Properties of Materials (Nature and Properties of Materials: III)

Professor Ashish Garg
Department of Material Science & Engineering,
Indian Institute of Technology Kanpur

Lecture 11

Elasticity of different materials, design of composites

So welcome again to the new lecture of the course, Properties of Materials, and let us do a brief recap of the previous lecture.

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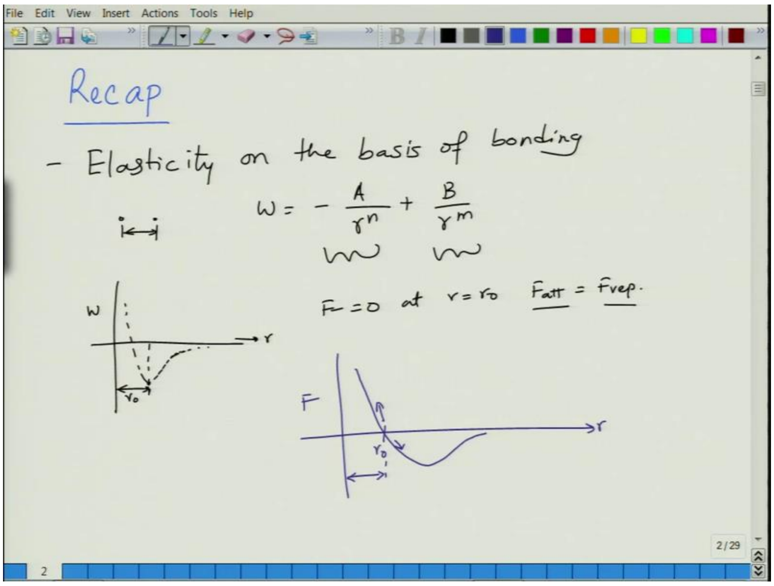

Recap

- Elasticity on the basis of bonding

$$W = -\frac{A}{r^n} + \frac{B}{r^m}$$

$F = 0$ at $r = r_0$ $\quad F_{att} = F_{rep}$.

So in the previous lecture, we learned about the elasticity on the basis of bonding. And we found that that for a pair of atoms, let us say, the equilibrium separation between them is determined by the potential energy and this potential energy can be written as minus of A divided by r to the power n plus B divided by r to the power m. So you have a attraction term, you have a repulsion term. And wherever the potential energy is minimum, so the overall potential energy, the sum of these two, wherever it is minimum, will show. So this is w, this is r, and the minima determines the r naught, that is the equilibrium separation.

And the, at this point where the potential energy is minimum, the force is equal to 0 at r is equal to r naught, which means the attractive and repulsive forces they outbalance each other completely as a result. So basically F attractive is equal to F repulsive. So these two terms they completely cancel each other and when you plot now, F versus r, then F versus r will be of similar nature, but except that this will be r naught.

So as you stretch the material beyond r naught, you basically lead to forces which are attractive in nature. So, basically, atoms tend to pull them, you have to overcome the attractive forces. And when you compress it, then the forces which are applied are the, repulsive forces basically, the tensile forces. So here it is compression as you separate the atom away from each other. And when you pull them together towards each other, then you apply tensile forces. And this modulus of elasticity is essentially related to bonding.

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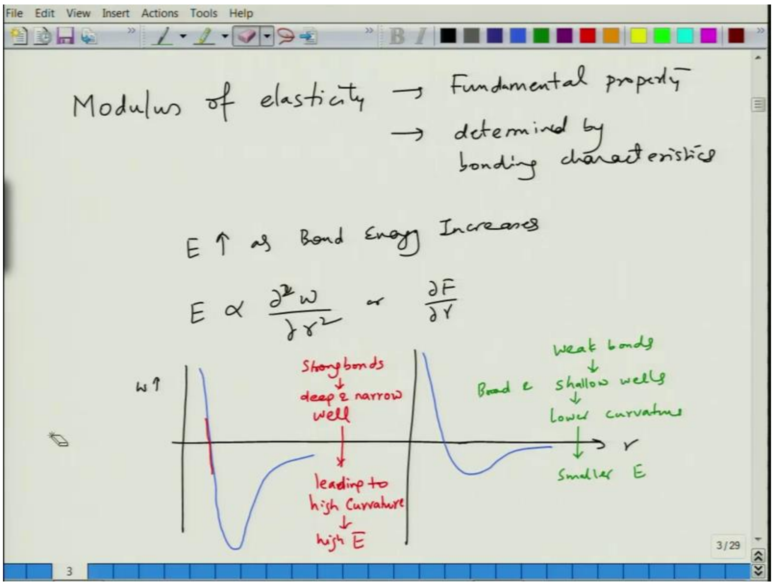

