Project of Materials (Nature and Properties of Materials: III)
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Lecture 23
Critical Resolved Shear Stress

So, welcome to the new of the course Properties of Materials. So, let us just briefly see the contents of last lectures, just a brief recap.

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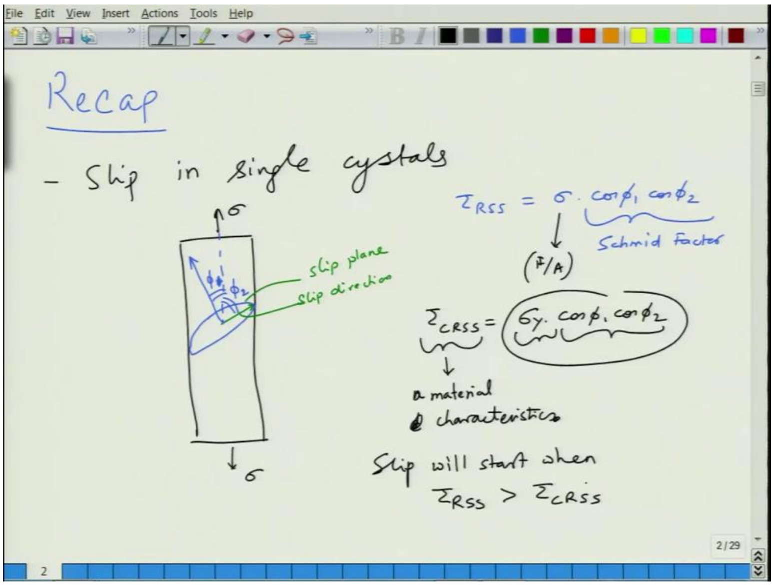

So, in the last lecture, we talked about the basically slip in single crystals. So, we consider the geometry like this. So, you have a crystal which is subjected to a uniaxial stress and let us say we have a slip plane which is oriented in this fashion. Now this slip plane as we discussed is a closed packed plane or it is a slip plane belonging to the slip system.

So, as a result it will have a slip direction. Let us say this is the slip direction and this is the slip plane and so this tensile axis makes angle phi 1 and phi 2 with respect to the tensile axis. So, these, so let us say phi 1 is the angle between, phi 1 is this and this is phi 2. So, it makes angle between.

So, the angle between the tensile axis and the plane normal is phi 1 and angle between the slip direction and the tensile axis is the phi 2. So, based on this we resolved what we call as shear stress tau RSS which is sigma into cos phi 1 into cos phi 2 and this product of cos phi 1 and cos phi 2 is called Schmidt factor and sigma is nothing but force divided by area. Now, this resolved shear stress has to exceed a critical stress which is called as tau CRSS, Critical Resolved Shear Stress which is sigma y into cos phi 1 into cos phi 2.

So, CRSS is a, is a material characteristics, characteristics. So, sigma y changes with the orientations. So, the product of sigma y and the orientation dependant parameters that are phi 1 and phi 2, these two determine a value called as CRSS. So, the slip will start, slip will start when tau RSS on a given plane will exceed tau CRSS.

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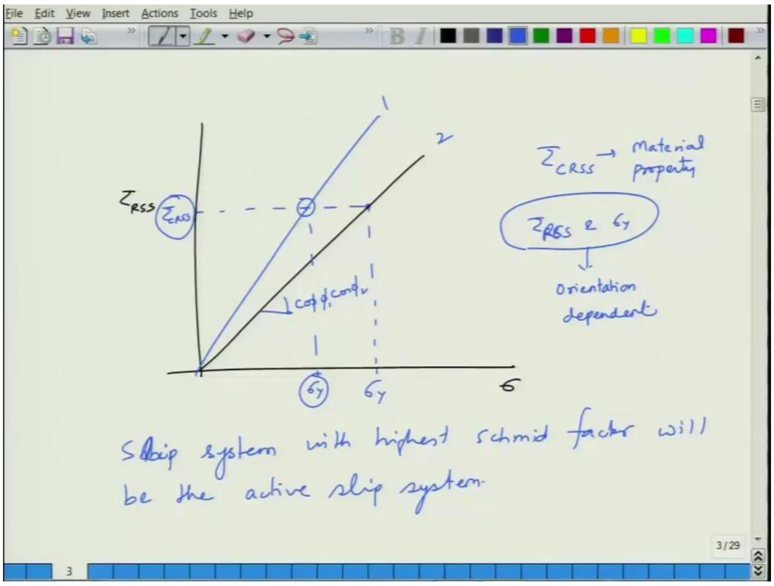

