Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
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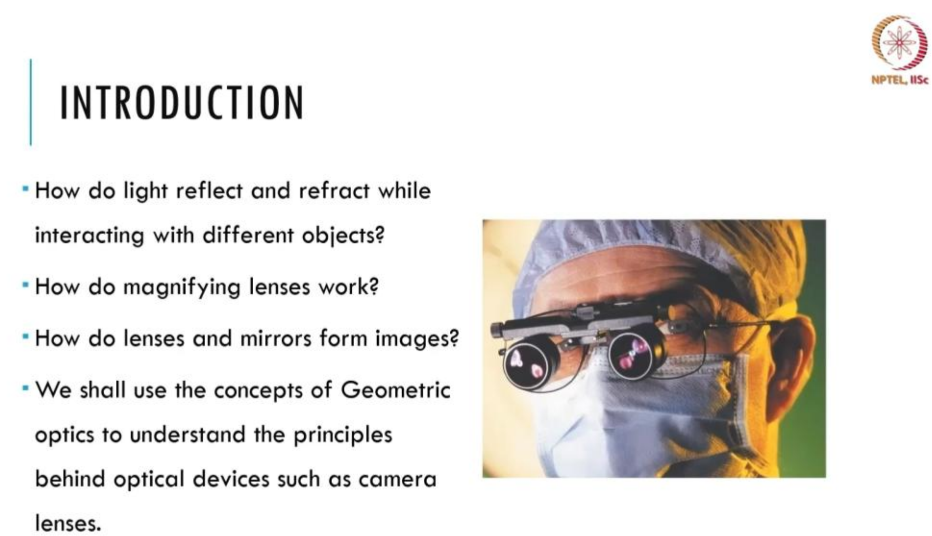
Week – 02
Lecture - 07
Geometric Optics – 1

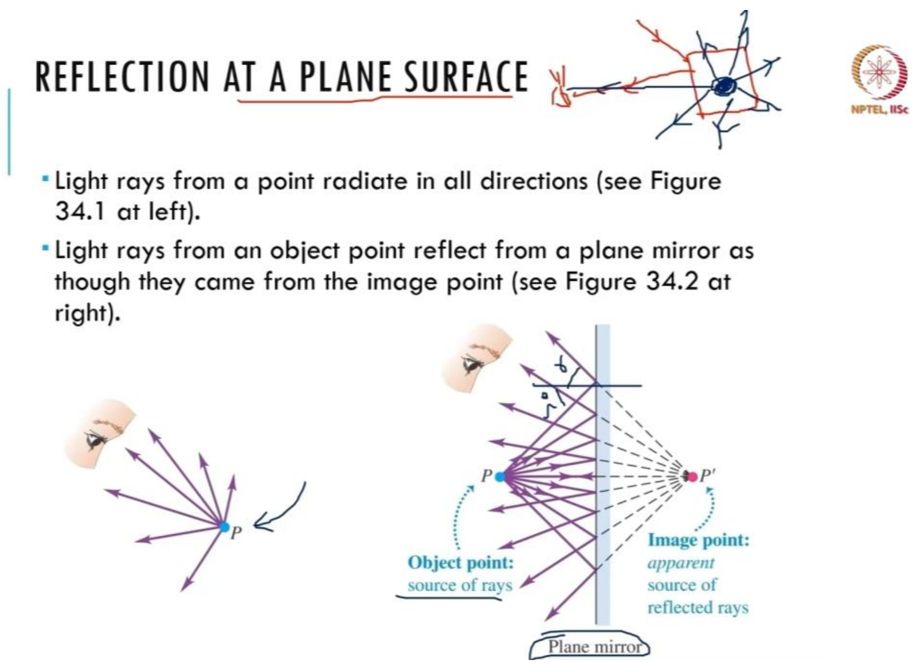
In this session, we are going to discuss the topic of geometric optics; in this lecture, we'll be discussing ray optics, how light reflects and refracts, and how light interacts with different lenses. These lecture slides are taken from the lectures by Wayne Anderson. In this lecture, we will address the following question. How do light reflect and refract while interacting with different objects? How do magnifying lenses work? How do lenses and mirrors form images? And finally, we shall use the concepts of geometric optics to understand the principles behind optical devices, such as camera lenses.



So, firstly, let's see how we can see different objects. If we have an object in front of us, we are able to see that object because the sunlight falls on that object, reflects off that object, and enters our eyes.

Now we will see how reflection occurs at a plane surface. For this, we will consider the simplistic assumption that the object is a point. If the object is a single point, then the light falling on the object is scattered in all directions, and when such a light ray enters our eye, we are able to see the object. Now let's see how the image of this object is formed when it is placed in front of a plane mirror.

