Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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Week – 02

Lecture - 08

Geometric Optics – 2

In part 1 of geometric optics, we discussed the basics of geometric optics, such as how reflection occurs at a plane mirror, how refraction occurs at a plane surface, and the characteristics of the image formed by a plane mirror. Later, we discussed how an image is formed using a spherical mirror. Discuss a concept such as focal point and focal length, and then we discussed how an image is formed using concave and convex mirrors. In the current lecture, we'll be discussing concepts such as refraction through a spherical surface. Later, we'll discuss both thin converging and thin diverging lenses, and finally, we'll conclude this lecture by connecting these basics with how a camera works. Now, let us see how refraction happens at a spherical surface.

Let us assume these two are two different mediums with n_a and n_b refractive indices. Now, let us assume this is air and that this is glass. Then, n_b is greater than n_a. Thus, when we see the ray optics diagram, the light rays emanating from point B incident on the surface of the spherical lens, refract, and transmit through the lens as they deflect towards the normal because n_a is less than n_b, which gives θ_b is less than theta from the law of refraction.

Now, this refracted light, when it meets the incident light along the optical axis, is where the image is formed. The location where this refracted light intersects the refracted light along the optical axis is where the image is formed, which is at P'. Similar to before, we have angle alpha, which is the angle subtended by the incident light, and angle β, which is the angle subtended by the reflected light. θ_b and θ_a, which correspond to the angle of incidence and the angle of refraction, and phi is the angle subtended by the normal with respect to the optical axis. Similar to before using geometry, we can determine that this angle is α and this angle is ϕ .

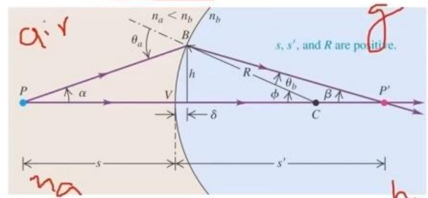
Thus, we get $\theta_a = \alpha + \phi$. Similarly, we get $\phi = \beta + \theta_b$, and the law of refraction $n_a \sin\theta_a = n_b \sin\theta_b$. Let us say this point is O. So, from the triangles PBO, BCO, and BP' O, we get these relations. Similar to before, we assume the paraxial approximation, which states that theta and theta b are very small.

This gives us these relations. Now, substituting these relations here, we get this relation; this is the object-image relationship for a spherical refracting surface.

REFRACTION AT A SPHERICAL SURFACE

- Look at the geometry of the figure. Angles α, β, ϕ, θ_a, θ_b have the following relationships:

$$\theta_a = \alpha + \phi \qquad \phi = \beta + \theta_b$$
$$n_a \sin\theta_a = n_b \sin\theta_b$$



- The three triangles with height h have these relationships:

$$\tan\alpha = \frac{h}{s+\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

- We make the "paraxial approximation for small θ_a, θ_b

$$\theta_a = \alpha + \phi \quad n_a\theta_a = n_b\theta_b \quad \Rightarrow \quad \theta_b = \frac{n_a}{n_b}(\alpha+\phi) \text{ and } n_a\alpha + n_b\beta = (n_b - n_a)\phi$$
$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

- Finally, we have

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$ object-image relationship for spherical refracting surface

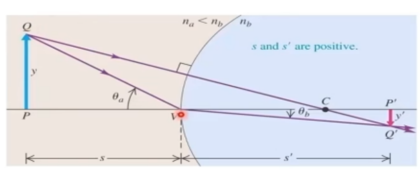
Now, let's see how the magnification changes when the object has a finite length. So, assuming the object PQ with length y, if we construct a ray diagram again, the refraction follows Snell's law, which is the law of refraction. That is, $n_a \sin\theta_a = n_b \sin\theta_b$.

Then, we will get the image at $P'Q'$. Using geometry, we get $tan\theta_a = {}^{y}/_{s}$. And $tan\theta_b = \frac{-y'}{s}$; here it is minus y dash because it is in the opposite direction. Using the paraxial approximation, that is, θ_a and θ_b are small, we can assume $tan\theta_a$ is similar to $sin\theta_a$and $tan\theta_b$ is similar to $sin\theta_b$; thus, we get this relation. As $\frac{n_a y}{s} = -\frac{n_b y'}{s'}$, we know $m = \frac{y'}{y}$.Thus, we get the relation for magnification while using a spherical refracting surface.

HEIGHT OF THE IMAGE FORMED BY A SPHERICAL SURFACE

- Look at the geometry of the figure. Triangles PQV and P′Q′V give:

$$\tan\theta_a = \frac{y}{s} \qquad \tan\theta_b = \frac{-y'}{s}$$



- From Snell's Law of refraction:

$$n_a \sin\theta_a = n_b \sin\theta_b$$

- We make the "paraxial approximation for small θ_a, θ_b

$$\tan\theta_a = \sin\theta_a \qquad \tan\theta_b = \sin\theta_b$$
$$\frac{n_a y}{s} = -\frac{n_b y'}{s'}$$