So, essentially what we are saying is that, if we plot tau RSS as a function of stress then your slip will began, let us say at this point, this is the corresponding yield stress and this will be the tau CRSS for that given configurations and when tau RSS exceeds this value of tau CRSS then only the slip will start.

So, this is sigma basically and the slope of this will be cos phi 1 cos phi 2. So, essentially what we are saying is that, tau CRSS is a material property a material constant whereas tau, tau RSS and sigma y are dependant up on the orientations. So, so these are we can say orientation dependant. So, if you have two plots let us say, one is this and another one is this. So, depending on what you have here. This system corresponding to tau CRSS this system will show.

So, in this case for a same value you have higher RSS. So, this system will start yielding first. So, you have 1 you have 2. So, 1 will show yielding before the 2 because, because of higher resolved shear stress. So, basically the slip system with higher, highest Schmidt factor you can say, slip system with highest Schmidt factor will be the active slip system and so they will, basically what it means is that, when the slip systems are oriented favourably with respect to tensile axis maximise, maximizing the Schmidt factor.

So, CRSS will be achieved even at lower yield stresses as compared to system with unfavourably oriented. So, you may have high yield stress but the Schmidt factor is low then your critical resolved shear stress will remain low. So, basically you want maximize the Schmidt factor to reach the CRSS early to initiate the yielding at lower yield stresses. So, now this is what the summary of slip systems, slip in crystal says.

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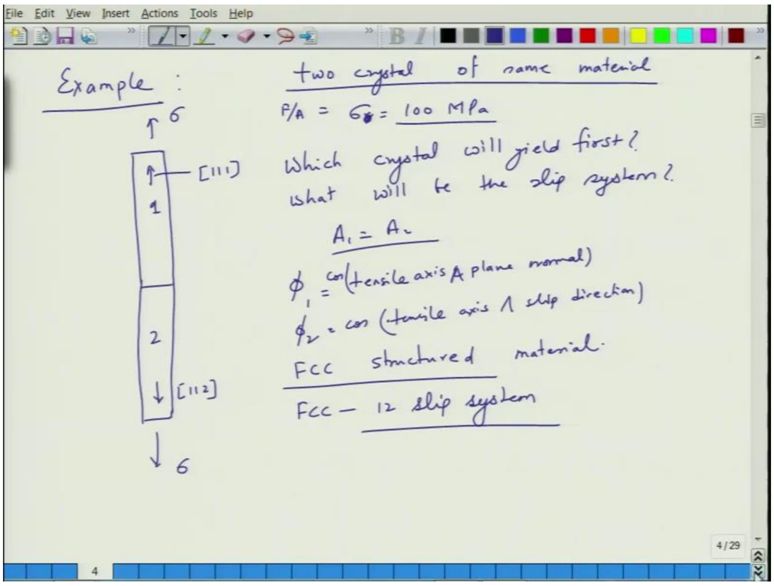

Let us take an example of this. Our example is let us say you have a crystal. Let us take a little kind of difficult problem. So, this is bi-crystal. Let us say crystal 1 and crystal 2 and this is subjected to a stress sigma in this direction. So, this direction for the crystal 1 is 1 1 1 and the direction for the crystal 2 is 1 1 2.