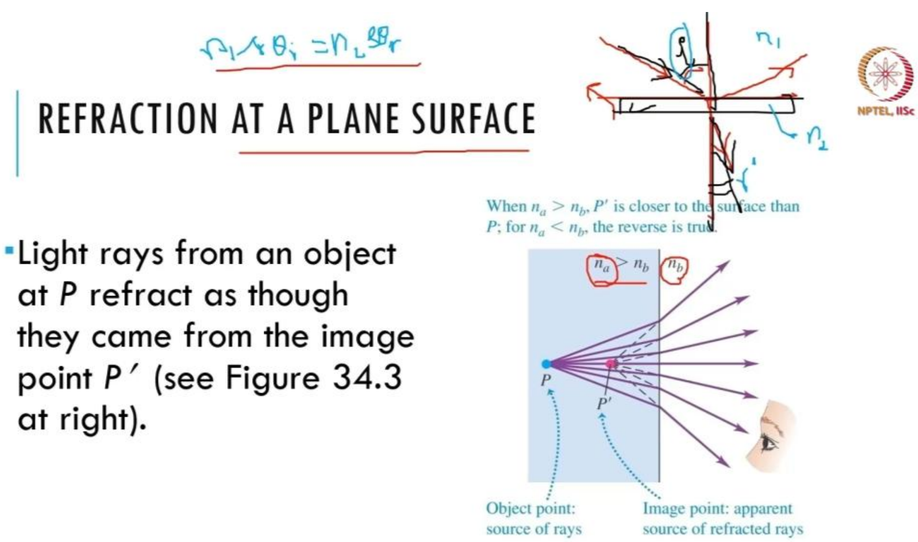
Now this point object is placed on one side of the plane mirror, and since it's a point object, different light rays travel radially outward from this point. And those light rays hit the interface of the mirror and reflect back. Now the reflection at the mirror occurs in such a way that the angle of incidence is equal to the angle of reflection; in that way, different light rays coming out from the point get reflected at the plane mirror. Now, if a person sees the reflection of this object, then they see that the light rays coming from this object are coming in this direction. If we extend these reflected light rays that the person is seeing backward, then they'll all converge at a single point, and this location is where the person will see the apparent reflection of this object, which is the image point. So, this point acts as a virtual apparent source of reflected light rays.



Now, let's see how refraction occurs at a plane surface. The main difference between reflection and refraction is that if we have a plane surface, the reflected light bounces back and propagates in the same direction as the interface. In case of refraction, the light ray gets transmitted through the object. Generally, for reflection, the object has to be translucent or transparent; in the case of refraction, if the light ray is incident on the glass lab in this direction, this is the angle of incidence, which is the angle made by the incident ray with the normal, whereas when the light ray transmits into the medium, which is refraction, this is the angle of refraction.

Here, the relationship between the angle of incidence and the angle of reflection is n_1 sine $\theta_i = n_2 \, sin\theta$ refraction, where n_1 is the refractive index of air and n_2 is the refractive index of the glass slab that we are using here. So now, if you see the example shown here. Where the object point is inside a medium with a refractive index of na and a person is seeing the object from outside, where the medium is air with a refractive index of n_b, it is given that n_a is greater than n_b. This means that, according to the formula for refraction, if n_a is greater than n_b, than θ_a . should be less than θ_b .

That means if we take normal here, then this is θ_a and this is θ_b. As we can see here, when the light rays are traveling from a higher refractive index to a lower refractive index, they deflect away from the normal. Originally, the light source had to go in this direction, but it has deflected away from the normal, and these light rays enter into the eye of the person to see the object. Thus, the apparent image that the person sees appears to be at lower depth compared to its actual depth. This is because the apparent refracted rays appear to be converging at a shallower angle because θ_b is higher than θ_a .

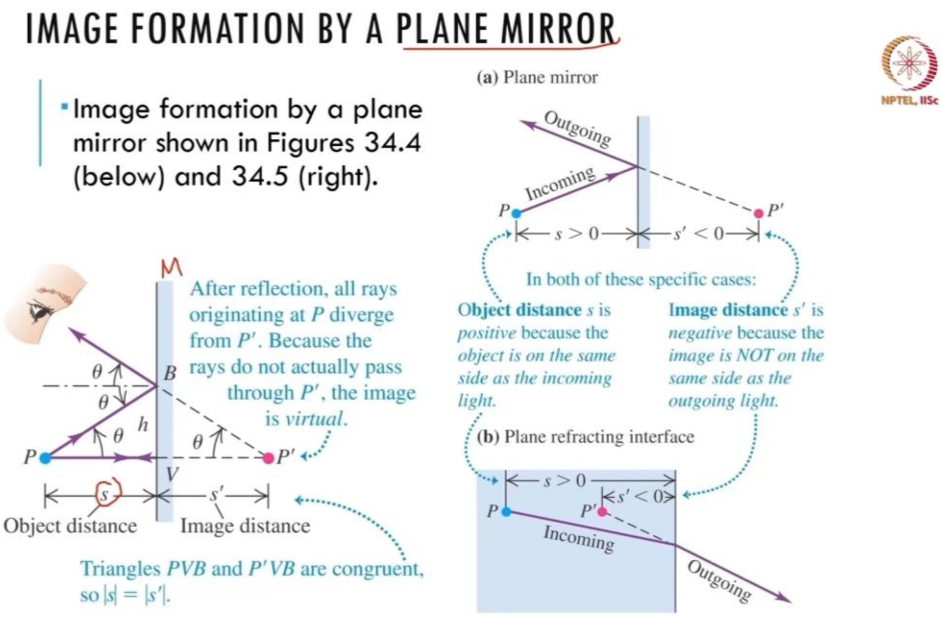


Now let's see how an image is formed using a plane mirror. If we look at this illustration on the left side, we can see that the object is present at a distance of s from the mirror, and the person is standing on the same side of the mirror as the object. Now, if the person wants to see the reflection of the object in the mirror, then if we construct a line diagram, the light ray.

