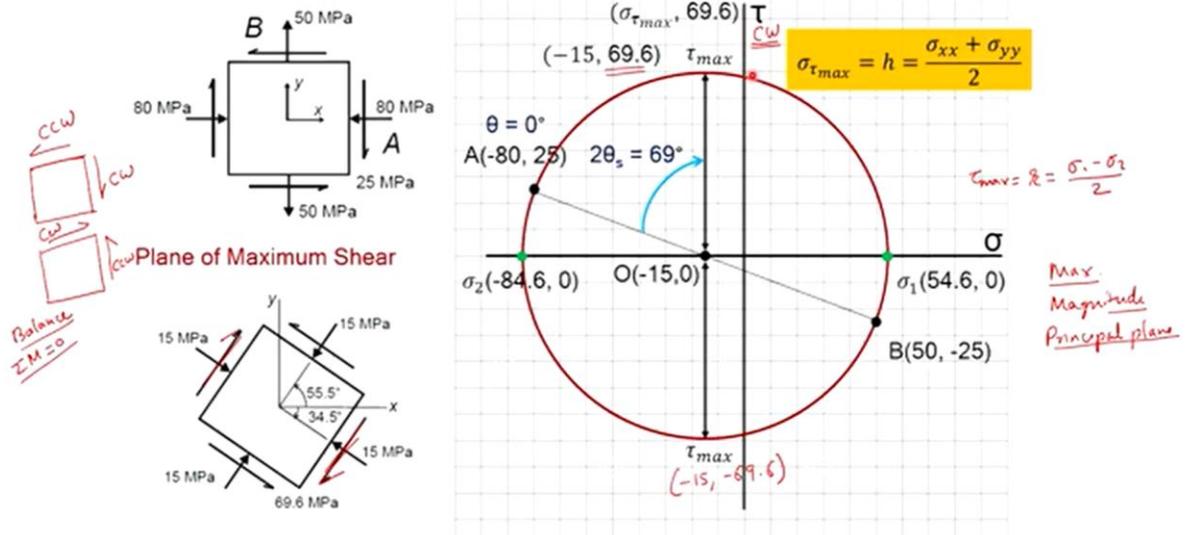
par state of stress aayega yahan par -15 aur yeh aa jayega -69.6, theek hai. Toh humne jab dekha tha abhi... abhi hum ye ye stress kaise mark karte hain dekhte hain yahan pe. Shear stress jo hai 69.6 hai aur humne dekha tha ki jo positive hai toh hum isko clockwise notion se mark karte hain. Toh yeh jo stress yahan par ye clockwise notion de raha hai humein yeh jo stress hai. Toh aur ya yeh stress hai yeh clockwise notion dega toh isliye hum iska direction is side rakhenge aur iska direction uske opposite rahega. Toh hum aur ek cheez main yahan pe mark karna chahta hoon yahan pe aapko bataana chahta hoon ki jab hum kisi bhi element ko mark...

karte hain 2D Stress State mein hum likh rahe hain toh agar mera shear stress is direction mein hai toh doosra stress jo iske plane ko perpendicular hai woh uske opposite direction mein rahega ya woh ek toh is direction mein hai ya is direction mein. Yeh isliye hai kyunki agar ye mera clockwise notion de raha hai ye counter-clockwise notion de raha hai, yeh mera clockwise notion dega aur yeh mera counter-clockwise notion dega. Toh yeh dono balance karne chahiye kyunki humein hamesha dekhna hai ki yeh Moment (ΔM) shunya hona chahiye. Is hamara Static Equilibrium ka condition hai isliye iska direction hum is tarah se hi mark karenge.

Toh ye ho gaya mera Mohr circle ek humne solved problem dekha. Aur ismein se ek aur ek important cheez main bataana chahta hoon. Agar hum hum dekhenge toh mere paas ye principal planes hain toh agar iski value dekhenge sirf magnitude dekhenge toh yahan par 54.6 hai aur yahan par 84.6 hai. Toh humein kisi bhi plane par agar hum is circle par koi bhi plane agar consider karenge toh maximum value magnitude ke baare mein agar magnitude dekhenge hum sirf toh woh principal planes par hi aati hai. Toh isliye principal planes hamesha important hote hain. Kuch kuch engineering application mein humein maximum shear nikaalna hota hai, maximum shear value isliye yeh planes jo...