Let us say this is, you know these are crystals of same material. So, you can say two crystal of same material. So, let us say we have a, and we have a yield stress 100 MPa or stress not the yield stress but stress is. So, basically we are saying F divided by is equal to 100 MPa. So, essentially if this is the, if this is the situation in which you we are in right now.

So, for first grain the stress is parallel to 1 1 1 axis and for the second grain or crystal the stress is parallel to 1 1 2 axis and they are joined in such fractions, so that they make a vertical rod. So, basically now the question is which crystal will, will yield first and what will be the slip system? This is the problem.

Assuming that cross sectional area A1 is equal to A2. So, cross sectional area is similar. So, let us see this problem. Let us say phi 1 is the, so we know what phi 1 and phi 2 are and so, phi 1 let say is the angle between tensile axis, so cos of angle between tensile axis and plane normal and phi 2 are is cos of angle between basically tensile axis and slip direction.

Let us say the material is FCC structured, material is FCC structured. So, basically we have twelve system for FCC. So, FCC has twelve slip systems and we need to figure out which of these slip system will have highest resolved stress and which of these and then compare the two of them which one will.

So, let us first take the case what will happen if it have only one crystal. Let us say we workout both 1 and 2 separately and we see which one have highest resolved shear stress for which slip system and then compare the two.

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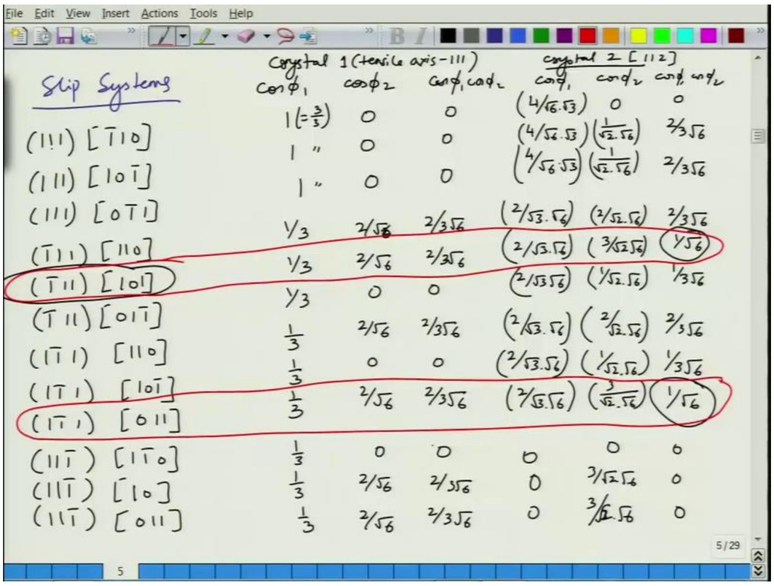