Modulus of elasticity → Fundamental property
→ determined by bonding characteristics

E ↑ as Bond Energy Increases

$$E \propto \frac{\partial^2 w}{\partial r^2} \text{ or } \frac{\partial F}{\partial r}$$

Strong bonds ↓ deep & narrow well ↓ leading to high Curvature ↓ high E

Bond e weak bonds ↓ shallow wells ↓ lower curvature ↓ Smaller E

So you can see it is a fundamental property of the material determined by bonding characteristics. And so, basically, E increases as bond energy increases, and this is basically because the modulus of elasticity is proportional to curvature of the potential energy curve or you can say it is proportional to del F by del r, the slope of force versus distance curve.

So what happens is that as your bond energy goes up, so if you make a distinction between two plots, let us say, the strong bonds. Strong bonds will mean that you have a potential energy curve which is very deep, like this. And on, in contrast, if you want to plot, let us say, another plot for weak bonds, so let us say this is r, this is w, the weak bonds will have a plot which is something like this.

So basically weak, this is strong bonds. Strong bonds will depict deep and narrow well. So as a result, the curvature is so, leading to high curvature, which means high modulus of elasticity.

On the other hand, this is the case for weak bonds. Weak bonds will lead to shallow wells, shallow and broad and shallow wells, which means lower value of curvature, which means smaller modulus. And this is what indeed it is true. So for ceramics, I am sorry, let me use the right color.

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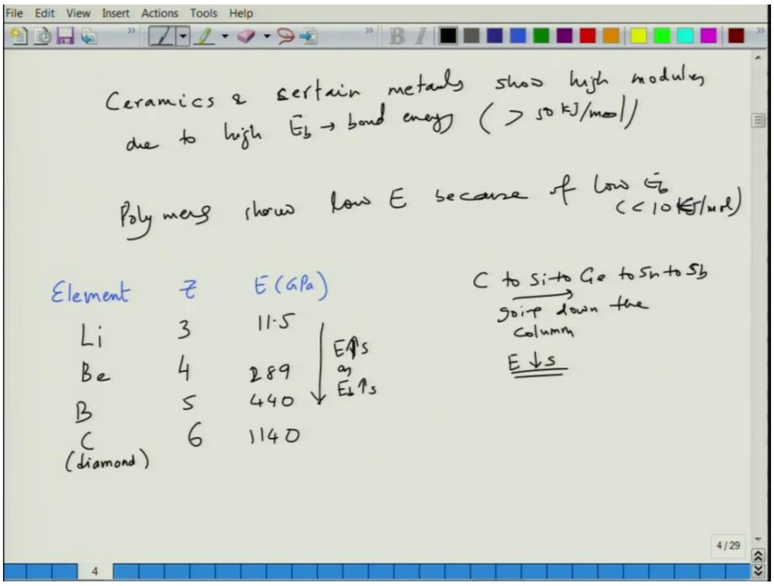

Ceramics & certain metals show high modulus due to high E_b → bond energy (> 50 kJ/mol)

Polymers shows low E because of low E_b (<< 10 kJ/mol)

Element	Z	E (GPa)
Li	3	11.5
Be	4	289
B	5	440
C (diamond)	6	1140

E ↑s as E_b ↑s

C to Si to Ge to Sn to Sb →
going down the column
E ↓s

So materials such as ceramics and certain metals show high modulus because of high, you can say, Eb that is bond energy. Whereas polymers, so here the bond energy is, let us say, 50 kilo joule per mole, and polymers show low E because of low Eb, which is less than generally 10 kilo joule per mole. So this is the difference between the two types of materials that.

So let me look, let me now give you the values of certain materials. So we go through the values. So last time we saw, for example, for, let us say, so this is, let us say, the element, as you go down, and then you have z value, then you have E value in GPa. So this is, for instance, for metals.

So let us say lithium, beryllium, boron and carbon. And you are going in row from left to right. So 3, 4, 5 and 6, and the values change from 11.5 to 289 to 440 to 1140, and this is diamond basically. So you can see that the bond strength increases and the modulus goes up.

And this, by the way, and this is also true for, as you go down in column, as we saw last time. So as you go from carbon to silicon to germanium to tin to antimony, as you going down the column, as you go down the column, the E value decreases and this is because of decreasing bond strength, because atoms get bigger as a result the forces between them are lower, as a result, the electrons are loosely bound, as a result, the bond strength is lower.