Mohr's circle: Solved example



47

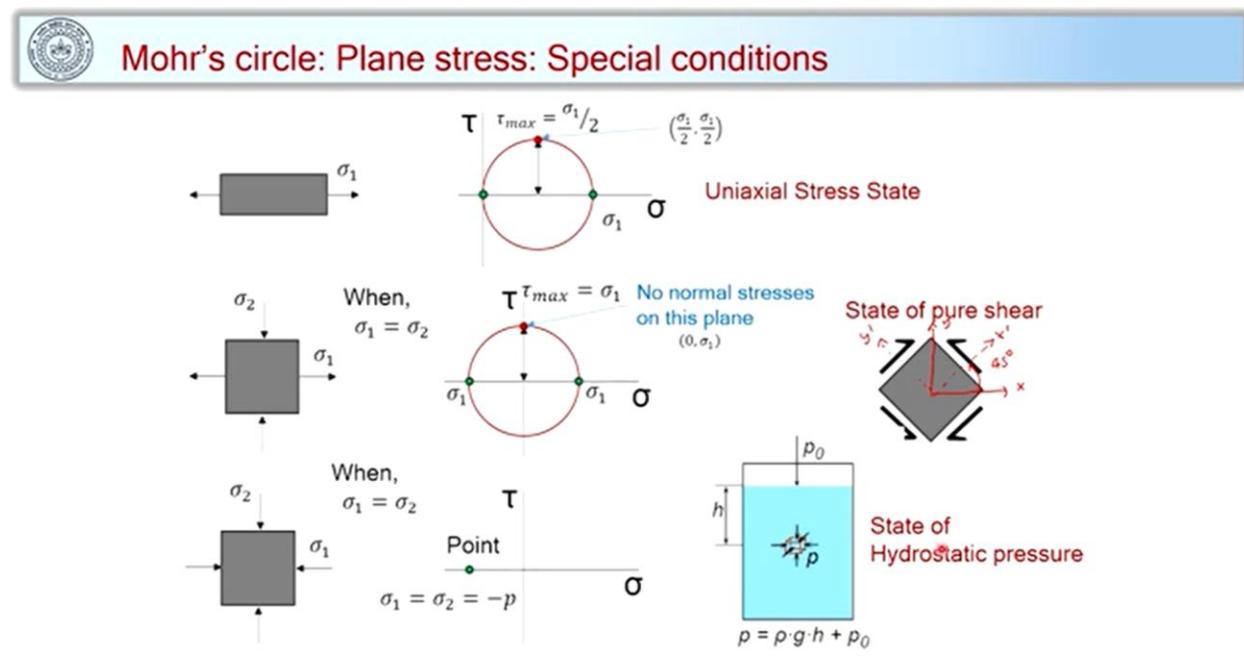
hain principal planes, major principal plane, minor principal plane aur plane of maximum shear ye isliye important hote hain calculations mein. Yeh saari jo cheezein humne ki hain, equations nikaale analytically ya Mohr circle nikaala, ye in conditions ko nikaalne ke liye: principal planes aur plane of maximum shear. Abhi hum aage badhte hain aur kuch interesting conditions dekhte hain plane stress ke around, jaise yeh Uniaxial Stress State hai jahan par sirf ek direction par stress laga hua hai. Is plane par koi stress nahi laga hua hai toh isko hum kaise plot karenge Mohr circle ke around? Toh σ mera yahan par shunya...

hai is plane pe. Toh yeh shunya yahan par aa gaya aur yeh jo stress hai σ_1 hai jo maine mark kiya aur hum abhi isko diameter (\varnothing) maan ke iska circle nikaal sakte hain. Yeh circle is tarah se aa jayega aur plane of maximum shear aa jayega yahan pe maximum value. Aur ye maximum value aa jayegi meri $\sigma_1 / 2$. Humne dekha tha ki iski radius jo hogi $(\sigma_1 - 0) / 2$, toh ye $\sigma_1 / 2$ ho jayegi. Aur iske coordinates agar hum dekhenge toh ye $(\sigma_1/2, \sigma_1/2)$ aane chahiye. Yeh jo condition hai yeh humari uniaxial stress state hai kyunki yahan pe ek hi stress lag raha hai jo normal stress hai aur ek hi direction mein hai. Toh yeh ho jayega humara uniaxial state of stress.

Ek Biaxial State dekhte hain jahan pe dono direction mein stress lag raha hai aur maan lete hain ki σ_1 magnitude barabar hai par direction opposite hai. Toh σ_1 yahan pe aani chahiye compressive

value aur tensile hai toh ye positive side mein aani chahiye. Toh ye $\sigma_2 = -\sigma_1$. Toh isliye maine isko σ_1 hi likha hai. Toh is condition mein hum dekhenge ki plane of maximum shear yahan par hai aur is point ke corresponding normal stress kitna hai yahan par shunya (0) hai. Toh is state of stress ko hum agar element ke through karenge toh mujhe is tarah se element milna chahiye aur is state of stress ko hum kahenge State of Pure Shear. Toh agar hum...

dekhenge humne bola bhi tha ki state of pure shear mera agar yeh mera reference axis hai X aur Y, toh yeh jo planes hain mere main dekhoonga yeh jo planes hain X', Y', state of pure shear ye hamesha 45° hone chahiye. Toh yeh yahan se bhi aap dekh sakte hain ki ye 45° hoga. Aur ek special condition dekhte hain, is case mein dono compressive hain aur dono ka magnitude same hai. Toh σ_2 bhi negative side pe, σ_1 bhi negative side pe aur dono ka value agar same hai toh isko maan ke chalta hoon is magnitude ko P. Toh ye Mohr circle pe ek point (·) aayega. Isko kehte hain hum State of Hydrostatic Pressure.