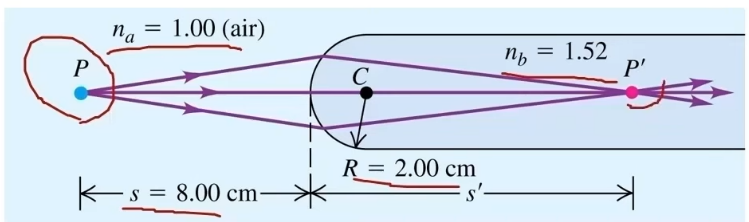
- Finally, we have

$$m = \frac{y'}{y} = \frac{-n_a s'}{n_b s}$$ magnification for spherical refracting surface

Now let's see how an image is formed using a glass rod when it is placed in air. Here, an object is placed at a distance of s from the glass rod. Our radius of curvature of the glass rod is given and Since we know n_b is greater than n_a, we will get an image inside the glass art, which is a real image because the light rays are actually passing through it. Now, using the image object relation for refraction, we substitute the values of n_a, n_b, S, and R to get the value of S'. Now that we have all the values, we can compute the value of magnification.

IMAGE FORMED BY A GLASS ROD IN AIR

- Example 34.5 for a glass rod in air. Find image distance s' and lateral magnification. Use Figure 34.24 below.



Start with $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$

$$\frac{n_b}{s'} = \frac{n_b - n_a}{R} - \frac{n_a}{s} \quad \Rightarrow \quad \frac{n_b}{s'} = \frac{s(n_b - n_a) - Rn_a}{Rs}$$

$$s' = n_b \frac{Rs}{s(n_b - n_a) - Rn_a} = \frac{1.52(2\text{ cm})(8\text{ cm})}{(8\text{ cm})(0.52) - (2\text{ cm})} = 11.3\text{ cm}$$

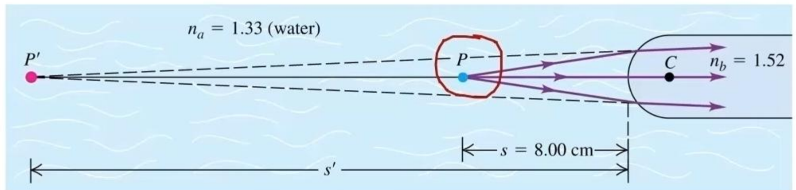
Start with $m = \frac{y'}{y} = \frac{-n_a s'}{n_b s} = \frac{-11.3\text{ cm}}{(1.52)(8\text{ cm})} = -0.926$

Now, let's see what happens when the glass rod is submerged in water and the object is placed inside the water. When the object is placed inside the water, then substituting the new values of $n_a$and $n_b$in this equation, we get the value of h', which is surprisingly minus 23 or 23 centimeters. Since h' is less than 0, that means the image is present on the opposite side of the interface from that of the outgoing rays. Since the outgoing rays are inside our glass, our image will be on the other side of the interface, which is in water.

Thus, the image is formed in the water at a distance of 21.3 cm. Interestingly, since the value of $h'$is less than zero and the refracted rays are not passing through the image, the image is formed by extending the refracted rays into the water, and the image is formed where those lines intersect; thus, the image is a virtual image. Now let's calculate magnification. Here magnification, here we can get $m = \frac{-n_a s'}{n_b s}$. Here we substitute h' with less than zero.

IMAGE FORMED BY A GLASS ROD IN WATER

- Follow Example 34.6 for a glass rod in water. Use Figure 34.25 below.



$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

$$s' = n_b \frac{Rs}{s(n_b - n_a) - Rn_a} = \frac{1.52(2\text{ cm})(8\text{ cm})}{(8\text{ cm})(0.19) - (2\text{ cm})(1.33)} = -21.3\text{ cm}$$

This is a virtual image!

$$m = \frac{y'}{y} = \frac{-n_a s'}{n_b s} = \frac{(1.33)(21.3\text{ cm})}{(1.52)(8\text{ cm})} = 2.33$$

So simply moving the experiment from air to water has a huge effect on the outcome.

This is another interesting example of refraction that we observe in our daily lives. When we go to a swimming pool or a body of water, the bottom of the water body appears to be at a shallower depth than its actual depth. This is due to refraction. Here, as we can see, the light rays coming from the bottom refract at the water-air interface.

Let's assume this is the light ray coming from the bottom of the swimming pool, and when this light ray refracts at the water-air interface, since the refractive index of water is greater than the refractive index of air, from Snell's law, we get theta refraction. If it is greater than theta i, that means it bends away from the normal, and a person seeing this light ray observes that this light ray is coming from a shallower depth. The person seeing this light ray observes that this light ray is coming at a shallower angle than the incident light ray. Thus, the The pool's depth appears to be shallower than it actually is due to refraction.

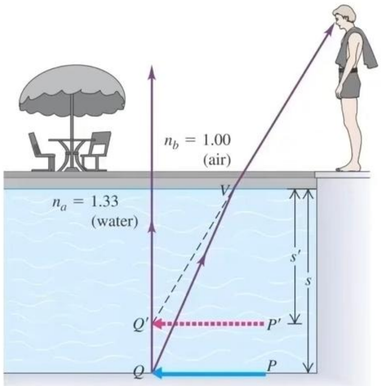
APPARENT DEPTH OF A SWIMMING POOL

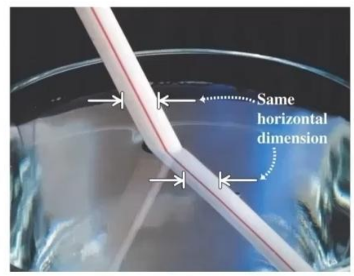
- Example 34.7 using Figure 34.26 at the left—how deep does the pool appear?

