CRYSTAL SYMMETRY, X-RAY DIFFRACTION, AND PHYSICAL PROPERTIES
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Lecture 25: Plane Groups - II: Glide Planes

In this lecture we continue developing the plane groups or the 2D space groups. In the last lecture we reached the point where we took a primitive rectangular cell and added a mirror plane to obtain the associated plane group. Now we begin by taking a centered rectangular lattice and adding a mirror to it. The centered cell has lattice points at the corners and a lattice point at the center of the cell.

We add a mirror to this by placing the mirror on one of the edges. Putting the mirror vertically, the translation brings the mirror to the opposite edge. There is also a centering translation, so a mirror will pass through the middle lattice point as well. This gives the complete set of mirrors for the group. If we now place motifs, for example a right-handed motif on one side, the mirror reflects it into a left-handed motif. The translations carry both right-handed and left-handed motifs to every lattice point. The centering translation also generates motifs at the central lattice point.

Let the first motif be motif 1, the reflected one motif 2, and the translated one motif 3. Motif 1 goes to motif 2 by a reflection; motif 2 goes to motif 3 by a translation. If $\overline{a}$ is one lattice vector and $\overline{b}$ is the other, then the translation taking motif 2 to motif 3 must be $\frac{1}{2}(\overline{a} + \overline{b})$ because it is the centering translation. The question we now ask is how motif 1 goes to motif 3 and what symmetry operation should take it there. The group must satisfy closure, so the combination of reflection followed by translation must produce a third symmetry operation.

Motif 1 is right-handed and motif 3 is left-handed, so no translation can take 1 to 3 and no rotation can do it either because rotation and translation do not change handedness. Therefore, the operation must be some kind of reflection. But we cannot put an inclined

mirror plane, because that would destroy the symmetry of the centered rectangular lattice. This means we must look for a type of symmetry operation not yet introduced.

To understand this, we draw the situation separately. We have a mirror and a translation T, which here represents the centering translation. Consider an asymmetric motif at a distance δ from the mirror. Upon reflection it becomes a left-handed motif also at distance δ on the opposite side. This is motif 2. The translation T then moves motif 2 horizontally to motif 3 without changing handedness. The question again is how to go directly from motif 1 to motif 3.

We enlarge the diagram for clarity. The translation T can be broken into two components: one component perpendicular to the mirror, called $T_{\perp}$, and one component parallel to the mirror, called $T_{\parallel}$. The translation taking motif 2 to motif 3 can be decomposed in the same way. Motif 2 is at distance δ from the mirror on the left side, so motif 3 must also be at distance δ from the mirror. Motif 1 is already at distance δ from the mirror on the right side.

$$T = T_{\perp} + T_{\parallel}$$

Since motif 1 is right-handed and motif 3 is left-handed, an operation taking 1 to 3 must change handedness. Suppose we try to use an ordinary mirror: motif 1 would reflect into a left-handed motif directly across the mirror, but the reflected motif would not coincide with motif 3 because it would not have the necessary parallel displacement. Thus, a simple mirror reflection is insufficient.

To make motif 1 coincide with motif 3, I can reflect motif 1 first and then translate it. I cannot leave the reflected image where it appears after reflection; I must slide it along the mirror plane. The amount of translation required is already known: it is the component of the translation vector parallel to the mirror plane, $T_{\parallel}$. Therefore motif 3 can be obtained by reflecting motif 1 and then translating it by $T_{\parallel}$. This gives an exact coincidence.

This means the operation involved is not an ordinary mirror reflection anymore. We draw this new type of mirror as a dashed line and label it G. The letter G stands for *glide*. A glide reflection is a two-step operation: first a reflection, and second a translation. Because a translation is involved, a glide is not a point symmetry. Point symmetries always have at least one invariant point, but when translation is present, no point can remain fixed. Therefore, a glide cannot exist in point groups; it only appears in plane groups or space groups.

Now we ask where the glide must appear in our centered rectangular plane group with a mirror. We have a mirror and a translation that is not perpendicular to the mirror. Therefore, a glide must appear at the correct symmetry position. By symmetry there will also be a glide at the opposite location. To find the exact position of the glide, we return to the earlier diagram. The glide is located at the midpoint of the perpendicular component of the translation. That perpendicular component is $T_\perp$, so the glide plane lies at $T_\perp/2$. In the plane group diagram this perpendicular component corresponds to $\overline{b}/2$. The parallel component corresponds to $\overline{a}$. So $\overline{b}$ is the perpendicular direction and $\overline{a}$ is the parallel direction.

The glide operation is written as a reflection followed by a translation. The symbol is written using the reflection symbol σ with a subscript τ, where τ is the glide translation. The glide is located at $T_\perp/2$, and the translation used in the glide is $T_\parallel$. The vector τ is the translation component parallel to the mirror plane.