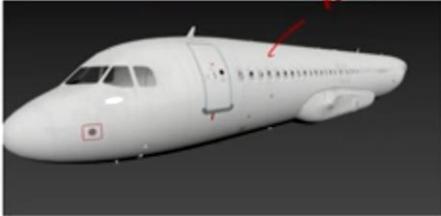
Yeh condition tab arise hoti hai jaise koi bhi material hai woh paani mein submerged hai yaane dooba hua hai. Toh uske face par agar koi hum ek element lenge toh uske jo infinitesimal cube jo hoga woh cube ke jo faces honge uspe same pressure padega, woh compressive rahega. Toh isko isliye is condition ko hum kehte hain State of Hydrostatic Pressure. Toh iska hum istemaal karenge

dono states ka, state of pure shear, state of hydrostatic pressure humare bhavishya ke analysis mein. Toh plane stress humne abhi tak baat ki toh yeh kahan kahan par important hai hum dekhenge ki plane stress maine bataaya tha ki plane stress tab bhi aata hai ki agar mera material...

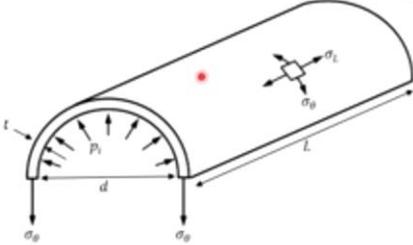
hai jo bhi component hai usmein length width bahut badi hai thickness (t) ki tulna mein. Thickness bahut small hai yahan par. Toh yeh aisi condition humein isi agar dimensions aise hain humare component ke toh humein state of plane stress milta hai. Hum dekhenge ki humare yeh kuch cans hain cold drinks ke. Toh yahan par hum dekhenge length aur width iski badi hoti hai par thickness kam hota hai aur yeh pressurized hai. Toh hum dekhenge ki state of stress yahan par 2D Stress State hoga ya plane stress hoga. Yeh jaise Fuselage hai jaise aero jo body hai isko fuselage kehte hain. Fuselage mein bhi humein state of...

Plane stress: Day to day examples

material → $L \times W \uparrow \quad \downarrow$ *Fuselage*





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stress 2D stress rate milega. Yeh kuch oil refinery mein jo pipes hoti hain jahan par thickness bahut kam hoti hai. Humara LPG Cylinder hota hai ya is LPG cylinder ke agar body hum dekhenge toh hum dekhenge ki state of stress meri is tarah se hogi, thickness direction par stress nahi hoga. Toh isko hum 2D stress state kehte hain. Isliye 2D stress state ko samajhna bahut aavashyak ho jata hai kyunki hum yeh humare roz marra ki zindagi mein hum isko istemaal karte hain. Toh abhi hum

aage badhte hain aur dekhte hain ki Principal Stresses. Toh principal stresses maine bataaya tha ki principal stresses important isliye hain kyunki bahut saare applications mein...

humein maximum stress jo hota hai aur maine bola tha ki woh principal stresses principal planes par milta hai. Toh principal planes humne nikaale thae Mohr circle ko construct karke. Abhi hum nikaalenge analytical way se ya Eigenvalues (λ) aur Eigenvectors (v) se. Isko thoda samajhte hain. Toh ye ek thoda alternate way hai sochne ka. Toh maan chalte hain humare paas ek 2D stress state hai σ_{11} , σ_{12} , σ_{21} aur σ_{22} . Toh ye jo stress state hai humare paas yeh symmetric hai kyunki $\sigma_{12} = \sigma_{21}$ yaane jo off-diagonal components hain woh equal hain. Toh isliye symmetric stress state hai aur isse humein principal stress state...

