Mechanical behavior of materials

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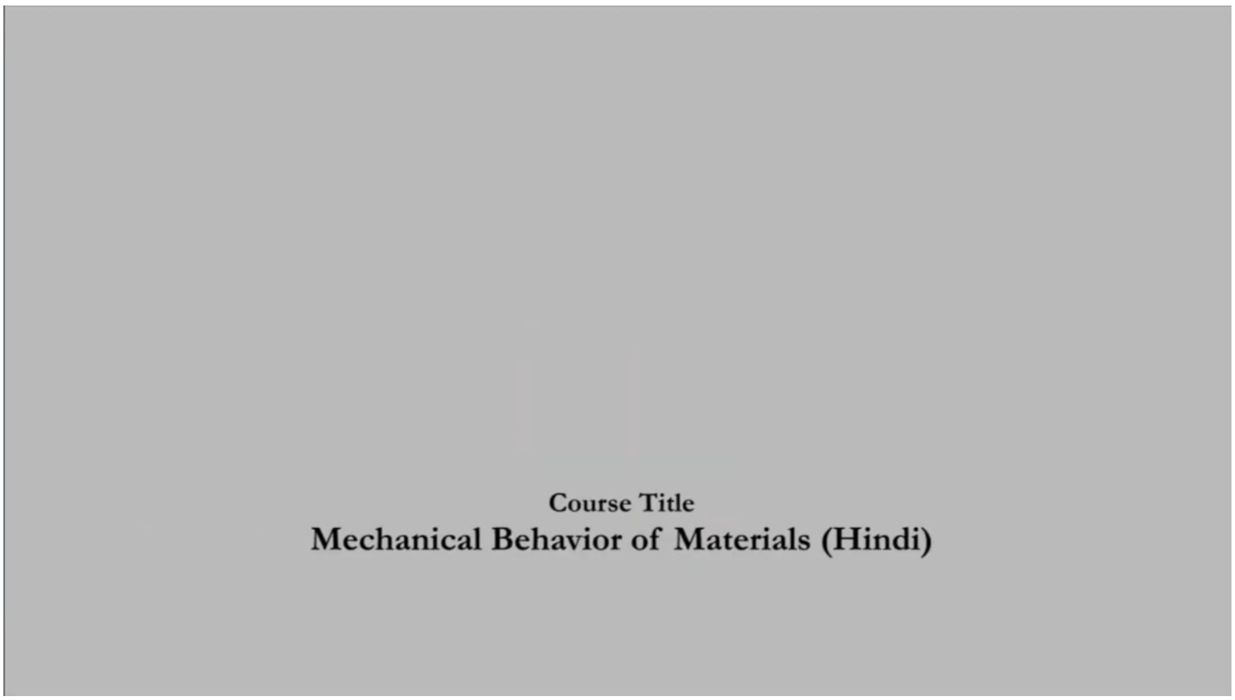
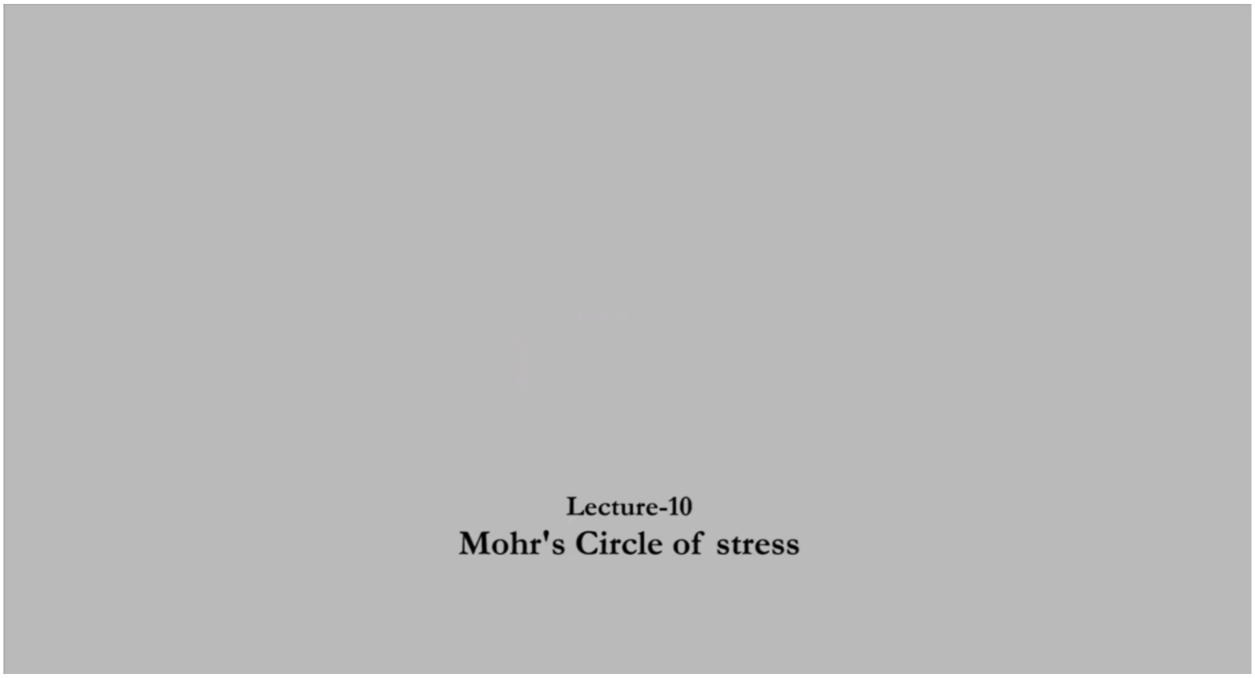
Week-2