Start with $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ What is the radius in this case?

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0 \quad \Rightarrow \quad s' = -\frac{n_b s}{n_a} = -\frac{2 \text{ m}}{1.33} = -1.50 \text{ m}$$

- This is a virtual image—the pool appears shallower
- Figure 34.27 (right) shows that the submerged portion of the straw appears to be at a shallower depth than it actually is.





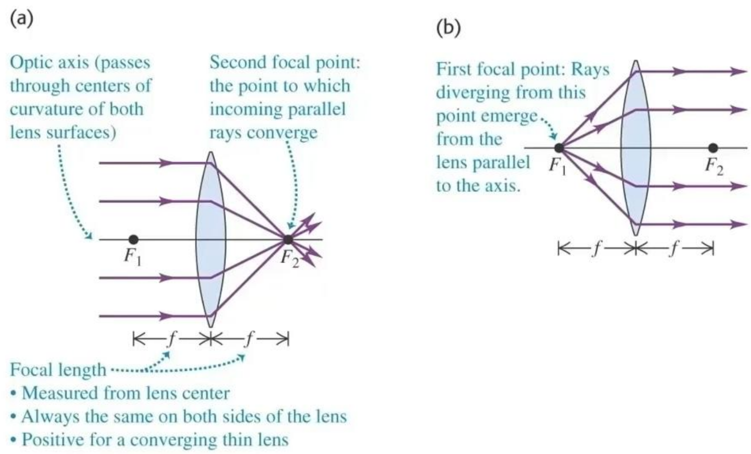
Now, let's discuss thin converging lenses, which are the magnifying lenses that we see in our daily lives.

These thin lenses form images in the same medium as the object because the object is also in the air and the image is also formed in the air. Thus, $n_a = n_b$. And we characterize these lenses based on their focal length. Here we can see that for this specific lens, the focal points on each side of the lens are shown having equal values. The focal length is taken to be positive for converging lenses, whereas it is taken to be negative for diverging lenses.

As we can see here, the rays diverging from an object placed at the focal point of a thin converging lens form parallel rays parallel to the axis. Whereas the parallel rays converge at the focal point on the other side when these thin converging lenses are used.

THIN CONVERGING LENS

- Thin lens => images are formed in the same medium as the object. The same rules apply, but $n_a = n_b$, and we use "focal length" to provide information about the lens curvature, index of refraction, etc.
- Figure 34.28 below shows the focal points F and focal length f of a *thin converging lens*. Note, F_1 and F_2 are equidistant from lens.



Let's see how an image is formed using a thin converging lens when the object has a finite length. So when the parallel light ray emanating from Q is converged, it passes through the focal point on the other side. Another beam passing through the center of the lens intersects this original refracted ray at point q', where the image is formed.

Here, as you can see, the image is inverted, similar to before. The angle subtended by the incident light is alpha, and the angle subtended by the refracted light is beta. From the similarity of the triangles, that is, opq and $op'q'$, we get $\frac{y'}{y}.\frac{y}{y'} = -\frac{s}{s'}$; here, $y'$is negative. Thus, we take $\frac{y}{s} = -\frac{y'}{s'}$.

Also, we have two more similar triangles, which are a o f 2 and 2 dash p dash f 2. These two triangles are also similar; thus, we get the relation of. $\frac{y'}{y} = -\frac{s'-f}{f}$ because y' is in the opposite direction. Thus, we get the second relation. Combining these two, we get the object-image relationship for thin lengths, and the magnification of this length, as same as before, is $\frac{y'}{y}$.

IMAGE FORMED BY A THIN CONVERGING LENS

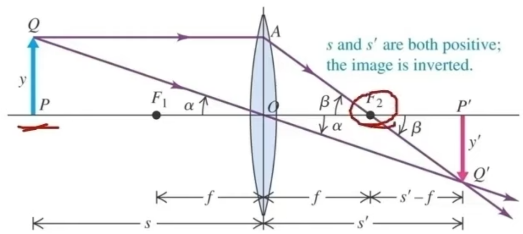
- Look at the geometry of the figure. Triangles OPQ and OP'Q' are similar triangles, so:

$$\frac{y}{s} = -\frac{y'}{s'} \quad \Rightarrow \quad \frac{y'}{y} = -\frac{s'}{s}$$

- Also, AOF2 and Q'P'F2 are similar, so:

$$\frac{y}{f} = -\frac{y'}{s'-f} \quad \Rightarrow \quad \frac{y'}{y} = -\frac{s'-f}{f}$$

- Equating and rearranging, we have the same result as earlier for mirrors:



and

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$ object-image relationship for thin lens

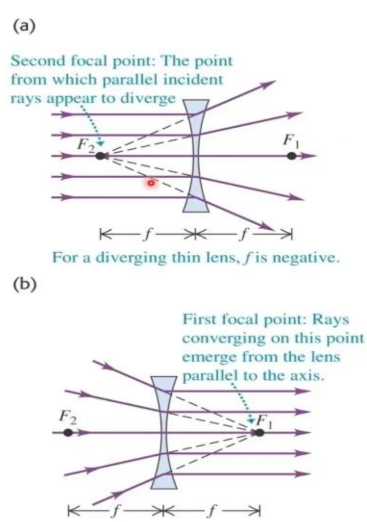
$$m = \frac{y'}{y} = -\frac{s'}{s}$$ magnification for thin lens

Let's look at how a thin diverging lens works here, similar to before. Similar to the thin converging lens, the thin diverging lens also has two focal points. When parallel beams are incident on this thin diverging lens, these rays refract and diverge, and they appear to be diverging from the focal point on the other side. Incident rays that appear to be converging at the focal point on the other side then get converted into parallel rays.