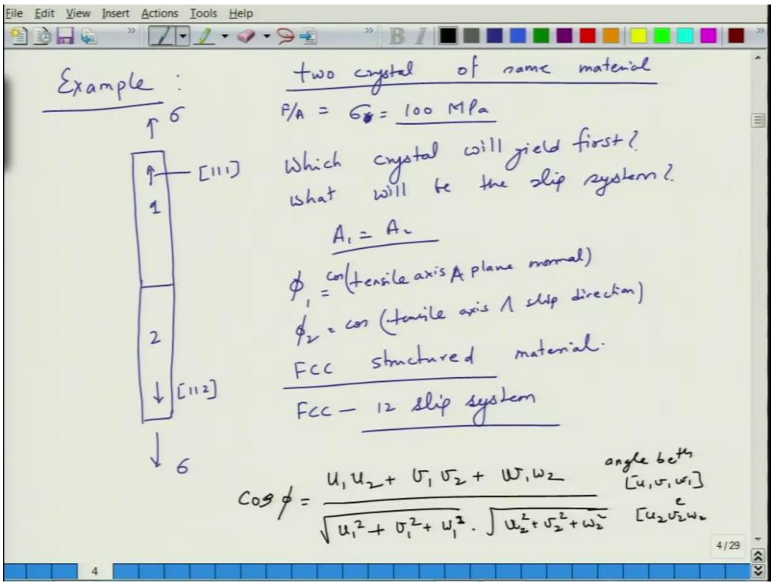

So, Let us do this. So, let us write down the slip systems for FCC. Slip system for FCC can be written as, let us say the first one is 1 1 1, bar 1 1 0, second one is 1 1 1, 1 1 1 and 1 0 bar 1. Third one is 1 1 1, 0 bar 1 1. Fourth one is bar 1 1 1, 1 1 0, bar 1 1 1, 1 0 1 and bar 1 1 1, 0 1 bar 1.

And then we have another direction and making sure that and these slip systems you can see you take a dot product, the dot product will be equal to 0. Then we have third one, third group of planes in direction that is, 1 bar 1 1, 1 1 0. Then we have 1 bar 1 1, 1 0 bar 1 and then we have 1 bar 1 1, 0 1 1.

Then we have 1 1 bar 1 and the three corresponding directions will be 1 bar 1 0, 1 0 1 and 0 1 1. So, these are slip systems, let us say, what is cos phi 1? So, crystal 1, tensile axis is 1 1 1. So, let us work out what is cos phi 1, cos phi 2. So, we can say that cos phi between two directions will be u1 u2 plus v1 v2 plus w1 w2 divided by square root of u1 square plus u2 square plus u3 square into, u1 v1 and w1 square. Just need this change here and u2 square plus v2 square plus w2 square.

So, this is the angle between you can say u1 v1 w1 and u2 v2 w2 in a cubic system. So, if that is a case, for the first one it will be 1 this will be 0, this will be 1 this will be 0. This will be 1 this will be 0. So, we can see that 1 and 1 1 1 are collinear. So, this is basically 3 divided by 3. So, this will be, so this is basically you can say for 1 1 1, the slip plane normal will also be 1 1 1. So, essentially you are calculating between the angle between 1 1 1 and 1 1 1 which is 1 1 1 basically 1 1 and 1.

So, basically it is nothing but 3 divided by 3. So, this is and between these, it will become 0, the dot product become 0, so this is 1, this is 0 0 0. So, essentially the product of these cos phi 1 cos phi 2 will be equal to 0 0 0. So, essentially these slip systems will not be active. Now, let us work out for others. So, if you do the maths, first one will be 1 by 3, 1 by 3, 1 by 3. This will be 2 divided by root 6, 2 divided by root 6 and 0. So, the product will be 2 divided by 3 root 6, 2 divided by 3 root 6 and 0. Now, let us work out the third group of planes.

So, 1 by 3, 1 by 3 and 1 by 3, if you take the dot product and this will be 2 divided by root 6, 0, 2 divided by root 6. So, if you take the cos of u1 u2, u1, u1 u2 plus v1 v2 and so on and so forth, this will come to be 1 by 3 for all three of them. Here, it will be 2 by root 6 for the first one, 0 for second one, 2 by root 6 for the third out. So, this will be 2 divided by 3 root 6, 0 and 2 divided by 3 root 6. Now, work out for the forth group of plane, that is, 1 by 3, 1 by 3 and 1 by 3. This will be 0, 2 divided by root 6 and 2 divided by root 6.

So, this will be 0, 2 divided by 3 root 6, 2 divided by 3 root 6. So, you can see that, out of these 12 slip systems for this orientation 1 1 1, 6 slip systems shows 0 0 stress which means they will not slip at all. Whereas, remaining 6 shows similar value of stress. Now, let us see the crystal 2. So, basically you can see, if it was crystal 1 alone, then either of these planes could have start either of this slip system could have start slipping because all of them have similar shear stresses or Schmidt factor.

