Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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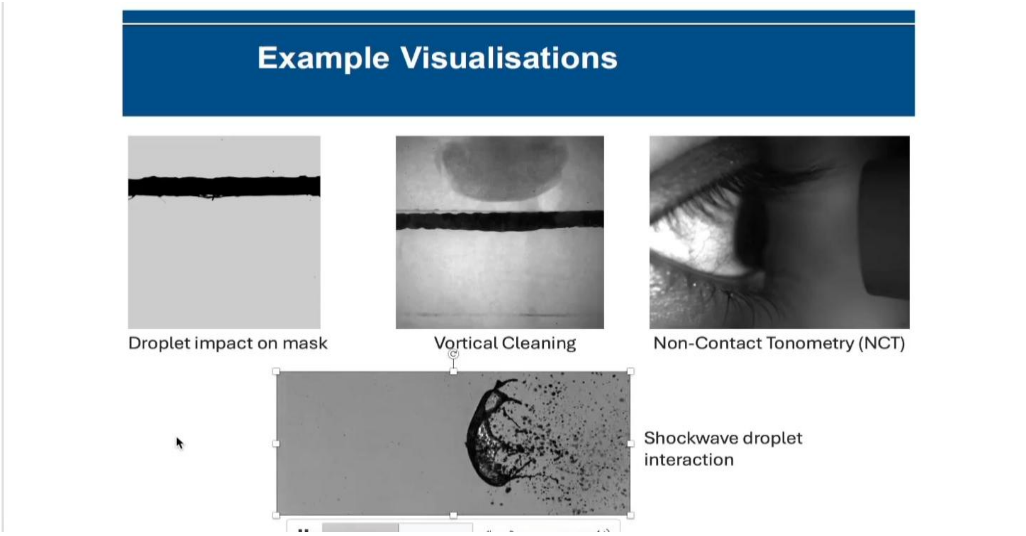
Week – 02
Lecture - 09
Imaging and Optics -1

All right, so we are going to do a little bit now on the fundamentals of optics and image processing. Again, my name is Saptarshi Basu. These lecture notes are once again courtesy of Professor Cameron Tropea, who is my long-term collaborator. And this is also part of a lecture that he delivered at IISc during a short course here. All right, so the majority of the materials are taken from him with a little bit of interjection from other places as well. So if you look at visualization in general, this is, for example, a droplet that is impinging on a mask or a porous substrate.

This is a vortex that is interacting with liquid-infused porous media. Here you can see that it is a vortex which is interacting with a tear film. Lastly, you can see that this is a shock wave droplet interaction. In all these things, you can see one common element: we are using optics and techniques like Schlieren, in some cases like particle image velocimetry. In some cases, it could also be LDV and other kinds of things. To basically visualize a flow field or visualize the events, be it the flow field or the structures, et cetera, et cetera. So this therefore requires, as the subject of this talk actually suggests, that we need to know the fundamentals of optics and image processing because this is what has come out after very careful optical arrangements and after very careful and rigorous image processing. So you need to know both in order to get an idea of what we are doing over here, all right? So this is of paramount importance if you want to get quality like this. These are all from my research work or the research work of my group, as you can see, and these are varied events at very high speeds.

This is, for example, very high-speed because there is a shockwave. On the other hand, this is, for example, a low-speed event, okay? This is also medium-speed. This is also medium-speed. You can see that you can imagine different types of events, from very high speed to very low speed as well, if you have your optics and imaging correct. So that is the whole point here.



So the contents of this particular lecture will follow what Professor Tropia did: there are some optical fundamentals, and then there is some visualization stuff like hardware. Then, of course, there is how light propagates and scatters. These are of vital importance for many measurements that will follow after this. For example, light is an electromagnetic wave; it has coherence, it can be polarized, and then, of course, Snell's law, which you might have seen earlier in your high school and undergraduate physics, and Fresnel equations, the Lorentz theory, geometric optics, and scattering from small particles—all of these are very crucial when you actually go on to study the more detailed diagnostic techniques. And then, of course, there are image processing fundamentals.

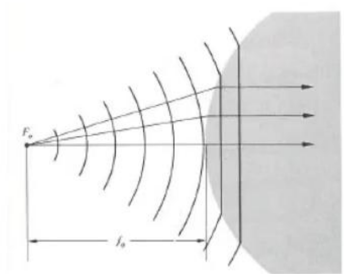
So, these are all optics. There are fundamental image processing concepts. So image processing fundamentals are also very important because once you get the images, how do you process them to recover information that may sometimes be embedded inside? All right. So that is the whole point of stating what the image processing fundamentals are. This will also include videos that showcase how the images are actually processed.