From the point source, travel towards the mirror, and it gets reflected and enters the person's eye. If we extend the reflected rays, just like before, into the mirror, and if we extend any other reflected ray into the mirror, either this or we can consider one more, and if we extend it, they all converge at one point, and this point is where the image appears to be present. Now here the point to be noted is that all the reflected rays that we see here, either this or this or this, don't pass through the image, but the image is formed at the intersection of the extended geometric lines from these reflected rays. So this kind of image is called a virtual image since the reflected light rays don't actually pass through the image. Now let's see two different scenarios: in one scenario, the person is looking at the mirror while standing on the same side as the mirror to the object, just like before, and in this configuration, the person sees the image to be behind the mirror.

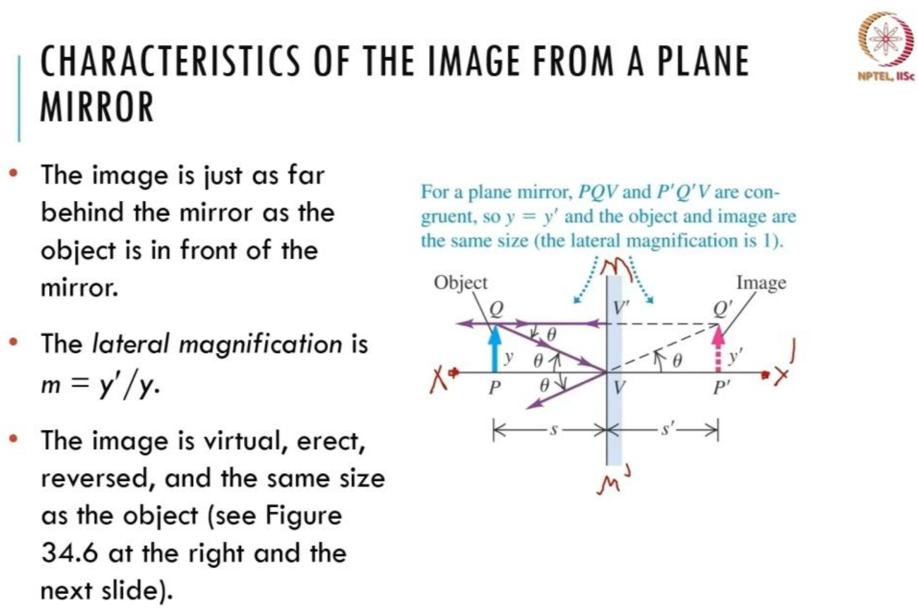
at the same distance as the object. The distance of the object from the mirror is denoted by s, and the distance of the image from the mirror is denoted by s'. It is to be noted that s is greater than zero if the object is on the same side as the incoming light, and s dash is considered negative because the image is not on the same side as the outgoing light, basically because it is a virtual image. Now let's see a different case where the object is on one side of the mirror, that is, inside the mirror, and the person looking at it is on the other side of the mirror. Now, an incoming light ray from the object refracts at the surface of the mirror, and then it reaches the person's eye.

Since, as I explained before, due to refraction from glass to air, which is a transition from a higher refractive index to a lower refractive index, the angle of refraction is higher, so the person sees the image at a shallower depth, which means h dash is less than s if n is greater than n outside. And s' is greater than s if n is less than n; outside here, n outside is the refractive index of air, and n is the refractive index of the transparent material; here, we are assuming it to be glass. Thus, n is greater than n_0. Now, even here, if we see the object is on the same side of the mirror as the incoming beam, thus s is greater than zero. However, the image and the outgoing light are on different sides of the mirror; thus, h dash is less than zero.

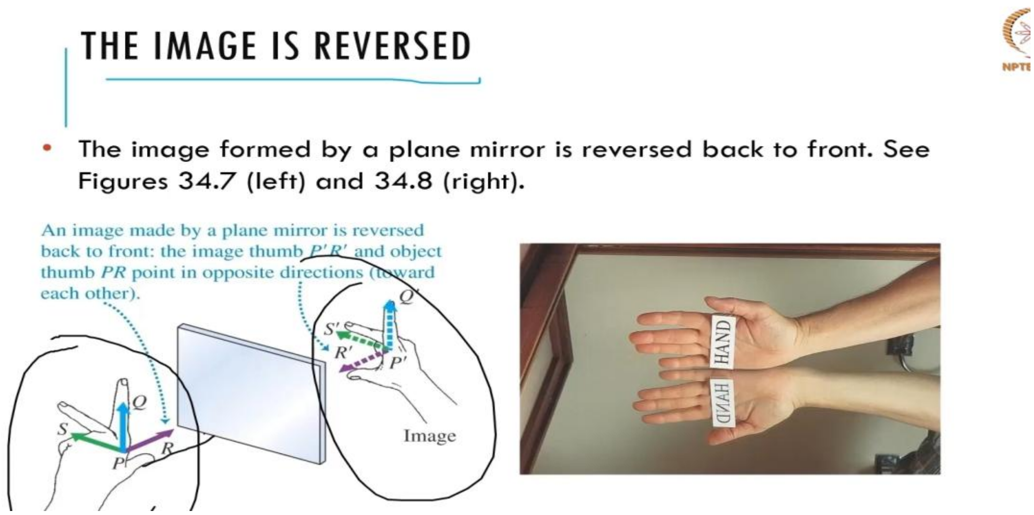


Now let's see the characteristics of the image from a plane mirror; let's look at this line diagram. This is the mirror MM', and this is the horizontal XX' connecting the bottom of the mirror to the center of the mirror on the object. Now let's assume that this is the height of the object Y, and the object is PQ. Now the light rays travel from the object in this direction, and they reflect back, and the light rays from the tip will also travel in this direction and reflect back like this. Now, if we intersect two reflected beams, we get the location of the image.