nikaalni hai. Toh maan chalte hain ki humare paas ek coordinate axis hai aur hum isko jab rotate karenge kisi element ko dhoondhne dhoondhenge ki ji ek element jo θ degree apart hai jahan par sirf humare paas normal stresses hain yaane sirf principal stresses apply ho rahe hain. Toh ye stress state humein kuch rotation ke baad milti hai. Toh yeh maan lenge humare paas σ_1 aur σ_2 jo mere principal stresses hain aur jab hum isko find karenge toh yeh agar isko matrix maanenge toh yeh yeh jo tareeka hai jaise hum sirf diagonal elements hain aur non-diagonal elements ko hum zero (0) kar rahe hain, toh isko kehte hain hum Diagonalization of Matrix. Yeh aapne padha hoga...

matrix algebra mein. Toh yeh agar symmetric hai jab humne baat ki thi ki state of stress humari symmetric hoti hai toh agar yeh symmetry hai toh yeh jo values jo milengi humein yeh humein Real Values milengi. Toh agar humara matrix symmetric hai toh diagonal value jo hogi woh real hogi aur stress matrix agar real hai toh principal stresses ki value bhi humein woh real honi chahiye. Toh hum linear algebra ya matrix algebra mein isko yeh jo transformation hai isko kehte hain ki finding out eigenvalues aur eigenvectors aapne padha hoga. Toh abhi tak aapne jaan liya hoga jo eigenvalues hain yeh humare principal stresses...

hain σ_1 aur σ_2 hain. Toh hum isko kis tarah se nikaalenge aur jo eigenvectors hain woh humare principal stresses ka plane determine karenge. Toh yeh jo matrix hai maine stress matrix isko consider kiya aur eigenvectors ko maine is tarah se note kar diya v_1 aur v_2 . Toh yeh v_1 aur v_2 mere principal planes jo hain unka location bataayenge. Hum isko dekhenge solved example bhi hum dekhenge. Toh agar isko solve karna hai toh ek matrix equation humein likhna padega. Toh

matrix equation is tarah se likha jaata hai jo yeh stress matrix hai aur iska multiplication eigenvector ke saath...



Principal Stresses: Eigen values and Eigen vectors

Alternate way of thinking principal stress Application Shows how principal planes

Symmetric

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$



$$\longrightarrow \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

Diagonalization of matrix

$\sigma_{12} = \sigma_{21}$

- If matrix is symmetric then diagonal values are real
- Stress matrix is real, i.e., Principal stresses are real

In Linear algebra, problem of finding Eigen Values and Eigen vectors

Eigen Values: λ_1 and λ_2 = Principal stresses σ_1 and σ_2

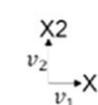
Eigen Vectors: Direction of Principal stresses

Stress Matrix $A = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$

Eigen vector $\underline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

Matrix Equation

$$A \underline{v} = \lambda \underline{v}$$



aur ek yahan par value aayegi yeh jo hai eigenvalue (λ) hai aur uska multiplication humare eigenvector ke saath. Yeh jo ho gaya humara matrix equation. Jab hum is matrix equation ko dekhte hain toh yahan par agar dekhenge toh yeh toh matrix multiplication hai par yeh jo value hai yeh just ek vector ke saath multiplied hai toh ye scalar product hai. Isko solve karne ke liye humein isko aur matrix tareeke se likhna padega. Toh hum is tarah se likhte hain isko ek Identity Matrix ([I]) se hum multiply karte hain. Toh ye identity matrix yahan par [1, 0; 0, 1] hoga. Aur agar main isko doosri side le auu aur main v ko common nikaalu yahan par... yeh jo zero hai mera...

Null Matrix hai aur v ko jab maine common nikaala toh mere paas yeh identity rehti hai. Tab yahan par iska solution do hi cheezon se ho sakta hai, ek toh mera v shunya (0) ho sakta hai ya jo yeh matrix hai ya ye jo matrix ka determinant ($|\cdot|$) hai yeh shunya ho sakta hai. Humne dekha tha ki humara jo stress matrix hai yeh real hai toh ye principal values aayenge, eigenvalue jo aayegi woh bhi real honi chahiye. Toh yeh ek solution hai ya ek solution hai toh isko hum discard kar denge kyunki humare paas real values aani chahiye. Toh hum isko consider karte hain. Toh jab hum isko solve karenge ab isko solve kaise karenge? Toh hum isko is tarah se solve karenge...