To find the magnitude of the glide translation, we look at a simple pattern. Start with a right-handed motif. Reflect it to change handedness, then translate it by τ to place it at its new position. Again, reflect and translate to the next position, and repeat. Along the glide plane, the pattern alternates handedness and moves forward by τ every time. The separation between identical handed motifs on the same side of the glide plane is 2τ. These distances are all 2τ along the glide direction. Therefore 2τ is the lattice translation

along the glide plane. This is the shortest lattice translation in that direction. *Hence the glide translation τ is exactly half the lattice translation along the glide plane.*

Returning to the centered rectangular plane group, the glide translation must therefore be $\frac{1}{2}\bar{a}$, since $\bar{a}$ is the lattice translation along the glide plane. So, the glide translation is half of a in this plane group as well.

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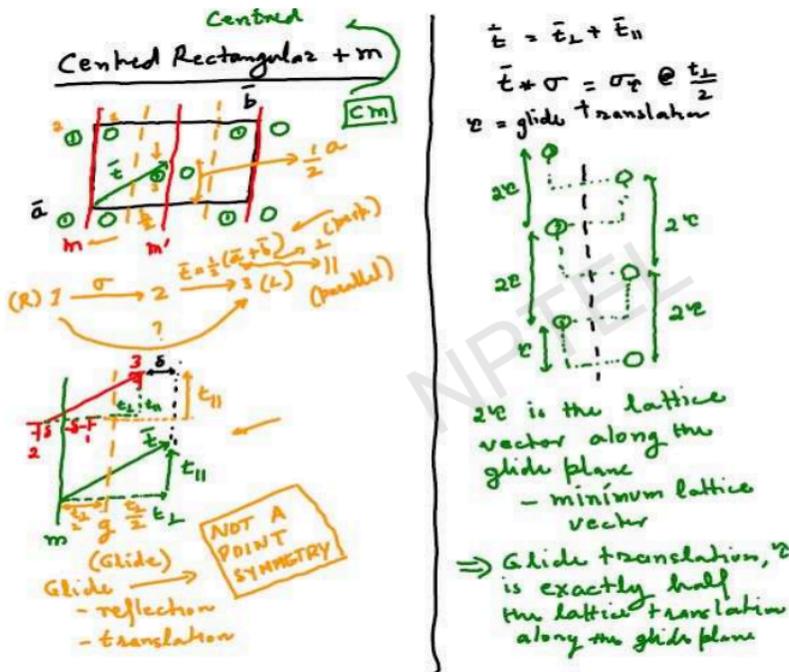
We can also look at the matrix representation of this operation, which combines reflection and translation. It turns out to be quite simple.

To construct the glide matrix, we begin by taking the mirror reflection matrix and then following it with the translation matrix. Let the x-direction be horizontal and the y-direction be vertical. Reflection followed by translation gives the operation σ_τ, where $\tau = T_\parallel$, and the glide plane is located at half of the perpendicular component, $T_\perp/2$.

$$T * \sigma_\tau = \sigma_{\tau=T_\parallel} @T_\perp/2$$

The mirror plane is normal to the x axis, meaning the mirror plane lies in the yz-plane. The reflection matrix for this mirror is

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

For the translation, using homogeneous coordinates, the translation matrix is

$$\begin{pmatrix} 1 & 0 & T_\perp \\ 0 & 1 & T_\parallel \\ 0 & 0 & 1 \end{pmatrix}.$$

Multiplying the two matrices gives

$$\begin{pmatrix} -1 & 0 & T_\perp \\ 0 & 1 & T_\parallel \\ 0 & 0 & 1 \end{pmatrix},$$

which is the glide matrix.

To interpret this matrix, recall that the glide plane is located at $T_\perp/2$. Compare this with the mirror matrix that is located at $(a, 0)$, written as

$$\begin{pmatrix} -1 & 0 & 2a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The comparison clearly shows that for a pure mirror we must have $T_\parallel = 0$, and the perpendicular translation must satisfy $T_\perp = 2a$, which implies $a = T_\perp/2$. Therefore, whenever $T_\parallel = 0$, the glide becomes a pure reflection, and the operation becomes a point symmetry because the translation component disappears.

Now we ask whether $T_\parallel$ can be a lattice translation. Consider the glide plane and the pattern of motifs generated by reflection and translation. Starting with a right-handed

motif, applying the glide repeatedly produces alternating handed motifs separated by $\tau = T_{\parallel}$. If τ is taken to be a lattice translation, then the lattice translation symmetry forces each motif to be translated by a full lattice vector. A motif at one position will move to the next lattice point, the second motif will also move to the next lattice point, and so on.

If we impose lattice translation on the pattern created by the glide, the resulting arrangement becomes identical to a pure mirror reflection pattern. The alternating handedness collapses into a normal mirror symmetry. This shows that when τ becomes a lattice translation, the glide operation loses its character and becomes an ordinary mirror. Therefore, τ can be zero, giving a normal mirror, and if τ equals a lattice translation, the glide again reduces to a pure reflection. With this the lecture ends.

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