Now let's look at the image being fundamentals, which is basically if you have waves that are essentially a point source, and you have these light waves coming out as spherical wave fronts. So, as they interact with a spherical interface, you can actually have plane waves propagating through it. You can also reshape plane waves. In this case, it is a plane wave. Say that what is coming out of this may be plane waves or highly collimated beams.

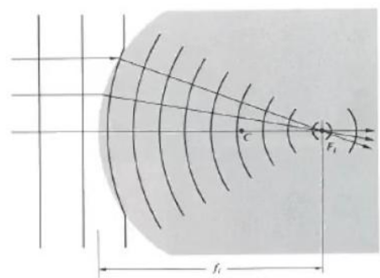
Now you can use a spherical interface, and you can focus the planes into spherical waves, plane waves into spherical waves, and you can focus it right at this particular point. So this is basically what lenses do. And this is also what lenses do. They also make collimated beams. So plane waves propagate beyond a spherical interface.

This is what you see. This is plane waves, which are coming and interacting with a spherical interface. It's converted into spherical waves and focused at the image focus. All these things, this is the lens, which you know you are very well conversant with what lenses are capable of doing. So this, you know, from your high school physics, is most likely not rocket science.

Imaging Fundamentals: I



Plane waves propagating beyond a spherical interface

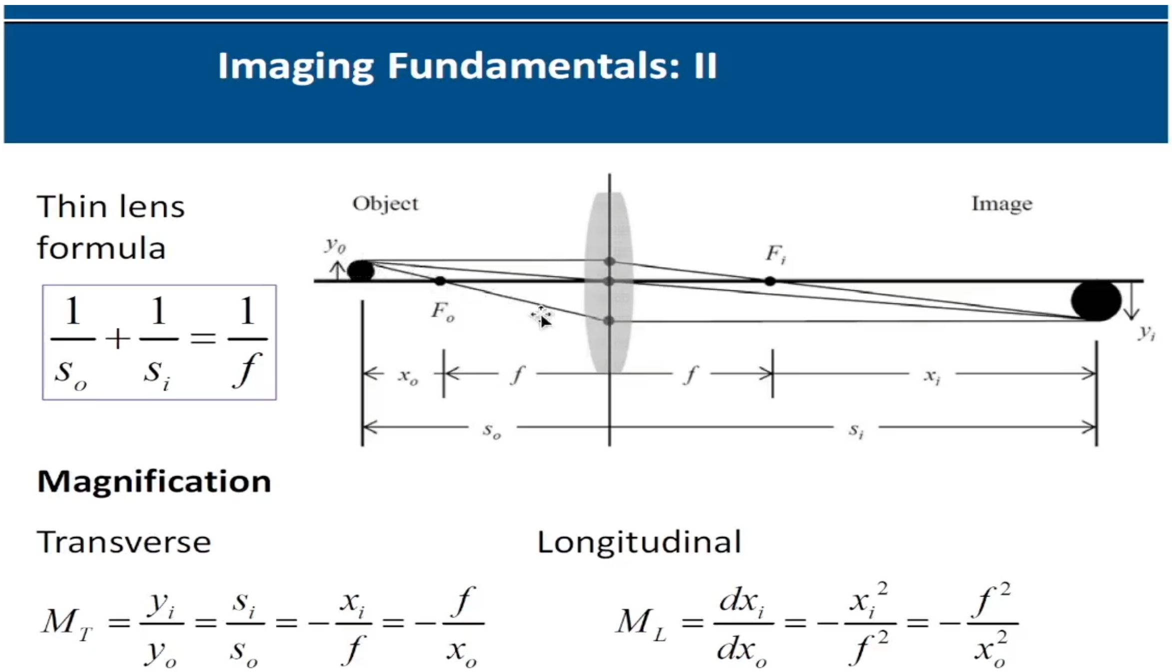


The reshaping of plane waves into spherical waves at a spherical interface – the image focus

Okay, so imagining fundamentals two. So if you have a thin lens over here, let's assume this is a thin lens. So there is an object that is given by that, which has a height of y naught. Now, this object is the imaging plane. So you form an image of the object; in this case, an inverted image.