kyunki jab λI ko hum likhengye toh agar λ ko main is identity matrix se multiply karunga toh ye mere paas ye matrix aa jayega aur main stress state se isko subtract karunga toh mere paas ye matrix ka equation aa jayega. Agar main isko solve karta hoon, just main isko solve karunga toh mere paas ek Quadratic Equation ($ax^2 + bx + c = 0$) aata hai. Ye jo quadratic equation hai iska agar hum solution nikaalenge. Quadratic isliye kyunki iska power do (2) hai. Ye quadratic equation ko hum solve karenge toh agar aapko yaad karenge mere paas simple hai $ax^2 + bx + c = 0$, agar ye quadratic equation hai toh iske roots aise aane chahiye:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Toh ye main jaanta hoon agar main yahan par isko apply karunga toh mere paas yeh solution milega. Jo λ rahegi yeh meri eigenvalue rahegi aur maine bola tha iske do values rahengi. Ek isko ye value ko hum plus (+) karenge toh ek value milegi λ_1 aur subtract (-) karenge toh milegi λ_2 . Toh mere paas do eigenvalues aayengi. Agar hum dekhenge acchi tareeke se aur isko Mohr circle ke saath compare karenge toh ye value thi, yeh mera Center (C) rahega Mohr circle ka aur yeh jo yeh jo value thi yeh mera Radius (R) rahega Mohr circle ka. Toh agar hum dekhenge agar main isko yahan...

par draw karna chahta hoon agar aapko... main agar Mohr circle draw karunga yahan par toh yeh σ hai ye τ hai. Toh center jo value rahegi woh meri a, 0 rahegi. Yeh h value yahan par likhi hai aur ye jo radius rahegi yeh rahegi meri yeh value. Toh agar hum values nikaalenge σ_1 aur σ_2 ki toh yeh mera σ_1 ho gaya ye σ_2 ho gaya. Agar hum dekhenge toh ye yeh equation ye eigenvalues ye isi ko determine bol rahi hai ki agar center se main add kar doon radius toh mujhe ek principal plane milega aur center se main agar subtract karoon toh mujhe doosra principal plane milega.

Agar hum dekhenge is equation ko toh yeh jo value hai $\sigma_{11} + \sigma_{22}$, yeh ek Invariant value hai. Agar main state of stress phir se likh deta hoon aapke liye. Humne ye state of stress consider kiya tha σ_{11} , σ_{12} , σ_{21} , σ_{22} . Isko main iske liye same likh raha hoon kyunki dono same hi hote hain σ_{xy} aur σ_{yx} yahan pe humne mark kiya. Toh agar hum dekhenge ye state of stress humne liya era, ye jo pehla invariant hai jo main bol raha hoon ye $\sigma_{11} + \sigma_{22}$, ye Summation (Σ) of normal stresses hai. Toh yeh jo summation hai isko hum Trace of Matrix (tr) bhi kehte hain, trace bhi kehte hain isko.



Principal Stresses: Eigen values and Eigen vectors

$$A \underline{v} = \lambda \underline{v}$$

To solve this equation, let's rewrite

$$A \underline{v} = \lambda I \underline{v} \quad \text{Identity matrix, } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A \underline{v} - \lambda I \underline{v} = \underline{0} \quad \underline{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

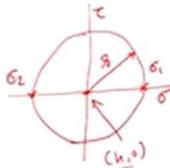
$$(A - \lambda I) \underline{v} = \underline{0}$$

This equation is satisfied, when $\underline{v} = \underline{0}$ or $|A - \lambda I| = 0$

$$\therefore \underline{v} \neq \underline{0}$$

$$\text{Thus, } |A - \lambda I| = 0$$

$$\begin{vmatrix} \sigma_{11} - \lambda & \sigma_{12} \\ \sigma_{12} & \sigma_{22} - \lambda \end{vmatrix} = 0 \quad \therefore \lambda I = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \text{ and } \sigma_{12} = \sigma_{21}$$



$$\begin{vmatrix} \sigma_{11} - \lambda & \sigma_{12} \\ \sigma_{12} & \sigma_{22} - \lambda \end{vmatrix} = 0 \quad \text{Invar} \quad \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

$$\lambda^2 - (\sigma_{11} + \sigma_{22})\lambda + (\sigma_{11}\sigma_{22} - \sigma_{12}^2) = 0$$

2 invariants
2D 3 invariants
IN VARIANT IN VARIANT

Quadratic equation, solution

$$\lambda = \frac{(\sigma_{11} + \sigma_{22})}{2} \pm \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + (\sigma_{12})^2}$$

Centre Mohr's circle Radius

Substitute, the values of λ to find Eigen vector

$$(A - \lambda I) \underline{v} = \underline{0}$$

Aur yeh jo aap value dekhenge yeh jo doosra invariant hai toh yeh hai humara Determinant ($|A|$). Agar hum determinant nikaalenge is matrix ka ye determinant hai, determinant of stress matrix. Toh humein do invariants milte hain kyunki hum consider kar rahe hain 2D stress state isliye humein do invariants milte hain. Jab hum 3D stress state consider karenge toh humein teen invariants milenge. Yeh hum aage dekhenge. Toh yeh ho gaya mera principal stresses ki values hum λ se nikaal sakte hain aur jab hum λ ko is equation mein rakhenge, ye equation mein rakhenge toh humein eigenvectors milenge. Solved problem dekhte hain humare paas ek stress state hai toh is...