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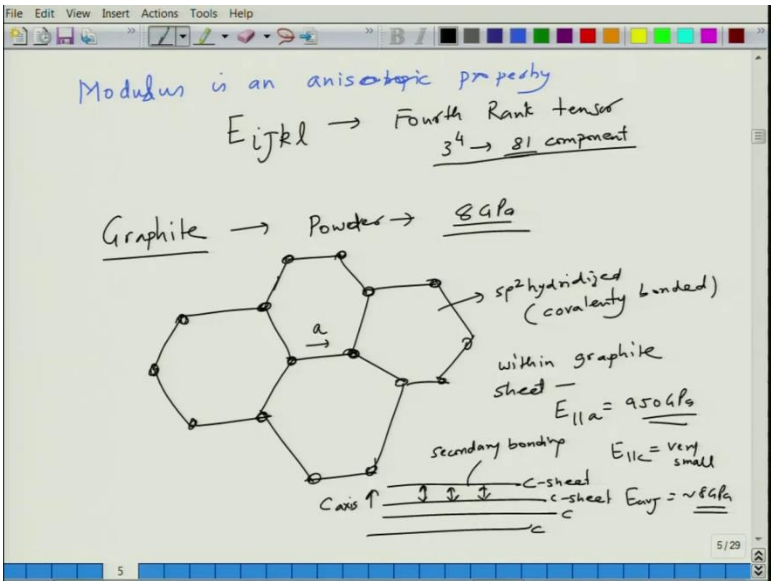

Modulus is an anisotropic property

$E_{ijkl} \rightarrow$ Fourth Rank tensor

$3^4 \rightarrow \underline{81}$ component

Graphite → Powder → 8 GPa

within graphite sheet —

$E_{\parallel a} = 950\,GPa$

$E_{\parallel c}$ = very small

$E_{avg} = \sim 8\,GPa$

Now this modulus is also an isotropic. And as we saw earlier, it is depicted tensorially as Eijkl. So it is basically a fourth rank tensor. So essentially it has 81 components in the matrix. However, because of symmetry and other consideration, we do not do generally have 81 components, we have much smaller number of components. We are not going to go into details of that, but it is a tensorial properties.

For example, if you look at graphite, graphite generally have, so if you look at graphite powder and you make a pellet out of it, then graphite will show a modulus of 8 GPa. However, as we know, graphite is basically has a structure like this, in which carbon atoms are covalently bonded within the plane. Something like that. So these are all carbon atoms.

So we are looking down the graphite layer. So within the plane, these are all sp2 hybridized and covalently bonded. So within graphite sheet, the modulus, so you can say, E parallel to, so let us say if this is a axis, E parallel to a would be 950 GPa, which is very similar to diamond, very close to diamond, not very far from diamond, almost 1000 GPa.

However, if you now look at the graphite sheets like this, parallel to each other. So this is one sheet of carbon, this is another sheet of carbon. So between them we have what we call a secondary bonding. And this leads to E, let us say parallel to c, if this is c axis. Then E parallel to c is very small, very small.

So E average turns out to be about 8 GPa for graphite. And this is mainly due to secondary bonding, which is perpendicular to the graphite layers and that is the reason why graphite acts well as a lubricant as well because of lower strength between the layers and they slide against each other very well.

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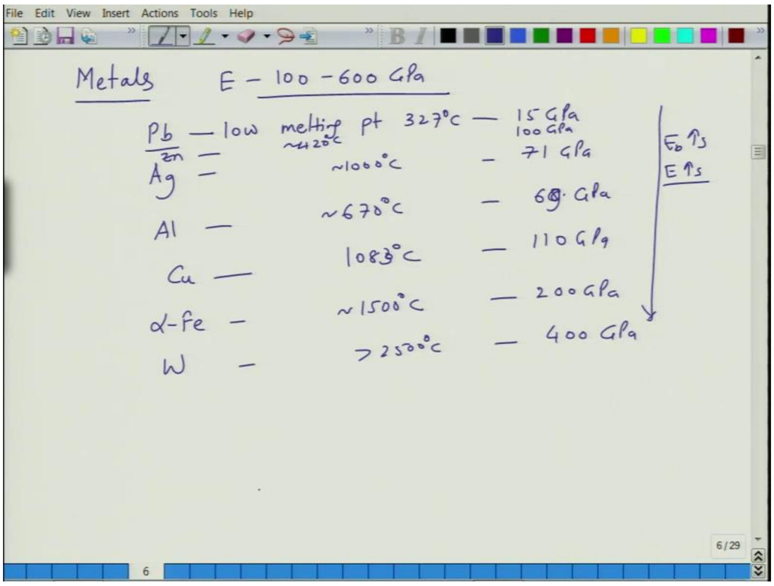

Metals E – 100 – 600 GPa

Pb	— low melting pt	327°C —	15 GPa
Zn	—	~420°C	100 GPa
Ag	—	~1000°C —	71 GPa
Al	—	~670°C —	69 GPa
Cu	—	1083°C —	110 GPa
α-Fe	—	~1500°C —	200 GPa
W	—	> 2500°C —	400 GPa

$E_b \uparrow$s

$E \uparrow$s

If you look at now metals, metal show modulus values which are from 100 to let us say 600 GPa. And if you look at various metals, so you have low melting point metal such as lead. Lead is a low melting point metal, 327 degree centigrade and modulus of this is nearly 15 giga pascal, very small.