Lecture-10

Mohr’s Circle of stress





Namaskaar is part mein hum Mohr Circle ka construction dekhenge. Last part mein humne dekha tha ki 2D stress state kya hoti hai aur 2D stress state ko hum jab doosre reference axis mein dekhte hain to hum us reference axis mein ya us plane mein kaise stress nikaalenge. Humne in teen equation ke dvaara dekha tha jo ki hamare paas yeh sigma $x'x'$ hai. Hamare paas sigma xx, sigma yy aur tau xy hai. Ye kisi state of stress mein hume pata hai aur hume theta pata hai, to hum doosre plane of reference ya plane mein state of stress kaise hoga yeh hum nikaal sakte hain.

Jaise main usko thoda brief explain kar deta hoon. Humne last class mein dekha tha ek reference mein hume pata hai x–y reference mein agar hamare paas ek state of stress hai sigma xx, sigma yy yeh hamare normal stresses ho gaye; tau xy, tau yx yeh hamare shear stresses ho gaye. Yeh humne 2D stress state mein dekha tha.

To agar hum isko transform karte hain, jaise hamare paas doosra kuch reference hai x'–y' aur yeh rotate hua hai theta ° se ya doosra koi element hai, to hum yeh state of stress nikaal sakte hain is equation ke dvaara. Yeh humne last class mein dekha tha aur yahi relations yahan par likhe hue hain.

To ek important cheez yahan par hum discuss karenge. Agar hum yeh do equations ko add karte hain sigma $x'x'$ aur sigma $y'y'$ to hume ek identity milti hai. Agar hum dono ko add karenge to hume milega ki sigma xx plus sigma yy ka addition aur sigma x′x′ plus sigma y′y′ ka addition same hai. To hum ek conclusion nikaal sakte hain: regardless of orientation yaani koi bhi angle change ho, koi bhi theta ho, koi bhi arbitrary theta ho yeh normal stresses ka summation hamesha invariant rahega. Invariant yaani wo badlega nahi, wo same rahega. Yaani sigma xx plus sigma yy ka jo addition hai, aur sigma x′x′ plus sigma y′y′ ka addition same rahega.

Aur ek example le sakte hain hum. Jaise humne aur teesre axis mein jaake dekha, jaise x_2–y_2 reference gaye. Yeh koi aur ek third maan lenge ya third maan lenge isko. Aur hume ek state of stress milegi sigma x_2x_2, sigma y_2y_2, tau x_2y_2, tau y_2x_2. To inka bhi summation, jaise sigma normal stresses ka summation x_2–y_2 plane mein sigma x_2x_2 plus sigma y_2y_2 yeh same rahega sigma xx plus sigma yy ke barabar, aur sigma x′x′ plus sigma y′y′ ke barabar.

To yaani koi bhi theta angle rahe, jo normal stresses ka summation hoga wo badlega nahi wo invariant rahega. Ek important conclusion hum nikaal sakte hain in equations ke dvaara.

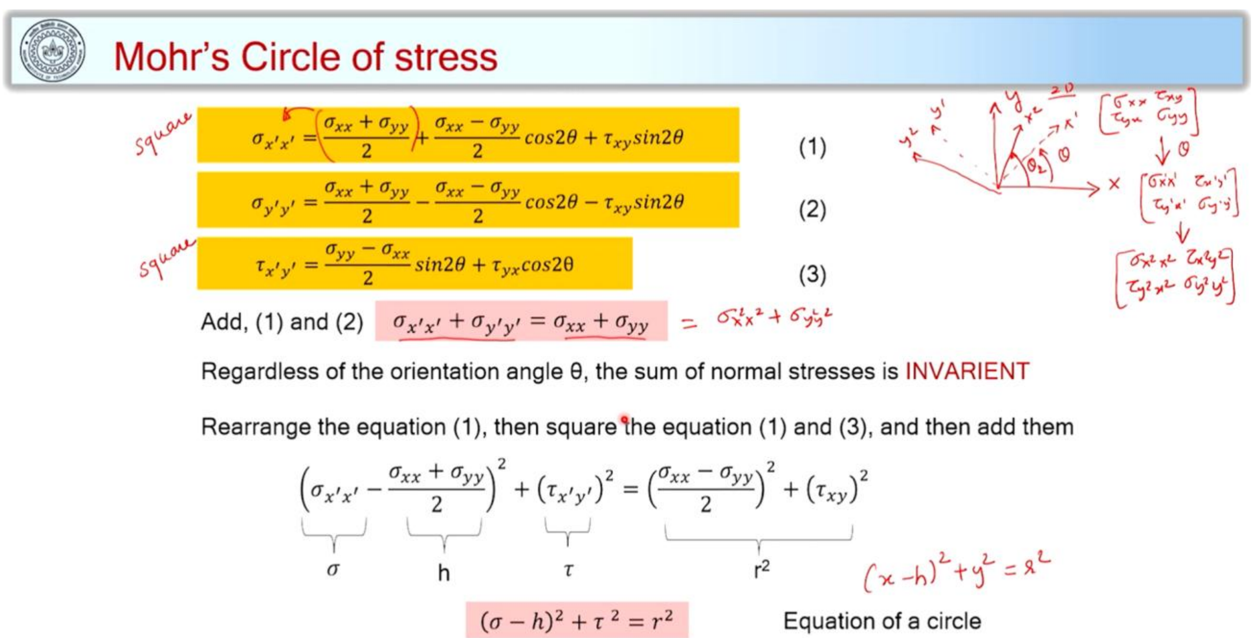
Aur kuch mathematical hum changes karenge. Jaise hum kya karenge jaise hum is equation 1 ko rearrange karenge. Rearrange karenge yaani yeh jo term hai, yeh term hum left-hand side le aayenge aur phir usko square karenge. Phir square karenge, aur yeh jo third equation hai isko square karenge. Square karne ke baad hum add karenge yeh do terms ko, to hume yeh mathematical identity milni chahiye.

