Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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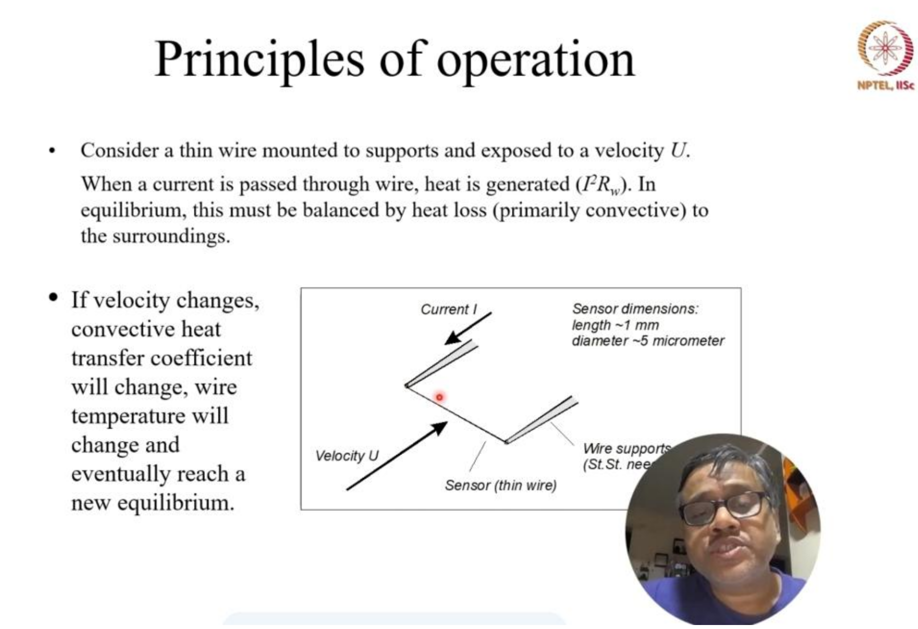
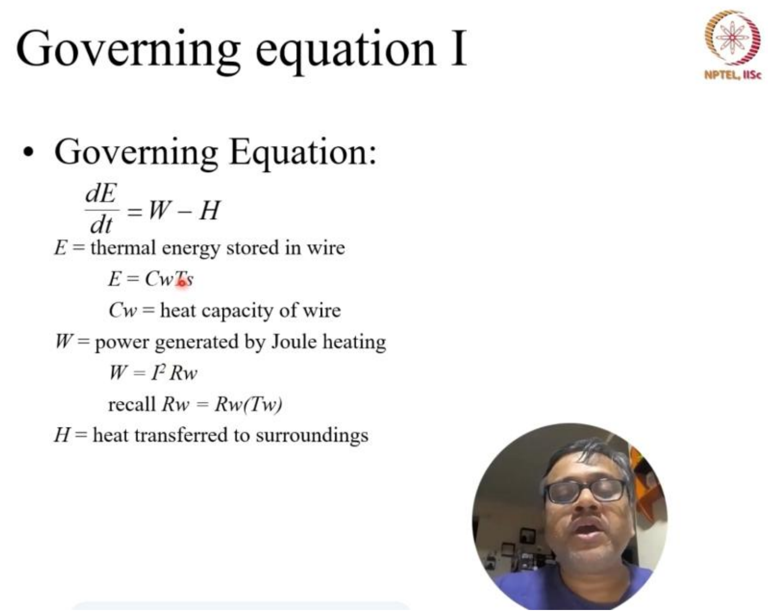
Week – 12
Lecture - 58
Hot Wire Anemometry – 2

So the principles of operation are very simple. So imagine that you have a thin wire that is mounted on two supports, right? When a current is passed through this wire, these are the supports; this is the wire. A current is passed through this wire, current I, for example, okay? The heat that is generated is nothing but I squared R. So in this equilibrium, this must be balanced by the heat loss, which is primarily convective to the surroundings. Alright, so for example, here the dimensions are the diameter is about 5 micrometers and the length is about 1 mm. So you pass a current, you generate what we call the resistive heating, so I^2R_w kind of heating; this is on the sensor.

But there is a flow now. Over the wire, the heat will be carried by the velocity, so whatever heat you are generating using the current is lost by the flow. If the velocity changes, the convective heat transfer coefficient will also change because H is a function of your Reynolds number. Therefore, the wire temperature will change and eventually reach a new equilibrium.



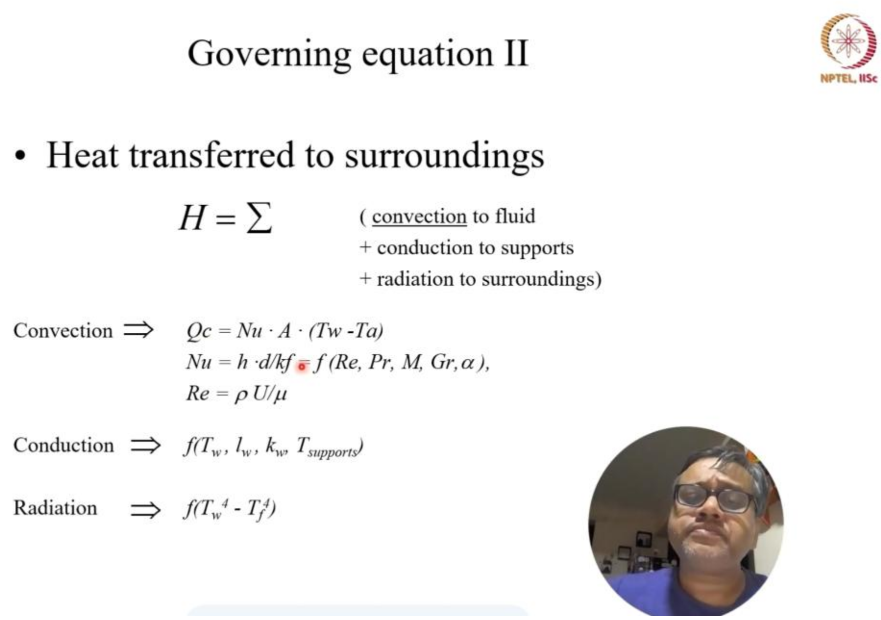
Okay, so velocity change is now, so when the velocity changes, you pull more heat; that means you actually draw more current, so you will reach a new equilibrium every time there is a shift in the velocity. Okay, so this sounds pretty simple: you have a wire, you are heating the wire, and the heat is being taken away by the velocity, and you are kind of comparing the two and inferring what the velocity will be because velocity is a variable that you ultimately are interested in, but you are doing it by passing a current through the wire. Okay, so the governing equation is this: the change in dE/dt, where E is the thermal energy stored in the wire, C_w is the heat capacity of the wire, and W is the power generated by the Joule heating, which is equal to I^2R_w. Recall that your R_w is a function of the temperature. Okay, so that is it.