If you go to silver again, silver is not a very high melting point material, close to 1000 degree centigrade and its melt, and this is 71 GPa. If you go to aluminium, aluminium is 670 degree centigrade approximately and this has a modulus of 68 or 69 GPa. If you go to another low melting point, moderate melting point metal copper which has 1083 degree centigrade. Then copper has a modulus of 110 GPa. If you go to something like alpha iron, alpha iron is nearly 1500 degree centigrade and its melting point is, its modulus is 200 GPa.

Somewhere in between you can fit in zinc as well. Zinc is again melting point of, I think, about 420 degree centigrade or so, and zinc's melting point is close to 100 GPa. And then we have, let us say, tungsten, tungsten has extremely high melting point, more than 2500 degree centigrade and tungsten has a bond energy modulus of 200 GPa. So by and large in within these metals as Eb increases, the modulus also increases. So there is a close correlation between the bond energy of metals and the modulus.

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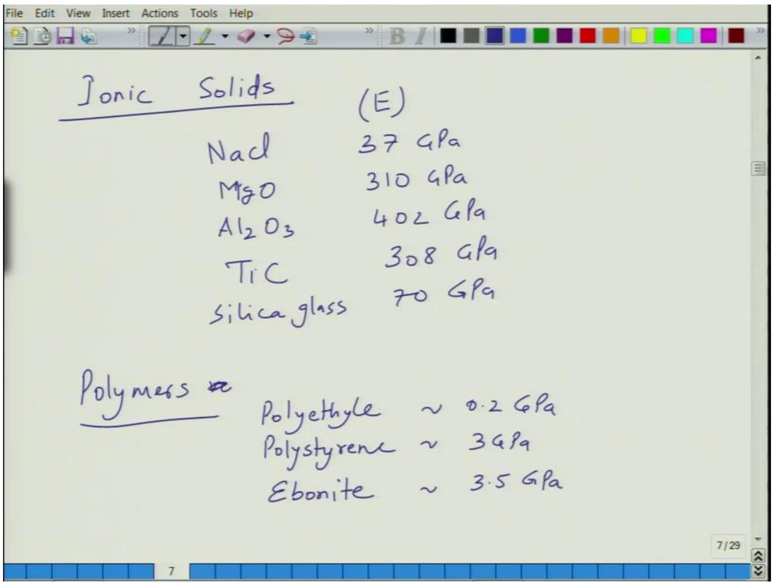

Ionic Solids	(E)
NaCl	37 GPa
MgO	310 GPa
Al_2O_3	402 GPa
TiC	308 GPa
silica glass	70 GPa

Polymers

Polyethyle	~	0.2 GPa
Polystyrene	~	3 GPa
Ebonite	~	3.5 GPa

So, and if you compare the values with ionic solids, for instance, within ionic solids, if you look at examples of NaCl, NaCl has a value of E which is 37 GPa. NaCl's melting point is lower. If you look at something like magnesium oxide, which is high melting point, its modulus is 310 GPa. If you look at Al2O3, alumina, it has a melting, it has a modulus of 402 GPa. If you look at titanium carbide 308 GPa. And if you look at silica glass, the value for this is about 70 GPa.

So among these silica glass and NaCl have lower melting points, but MgO, Al2O3 and TiC they have higher melting points or higher bond energy. As a result, they have higher elastic modulus. So generally covalent and ionic solids have very high modulus, metals have medium to, moderate to high modulus depending upon the bond energy. And if you look at polymers, they have even lower values, very low values.

So if you look at polyethylene, for instance, the modulus is about 0.2 GPa. If you look at polystyrene, it is about 3 GPa. If you look at ebonite, which is in thermoset polymer, which is supposed to be hard polymer, it is also not very high, it is about only 3.5 GPa.