Now QP' is a reflection or the image of the object PQ. And if you see the distance from P to the mirror is s and the distance from the mirror to the image is s', and s is equal to s' in the case of plane mirrors, the angle of incidence is the same as the angle of reflection, and because of symmetry, if we see the lateral magnification which is equal to m equal to. The height of the image by the height of the object is one in the case of plane mirrors, so for plane mirrors, we can say that the image is virtual because the image is on the opposite side of the reflected ray. The image is erect because, as we can see, the orientation of PQ and P'Q' is pointing in the same direction; thus, the image is erect and it is of the same size, that is, magnification one.



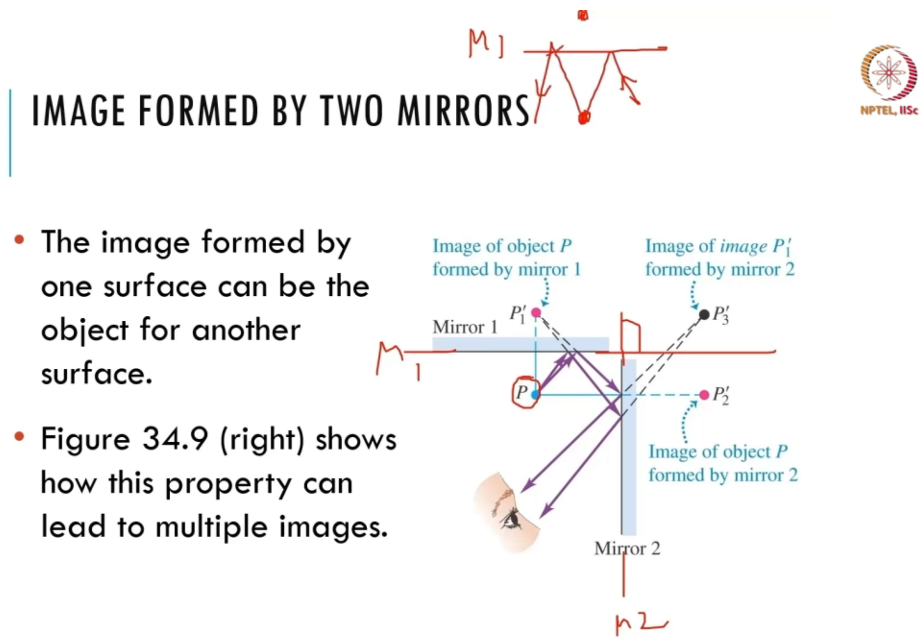
Now let's see why the image is reversed. Here we can see the object, which is a hand, and the image is a reflection of the hand. If we look at both the object and the image, for the image to be the reflection of the object, in both cases the thumb has to point towards the mirror. As we can see, the pr is pointing towards the inside, whereas $P'R'$ is pointing towards the outside. Thus, this is what we call reversing the image in a mirror. This can also be seen in this image where the hand is reversed and even the letters written on the text are also reversed.



Now, let's discuss how the image is formed if two mirrors are present. So let's consider a point object present at this location between two mirrors. The mirrors are placed orthogonally to each other at 90 degrees. Now, if we consider only one mirror M1 with respect to the object, then we'll have the image formed here as we consider the reflected rays and the extension of the reflected rays intersecting at this location. And if we consider only mirror 2 (m2) and the object, then the image of this object is formed here with respect to mirror 2, as we construct a ray diagram for the reflected rays, and those reflected rays will converge at this location where the image is located with respect to mirror 2.

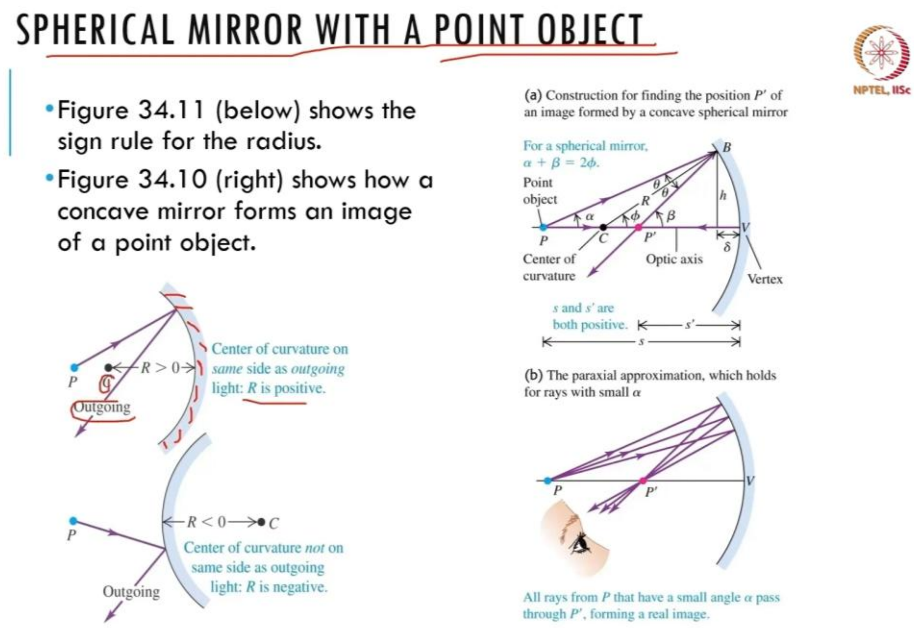
Let's consider the image with respect to mirror 1 as P_1' and the object as P. So, we are getting two different images when we consider two individual mirrors. But when we combine these two mirrors, something interesting happens. The image in mirror 1 acts as an object with respect to mirror 2. That means if we consider the reflected rays that are apparently originating from the image in mirror 1 and they get reflected on the surface of mirror 2, then if we extend these reflected rays from mirror 2 inside mirror 2, they will converge at point p3, which is the image with respect to mirror 2.