To yahan pe aap dekhenge yaani yeh jo, yeh to saral part hai. Yaani yeh agar humne is side laaya aur iska square kar diya to yeh term mil jaayega. Yeh jo term hai, yeh direct square ho jaayegi yahan pe. Aap solve karke dekhenge agar yeh term ka square karke aur is term ka square karke add karenge, aur rearrange karenge, to yeh part hume milna chahiye right-hand side pe jo hai. Abhi aap karke dekh sakte hain.

Par is equation se main ek point batana chahta hoon. Jaise is term ko main maan lu sigmā aur is term ko main maan leta hoon h. Yeh jo sigma xx plus sigma yy humne dekha tha yeh invariant hai, to yeh jo term hai ek constant term ho jaayegi. Aur yeh tau x′y′ hai yeh jo term hai isko maan lenge tau. Aur yeh jo term hai isko maan lenge hum r^2.

To agar aap dekhenge is equation ko, to main is tarah se likh sakta hoon: sigma minus…

“$(\sigma-h)^2 + \tau^2 = R^2$ to ye yah jo relation aaya hai, ye ek equation of circle hai. Main isko is tarah se bhi likh sakta hoon aapne mathematics mein padha hoga: $(x - h)^2 + y^2 = R^2$. Ye equation of circle hai jiska centre jo hai, ye circle ka (h, 0) hai. To to yahi concept hai hamare Mohr’s Circle ka. To hamne Mohr’s Circle ka equation nikaala hai is tarah se, aur ye Mohr’s Circle ka equation humein mila hai. Ye jo identities hain, in tino identities se humein ye Mohr Circle ka equation mila hai. To abhi hum ye jo state of stress hai kisi bhi plane pe, ye hum geometry kaise plot karte hain, geometric kaise dekhte hain isko hi hum Mohr Circle kehte hain. To ye hamara…”



“Equation ho gaya, jo equation of circle hai. Aur ye humne maan liya mera origin hai,aur ye x–y axis mein plot kar raha. Aur x-axis par main normal stress (σ) plot karoonga, kyonki ye maine bataya tha ki equation of circle hai. To ye $(x - h)^2$ isko aap aise relate kar sakte hain. To normal stress aayega mera x-axis par, aur shear stress (τ) main plot karoonga y-axis par.

To ye mera shear stress (τ) y-axis par aa gaya. Aur ye mera centre ho gaya h_0.

Centre ki value jo rahegi:

$h_0 = (\sigma_{xx} + \sigma_{\gamma\gamma}) / 2$, jo ki yahan se aapko milegi. So ye ho gaya h,0 ye mere coordinates ho gaye centre ke. Abhi main radius agar dekoonga,to ye meri radius ho gayi: $R = \sqrt{[((\sigma_{xx} - \sigma_{\gamma\gamma})/2)^2 + \tau_{x\gamma}^2]}$. Aur agar radius mujhe pata hai,to main ek circle draw kar sakta hoon.

Aur ye circle main is tarah se draw karoonga mujhe centre pata hai,mujhe radius pata hai, to main ek circle draw kar sakta hoon.Ye ho gaya mera Mohr Circle. Isko kehte hain Mohr Circle of Stress. Ek Mohr Circle of Strain bhi hai,jo hum baad mein dekhenge jab hum strain ko discuss karenge.