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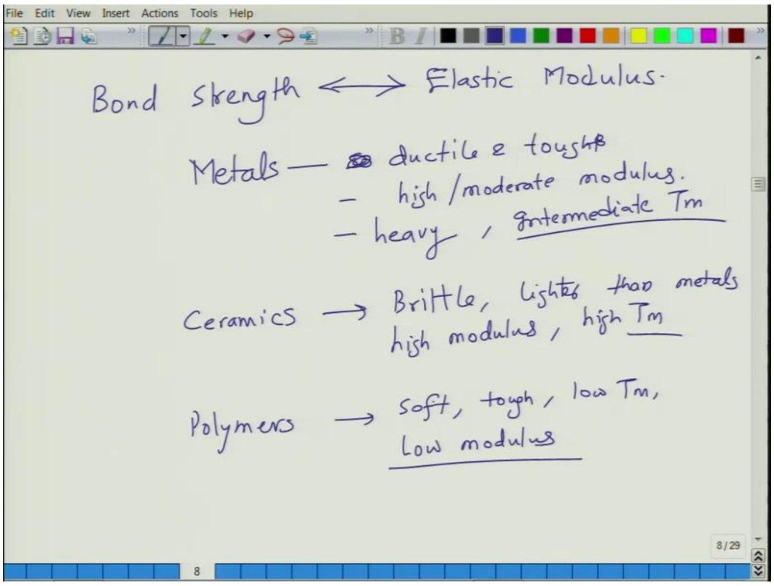

Bond Strength ⟷ Elastic Modulus.

Metals — ductile & tough
- high / moderate modulus.
- heavy / intermediate Tm

Ceramics → Brittle, lighter than metals
high modulus, high Tm

Polymers → soft, tough, low Tm,
low modulus

So there are, so materials can be classified based on the bond strength. So basically you can say, bond strength has close correlation with elastic modulus. So we see that metals which are ductile and tough, they have generally high or moderate modulus, but metals are also heavy. If you look at ceramics, which are either covalently bonded or ionically bonded, they have, they are brittle, they are lighter than metals, but they are, but they have high modulus by and large.

And then we look at polymers, polymers are soft. They can be tough, but they have low, so metals also have intermediate melting points by and large, ceramics have high melting point by and large, metals have low melting points, polymers have low melting points, but they have low modulus. But polymers can be soft and tough and they can be deformed.

So this difference between the materials based on their bonding characteristics leading to different properties of bonding leads to, can allow us to design certain materials with high modulus depending upon what we want to do with them.

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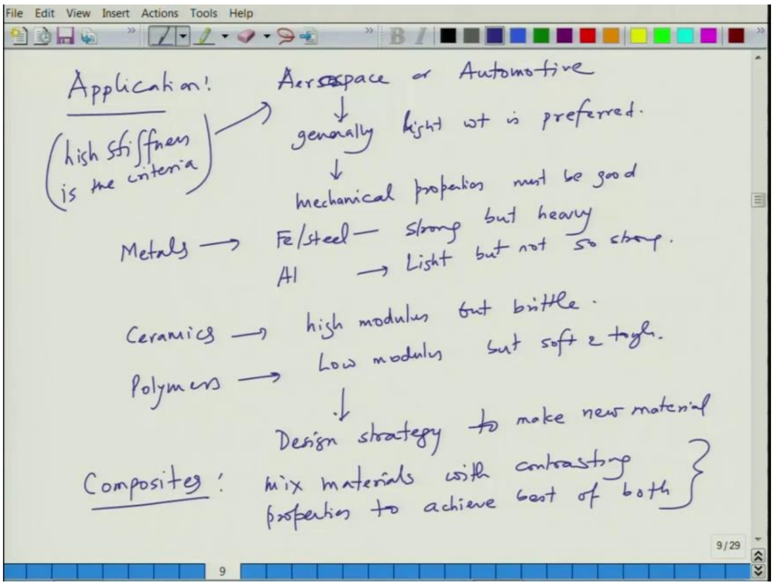

Application!

(high stiffness is the criteria) → Aerospace or Automotive

↓

generally light wt is preferred.

↓

mechanical properties must be good

Metals → Fe/steel — Strong but heavy

Al → Light but not so strong.

Ceramics → high modulus but brittle.

Polymers → Low modulus but soft & tough.

↓

Design strategy to make new material

Composites: mix materials with contrasting properties to achieve best of both

So, for example, if you look at, so applications such as, aerospace or automotive. In these applications what we require generally is, generally light weight is preferred. Because we require, we can say fuel, if your weight is light. Now light weight cannot, but mechanical properties must be, must be good.

So among metals the options are iron or steel which is strong, but heavy. If you look at aluminium, it is light, but not so strong. So, for example, our criteria is let us say stiffness, we want high stiffness. Aluminium does not have very high modulus. So it is not very stiff. But it is lighter. Steel, on the other hand, is cheaper. It is stronger, but it is very heavy. It has high modulus, but it is very heavy.