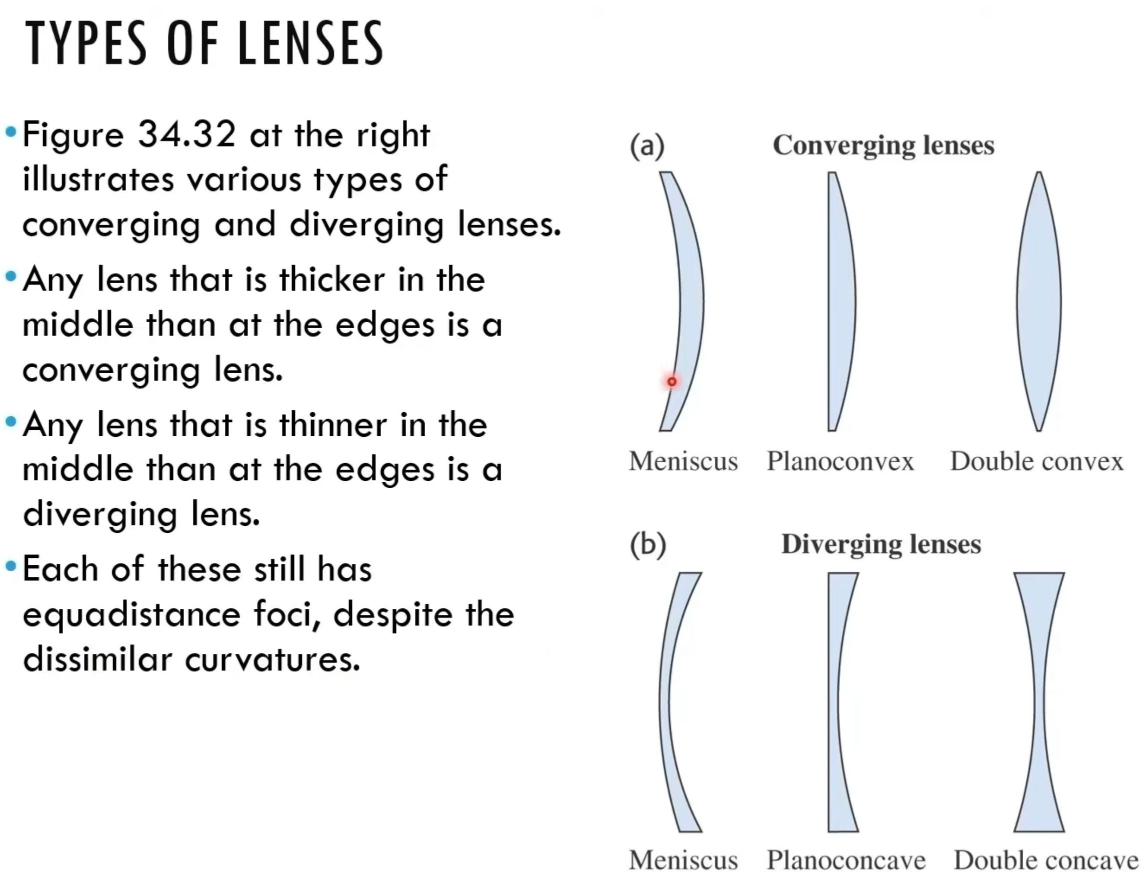
THIN DIVERGING LENS

- Figure 34.31 at the right shows the focal points and focal length for a thin diverging lens.

- The results for a converging lens also apply to a diverging lens.



This slide shows the different types of converging and diverging lenses. Any lens that is thicker in the middle than at the edges is a converging lens, while any lens that is thinner in the middle than at the edges is a diverging lens. Each of these still has equidistant foci despite the dissimilar curvatures, and all these are formed by various combinations of curvatures on either side.



Now, let's see how we can determine the focal length of a thin lens using the radii of curvature. So here we are considering a meniscus lens with the convex side on the left side and the concave side on the right side. Let's assume the object is placed on the left side of this lens, as shown here.