Of the image inside mirror 1, if we do the same in the opposite direction, where if we consider the image in mirror 2 to be the object with respect to mirror 1, and if we consider the line diagram of reflected rays from this point, and if we extend these reflected rays, they converge at the same point. Thus, by the addition of a mirror, we not only get the images with respect to each mirror, but the combination of mirrors will create extra images as well.



Let's see how an image is formed with respect to a spherical mirror. Firstly, we will consider a point object. In the case of spherical mirrors, there are two types. The first type is a concave mirror, where the reflective side is on the outer side. That means the center of curvature and the outgoing light are on the same side. For this case, we consider the radius R to be positive. Whereas if the reflective coating is on the inside of the spherical mirror, that means the outgoing eye tray and the center of curvature are on either side of the mirror; then the radius is considered negative here r is the radius of curvature. Now let's see how an image is formed for a point object with respect to a concave spherical mirror. In the case of a concave spherical mirror, if an object is placed on the optical axis of the mirror at a point B, let's consider an incident light ray going from the object towards the mirror. It intersects the mirror at point B. Now, if we draw a normal at point B, because of the geometry of the mirror, the normal passes through the center of curvature. If we draw a perpendicular that is tangent to the spherical mirror through point B, then we can assume that the tangent is similar to a plane mirror, and the reflection of this incident light at point B is similar to a plane mirror along this tangent.

Thus, we get a reflected beam from point B, which intersects the optical axis at point P'. This is the point where the image of object P will be formed. Interestingly, as we can see, unlike before, the reflected ray is actually passing through the image. Thus, this image is a real image, and it can be captured on a screen.



Now let's see the geometric aspects of this reflection. Here on the right side, we can see the schematic of the frame where p is your object, c is the center of curvature, p' is the image, p is the point of incidence, and v is the vertex of the mirror. If you drop a perpendicular from point b onto the optical axis, the horizontal distance between that vertical and the vertex is delta, and the height of b from the optical axis is h. Now, in this construction, we find many different angles, alpha, which is the angle subtended by the incident ray with the optical axis. Beta, which is the angle subtended by the reflected ray with the optical axis; ϕ, which is the angle subtended by the normal with respect to the optical axis; and θ, which is the angle of reflection and refraction with respect to the normal. Now, considering the geometry, this angle is α, and this whole angle from the normal to the horizontal is ϕ. Thus, we get $\alpha + \theta = \phi$. Similarly, we get $\beta = \phi + \theta$. Eliminating theta from these equations, we get $\alpha + \beta = 2\phi$. Now let's see, we have three right-angled triangles here. The three right-angle triangles are PBO, CBO, and P'BO. Then the $tan\, \alpha = \frac{b0}{p0}$, which is h by the distance of the object from o, that is s minus delta. Similarly, the $tan\, \phi = \frac{h}{R-\delta}$. As shown here, we now take an approximation called the paraxial approximation, where we assume small values for alpha, beta, and phi; that is, alpha, beta, and phi are 10 to 0, and delta is far less than s, h dash, and r. Then these relations transform into $\alpha = \frac{h}{s}$, $\beta = \frac{h}{h'}$, and $\phi = \frac{h}{r}$. Substituting these simplified relations into $\alpha + \beta = 2\phi$, we get this relation, which gives the relationship between s and s', that is, the distance of the object and image. Thus, we obtain the object-image relationship for a spherical mirror, which is $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$.

PARAXIAL APPROXIMATION

NPTEL, IISc

- Look at the geometry of the figure. Angles $\alpha, \beta, \phi, \theta$ have the following relationships:

$$\phi = \alpha + \theta \qquad \beta = \phi + \theta$$
$$\alpha + \beta = 2\phi$$

- The three triangles with height h have these relationships:

$$\tan\alpha = \frac{h}{s-\delta} \qquad \tan\beta = \frac{h}{s'-\delta} \qquad \tan\phi = \frac{h}{R-\delta}$$

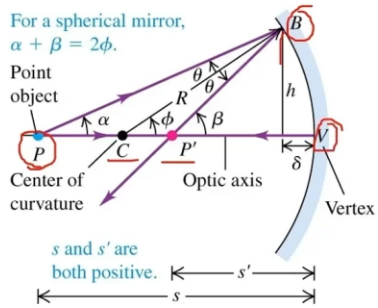
- We make the "paraxial approximation for small α, β, and ϕ