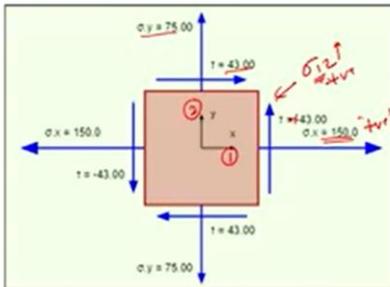
stress state ko hum pehle likh lete hain. Toh hum isko likhenge σ_{11} , σ_{12} , σ_{21} aur σ_{22} . Toh isko main is tarah se mark kar leta hoon yeh meri ek (1) direction hai ye do (2) direction hai. Toh σ_{11} mera 150 hai ye tensile hai toh isko main positive se mark kar raha hoon. Yahan par ek galti hai toh yeh positive honi chahiye kyunki positive... yeh mera σ_{12} hai kyunki mere plane ek pe act kar raha hai aur along do direction act ho raha hai toh yeh positive value honi chahiye toh isliye maine isko 43 likha hai aur similarly yeh 43 hai toh yeh bhi 43 ho jayegi aur yeh positive stress hai jo plane do par lag raha hai toh ye 75 jo σ_{22} ki value hai.

Toh mera stress matrix ho gaya is tarah se aur abhi hum dekhenge ki isko solve kaise karenge. Toh humne yeh determinant ki value shunya (0) honi chahiye jahan par λ mujhe eigenvalues dega. Toh agar hum isko solve karenge toh yeh equation ko solve karenge humein yeh identity mil jayegi aur hum roots nikaalenge jo humne dekha tha ki agar hum direct iske roots nikaalte hain toh hume ye equation se hum iske roots nikaalenge. Hum ye values put kar dete hain σ_{11} , σ_{22} aur σ_{12} toh humein λ is tarah se milega aur hum λ nikaalenge toh humein milega ye 169.5 ek principal stress milega doosra principal stress milega humein...

55.45. Ye yeh jo units honge yahan par stress ke yeh hum ye principal stresses ko de denge. Toh abhi hum kya karte hain ismein yeh equation mein... yeh jo equation tha humne dekha tha humein principal planes ke location nikaalne hain toh humein vectors nikaalne hain, eigenvectors nikaalne hain. Toh hum kya karenge is equation mein λ ki jagah λ_1 rakhenge toh humein yeh equation mil jayega phir se. Agar hum isko solve karenge toh humein yeh do equations milenge v_1 aur v_2 ki tarah. Agar hum iska product lete hain toh humein yeh do equations milenge. Agar hum isko solve karne ki koshish karenge toh hum dekhenge ki yeh dono identical equations hain. Toh hum ek ek equation consider karte hain...



Principal Stresses: Eigen values and Eigen vectors: Solved example



$$A = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} 150 & 43 \\ 43 & 75 \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 150 - \lambda & 43 \\ 43 & 75 - \lambda \end{vmatrix} = 0$$

$$\lambda = \frac{(\sigma_{11} + \sigma_{22})}{2} \pm \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + (\sigma_{12})^2}$$

$$\lambda = \frac{(150+75)}{2} \pm \sqrt{\left(\frac{150-75}{2}\right)^2 + 43^2}$$

$$\lambda = 112.5 \pm 57.055$$

$$\lambda_1 = 169.55 \text{ and } \lambda_2 = 55.45$$

$$(A - \lambda I)\underline{v} = \underline{0}$$

$$\text{Substitute, } \lambda = \lambda_1$$

$$\begin{pmatrix} 150 - 169.55 & 43 \\ 43 & 75 - 169.55 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$-19.55v_1 + 43v_2 = 0 \quad (1)$$

$$43v_1 - 94.55v_2 = 0 \quad (2)$$

$$\text{Using eqn 1, when } v_1 = 5 \Rightarrow v_2 = 2.27$$

You will get the same answer using eqn (2)

aur ismein ek v_1 ki value maan lete hain ki 5 hai aur yahan par paanch (5) likh dete hain aur v_2 ki value nikaalte hain ye aayegi 2.27. Agar hum equation do bhi istemaal karte agar hum yahan par paanch rakhenge toh humare paas v_2 ki value 2.27 hi aani chahiye. Similarly hum kya karenge abhi lambda mein λ ki jagah λ^2 ki value rakhenge toh yeh value aa jayegi. Yeh jo yeh jo identity aayegi isko hum agar solve karenge toh humare paas aur do equations aayenge v_1 aur v_2 mein. Agar hum dono equation ko solve karenge toh ye dono equation same hi hain. Agar hum maan lete hain v_1 ki value yahan par hum 1 rakhte hain toh humare paas v_2 ki value aa jayegi -2.19. Toh aapko agar v_1 ki value yahan par bhi...