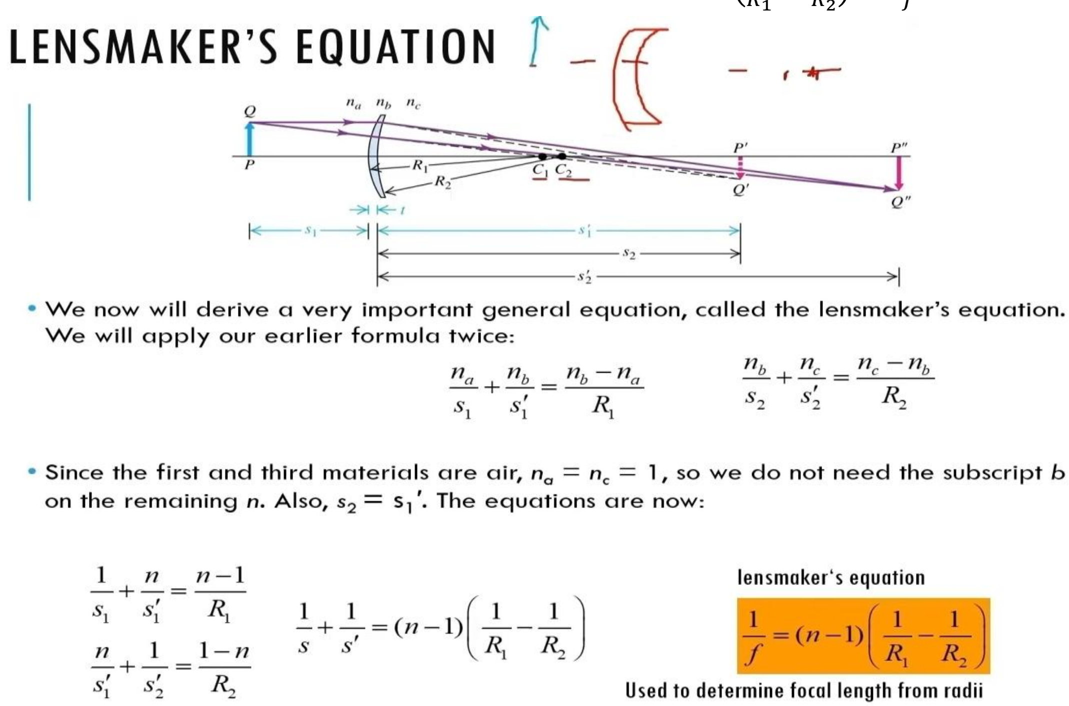
Now, as we can see here, there are two curvatures: the convex curvature on the left side, which I have shaded with black, and the concave curvature on the right side, which I have shaded with green. Now, using the earlier relation that we have derived for the left side, which is the face shaded with black, let's assume the refractive index of the medium on the left side is n_a, the refractive index of the lens is n_b, and the refractive index of the medium on the right side is n_c. So, when the light rays are passing from the left side, Refracting at the left face of the lens, which is shaded in black, then considering the previous relation, we have n_a, which is the left side refractive index, n_b, the refractive index of the lens, and

s_1 is the distance at which the object is placed. And R_1 is the radius of curvature of the left face, which is shaded in black. Using this, we will obtain the value of S_1, which is the image of the object PQ with respect to the left face of this thin lens.

So, this is the image that is formed by the left face of this thin lens. Now this image from the left face of the lens acts as an object with respect to the right face of the lens. Here, the light rays are going from n_b to n_c. Thus, using the same relation, we substitute n_b by s_2. s_2 is equal to s_1' that we obtain here.

So, $\frac{n_b}{s_2'} + \frac{n_C}{s_2'}$, that is the refractive index on the right side, divided by s_2', that is the final image that we want, is equal to $\frac{n_c - n_b}{R_2}$. R_2 is the radius of curvature of the right face of the lens. So, these two are the general equations for finding the location of the final image. Now, since the first and third materials are air, we set $n_a = n_c = 1$. And substituting that into these two relations, we get $\frac{1}{s}, + \frac{n}{s_1'} = \frac{n-1}{R_1}$.

Combining these two, now eliminating s_1' from these relations, we finally get this relation which is $\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Here, we can rewrite $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. Thus, we get the lens maker's equation as shown here, which is $(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$. In this slide, we will see how to determine the focal length of a lens using the relation that we derived earlier. The relation was $(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$.



So, it has been given that it is a converging lens with both radii of 10 centimeters and refracted at 1.52. So, to determine the focal length, we have to get the sign convention of the radius of curvature correct; for this specific lens, the radius of curvature of the left-hand side lies on the right-hand side, and the radius of curvature of the face on the right-hand side lies on the left-hand side. Thus, this is R_2 and this is R_1. As per our convention, if the center of curvature is on the same side as outgoing light, then the radius is considered positive.

Here, if we consider the object on the left side, then the outgoing light rays are traveling towards the right side; thus, c_1 is on the same side as that of the outgoing rays, whereas c_2 is on the opposite side of the outgoing light rays. Thus, we consider r_2 to be less than 0 and r_1 to be greater than 0. Thus, we substitute r_1 equal to 10 and r_2 equal to -10. And by using this relation, we can find the focal length of this lens. Now, suppose it is a diverging lens; what happens? In the case of a diverging lens, as you can see here, the center of curvature of the blue side is on the right side, and the center of curvature of the red side is on the left side.

Now, if we assume the object to be on the left side, then the outgoing rays will go towards the right side. So, in this case, C_2 is on the same side as the outgoing race, whereas C_1 is on the opposite side of the length compared to the outgoing race; thus, we substitute R_2 to be positive and R_1 to be negative to obtain the focal length of the diverging lengths. As you can see, for converging lengths, the focal length is positive, and for diverging lengths, the focal length is negative.