Now, let us look at the crystal 2. Crystal 2 again we calculate cos phi 1 cos phi 2 and cos phi 1 cos phi 2. So, here this will work out to be, now crystal 2 has orientation which is 1 1 2. So, if you take the 1 1 2 cos, 1 1 2 versus 1 1 1 angle, then it will be 4 divided by square root of 6 into square root of 3. This will be 4 divided by square root of 6 into square root of 3 and this will again be 4 divided by square root of 6 into square root of 3.

And for cos phi 2, this will be 0, this will be 1 divided by square root 2 into square root 6 and this will be 1 divided by square root 2 into square root 6. If you take the Schmidt factor, this will be 0, this will be 2 divided by 3 root 6 and this will be 2 divided by 3 root 6. So, you can do the maths again for this. This will be 2 divided by square root 3 into square root 6. This will be 2 divided by square root 3 into square root 6. This will be 2 divided by square root 3 into square root 6.

This will become 2 divided by square root 2 into square root 6. This will be 3 divided by square root 2 square root 6 and this will be 1 divided by square root 2 into square root 6. If you now multiply, this will become 2 divided by 3 root 6, this will be 1 divided by square root 6 and this will be 1 divided by 3 root 6. So, this is how it will turn out to be. For the next one now this is 2 divided by square root 3 into square root 6, this will be 2 divided by square root 2 into square root 6 this will become 2 divided by 3 root 6.

For the next one its 2 divided by square root 3 into square root 6 and other one will be 1 divided by square root 2 into square root 6 and the product becomes 1 divided by 3 root 6, this will be 2 divided by square root 3 6, 3 divided by square root 2 into square root 6. This becomes 1 divided by square root 6 and this will be 0, these two will be 0, these will be 0 as well.

This will be 3 divided by square root 2 into square root 6 into 3 divided by square root 2 into square root 6. So, these 3 will show 0 Schmidt factor. So, we can see that here, first of all if it was only crystal 1 then, these could have slipped. Now, we have crystal 1 attached with crystal 6. We can see that the highest value is shown by this or this.

So, basically crystal 2 shows higher shear stress in the crystal 1. So, crystal 2 will slip first on the slip systems which are basically bar 1 1 1 1 1 0 and then it will be, not this one. It would be the next one bar 1 1 1 1 0 1. So, essentially we can say it is this slip system which shows higher shear stress and then we have this slip system which shows higher shear stress.

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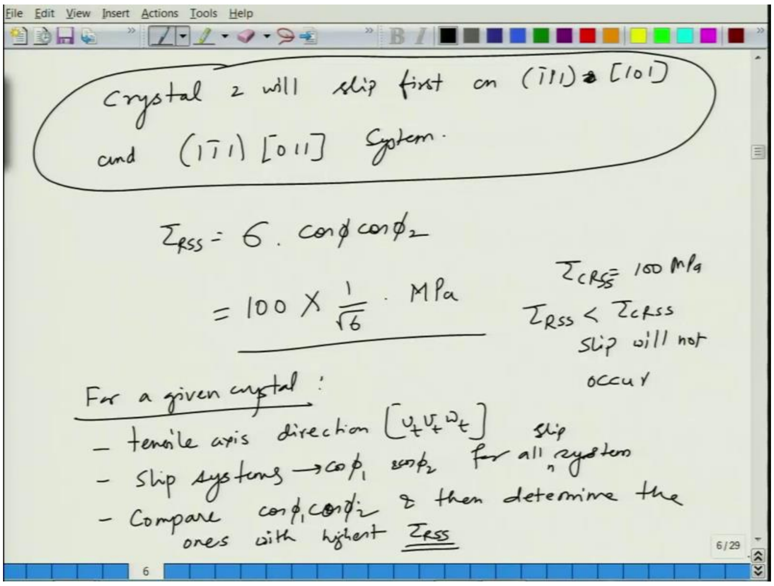