ek rakhte hain chauthi equation mein toh v_2 ki value -2.19 aani chahiye. Toh aap ye solve karke dekh sakte hain. Yeh isliye kar rahe hain kyunki jab hum principal plane nikaalne ki koshish kar rahe hain tab humein ye coordinates mark karne padenge. Jab humne pehla equation pehla principal stress ki value agar rakhi thi toh humare paas v_1 jab humne 5 रखा था v_2 humare paas 2.27 aaye. Toh yeh point hum iske corresponding mark kar sakte hain origin se aur yeh ek seedhi line jayegi isko hum mark karenge aur isko hum kahenge X_1' . Similarly hum yeh doosri value rakhenge $v_1 = 1$ rakhenge $v_2 = -2.19$ aayega. Toh humare paas yeh point aa jayega aur yeh line hum kheench denge origin se toh ye humari aa jayegi X_2' .

Toh agar humein principal planes ka rotation dekhna hai toh humein sirf ye angle abhi nikaalne padenge. Yeh ho jayega mera θ_1 jo mere X_1 se rotated hai aur yeh ho jayega doosra plane doosra principal plane yeh ho jayega humara θ_2 jo X_2 se rotated hai. Toh ek humara ek alternate way hai nikaalne ka. Toh agar hum element draw karenge toh humare paas yeh element is tarah se rahega jo ki ye is principal jo eigenvalues hain, yeh meri eigenvalues ho gayi, yeh eigenvectors ho gaye. Eigenvectors ko perpendicular (\perp) jo rahega woh mere principal planes ki location hogi. Agar hum is se angle agar nikaalenge, yeh agar angle agar hum nikaalenge toh humein iski location hum...

geometrically bhi plot kar sakte hain. Toh humne dekha ki eigenvalues se... eigenvalues humein do eigenvalue mili kyunki humare paas 2D stress state tha toh eigenvalue λ_1 aur λ_2 ye humari nothing but humare principal stresses hain σ_1 aur σ_2 . Aur eigenvectors humne nikaale v_1, v_2 . Toh ye v_1, v_2 humein bataayenge location kya hai θ_1 aur θ_2 with respect to humara original axis X_1 ya X_2 . Toh yeh utility hogi humari principal planes aur principal direction nikaalne ki based on

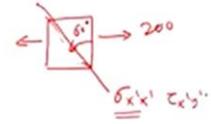
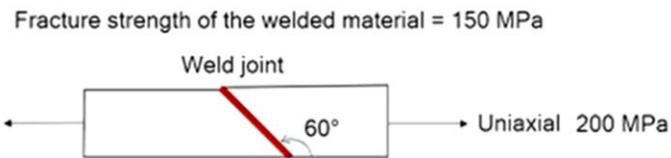
eigenvalues aur eigenvectors. Toh humne ek solved problem dekha. Abhi hum aage badhte hain aur ek problem main aapko solve karne...

ke liye deta hoon. Toh let's say humare paas do plates hain aur ye tooti hui plates hain aur isko hum abhi yahan pe weld karna chahte hain. Isko hum weld karenge toh is isko kahenge Weld Joint. Yeh kisi bhi arbitrary angle pe hai, maan lijiye 60° pe hai. Toh ye 60° pe humne weld kiya aur abhi abhi is member ko jo humne join kiya hai hum isko ek stress dete hain 200 MPa uniaxial tensile stress. Yeh hamesha hum dekhte hain jab weld karte hain kisi bhi member ko hum join karte hain toh hum usko load (\downarrow) apply karte hain. Maan lijiye saral member hai aur isko hum dono members ko join karke uniaxial stress diya 200 MPa. Abhi aapko find out karna hai ki ye weld joint sustain kar paayega is stress ko...

ya nahi jabki humari Fracture Strength diya hai 150 MPa. Theek hai? Toh aapko dekhna hai ki yeh weld joint pe 150 MPa pe pahunchega tab yeh yeh members alag ho jayenge, toot jayenge. Agar yeh stress 150 MPa se neeche hai toh yeh sustain kar lega. Toh aapko yeh find out karna hai. Toh iske liye ek simple sa hum isko state of stress aapko define karna hai. Maine ek uniaxial stress state diya hai aapke paas 200 MPa hai aur ek angle hai jo isse 60° hai counter-clockwise. Abhi aapko state of stress is plane pe nikaalna hai. Jab state of stress is plane pe nikaaloge aap toh aapke paas main maan leta hoon ye state of stress aata hai aapke paas $\sigma_{x'x'}$. Theek hai? Yeh...