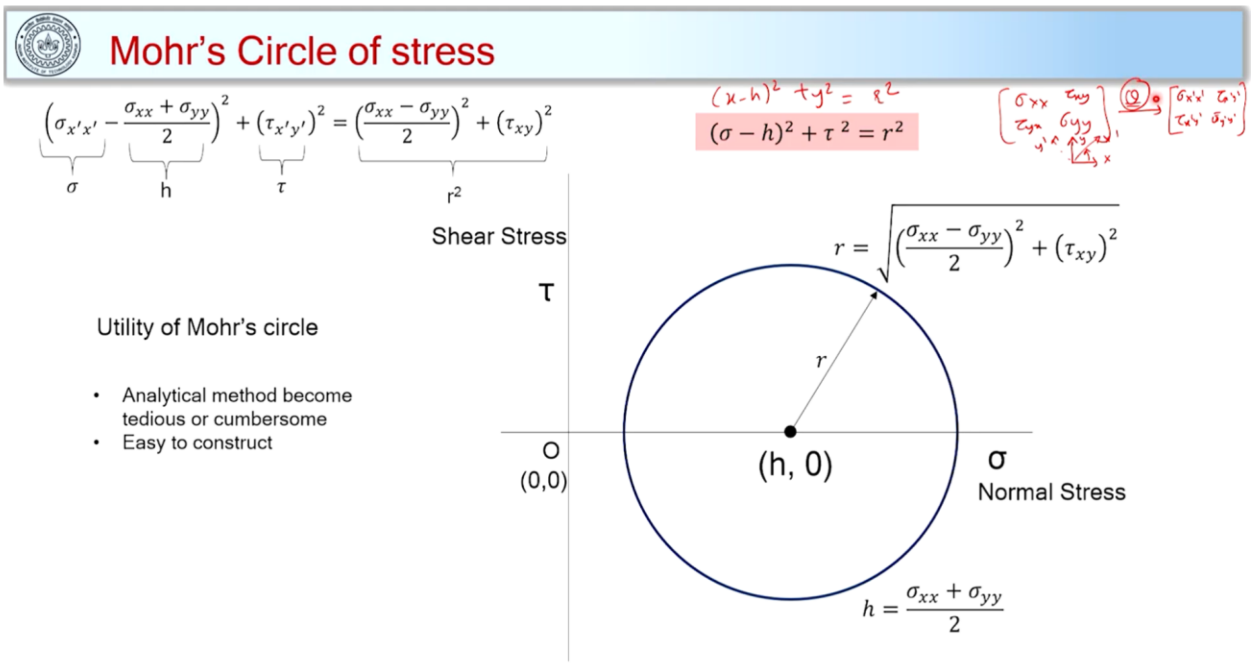
Par abhi ye jaanenge ki ye Mohr Circle of Stress aur iski utility kya hai yaani upayogitā kya hai. To Mohr Circle ki upayogitā ye hai ki agar hume bahut saare planes pata hain jaise humne bataya ki ye jo $\sigma_{x'x'}$ hai, ye hum kisi aur…"

"Ek plane par, jo θ-° inclined hai ye maan ke chala hai humne. Jaise humne σ_{xx} aur $\sigma_{\gamma\gamma}$ ko change kiya change kiya in the sense humne isko transform kiya jab hum θ-plane par dekhte hain is reference axis ke. To hume pata chalega ki hamara ye jo stress hai $\sigma_{x'x'}$, ye kisi plane ke around hai jo θ-° oriented hai mere original plane se. $\sigma_{\gamma'\gamma'}$ aur ye $\tau_{x'\gamma'}$. To agar mujhe doosre plane par karna hai, to mujhe phir se equations solve karne padenge. Par agar mere paas ye circle hai, to is circle se main kisi bhi plane ka state of stress nikal sakta hoon."

"To yahan par hum dekh sakte hain ki koi bhi anagint planes hum yahan se is jo circle hai circle par jo points hain, wo represent karenge. Ye aur ye easily construct bhi ho jata hai.

To agar mujhe ek state of stress pata hai, aur doosre plane pe jo θ-° oriented hai, us par agar mujhe state of stress nikalna hai, to main Mohr circle se aasaanee se nikal paaunga.

To ye ho gayi meri advantage Mohr circle ki advantage ho gayi.To abhi dekhte ki Mohr circle ko construct kaise karte hain aur kya convention hai, sign convention kya hai. To ye hum abhi Mohr circle ke construction ko dekhenge.



"To humne dekha ki Mohr circle jo σ hai vah x-axis par plot karenge,aur y-axis yaani ordinate par hum shear stress τ plot karte. Ye mera origin hai. To jo normal stresses hote hain, σ mere normal stresses to inko main plot karunga jo positive normal stresses ya tensile stresses hain, inko main plot karunga positive x-axis ki taraf. Aur jo negative values hain ya negative compressive stresses, ya compressive stresses negative normal stresses unko main plot karunga negative direction.

To ye to saral hai. Abhi hum dekhte hain ki shear stresses (τ) ka convention kya hai. To humne baat ki thi ki ye jo shear stress hai, ye shear stress τ hai ye mera $\tau_{x\gamma}$ hai kyonki ye mera x- lane par, agar main is tarah se coordinates mark karta hoon x aur y stress ke liye,to ye mera x-plane par act ho raha hai along y-direction. To ye positive x hai, aur ye along positive y-direction. To ye shear stress mera positive ho jayega. Ye mera positive shear stress $\tau_{x\gamma}$.

Humne sign convention dekha tha shear stress ke liye. Par yahan par jab hum plot karenge, tab hum dekhenge ki ye shear stress, ye jo direction hai, agar hum yahan par dekhenge, ki kis tarah se rotate kar raha hai

To ye is tarah se rotate kar raha hai. Yeh shear stress (τ) to yeh shear stress (τ) is tarah se rotate kar raha hai. Isko hum bolenge counter-clockwise. To is shear stress (τ) ko hum plot karenge yeh counter-clockwise rotation de raha hai is element ko to shear stress (τ) ko hum plot karenge negative side pe, negative y-side pe. To similarly y yeh jo shear stress (τ_{yx}) hai, yeh shear stress mera τ_{yx} hai kyonki y-plane pe act ho raha hai aur along x-direction. To yeh bhi positive shear stress ho gaya. Par yeh jo notion de raha hai, yeh jo notion…

…de raha hai is element ko, yeh is tarah se rotate kar raha hai. To hum dekhenge ki yeh jo shear stress (τ) hai, yeh clockwise notion de raha hai, ya is element ko clockwise rotate kar raha hai.