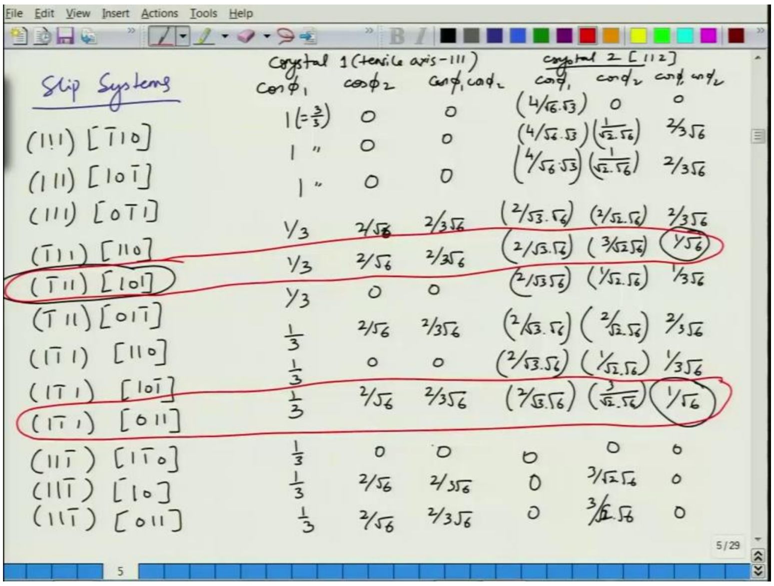

So, we can summarize from this, the answer will be crystal 2 will slip first on bar 1 1 1 and 1 0 1 and on 1 bar 1 1 0 1 1 systems and that is because you can see because these shows higher value of, because tau RSS will be equal to sigma into cos phi 1 cos phi 2. So, you can see that, the stress will be equal to 100 MPa multiplied by 1 divided by root 6, 1 square root 6 MPa.

So, this will be the value of resolved shear stress for this particular system. All other slip systems will show 0 stress. So, essentially what you are going to do? So, the procedure basically is, for a given crystal, you need to first work out what tensile axis is? Tensile axis direction u v w, let us say, u t w t, u t v t w t. Then you need to work out for all the slip systems. For all the slip systems, the phi 1 and phi 2, basically cos phi 1 and cos phi 2 for all the systems, slip systems.

Compare cos phi 1, product of cos phi 2 and then determine the ones with highest tau RSS. So, let us say, even if the value of tau RSS is this. If tau CRSS was let us say, 100 MPa, then we can see that even with this, even with this orientation, tau RSS is lower than tau CRSS, slip will not occur.

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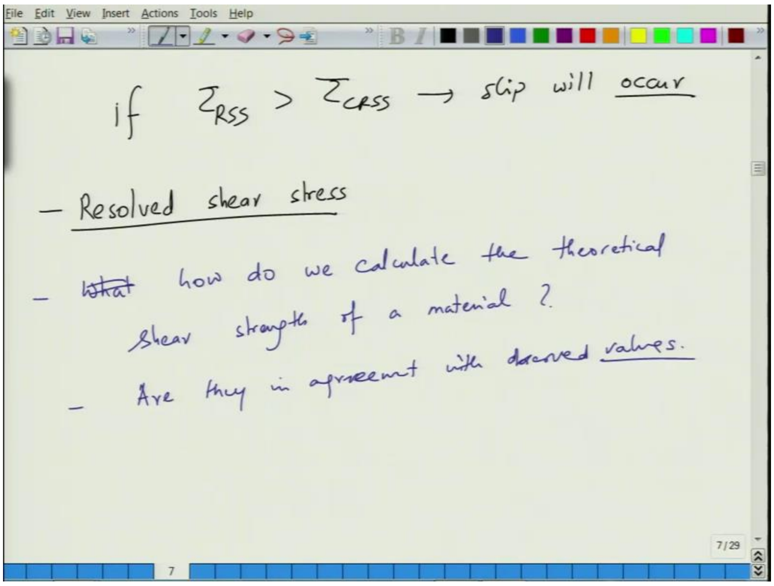

However, if tau RSS was more than tau CRSS, then slip will occur. So, which means you need to increase the value of sigma for that to happen. So, this is what we have done in, what we have seen is basically a simple example of how one can calculate the resolved shear stress for a given orientation of crystal with respect to the kind of slip systems that are present in the in the. Now, what happens, now we need to work out. So, this is the, this is the calculation of resolved shear stress.