What happens is that this is the lens that basically forms that image. The image has a dimension of one. Now the thin lens formula is known to most of you, which is basically one over S_0, where S_0 is the distance of the object from the lens, and one over SI, which is the distance of the image plane from the lens. The summation of the inverses of these two quantities is equal to one over the focal length of the lens, which is the focal length right here that you can see. So this is the focal length of the leg.

Now, this is the thin lens formula. This is very common; you know this. Now, if we look at that, there can be two possible magnifications. What is transverse magnification? And one is a longitudinal magnification. So, the transverse magnification is rather easy. It is basically given as $\frac{y_i}{y_0}$. So basically, the dimension or the vertical height of the imaged object compared to the actual object. And that is given as a ratio because if you just follow the ratio, it is given as S_i over S_0. So S_i over S_0. That is given as $\frac{-x_i}{f}$is equal to $\frac{-f}{x_0}$. And X_0 is this. X_i, as you can see, is this. With f being the focal length. So this is the transverse magnification that you have. So this is how much it is removed from the focal point. How much is it removed from the focal point? So the longitudinal magnification, on the other hand, is given as dxi by dx naught. Okay, in this particular direction, in the horizontal direction, that is given as $-\frac{f^2}{x_0^2}$. So the transverse magnification and the longitudinal magnification are given in this particular form. Based on this, this comes from the thin lens formula and this very basic arrangement you already know, I suppose.



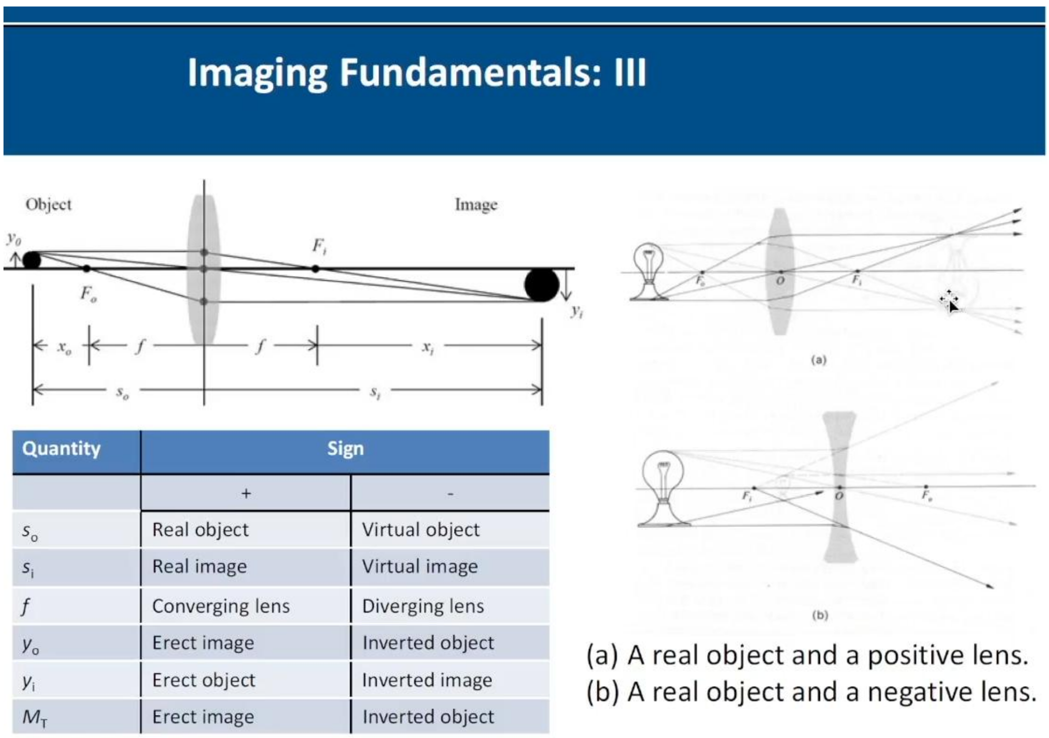
Then imaging fundamentals three. So you can see that you have an object and you have an image. Now, if you have, for example, a light bulb here, this passes through this focal point.

So what this lens actually does is divert some of these beams in this particular fashion, and you form an inverted image. of this light bulb. That is what you get to see over there. This is a convex lens. And you can also have a concave lens over there.

And there, of course, you form the image on the same side, not on the other side; same side, and it's not inverted. It is not an inverted image, but this is the image that you actually see here. So what you can see is a real object and a positive lens and a real object and a negative lens. These are the two examples that are shown here. And here are some of the quantities, you know, that, you know, S_0, you know, real object and then your virtual object; all these signs you can actually put in the form of our table.