DETERMINING THE FOCAL LENGTH OF A LENS

- Example 34.8: (a) Determine focal length of converging lens below if both radii are 10 cm and index of refraction is 1.52. (b) Determine focal length of a diverging lens of the same curvatures. (a)

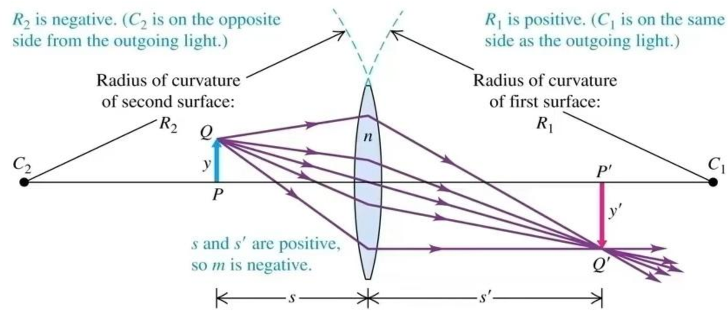
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f} = (1.52-1)\left(\frac{1}{10\text{ cm}} - \frac{1}{-10\text{ cm}}\right)$$

$$f = 9.6\text{ cm}$$

(b)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f} = (1.52-1)\left(\frac{1}{-10\text{ cm}} - \frac{1}{+10\text{ cm}}\right)$$

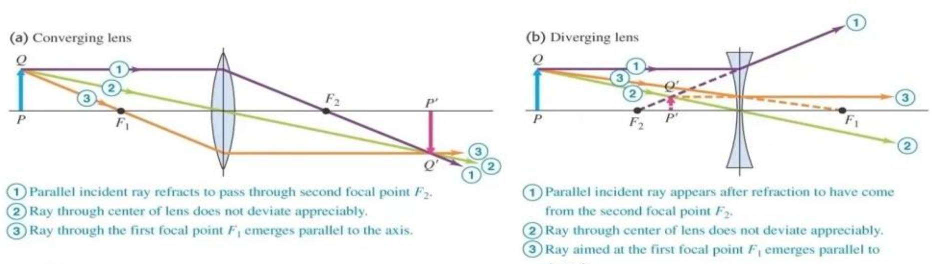
$$f = -9.6\text{ cm}$$



This slide shows the summary of the principles of ray optics using thin lenses. The left side shows the converging lens; the right side shows the diverging lens. In a converging lens, an inverted real image is formed, whereas in a diverging lens, an erect virtual image is formed.

GRAPHICAL METHODS FOR LENSES

- Follow the text summary of the three principal rays.
- Figure 34.36 below illustrates the principal rays for converging and diverging lenses.

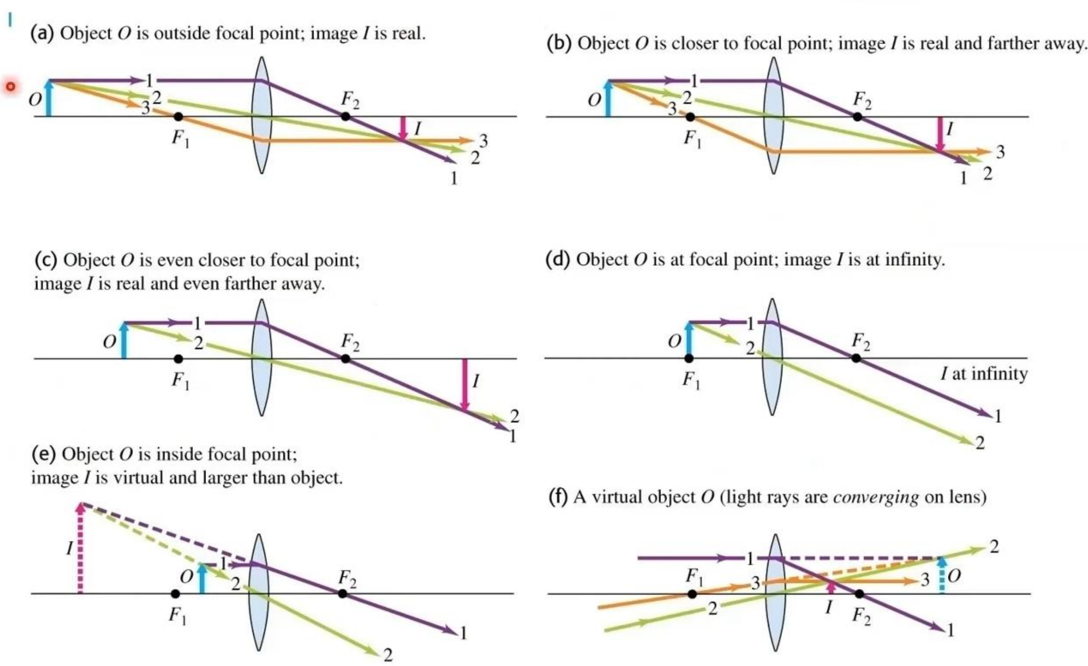


Let's see how the image is formed when an object is placed in front of a thin converging lens. When the object is placed beyond the focal point on the left side, the image is formed beyond the focal point on the right side, which is inverted but real. As we move the object closer to the focal point on the left side, the image, which is real, moves farther and farther away from the lens towards the right side, as shown here from A, B, and C, and the size of the image increases. Now, if you bring the object to the location of the focal point on the left side, then the image is formed at infinity on the right side; that is, the image cannot be captured on a screen.