- Finally, we have

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$ object-image relationship for spherical mirror

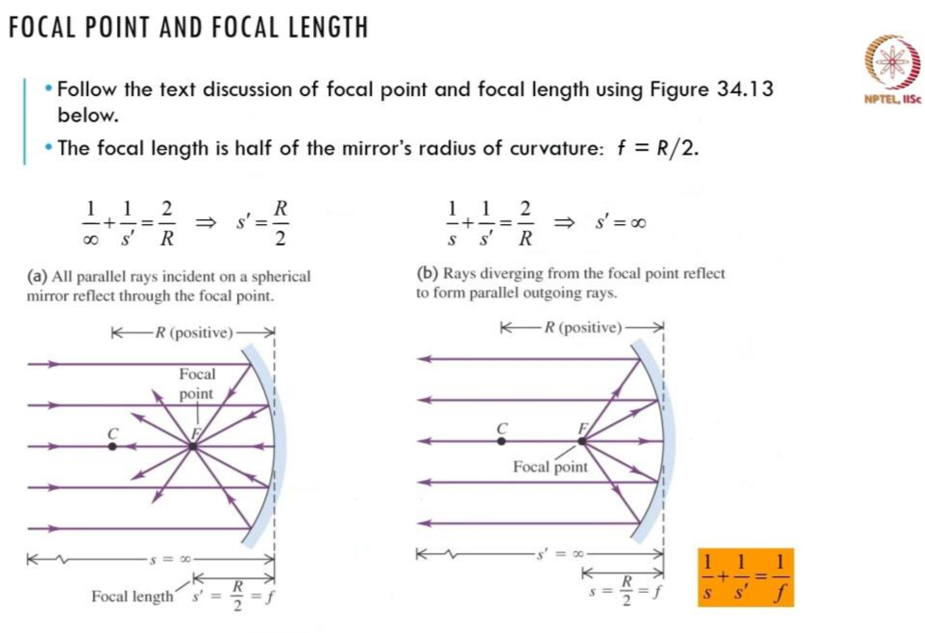
(a) Construction for finding the position P' of an image formed by a concave spherical mirror



Now, let's see the concepts of focal plane, focal point, and focal length. In a concave mirror, let's assume parallel beams are incident on this concave mirror; that means the object is at infinity, which means s is equal to infinity. Then, if we substitute s as infinity in $\frac{1}{s}, + \frac{1}{\infty} = \frac{2}{R}$, where s tends to infinity, then s' is obtained to be $\frac{R}{2}$.

That means all these rays reflect off the mirror and converge at a single point, which is at a distance of r/2. This point where all the parallel beams intersect is known as the focal point, and the distance between the focal point and the vertex is the focal length, which is equal to r/2. Thus, the focal length is half of the mirror's radius of curvature, which is f = r/2. Thus, as explained before, if we have parallel beams incident on the spherical mirror, the object is at infinity.

So, s is at infinity. Thus, these rays converge at a point called the focal point, which is at a distance of r/2 from the vertex. Conversely, if you have an object at the focal point, then all the light rays that travel outward from the focal point are reflected and converted into parallel light beams, implying the image at the focal point, that is h', is at infinity. Substituting f = r/2 in the image object relationship, we get this relation, which is $\frac{1}{s}, + \frac{1}{s} = \frac{1}{f}$.

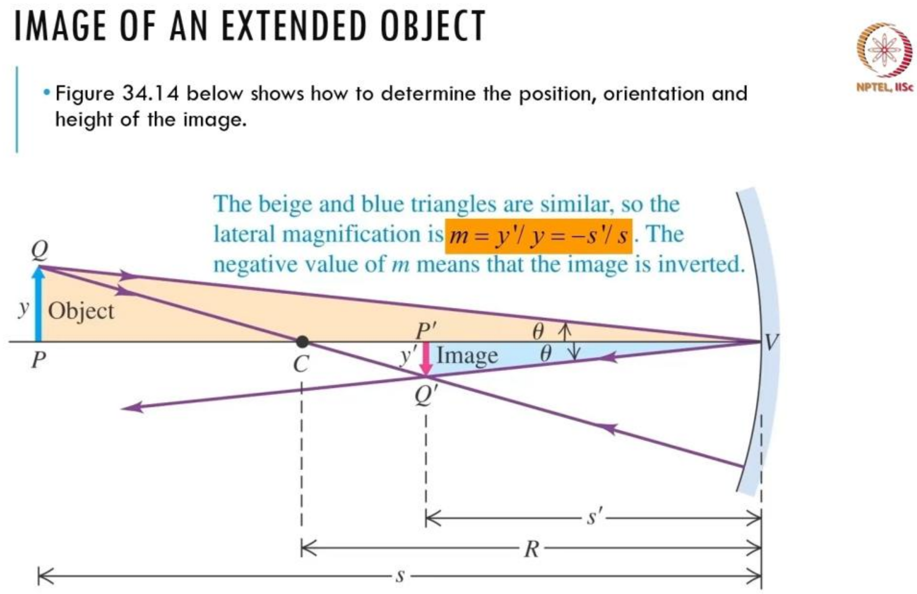


Now let's see the case where the object has a finite length. So far, we have only seen the object as a point object, which doesn't have any lateral dimensions.