If you look at ceramics, so let us say, our criteria is high stiffness is the criteria for certain parts, especially in aerospace applications. Ceramics, on the other hand, they have high modulus, but brittle, they break very easily, little bit of shock, they break. So what we do is that by, and the polymers, on the other hand, we are saying, they are low modulus, but soft and could be tough. So this gives us to a design strategy to mix, to make new materials called as composites.

Composites means you mix materials with contrasting properties to achieve best of both. This is the objective. So this is the objective. You may, you can mix ceramic and polymer or you can mix metal and ceramic to make materials which are, which have compromised properties, but they maximize certain properties, especially their specific properties, their strength per unit weight or yield strength per unit weight or fracture strength per unit weight or whatever, especially the properties which are enhanced by increase, by lowering. So you can have lower mass of the material yet you achieve the similar value of performance and this is done by making composites or mixture of these materials.

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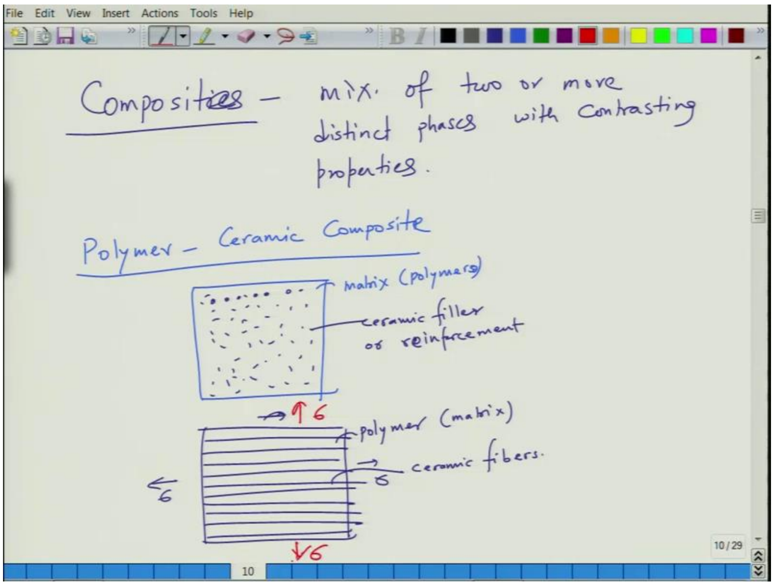

Composites – mix. of two or more distinct phases with contrasting properties.

Polymer – Ceramic Composite

So what are these composites? So composites are generally are mix of two or more distinct phases with contrasting properties. So, for example, if you like, let us say, we make a composite, which is polymer, ceramic composite, which means what might happen is that you may have a polymer matrix, matrix is of polymer, which means the bulk of the material is of polymer. In this you sort of add these small particles of ceramic. So this is called as ceramic filler or let us say reinforcement.

You can also have another design strategy where you can have polymer as a base or matrix and ceramics can be loaded in the form of fibers in a very well defined orientation. So these are ceramic fibers.

Now depending upon how you apply the load, whether you apply the load in this direction, so whether you apply the load in this direction or you apply the load in this direction, they can have different loading conditions and they define how do you change the, how do you write the expression for modulus. So when you make these composites, you can modify the modulus.