So whether the image will be erect or inverted is what you can actually see over here. Okay, so in this case, for example, this is an inverted image. All right, you can just do basic ray optics to trace the beams, and you can get an idea of where. The image will actually form, and what will be the magnification of these images? Okay, here you can see clearly it is magnified beyond one; here it is magnified below one. Okay, so it is actually a diminished image, so to say.

Okay, so these are things I think are well covered in your undergraduate studies or in your other types of studies as well. So the images of real objects are formed by thin lenses, correct? So when the object location is between infinity and infinity, S_0, I just go back over here; this is S_0, the location of the object from the lens. If that is the case, okay?



Quantity	Sign	
	+	-
s_o	Real object	Virtual object
s_i	Real image	Virtual image
f	Converging lens	Diverging lens
y_o	Erect image	Inverted object
y_i	Erect object	Inverted image
M_T	Erect image	Inverted object

(a) A real object and a positive lens.
(b) A real object and a negative lens.

So if it is, if $S_0$is greater than 2f but less than infinity, you get, and if you use a convex lens, you get a real type image. The location of the image is between the two focal lengths, that is S_i. We look at it; this is S_i, this is the location, it is inverted, and the size is minified.

Now, if S_0 is exactly equal to 2f, the location of the image is also at 2f, it is inverted, and it is of the same size. On the other hand, if the object is located between f and 2f, it is real, the image is formed somewhere beyond 2f, it is inverted, and it is magnified. All right. And similarly, if S_0 is less than f, then the type is virtual, and you get an erect image and a magnified image. Similar things happen for your concave lens as well.

And there are certain rules that you have probably learned in high school, but this is a table that tells you, so when you look at this table, please consult this particular figure, which will tell you where the object is placed. And based on that, where the image will be, whether it will be magnified or minified, and whether it will be inverted or erect. These are the decisions that you can make.

Imaging Fundamentals: IV

Images of real objects formed by thin lenses

Convex				
Object	Image			
Location	Type	Location	Orientation	Relative size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\lvert s_i \rvert > s_o$	Erect	Magnified

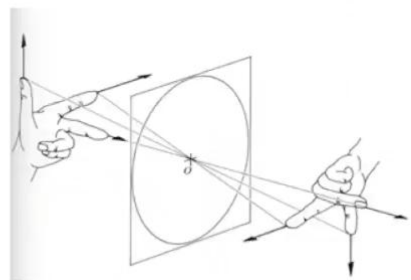


Image orientation for a thin lens

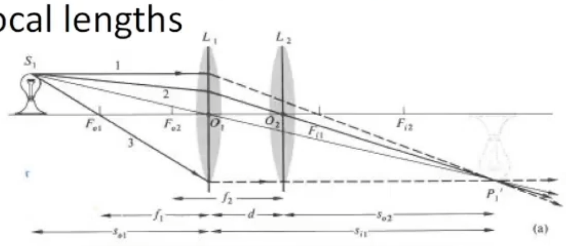
Concave				
Object	Image			
Location	Type	Location	Orientation	Relative size
Anywhere	Virtual	$\lvert s_i \rvert < \lvert f \rvert$	Erect	Minified

So when two lenses are separated by a distance that is smaller than the sum of their focal lengths, as you can see over here. So this is the image that has been formed.

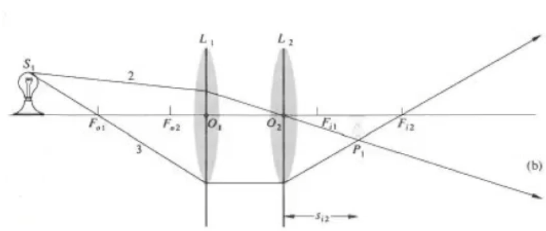
It's a rather complicated formula that you have because you have two focal lengths now, f_1 and f_2, and you have two S_0s as well, S_{01} and S_{02}, and then you have S_{i1} and S_{i2} as well. So you can see that these two thin lenses are separated, but the distance of separation between these two lenses, which is D, is smaller than the sum of their two focal lengths, so it is less than f_1 plus f_2. In these kinds of cases, if you read this particular book, there is a catch: the total magnification in the transverse direction would be the multiplicative product of the magnification of the individual lenses. So these are only for two thin lenses separated by a distance smaller than the sum of their focal lengths.