Some interesting examples: Welding

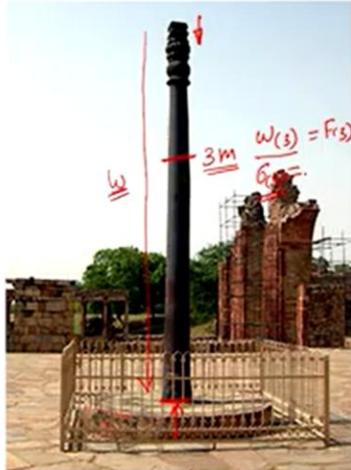


Will the weld withstand this condition?

ek aur state of stress aa gaya ek ek shear stress (τ) bhi aayega aapke paas iska. Toh aap is state of stress ko jabanikaalenge toh yeh jo stress hai ye aap dekhenge agar yeh cross karta hai 150 MPa se toh yeh weld break ho jayega. Aapko ye find out karna hai, aapko solve karna hai as homework. Ek doosra important example deta hoon, ye bahut hi interesting example hai, ye bhi aap ghar mein solve kar sakte hain. Ye Delhi Iron Pillar hai jo ki hum kehte hain Rustless Wonder yaani abhi tak itne saal ho gaye phir bhi rust nahi hua hai. Aur iske upar kaam kiya humare department ke professor Balasubramaniam ne. Unhone is pe corrosion aspect pe kaam kiya hai. Toh yeh abhi tak pata...



Delhi Iron Pillar: The rustless wonder



Late Prof R. Balasubramaniam
MSE, IIT Kanpur (Corrosion aspect)

Image courtesy: Wikipedia

Images are for educational and teaching purpose only

Height: 7.21 m
Diameter: 0.406 m

Density: 7800 kg/m³

Find compressive stresses and compressive strains at the cross-section located at 3 m from the top?

Relate normal force acting on the cross-sectional area of the pillar as a function of its length

nahi chal paa raha hai ki rustless wonder kyun hai. Kuch insights humein professor Balasubramaniam ne diye. Yeh yeh humari dharohar hai, bharat ki dharohar hai. Toh agar hum dekhenge agar humare mechanical behavior course ke liye hum isko consider karte hain, iski height (h) jo hai 7.21 m hai aur diameter (d) hai 406 mm aur iski density (ρ) agar maan ke chalte hain 7800 kg/m³ ye density maan ke chalte hain. Toh aapko kya karna hai? Aapko jaise agar hum is problem ko main is tarah se define karta hoon ki iska jo weight (W) hai ye act hoga is pillar pe aur maximum weight ye end pe act hoga is pillar ka khud ka hi weight. Toh iske wajah se kya hoga ye jo reaction aa...

raha hai ye is direction mein aa raha hai aur weight ka reaction is direction mein aayega. Toh ek Compressive Stress State yahan par taiyaar hogi is pillar mein. Toh aur iske wajah se compressive strains (ϵ) bhi aayenge. Toh humein find karna hai ki compressive stress let's say top se 3 meter pe kya compressive stress hai aur kya compressive strength hai ye humein nikaalni hai. Toh humein aap compressive stresses nikaalein hum compressive strains nikaalenge jab hum strains ke baare mein baat karenge. Abhi aap compressive stresses nikaalein. Toh isko aap is tarah se bhi dekh sakte hain, ek hint de raha hoon main ki aap dekhenge ki yeh jo weight hai ye is Cross Section Area (A) ko...

normal (\perp) hai. Toh aur yeh jo weight hai across its length yaani is length ke around woh change ho raha hai. Toh agar aap iska ek function nikaalte hain agar aapko weight pata hai toh aap weight kisi bhi is position weight pata hai aapko 3 meter pe weight pata hai toh aap ispe force (F) kitna lag raha hai yeh nikaal sakte hain. Cross section area aap diameter se nikaal sakte hain ($A = \pi r^2$). Toh aap yahan par stress ($\sigma = F/A$) bhi nikaal sakte hain. Jaise stress nikaalega toh aapko ek compressive stress मिलेगा. Toh ek interesting problem hai aap isko solve kar sakte hain. Next part mein hum dekhenge ki abhi tak humne 2D Stress State ke baare mein baat ki thi. Next part se hum dekhenge ki 3D Stress State kya hoti hai aur 3D stress...

36:24 - 36:42 ...state mein kaun se invariants milte hain, Mohr Circle (\bigcirc) kya ho sakti hai aur isko aur acchi tareeke se jaanenge agle bhaag se. Abhi main yahan pe rukta hoon. Yeh problems aap solve kariye aur kuch doubts aate hain toh hum discuss karenge. Dhanyavaad!