Now, the question is, how do we calculate, how do we calculate the theoretical shear strength of material and then we want to see are they in agreement with observed values? So, this is what we are going to do and next, so let us see how we do that.

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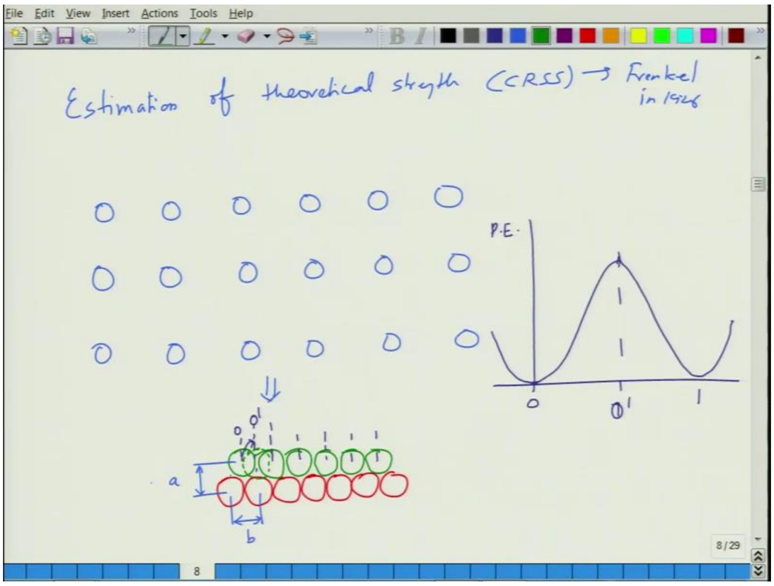

Let us say, estimation of CRSS was first given by Frenkel in 1926. So, let us see what happens in a material? So, in a material you can see the atoms are placed in this fashion. They are placed in ordered fashion in a lattice and if you take the touching a sphere model, then touching sphere model would be something like this. We have first row of atom.

So, they are touching in the sense they actually do not touch because of, because of electrostatic repulsion but nevertheless, for the sake of convenience let us consider that these are touching each other to the closest approximation. So, this is how you have first row of atom.

Then you have second row of atom and the spacing between of them between them let us say is, so there is one spacing here. So, let us say, this is b and this is a and these are theoretical spacing and basically what it means is that the potential energy of the crystal, potential energy of the crystal is minimum when the atoms are located at these positions.

So, potential energy goes like this. So, at equilibrium positions 0 and 1, potential energy is minimum. So, let us say this is the position 0 1 2 3 4 5. So, this is the 0 position. So, 0 position, 1 position, 2 position, potential energy is minimum but when it comes to this position, let us say. So, this atom when it moves to this position, then it comes on top of the next atom and that is where the potential energy is maximum.

So, let us say, this is 0 prime. So, this is 0 prime, this is 0, this is 1. So, when the green atom let us say moves here, then the potential energy is. So, let us this is the case of sought of a closed packed. So, you can see the shear stress of a material also varies in this sinusoidal fashion.