Dimension, however, if you consider an object with lateral dimensions that is an image of an extended object, then we get another factor called magnification, where the lateral length or lateral dimension of the object can be magnified or diminished. So, let's assume the object to be PQ, where P is the point on the optical axis of the down cave mirror and PQ is the height of our object; Q is the topmost point of our object. Considering two incident beams from the object, the light ray that is incident on the mirror at the vertex gets reflected back at the same angle theta below the optical axis, whereas the light ray that passes through the center retraces the same path after reflection because of the geometry of the sphere. Thus, the point of intersection of these two light rays occurs at point Q' and thus P'Q' is our image of our object PQ. It is interesting to know that the distance of the image S' is less than S, as we can see here at the same time.

Because of this, since θ is the same geometrically, the height of the image, which is Y', is found to be less than Y. That is the height of the object. Thus, our magnification, which is the height of the image divided by the height of the object, is less than 1. That means the height of our image is diminished with respect to our object. When our object is placed beyond the center of curvature of a concave mirror, It is important to note here that when we say m is less than 1, it means that the magnitude of the magnification m is less than 1; that is, the magnitude of the image size is less than the magnitude of the object size.

However, in actuality, we have to consider y' as negative because it is in the opposite direction; thus, y' is equal to y. y dash by y is actually between minus 1 and 0; that means m is less than 0, which means the object is inverted.

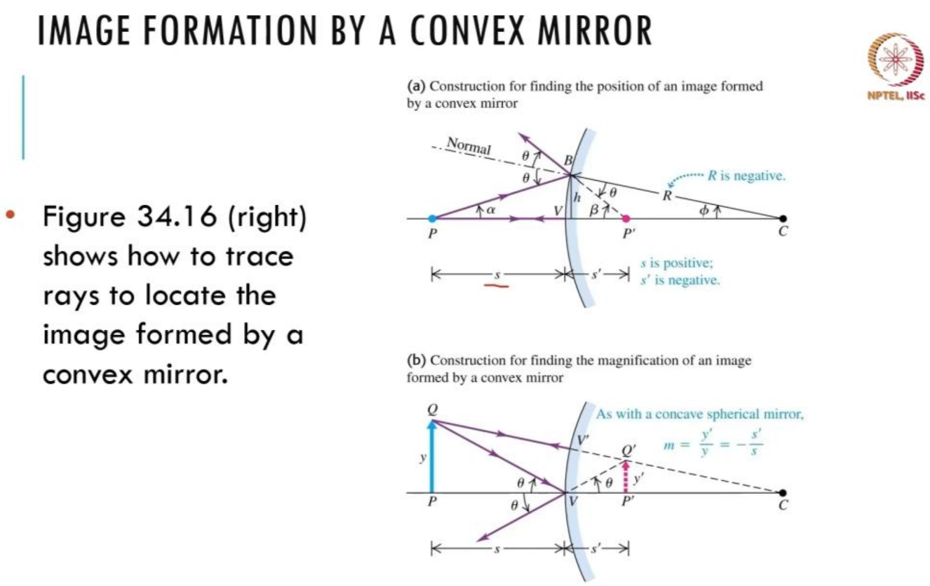


Now let's see how an image is formed if it is a convex mirror. In the case of a convex mirror, when a point object is placed along the optical axis at a distance s, we assume two light rays. One along the optical axis, which retraces back as it reflects, and another that gets reflected at a point B on the spherical mirror.

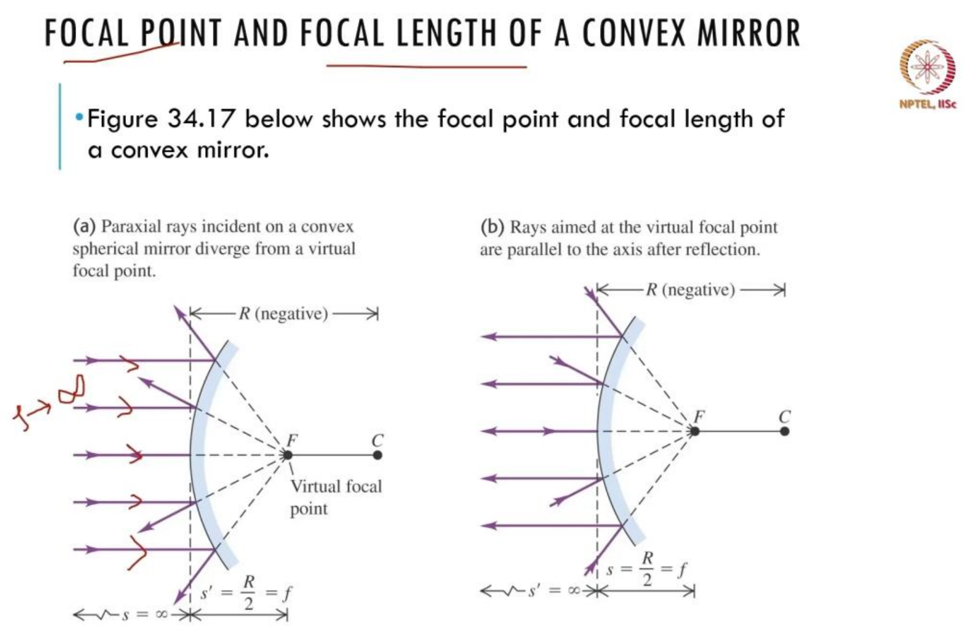
Assuming the tangential line at point B is similar to a plane mirror and using the law of reflection, we get a reflected ray. Now, extending this reflected ray back and extending both reflected rays back, both of them intersect at point P', which is where the image is formed. Since the reflected rays do not pass through the image, it is a virtual image. Now, looking at the geometry of the ray diagram, we have the angle alpha, which is the angle subtended by the incident ray with respect to the optical axis. Angle beta, which is the angle subtended by the reflected ray with respect to the optical axis, and angle phi, which is the angle subtended by the normal at the point of incidence that passes through the center of curvature.

