

CRYSTAL SYMMETRY, X-RAY DIFFRACTION, AND PHYSICAL PROPERTIES

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Lecture 26: Plane Groups - III: Glide Planes

In this lecture we continue from where we left off, but before proceeding further it is helpful to clarify the notation used for plane groups. Just as point groups have standard notations, plane groups also follow a systematic convention, and we will use the international notation.

Let us refer back to one of our earlier diagrams where we started deriving the plane groups. Consider the first plane group obtained by adding no symmetry to the oblique lattice. This is simply written as $p1$. The letter “p” denotes a primitive lattice, and the number “1” indicates that no symmetry has been added. We do not need to explicitly mention the word “oblique” because no other plane group corresponds to this notation.

Next, when we take the same oblique lattice and add a two-fold rotational symmetry (point group 2), the notation becomes $p2$, again indicating a primitive lattice with a two-fold axis. Adding a four-fold symmetry gives $p4$, and adding a three-fold or six-fold symmetry to the hexagonal lattice gives $p3$ and $p6$, respectively.

Moving to rectangular lattices, a primitive rectangular lattice with a mirror plane added (point group m) is written as pm . For the centered rectangular lattice, the letter C replaces P to indicate centering. Thus, adding a mirror plane gives cm .

Up to this point, the plane groups we have constructed are $p1$, $p2$, $p3$, $p4$, $p6$, pm , and cm . These correspond to the point groups 1, 2, 3, 4, 6, and m applied to the appropriate lattices. However, several point groups still remain unused. The remaining ones are:

- $2mm$
- $4mm$
- $3m$

- 6mm

Let us begin with the point group $2mm$. This point group must be added to the primitive rectangular net. Starting with a primitive rectangle, we place the $2mm$ motif at the corner. The motif contains a two-fold rotation with two mutually perpendicular mirror planes. Under lattice translations these mirrors repeat across the cell. Because a mirror combined with a translation produces another mirror at the center, additional vertical and horizontal mirrors appear automatically, and a two-fold rotation also arises at the center of the unit cell.

Thus, the pattern that emerges corresponds exactly to the addition of $2mm$ to a primitive rectangular lattice. It is also clear that no glide planes appear here, because any attempt to combine a mirror with a translation would generate a glide whose parallel component becomes a lattice translation, and such a case reduces to a mirror rather than a true glide.

Next, consider adding $2mm$ to the centered rectangular lattice. The process begins in the same way: we place the $2mm$ motif on the lattice, include the centering point, and examine the resulting symmetries. The interaction of the centering shift with the mirrors and the two-fold symmetry modifies the arrangement of symmetry elements, and the final pattern must be constructed carefully by following the translation and reflection operations throughout the cell.

In the centered rectangular lattice, there is also a lattice point at the center, and when we add the $2mm$ point group we begin by placing the two-fold rotational symmetry at the corner, just as we did earlier. All symmetry elements, two-fold rotations and mirror planes must now be transferred to every lattice point, including the centered one.

Once the two-fold is placed at the center, we can introduce motifs to understand how the mirrors act. A motif placed near a mirror will reflect across it, reversing its handedness. The next mirror reverses it again, and the horizontal mirror flips it once more. After the sequence of reflections, the motif returns to its original handedness, and all these motifs get repeated across the entire lattice, including the centered position.

Next, we examine what additional symmetry elements appear in this arrangement. As in the earlier case of the centered rectangular lattice with only a mirror, several glide planes are generated automatically. For example, if we take a vertical mirror and combine it with the centering translation from the top corner to the center, that combination produces a glide plane. A corresponding glide plane appears symmetrically on the other side. Similarly, taking the horizontal mirror and the same centering translation produces additional glide planes. These appear naturally because a mirror combined with a centering translation always generates a glide plane.

We must also inspect the two-fold rotational symmetries. A two-fold, when translated horizontally, produces additional two-folds in the expected positions. But we are not finished: if we take the same two-fold at the upper left and apply the centering translation, another two-fold appears at the midpoint of that translation. This completes the construction of the centered rectangular lattice with the $2mm$ point group. According to the notation, the primitive rectangular lattice with $2mm$ becomes $p2mm$, while the centered rectangular with $2mm$ becomes $c2mm$. The notation alone tells us that these lie in the rectangular crystal system.

We now return to the primitive hexagonal net. Earlier, we examined what happens when we add a three-fold or six-fold rotation to it. Now we add the point groups $3m$ and $6mm$. Let us begin with the primitive hexagonal lattice combined with $3m$. First, we draw the hexagonal cell, and then place the $3m$ point group, which consists of a three-fold rotation and three mirrors passing through it. These mirrors divide the plane symmetrically around the three-fold axis.

There are actually two distinct ways to add the $3m$ point group to this lattice. To illustrate this clearly, we draw the cell again to show the second option. In the first method, the $3m$ motif is oriented such that its mirror planes lie either exactly along the cell edges or perpendicular to them. Let us begin by examining the case where the mirror planes are aligned along the cell edges.

If the 3m point group is added with its mirrors aligned along the cell edges, we begin by placing the three-fold rotations at the lattice corners and then positioning the mirrors so that one mirror lies along a cell edge, the second mirror lies along the other cell edge, and the third mirror lies along the diagonal. Lattice translations then copy these mirrors to every cell edge, so each corner ends up with three mirrors meeting at the three-fold rotation. The corresponding three-folds also appear at all translated positions, consistent with the primitive hexagonal lattice.

To identify the glide planes, choose one mirror, say M_1 , and consider the lattice vectors \bar{a} and \bar{b} . Take the reflection σ_1 and follow it with the lattice translation \bar{b} .

$$\bar{b} * \sigma_1$$

Since \bar{b} is oblique to mirror M_1 , it has a component parallel to the mirror, T_{\parallel} , and another component perpendicular to it, T_{\perp} . Imagine going from point A to B first along the mirror direction for a distance of $a/2$, and then along the perpendicular direction; this decomposition reveals T_{\parallel} and T_{\perp} . The perpendicular component determines the location of the glide: halfway along T_{\perp} we obtain a glide plane parallel to mirror M_1 . By symmetry, a partner glide plane appears on the opposite side.

A similar analysis holds for mirror M_2 . Take the reflection σ_2 and follow it with translation \bar{a} .

$$\bar{a} * \sigma_2$$

This combination produces another glide plane, with its symmetric counterpart appearing automatically. Symbolically these operations appear as $\sigma_{1\tau}$ and $\sigma_{2\tau}$, and the three-fold symmetry generates the rest of the glide planes. This entire configuration corresponds to one distinct plane group.

In the second configuration, the mirrors of the 3m point group are placed perpendicular to the cell edges instead of along them. The three-fold rotations remain at the lattice corners, but each mirror is now drawn perpendicular to a cell edge and placed at its midpoint. Translations generate additional mirrors, and one mirror lies along the long diagonal of the cell. For clarity, the hexagonal cell is often divided into two equilateral triangles in such diagrams.

To determine the glide planes here, you may examine combinations such as σ_1 followed by translation \bar{b} or σ_2 followed by translation \bar{a} . Each mirror-translation combination produces a glide plane, and symmetry generates the remaining ones. This second arrangement produces a different plane group, so it cannot share the same notation as the first. Their correct international symbols are introduced in the next lecture.

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