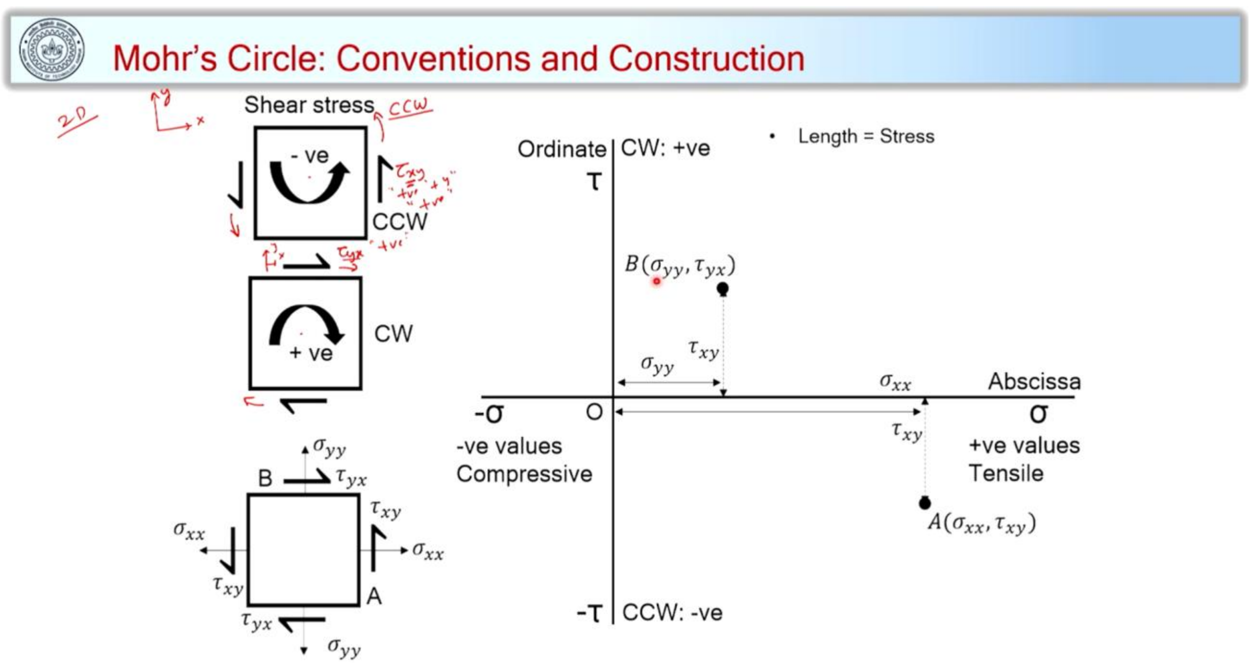
To isko hum kahenge yeh mera counter-clockwise ho gaya jisko main negative side pe plot karoonga,

aur yeh mera clockwise ho gaya jisko main positive side pe plot karoonga. To jab hum Mohr Circle plot karte hain, tab yeh sign convention bahut important hai, aur isko aapko yaad rakhna hai. Aap ulta bhi kar sakte ho ki counter-clockwise idhar bhi plot kar sakte ho, clockwise idhar bhi plot kar sakte ho. Par aapko ek hi convention se stick rakhna hai, yaani change nahi karna hai.

Ye convention jo aap follow kar rahe ho, usko continuously wahi sign convention aapko follow karna hai. Main clockwise ko positive maan ke chal raha hoon yahan par, aur positive side par plot karta hoon, aur counter-clockwise ko negative side par ye main sign convention maan ke chal raha hoon. Kisi books mein ulta bhi diya rehta hai, to dono bhi sahi, but aap ek convention ko follow kariye. To abhi dekhte hai ki Mohr's Circle construct kaise karte hain. To ek mera element maine le liya, stress state le liya, jahan par mere paas do planes hain jo maine mark kiye A plane aur ye B plane.

Aur ye B-plane, to A-plane mera x-plane hai aur B-plane mera y-plane hai.To agar maine dekha yahan par A-plane par mera σ_{xx} (sigma-x-x) hai, jo ki positive stress hai ya tensile stress hai. To main plot karunga isko positive x-axis ke direction mein. Aur ye τ_{xy} (tau-x-y) hai. τ_{xy} dekhenge ki ye aapko de raha hai is element ko counter-clockwise rotation. To main plot karunga** usko negative y-direction pe, ya is side pe. To isko plot karte hain. Jab plot karte hain, tab hum maante hain ki ye jo length hai ye proportional to stress hai. To geometrical representation hai. Aur jab bhi hum dekhenge construction mein, to hum length measure karenge yaani length measure kar rahe, to iske barabar jo hoga, wo stress hoga hamara. To ye ho gaya mera σ_{xx}. To maine proportional stress plott kiya length se. To ye mera σ_{xx} ho gaya positive x-direction pe. Aur τ_{xy} ho gaya mera negative y-direction pe. To ye mera point A aa gaya. Ye jo point A hai, ye point A kya represent kar raha hai? Mera state of stress A-plane pe. Ye A-plane ka state of stress ye point represent kar raha hai.

Abhi hum dekhenge ki point B. To point B mein kya hai? Mere paas positive σ_{yy} (sigma-y-y) stress hai, ya tensile stress hai. Aur ye jo stress hai, ye is element ko clockwise rotation de raha hai. To main isko plot karunga positive y-direction pe. Maan lijiye $\sigma_{yy} < \sigma_{xx}$, to main yahan pe plot kar raha hoon. Aur ye mera point ho gaya (σ_{yy} , τ_{yx}). To mera point B ho gaya.