Imaging Fundamentals: V

Two thins lenses separated by a distance smaller than the sum of their focal lengths



$$s_{i2} = \frac{f_2 d - f_2 s_{o1} f_1 / (s_{o1} - f_1)}{d - f_2 - s_{o1} f_1 / (s_{o1} - f_1)}$$



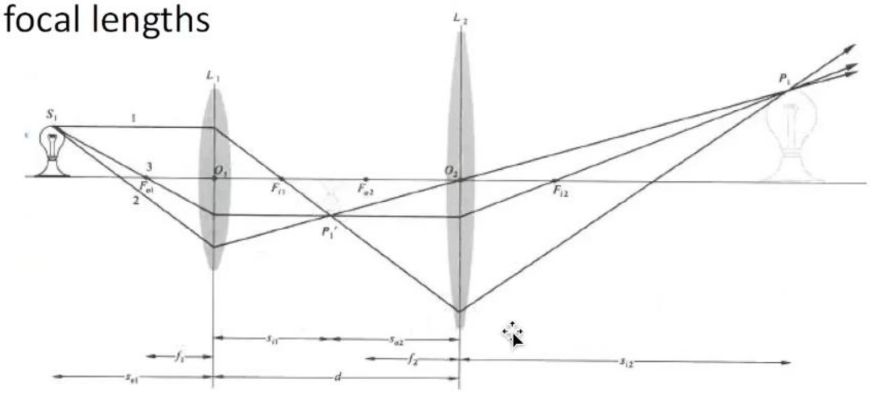
$$M_T = M_{T1} M_{T2}$$

Okay, now if two thin lenses are separated by a distance that is greater than the sum of their focal lengths, then the distance between these two lenses, which have very different focal lengths, for example, is greater than the sum of their focal lengths, which is again d.

Okay, so in that particular case, the effective focal length is given as $\frac{1}{f}$, as $\frac{1}{f_1} + \frac{1}{f_2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \frac{1}{f_n}$, the number of lenses that you can stack up, okay? And when D actually starts to become smaller and smaller, all right?

Imaging Fundamentals: VI

Two thins lenses separated by a distance greater than the sum of their focal lengths



if $d \to 0$ $$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\cdots+\frac{1}{f_N}$$ ➡ Effective focal length

Now we come to the concept of, you know, aperture. So, the aperture basically refers to these images that you can see. So, the aperture refers to the opening through which light passes before hitting our camera sensor. So it basically refers to the opening, the opening that you have.

This is the opening. This is the opening. This is an opening. This is the white center that you see. The *f* number, which is shown as $\frac{f}{\#}$, is pronounced "*f* number." Setting a lens controls the overall light throughput. That means how much light actually gets in, the depth of the field, and the ability to produce a contrast at a given resolution.

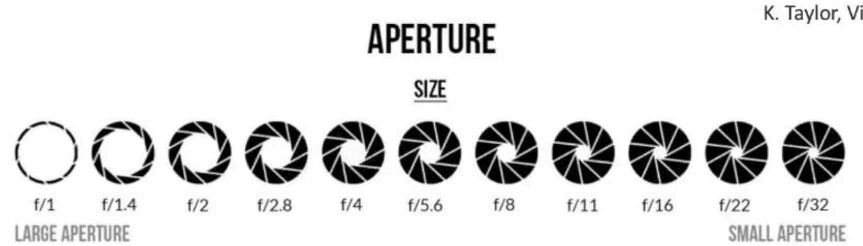
So these are three important points: the overall light throughput, the depth of field, and the ability to produce contrast at a given resolution. Fundamentally, the *f* number, or $\frac{f}{\#}$, is $\frac{f}{\phi_{EA}}$. So, the *f* number is nothing but $\frac{f}{\phi_{EA}}$, ϕ_{EA} is the effective aperture diameter. For example, this is a large aperture; okay, so effectively it is a small, uh, you know. Uh, the *f* number is such that as you go on decreasing the ϕ_{EA} size, you get to lower and lower apertures.

This is, for example, a very small aperture which is given as f/32. Okay, so you are going on reducing the effective aperture diameter, and therefore you are increasing, or rather, your F number actually becomes f/32, for example. So, in essence, aperture refers to the opening through which light passes before it hits the camera. This is the most important part. Then fundamentally, this also controls, which is given as an f number.

It is a lens control parameter that shows the light throughput, depth of field, and the ability to produce contrast at a given resolution, all right?

Imaging Fundamentals: VII

K. Taylor, Visual Education



Aperture refers to the opening through which light passes before hitting your camera's sensor.