And phi is the angle between this normal and the optical axis. Since the center of curvature and the outgoing ray are on opposite sides of the mirror, r is considered negative in this case. Similarly, if we construct a line diagram for an object with finite length pq equal to y, we get an image $q'p'$. With height y' here, as we can see, y' is less than y; that means for a convex mirror specifically, we can see that both the object and the image are pointing in the same direction. That means the image is erect and the image is not inverted, as we have seen in the concave mirror.

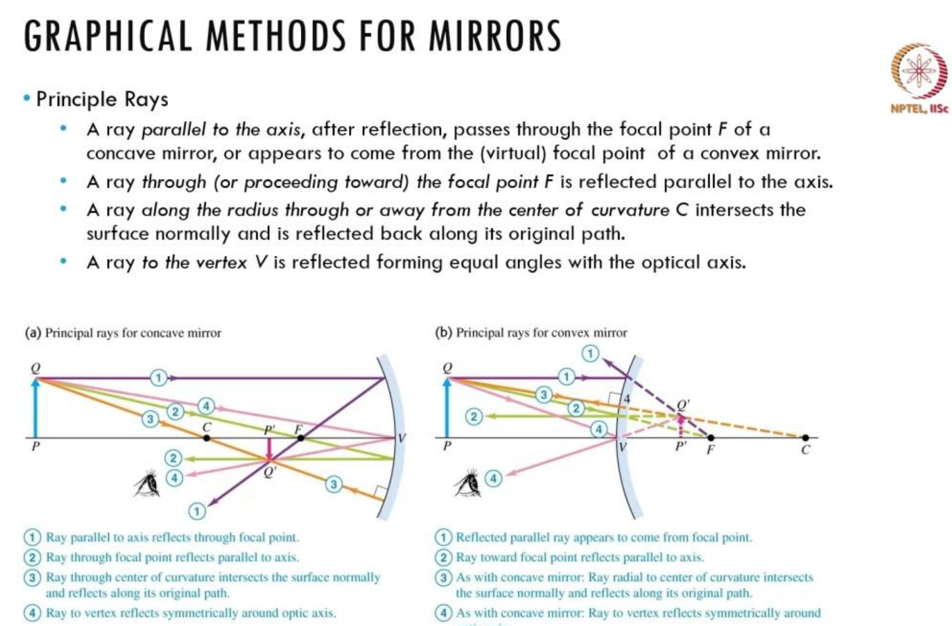
Thus, m equal to $\frac{y'}{y}$ is greater than zero. which corresponds to an erect image, and the magnitude of m is less than 0 because the size of the image is less than the size of the object. Thus, the actual value of m = (y' / y), which lies between minus 1 and less than m and less than 0. This also satisfies the ratio of $m = -\frac{h'}{f}$, where since the outgoing ray and the image are in opposite directions of the mirror, h' is negative, so we substitute s' less than zero; then m becomes positive, similar to a concave mirror.



If we consider the focal point and focal length for a convex mirror, doing the same exercise as before, where we assume parallel beams hitting the convex mirror, that is, h tends to infinity, then all these rays get reflected and diverge. However, if we extend these reflected rays inside the mirror, they all intersect at the focal point F, which can be obtained from the object-image relationship, which is $\frac{1}{s'} + \frac{1}{s} = \frac{2}{R}$, where when S tends to infinity, $s' = \frac{R}{2}$.



This slide shows the different concepts that we have discussed so far regarding ray optics and the graphical methods used in it. To obtain the location of the image corresponding to an object in the case of a spherical mirror. Both of these figures show the formation of an image corresponding to an object when the object has a finite length. The schematic on the left-hand side shows the reflection of a finite-length object using a concave mirror, whereas the schematic on the right side shows the reflection of a finite-length object using a convex mirror.



This slide shows how an image is formed when an object with a finite length is placed at different distances along the optical axis of a concave spherical mirror.

When the object is placed beyond the center of curvature, an image is formed between the center of curvature and the focal point; here, the image is inverted and real. And the size of the image is diminished, which means the magnification is less than 1. Now, when we move the object to the center of curvature, as shown here, the image is formed at the same location, which is the center of curvature, having the same size but inverted. Here, the image is real, and the magnification is 1. When we move this object between the center of curvature and the focal point, the converse of this case happens, where the image is formed beyond the center of curvature, which is inverted but real.

Here, the magnification is greater than 1, as the image is enlarged. Now, when we move the object to the focal point, all the light rays emanating from the object are converted into parallel beams. Thus, they won't intersect to form an image; thus, the image is formed at infinity when the object is placed at the focal distance. Now, when we move the object further, closer to the mirror, even closer than the focal length, then a virtual image is formed behind the mirror.