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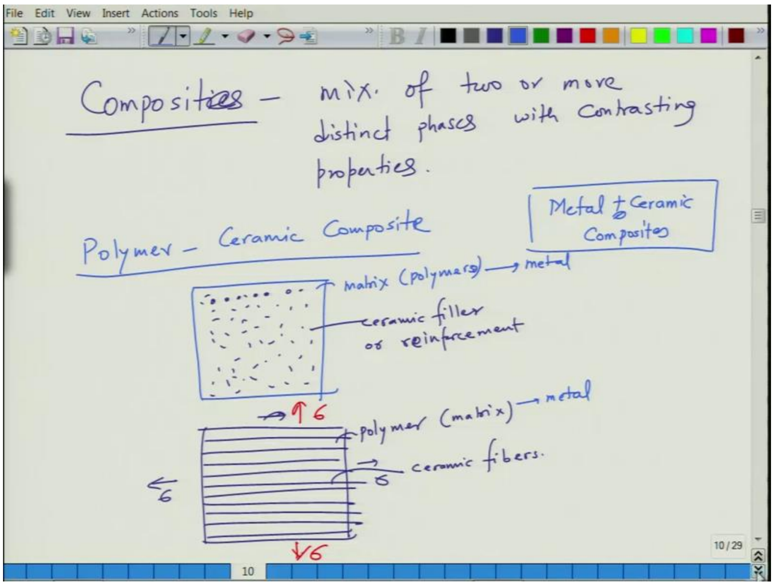
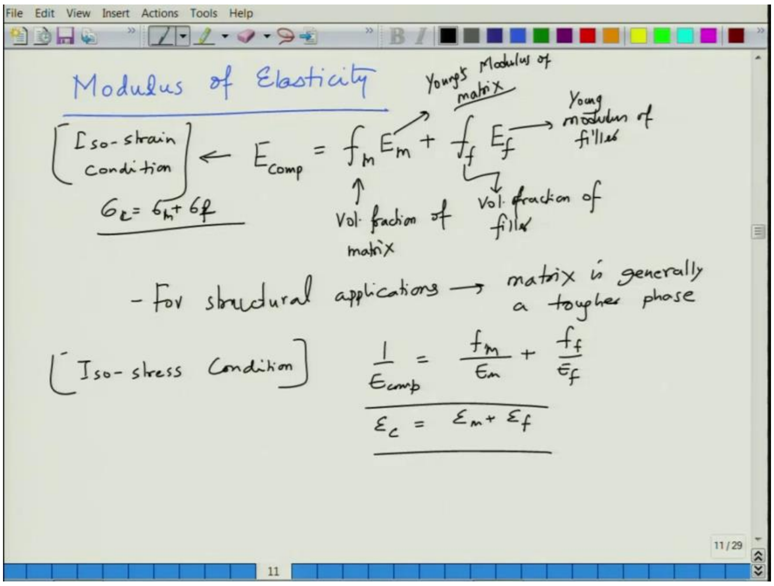


Composities – mix. of two or more distinct phases with contrasting properties.

Polymer – Ceramic Composite

Metal $\frac{+}{or}$ Ceramic Composites



Modulus of Elasticity

[Iso-strain condition] ← $E_{comp} = f_m E_m + f_f E_f$

Young's Modulus of matrix

Young modulus of filler

Vol. fraction of matrix

Vol. fraction of filler

$\sigma_c = \sigma_m + \sigma_f$

- For structural applications → matrix is generally a tougher phase

[Iso-stress Condition] $\frac{1}{E_{comp}} = \frac{f_m}{E_m} + \frac{f_f}{E_f}$

$\varepsilon_c = \varepsilon_m + \varepsilon_f$

So, modulus of elasticity of composites, so not only you can make polymer ceramic, but you can also make metal ceramic, metal plus ceramic composites. So instead of polymer, you can have metal here.

So here what happens is that the modulus of elasticity varies as E composite is equal to f1 E1, let us say, fm Em plus ff Ef. So this is basically you can say volume fraction of matrix and this is volume fraction of filler, this is modulus, Young's modulus of filler, and this is Young's modulus of matrix.

In general, the matrix for a structural application is chosen of a material which is tough. So for structural applications, where a load bearing is important, matrix is generally a tougher phase, because you do not want a crack to run through the matrix. So, generally, matrix is chosen as a tougher phase. So now, but this kind of formula is valid only for certain loading condition and that loading condition essentially is iso-strain condition. We will do the derivation of this later on in this course.

You have another condition which is called as iso-stress condition, which is basically 1 over E composite is equal to fm over Em plus ff over Ef. This is iso-stress condition. So basically what happens here is in iso-strain condition, the sigma of composite is equal to sigma of 1 phase plus sigma of, let us say, sigma m plus sigma F, and here it will, strain will be distributed. So sigma of, epsilon of composite will be equal to epsilon of matrix plus epsilon of fiber and you can then just plug in the value. So this is, and this is valid only in the elastic region, not in the plastic region.

So when you put in the value of stress and strain, so, for example, you can see there is, so let me derive it here only.

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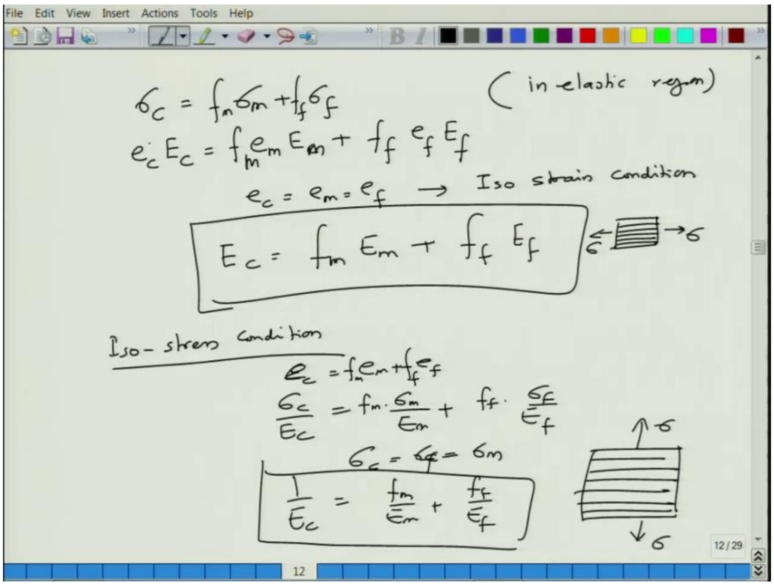