The $f/\#$ (pronounced "F-number") setting on a lens controls overall light throughput, depth of field (DOF), and the ability to produce contrast at a given resolution. Fundamentally, $f/\#$ is the ratio of the focal length (f), of the lens to the effective aperture diameter ($\emptyset_{EA}$):

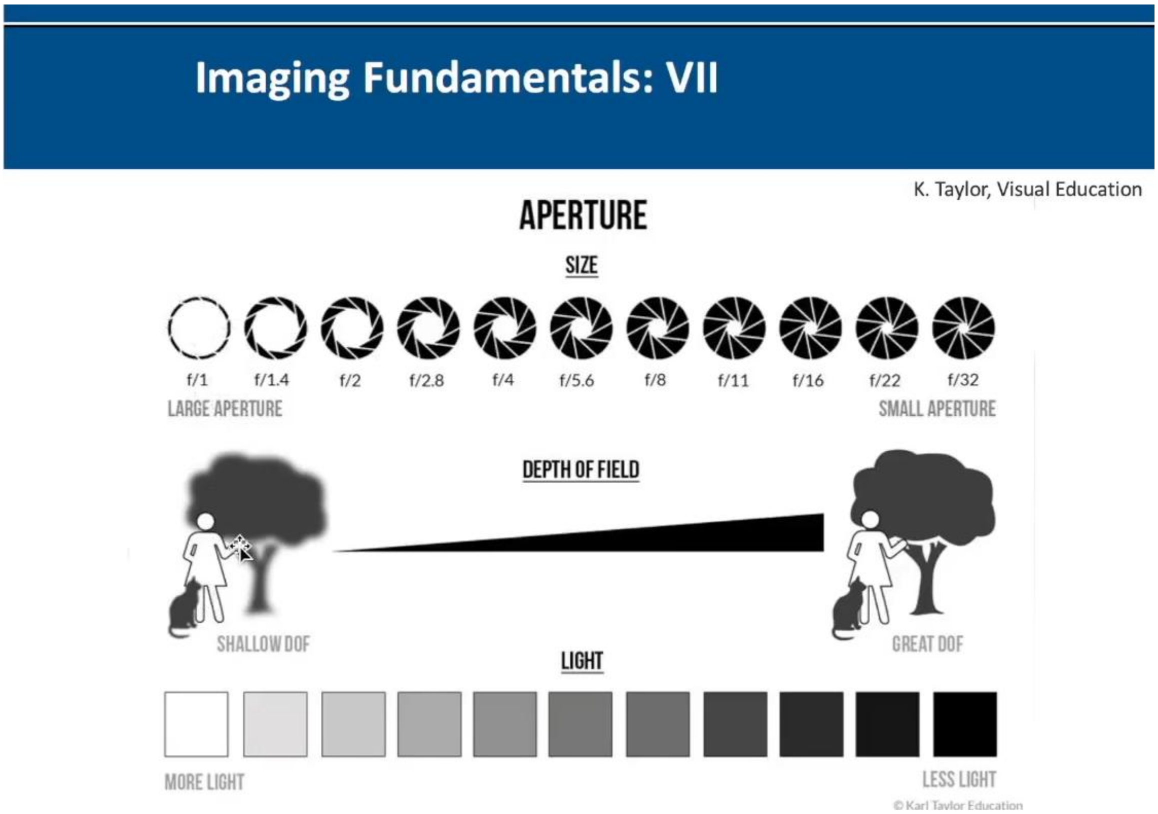
$$f/\# = \frac{f}{\emptyset_{EA}}$$

So if you look at these images, for example, here is a lady with a cat, and there is a tree; they are all at different depths of field. They are not in the same plane, so to say. Okay, so when you are using a large aperture—this is a large aperture—you have a very shallow depth of field. That means if you are focusing on this woman, she will be in sharp focus, not necessarily the background. As you continue to control the light, less and less light is getting in, and as you decrease the aperture, you go from a large aperture to a very small aperture, which is f/32 over here.

What you see is that you have great depth of field. Okay, so you can see the woman, the tree, and the cat. Okay, so a large aperture means more light. And as you decrease the aperture size, you get less and less light. So, great depth of field comes with a small aperture, while shallow depth of field comes with a large aperture.

This is important to record because in some cases, you may need to image something with a very shallow depth of field, so you don't want light from... You don't want other objects to be in focus, particularly important, for example, in micro particle image velocimetry. In that case, you use a shallow depth of field; therefore, you need a very large aperture.