Ye jo point B hai, ye kya represent kar raha hai? State of stress B-plane pe. Aur hum dekhenge ki ye jo planes hai A aur B A nothing but mera x-plane hai (positive x-plane), aur B mera positive y-plane hai. To ye dono 90° apart hain. Par hum Mohr circle par dekhenge to ye 180° apart hote hain. To hum construction karte waqt kya karenge? In dono points ko join karenge. Jab hum join karenge, to ye line yahan par intersect karegi x-axis ko. Ye ho jayega centre of Mohr circle (C). To ye ho gaya mera centre. Aur is centre se main radius dekhunga. Ye radius kya hogi? BC ya AC. To is radius ko maan kar main ek circle draw karunga. To mujhe Mohr circle mil jaata hai. Aur is circle pe jo bhi points hain, wo state of stress represent karenge. Aapne dekha ye point A hai aur point B hai ye state of stress represent** kar rahe hain apne-apne planes pe. Yaani: Point A represent kar raha hai state of stress A-plane pe

Point B represent kar raha hai state of stress B-plane pe To ye ho gaya mera Mohr circle construction.

Abhi hum dekhenge ki isko agar mujhe kisi aur plane par state of stress nikaalna hai, to hum kaise nikaalenge. To maan lijiye main point A ko mera reference plane maan raha hoon yaani is plane se main doosre plane ka angle measure karna chahta hoon. Jaise mera blue plane hai, aur ye $\theta°$ oriented hai A-plane se. Aur θ kis direction mein hai? Ye clockwise direction mein hai. To maan lijiye main A-plane ko reference maan raha hoon.

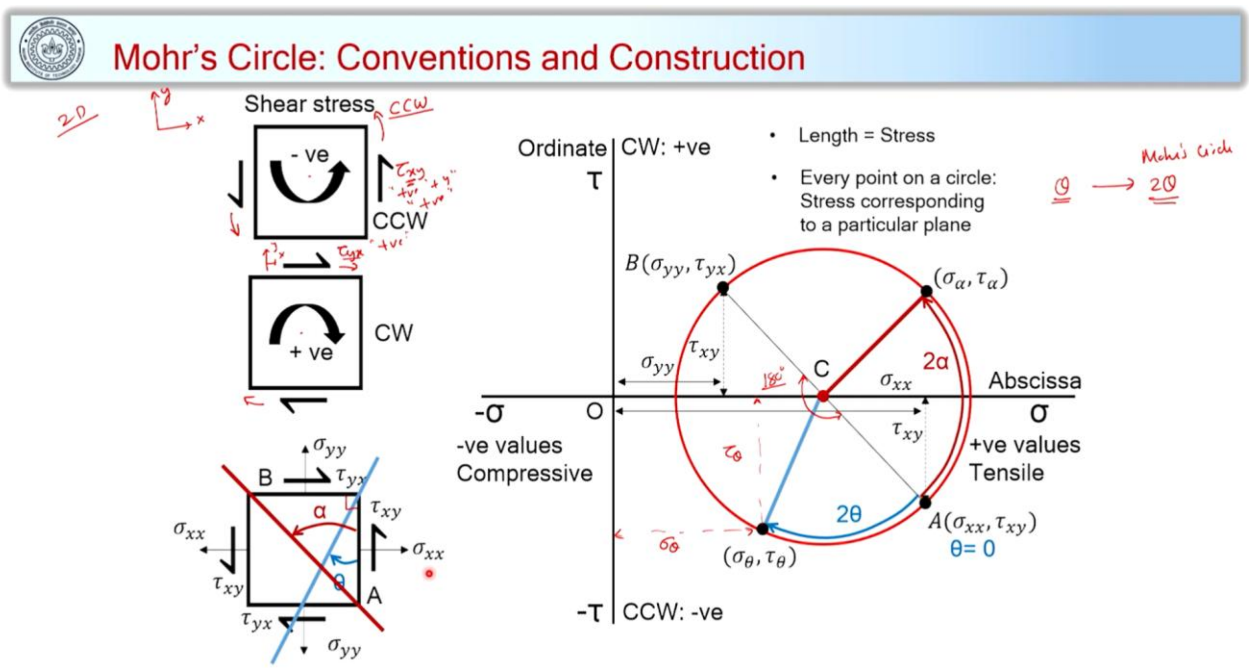
To is point ko main reference maanunga. To isko main $\theta = 0$ (theta zero) maan loonga. Aur main abhi clockwise move karunga. Yahan par physically kya hai? Ye plane θ oriented hai.