Now, when we move the object even closer than the focal point on the left side, a virtual image is formed on the left side of the lens as the rays start to diverge on the right side, and we have to extend the diverging rays such that the image appears to be emanating from the left side of the lens. Here the image is virtual and appears to be larger than the object. When two light rays coming from far away tend to converge on the right side, that is, if we have a virtual object on the right side, then we'll have a real image on the right side of the lens.

THE EFFECT OF OBJECT DISTANCE

- The object distance can have a large effect on the image (Figure 34.37 below).

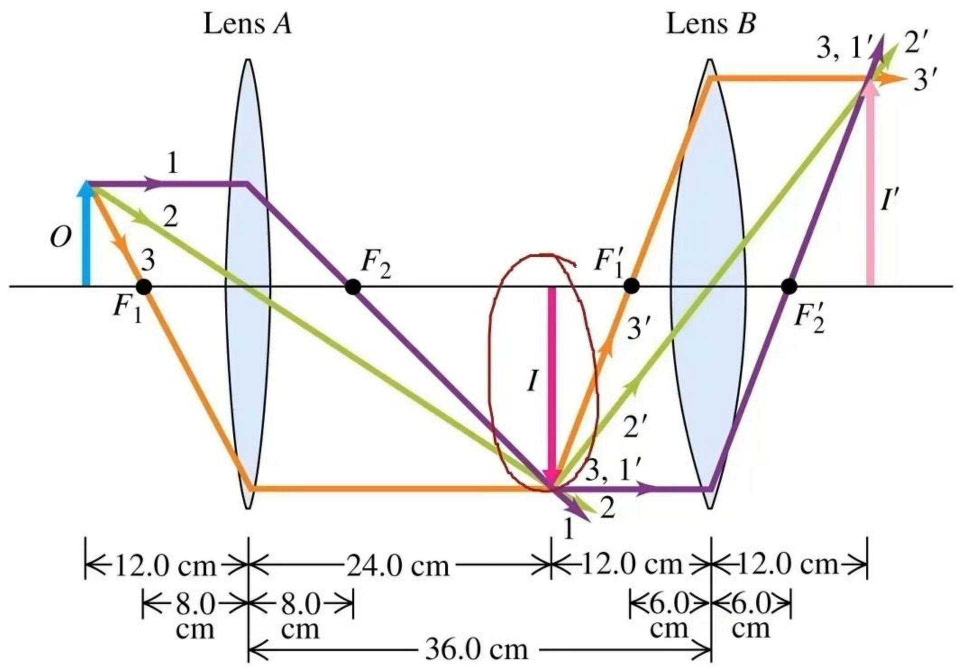


Now, finally, let's see how we can calculate the effective focal distance and effective distance of the image when we have two lenses. Here, using the first lens, we can estimate the location of the image to be here using the distance of the object from the lens, which is s_1, and the focal length of lens one, f.

Now using this image of the first lens, considering this image of the first lens as an object with respect to the second lens, if we compute the image, the distance of this image from lens B is considered as h_2, and then h_2' can be found using the focal length of the second lens, that is F_B. That is how we compute the size and orientation of the image by using multiple lenses.

AN IMAGE OF AN IMAGE

- Follow Example 34.11 using Figure 34.39 below.

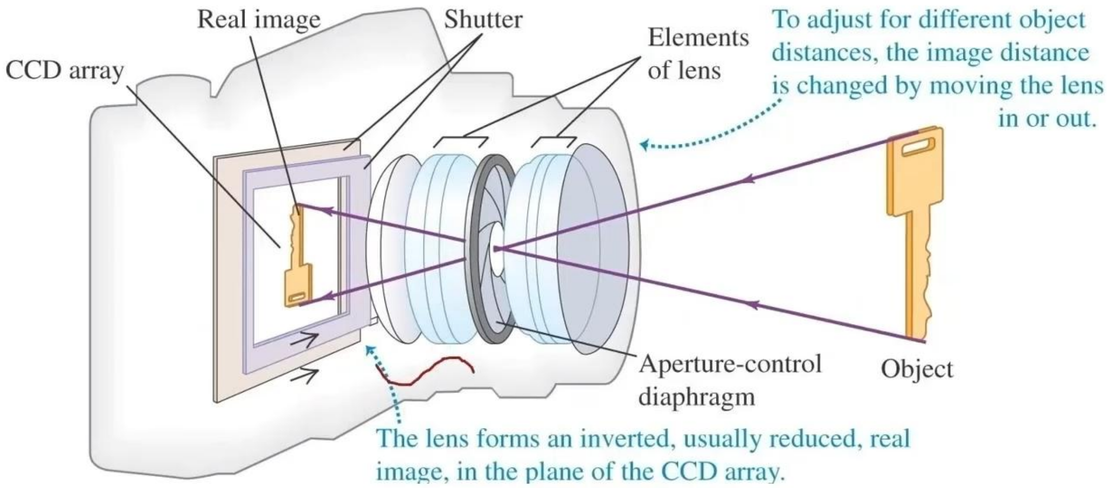


Now let's look briefly at how cameras work. In cameras, we have a series of lens systems, as shown here. Where these individual lens elements try to converge and diverge the light rays such that a real image is formed at the location of the sensor inside the camera, as we can see, this key is an object, and the camera is used to capture the image of this object; then the light rays pass through the complex lens system.