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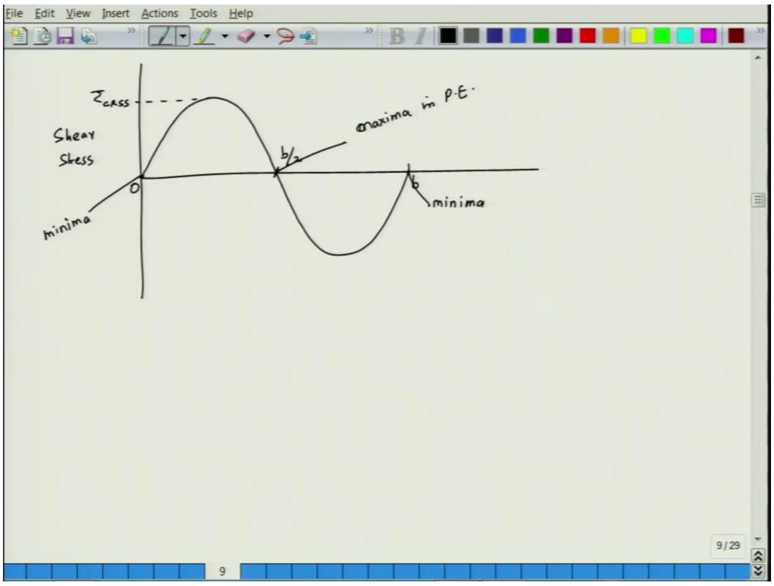

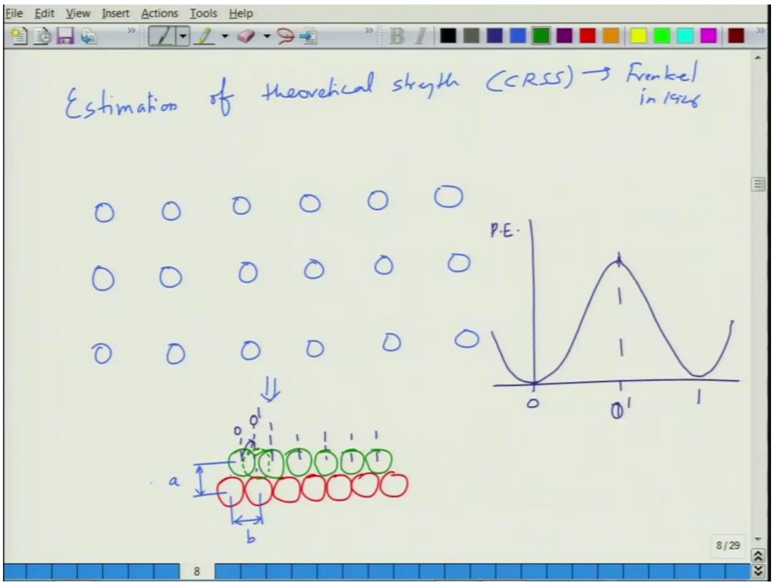

So, the shear stress varies, if you plot the shear stress, the shear stress varies in this fashion which is a sinusoidal variation. The maximum amplitude that we want to apply is tau we can apply tau CRSS and let us say this is overall, so this is where we have the change of slope. So, this is where your potential energy is maximum.

So, this is your b, this is 0 and this distance will be b by 2. So, if look at this distance, this will be basically you can say, this is, this is b 0, this is b, this will be 0 b by 2. So, this is what so this is what so this point corresponds to minima, this point also corresponds to minima and this point corresponds to maxima in potential energy.

So, what we want to do is that, we want to now develop a frame work of, from this variation of shear stress and from this sort of schematic diagram of how atoms are placed in a solid, considering the model, we want to develop a for a simple mathematical framework of calculating resolved shear stress and we will do that sin the next lecture because we are running out of time now.

So, what we have done in this lecture is basically we have looked at how do you work the resolved shear stress for given orientation of crystal and the type of crystal in terms of materials. So, if it is FCC, for all the 12 slip systems you work out the phi 1 and phi 2 values and then you work out the product of cos phi 1 cos phi 2 and see on which of the system the shear stress is a Schmidt factor is maximum which will basically maximise the shear stress.

But even then the resolved shear stress has to exceed the critical resolved shear stress otherwise the yielding will not occur. So, we will continue our discussion of theoretical shear strength in the next lecture. Thank you.