16:35

θ ° apart hai. Humne dekha tha ki plane A aur plane B ka physically kitne ° apart hai 90 ° apart. Par Mohr's circle mein yeh 180 ° apart hai. Isko likh lete hain. To yeh Mohr circle mein yeh jo distance hai, jo angle hai, yeh 180 ° hai. To physical jo θ (theta) hai humare element ka, woh Mohr circle meinMohr circle mein jaayega 2θ. To yahan par agar humein is plane jo blue plane hai, iska agar point nikaalna hai is circle par, to humein 2θ ° move hona padega A plane se, jo ki clockwise movement hai. To main clockwise direction mein move karunga. To yahan par main 2θ aaya. Yeh mera point ho gaya jo…

…is plane ke correspondingly mark karega state of stress is blue plane pe. Isko main mark karunga $\sigma\theta$ (sigma-theta), $\tau\theta$ (tau-theta).To $\sigma\theta$ kaise nikaalenge? $\sigma\theta$ mera distance ho jayega yeh. Yeh mera $\sigma\theta$ ho jayega. Aur $\tau\theta$ y-axis direction pe. Aur ek example lete hain: ek plane consider karte hain jo ki α (alpha) ° rotated hai mere A plane se. Yeh A plane ko reference maan raha hoon. To yeh α ° rotated hai. Yeh kis direction mein rotated hai? Yeh mera counter-clockwise direction mein rotated hai. To yahan pe physically yeh…

α ° rotated hai. To A se Mohr circle pe 2α ° rotated hoga. To hum 2α ° jaayengekis direction mein? Counter-clockwise. To yeh ho jayega mera α pointα plane ka point. Aur yeh jo point represent karega, yeh state of stress represent karega is plane pe. Humne jaana ki mere paas agar do state of stress haiyeh agar ek reference state of stress hai: σ_{xx}, σ_{yy} aur τ_{xy}yeh agar mere paas hai, to main kisi bhi plane ka, kisi bhi plane par jo state of stress hai, woh nikaal sakta hoon by just simple construction of Mohr circle. Just simple construction of Mohr circle karke kisi bhi plane ka main state of stress nikaal…



…sakta hoon. Abhi dekhte hain kuch special results jo circle se humein praapt ho sakti hain.

To humne dekha tha ki x-axis par σ hai aur y-axis par τ hai. Aur yeh mera center ho gaya, yeh mera Mohr's circle ho gaya. To agar aap dekhenge yeh do pointsyeh do points A aur B jo maine mark kiyeyeh kuch special points hain. Yeh jo points circle par hain, yeh represent karenge kuch planes ko. Humne dekha ki koi bhi point is circle par ek plane represent kar raha hai.

To yeh do planes ho gaye jahan par shear stress ki value shunya hai ya zero hai. Yahan par koi shear stress nahi hai. Aur in do planes ko hum kehte hain principal planes.

To jis plane par shear stress shunya hota hai, us plane…

…ko hum kehte hain principal plane. To agar hum iska element nikaalenge correspondinglyyeh mera A plane hai, aur yeh mera B plane hai to yahan par sirf normal stresses hi rahenge, yahan par shear stresses ki value zero.
To yeh mera state of stress ho jayega. Jo yeh do planes hain, yeh principal planes hainyahan par shear stress ki value shunya hai. Agar hum dekhenge ki in dono ka angle kya hai in dono ka angle hai 180 °, yaani Mohr circle par 180 ° apart hai. Par actually kitne ° apart rahenge? Yeh rahenge 90 ° apart. To principal planes hamesha orthogonal hote hain.
Aur yeh mark kar diya maine σ_1 A plane ke correspondingly, aur σ_2 B plane ke correspondingly.

Ab hum dekhenge ki diameter iska jo aayega. Haan, to main ek aur cheez mark karna chahta hoon yahan pe. Jo σ_2 hai, yeh magnitude ka minor principal stress kahenge. To yeh do terminologies bhi aap dekhenge books meinmajor principal plane aur minor principal planejo represent karega state of stress.Jaise yeh σ_1 haiyeh radius ho jayega.To agar hum dekhenge plane of maximum shear, to yahan par hum dekhenge is circle pe maximum shear value kahan pe aa rahi hai. To yeh is point pe aa rahi hai maximum shear value, ya is point pe aa rahi hai.

Magnitude pe.
To agar yeh τ_{max} hai, agar hum dekhenge, to yeh point A ya point B se 90 ° apart hai.
To hum kahenge plane A ya plane B jo mere major aur minor principal plane hain, wahan se 90 ° apart hai.
Aur iski jo value hogi τ_{max} ki, yeh radius ke barabar hogi.
Aur radius ki value humne dekhi: $(\sigma_1 - \sigma_2)/2$.
To τ_{max} ki value mujhe milegi: $(\sigma_1 - \sigma_2)/2$.
Ussi tarah se jo τ_{max} ki value hai magnitude rahegi $(\sigma_1 - \sigma_2)/2$, kyunki radius ke barabar hoti hai.
Aur maine abhi just bataya ki yeh jo plane hai, yeh 90 ° apart hai Mohr circle par.
To physically kitna ° apart rahega?
Yeh uska aadha rahegayaani 45 ° apart.

To agar main plane of maximum shear nikaalu, yahan par agar yeh mera element hai, to yeh hamesha 45 ° rahega.

Yeh jo angle rahega, yeh 45 ° rahega mere principal planes se.

Principal planes yaani kya?

Jahan par shear stress ki value zero hai.

To ek observation ho gaya.

Aur ek important observation abhi bata deta hoon:

Yahan par $\sigma 1$ plane A par, $\sigma 2$ plane B par hai, aur shear stress ki value zero hai.

Par agar hum point D mark karengeya plane Djo hai yahan par, jo ki mera plane of maximum shear hai, wahan par hum dekhenge σ ki value shunya nahi hai.

Yaani normal stress yahan par hai.

Yaani is plane par shear stress to shunya hai, par plane of maximum shear par normal stress shunya nahi hai.

Normal stress ki value mere is corresponding plane par shear stress zero hota hai, par plane of maximum shear par normal stress ki value shunya rahe yeh zaroori nahi.

To yeh ek bahut hi important point hai.