And finally, it forms a real image on the sensor of the camera, and the sensor captures the photon values using the CCD array. Now, depending on the distance of the object from the camera lens, we can adjust the distance between the different lens elements inside the camera lens so that a real image is formed at this specific location of the camera sensor by moving the lenses in and out. The camera has an iris with an aperture at the center, and this can be used to adjust the amount of light that can enter the camera. The more the aperture is open, the more light enters into the camera, and the smaller the aperture is, the less light enters into the camera. The lenses in the camera system form an inverted, usually reduced-size real image in the plane of the CCD array, which is the sensor, as shown here.

CAMERAS

- Figure 34.40 below shows the key elements of a digital camera.



Now let's look at some basics of the camera lens. In the camera, as we have shown, there is a central opening, and this opening is called the aperture. This aperture size determines how much light intensity goes inside the camera. Now, the f-number is the focal length divided by the aperture size, also known as the f/d ratio. A 50 mm lens with an aperture size of 25 mm in diameter has an f-number of 50 by 25, which is f by d equal to 2.

So, it would be said to have an f-stop of f/2. Since the exposure time depends on the area of the aperture, the f-stops changing by the square root of 2 can change the exposure time by a factor of 2. Typical f-stops are f/2, f/2.8, f/4, f/5.6, f/8, f/11, and f/16. Here, changing the diameter by a factor of the square root of 2 changes the intensity by a factor of 2 because the area changes by the square of the diameter.

So, to say the larger f numbers correspond to a smaller aperture, that is, lesser intensity of light. So, just to give an example, a common telephoto lens with a 35mm camera has a focal length of 200mm. Its f-stop ranges are given between f/2.8 and f/22. So, what is the range of apertures? So, using the relation f/D = f number, we have d = f/(f number), and we have f given the focal length as 200 and the minimum f number as 2.

8, which gives the diameter to be 71, whereas the diameter corresponding to f number 22 gives the diameter as 9.1 mm. So, to get the intensity of the image for these two f-stops. Intensity is proportional to d squared, so the ratio of intensity is the square of the ratio of the diameter. Thus, taking the square of the ratio of the diameter 71 by 9.

1 whole squared, we get 62. So, for an exposure time of 1 by 1000 seconds at f/2.8, you would have to expose for 62 by 1000 seconds, that is, 1 by 16 seconds at f/22. That means F2.8 has a 62 times brighter image compared to F22 while using the same exposure type.

CAMERA LENS BASICS

- The f-number is the focal length divided by the aperture size, also called the f / D ratio. A 50 mm lens with an aperture size D = 25 mm has an f-number

  f/D = 50 mm / 25 mm = 2

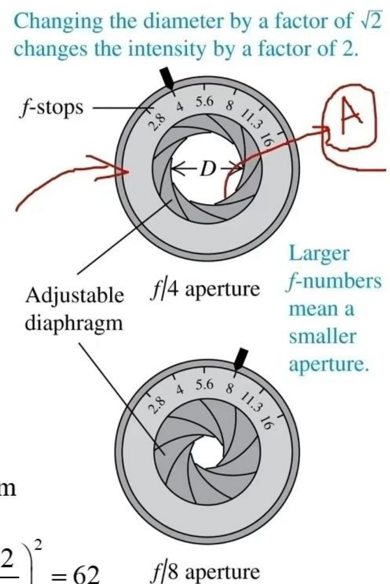
  so it would be said to have an f-stop of $f/2$. Since exposure time depends on the area of the aperature, f-stops changing by square-root of 2 change the exposure time by a factor of 2. Typical f-stops are $f/2$, $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, $f/16$.

- Example 34.12: A common telephoto lens for a 35-mm camera has a focal length of 200 mm; its f-stops range from $f/2.8$ to $f/22$. (a) What is the range of apertures? (b) What is the corresponding range of intensities on the film?

$$D = \frac{f}{f\text{-number}} = \frac{200\text{ mm}}{2.8} = 71\text{ mm and } D = \frac{200\text{ mm}}{22} = 9.1\text{ mm}$$

$$\text{Intensity is proportional to } D^2 \text{ so the ratio is } \left(\frac{71\text{ mm}}{9.1\text{ mm}}\right)^2 = \left(\frac{22}{2.8}\right)^2 = 62$$

- For an exposure of 1/1000 s at $f/2.8$, you would have to expose for 62/1000 s ~ 1/16 s at $f/22$.



Finally, before ending the discussion, let's look briefly at how the eye works. The optical behavior of the human eye is similar to that of a camera. In this figure, as we can see, there is an iris in our eye. which opens and closes and controls the amount of light that enters our eye, and there is a crystalline lens in our eye that adjusts the focus so that whatever object we are seeing, the object is focused on the backside of the retina. The image formed in our eyes is also inverted, and our brain corrects it. Also, as you can see, there are muscles that are attached to the lens called ciliary muscles.

These muscles contract and cause the lens to become more convex, decreasing its focal length to allow near vision. Thus, the human eye also functions similarly to a camera, where the lens is adjusted so that the object at a given distance can be focused onto the sensor, which is the retina in RI.

THE EYE

- The optical behavior of the eye is similar to that of a camera.
- Figure 34.44 below shows the basic structure of the eye.