Okay, but when you are going for a large depth of field or a great depth of field, you can deal with a very small aperture. Okay, so you understand what aperture is, what f-number is, and how large and small apertures are related to the depth of field of the image.



It is also convenient to represent the overall light throughput as a cone angle, as you can see.

So this is kind of an aperture. This is the cone angle. As a cone angle, it is also called the numerical aperture of a lens. The numerical aperture of a lens is defined as the sine of the angle made by the marginal ray and the optical axis in the image plane. So this black line is the optical axis. This red line is a marginal ray. It is defined as a sign of the angle between the two, okay? Sine of this angle, which is theta.

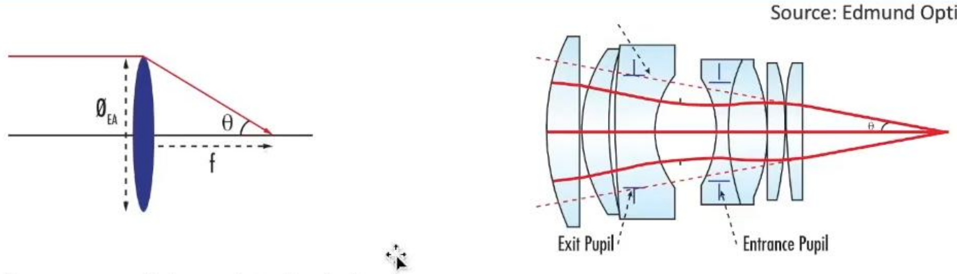
So, the f number and the numerical aperture are inversely related. That means the numerical aperture is $\frac{1}{2f\ number}$, all right? So $\frac{1}{2f/\#}$ are actually the numerical aperture. So, what is it once again? It is basically convenient to represent the overall light throughput as a cone angle. And the numerical aperture, this is called the numerical aperture.

It is defined as a sign of the angle. This is the angle sign, the sign of this angle made by the marginal ray and the optical axis of the image space, right? So they are inversely related.

So you know the *f* number, your numerical aperture, you can find out. So this is another way. They are basically one and the same. So this is kind of a convenient way of representing the same.

Imaging Fundamentals: VII

Source: Edmund Optics



Projection of image space marginal ray angle to edge of exit pupil

It is convenient to represent the overall light throughput as the cone angle, or the numerical aperture (NA), of a lens. The NA of a lens is defined as the sine of the angle made by the marginal ray and optical axis in image space.

$f/\#$ and NA are inversely related.

$$NA = \frac{1}{2f/\#}$$

Now the depth of field is taken from Ron Adrian's annual review of fluid mechanics. So the depth of field, as you can understand, is the distance between the nearest and the furthest objects that appear acceptably sharp in an image. So when you are imagining something, say the image has a certain depth of field because objects are never in a single plane, so to speak. So they have certain dimensions. So it is, for example, the droplet images that I showed when they interact with a shark, or actually this particular video that I showed.

Let's take a look at this one more time, for example. You can see that some of the droplets are very sharp, while others are not. So the idea is that some objects within a certain distance, given as, say, $DOF = D_r + D_f$, are around a certain focusing plane. So this is like the image plane according to which the lens assembly actually works. But around that lens assembly, at a certain depth inwards or outwards, perpendicular to this object plane, you will have a reasonably sharp or acceptably sharp image, as we showed in the case of those droplets. Some droplets that are within this band will appear acceptably sharp.