Kisi-kisi condition mein normal stress ki value shunya ho sakti hai, par yahan par nahi hai.

Abhi humne Mohr circle ke baare mein dekhaMohr circle ka construction dekha, kuch important points dekhe.

Abhi yahi hum analytical way se solve kar sakte hain.

Yeh mere paas teen equations hain, humne derive kiye the yeh equations.

Abhi isko analytical way se dekhenge.

To plane of maximum shear kis tarah se nikaalenge?

Agar main isko differentiate karuplane of maximum shear ko nikaalne ke liye analytically, yaani mathematically to isko hum differentiate karke solve karenge.

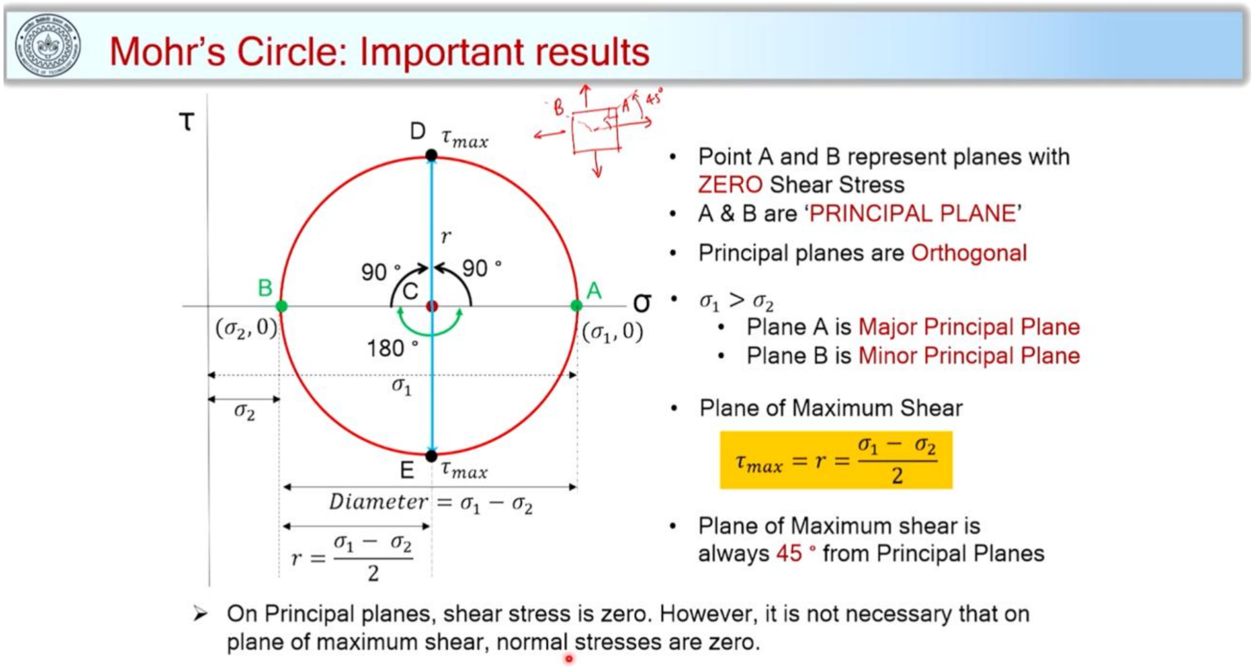
Aur hum kya nikaal sakte hain?

θ ki value nikaal sakte hain.

Aur principal stresses ki value nikaal sakte hain.

Aur jab hum θ ki value milegi to hum principal planes ka location bhi nikaal sakte hain.

To yeh main aapke liye homework ke liye chhod raha hoon…



…ki aapko yeh analytically solve karke dekhna hai, aur θ ki value nikaal ke dekhni hai, aur principal planes aur principal stresses ki value nikaal ke dekhni hai.

Aur yeh dikhana hai ki principal planes aur plane of maximum shear 45 ° hai by solving this equation.

Mohr's circle: Analytical way

$$\sigma_{x'x'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} cos2\theta + \tau_{xy} sin2\theta \quad (1)$$

$$\sigma_{y'y'} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} cos2\theta - \tau_{xy} sin2\theta \quad (2)$$

$$\tau_{x'y'} = \frac{\sigma_{yy} - \sigma_{xx}}{2} sin2\theta + \tau_{yx} cos2\theta \quad (3)$$

Christian Otto Mohr
German civil engineer
Image courtesy: Wikipedia

Location of the plane of Maximum shear and its value?

$$\frac{d\tau_{x'y'}}{d\theta} = 0$$

Show how much rotation needed to arrive at locations of Principal planes?

Prove that the angle between Principal planes and the plane of maximum shear is 45° ?

To jab hum baat karte hain Mohr circle ki, tab Mohr circle ek German civil engineer the. Unhone yeh equations ko geometry form mein laaya. Unka naam tha Christian Otto Mohr.

Aur agar aapko iski utility jaan ni hai to aap inke work ko padh sakte ho.

Yeh humne 2D stress state ke liye jaana tha.

Abhi hum dekhenge Mohr circle 3D stress state ke liye kaisa hota hai.

Abhi ke liye main yahi rukta hoon.

Phir next class mein hum dekhenge ek solved problem Mohr circle ke liye, aur principal planes aur principal stresses kya hoti hain, aur unka importance kya hota hai.

Yeh hum next part mein jaanenge. Dhanyavaad.