$\sigma_c = f_m \sigma_m + f_f \sigma_f$ (in elastic region)

$e_c E_c = f_m e_m E_m + f_f e_f E_f$

$e_c = e_m = e_f \rightarrow$ Iso strain condition

$$\boxed{E_c = f_m E_m + f_f E_f}$$

Iso-stress condition

$e_c = f_m e_m + f_f e_f$

$\frac{\sigma_c}{E_c} = f_m \cdot \frac{\sigma_m}{E_m} + f_f \cdot \frac{\sigma_f}{E_f}$

$\sigma_c = \sigma_f = \sigma_m$

$$\boxed{\frac{1}{E_c} = \frac{f_m}{E_m} + \frac{f_f}{E_f}}$$

So if you have sigma c is equal to sigma m plus sigma f, you can write sigma as ec into Ec plus and this would be equal to em into Em and this is how much is the volume fraction fm and this will be ff ef and Ef. Since ec is equal to em is equal to ef, which means it is iso-strain condition and this is in elastic region. One can write.

So you can see here, the es will cancel here. So Ec will be equal to fm Em plus ff Ef. So this is your iso-strain condition formula. And this is generally when your fibers are, so if you have a composite like this, then you apply stress in this fashion. So this is what fibers and matrix will experience same strain as a result, they will have iso-strain condition.

And iso-stress condition will be, epsilon c or ec, let us say, is equal to em plus ef and this can be, then you can write this as, so you can, ff, in fact here also fm and ff. So this will be equal to sigma c divided by Ec plus fm into sigma m divided by Em plus ff into sigma f divided by Ef.

And since sigma c is equal to sigma f is equal to sigma m, what we will get is 1 over Ec is equal to fm over Em plus ff over Ef. This is the formula that we will. And this is generally valid for this configuration when fibers are loaded like this and the loading condition is this. So this is sigma here. This is sigma, so, transverse loading condition. But for particulate reinforced composite, generally we use the formula which is iso-strain composite.

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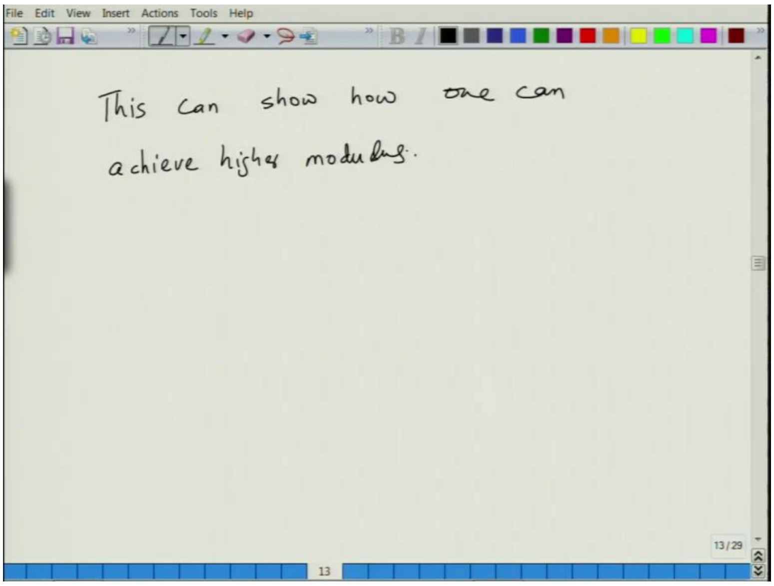

This can show how one can achieve higher modulus.

So when you do this, so as we will do in the next lecture, this can show you how one can achieve higher modulus and the exercise we will do in the next class.

So basically what we have seen in this lecture is, we have seen that on the basis of bonding, what are the differences in the values of modulus of elasticity of different materials and then by using these differences between, so by using these differences between these materials, now that these differences are there, but you want to design better materials. By mixing these materials appropriately in a scientific manner, one can design materials which use two materials of contrasting properties in appropriate weight or volume fractions and get the values especially from the prospect using the weight, which can lead to design a better material. So we will do more analysis on this in the next class. Thank you.