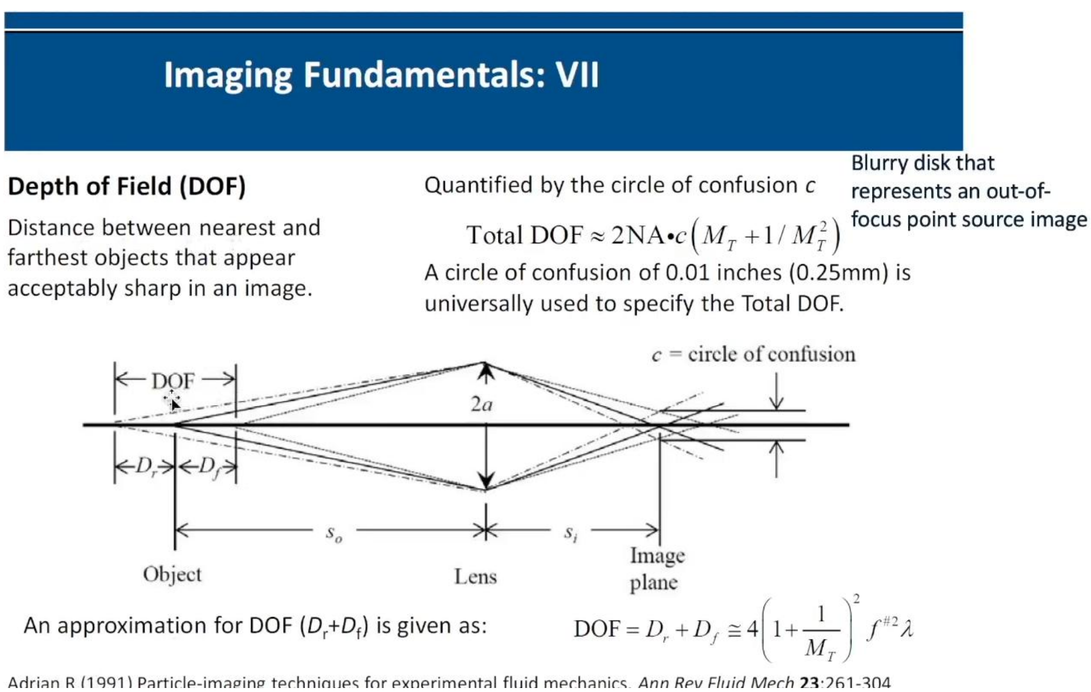
The droplets that are away will appear blurry. So the actual object plane is a single object plane on which the lens works, but there is a little bit of distance across which the image will appear moderately sharp. So this is quantified by what we call the circle of confusion.

So this is basically nothing but the blurry disk that represents an out-of-focus source image. The source image is a blurry disk.

This is called the circle of confusion. Okay, so the total depth of field is *2*NA*c*, which is basically the circle of confusion, multiplied by a function of the transverse magnification. The circle of confusion, which is approximately 2.25 mm, is universally used to specify the total depth of field. So, the total depth of field is given by this particular equation.

To say where *c* is about 0.25 mm, what is this circle of confusion? Once again, it is a blurry disk. You can see this is a blurry disk, which represents an out-of-focus point source image, which is exactly what it is because these points are out of focus since they are not on the image plane directly. As a result of that, they represent an out of focus point source image on the imaging side of things. So this is what you can see very clearly over here: where the rays actually converge, you can see that is the circle of confusion. So this can also be represented as $DOF = D_r + D_f$, and it can be approximated.

This is an approximation. This is given as $4\left(1+\frac{1}{M_T}\right)^2 f^{\#2}\lambda$. So this is the depth of field that has been given. So you can understand why the circle of confusion occurs, as the images are not exactly in focus because the focal plane is a single plane here. So these are around the focal plane, but they form an out-of-focus image. As a result of that, there is a degree of blurriness that should be acceptable to us.



So this is what the image fundamentals actually dictate. Okay, so if you look at, you know, the scenario, you learned quite a few things up to this particular point in this presentation; you saw some very cool images, which represent a lot of optics and a lot of image processing. You saw what a lens can actually do and what kind of magnifications it can provide. And you also found out that you know, yeah, if you do this imaging properly, these are the different parameters, and based on that, the image is quantified. How they are quantified, what will be the magnification that we learned, but the two important parts that we learned are that when the two lenses are separated by a distance.

Which is how their sum is the inverse summation of the focal lengths. The most important concept that you probably learned here was the aperture and how the aperture is related to something called an *f* number. The *f* number is nothing but *f* divided by the effective aperture diameter. So as we close in on the aperture diameter, the *f* number actually becomes *f*/32, for example. And in order to achieve a shallow depth of field, you need a large aperture.

To achieve a large or great depth of field, you require a small aperture. Light is greater when you have a large aperture because that is what it is. And light is less when you actually have a smaller aperture. Okay, the numerical aperture is just the inverse of the F number, and it is given by the sine of the angle that the marginal ray makes with the optical axis. And now we also know one other thing, which is basically the depth of field and how the depth of field is given by the magnification number and the *f* number. And we also learned a concept called the circle of confusion, which is a blurry disk that represents an out-of-focus point image.

So these are some of the important concepts before we go on to something called diffraction. That is also very important, as we will learn later. But this kind of covers what the normal imaging is that you will come across in this particular course. Right. So again, read through this very carefully and get your concept organized before we move on to what we call diffraction.