And H is nothing but the heat that is transferred to the surroundings. So, it's a simple energy balance. So the rate of change of thermal energy that is stored in the wire is equal to the power generated minus the joule heating. So this will change the temperature of the wire, therefore. Okay, because C_w is the heat capacity of the wire and T_s is the temperature, right? So, W is the power that is generated by the joule heating, and H is the heat that is transferred to the surroundings.

All right, so this is the governing equation that we have; that is the heat that is transferred to the surroundings. Okay, that is what we are. That is what. It is dependent on the velocity, so this is convection to the fluid; the principal part of it is convection to the fluid, plus there is conduction also to the support. Remember, it is on the two supports, so the heat is conducted through the support, and there can also be radiative influences from the wire; you may actually lose heat by radiation.

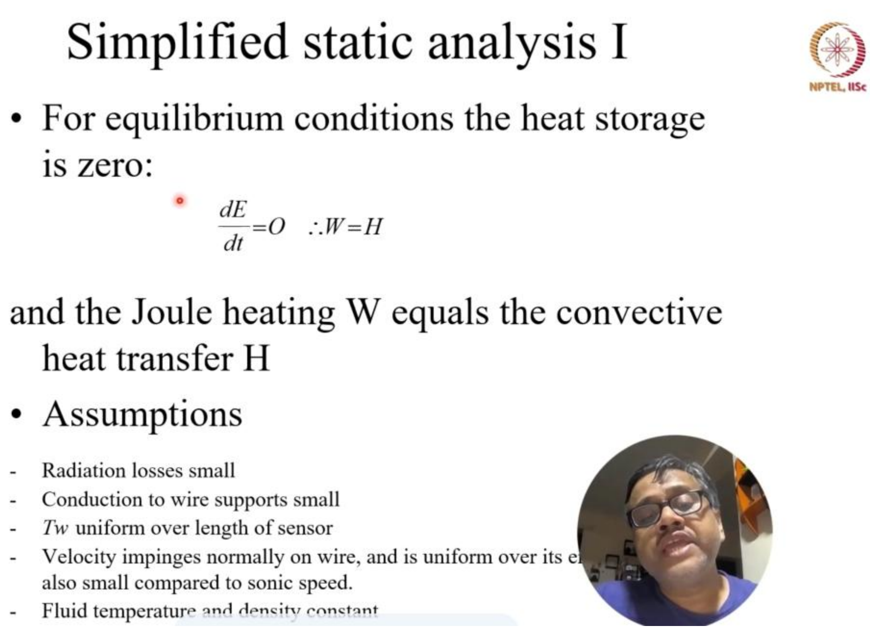
The convection, as we know, is given by the Nusselt number multiplied by the area. And there is a temperature differential, which is the temperature of the wire minus the ambient temperature. The Nusselt number is nothing but the heat transfer coefficient multiplied by d divided by K_f , where this heat transfer coefficient is a function of f, and this f is not the same as this; it is a function of the Reynolds number, Prandtl number, Mach number (if you have natural convection), Grashof number, and a whole lot of other parameters, and the Reynolds number is a function of the flow velocity. So directly, what happens is that the Nusselt number is a function of the flow velocity. Heat that is lost to convection is therefore also a function of the flow velocity, and this is the heat that is transferred to the surroundings.



Conduction is, of course, dependent on the wire temperature, the temperature of the supports, and all these parameters, and radiation is, of course, dependent on the fluid temperature and the TW, so it is given by the Stefan-Boltzmann law. Law, so you have a lot of heat loss sources. The principal component of that is the convection to the fluid. Okay, so for equilibrium considerations, there is no change in the thermal energy of the wire; therefore, this is equal to zero. This mandates that your w has to be equal to h, or in other words, the joule heating equals the convective heat transfer h.

So, more assumptions are: The radiation losses are small, the conduction to the wire supports is small, and T_w is uniform over this, which means the wire is uniform because it is a very thin wire; thus, you are assuming that there is no temperature gradient across the wire, either along the length or along the cross-section, and the velocity impinges normally on the wire, so the wire is kept like this. The velocity is impinging normally to the wire, and it is uniform over its entire length. That means in this particular length, if there is a variation of velocity, it is averaged out. Okay, the flow velocity is small compared to the sonic speed, and both the fluid temperature and the density are actually constant. Okay, so these are the factors; if you have all these things ironed out, then these are the assumptions that you are making for this particular case.

Now consider a wire that is immersed in a fluid flow, and assume that the wire is heated by an electrical current and is in thermal equilibrium with its environment.



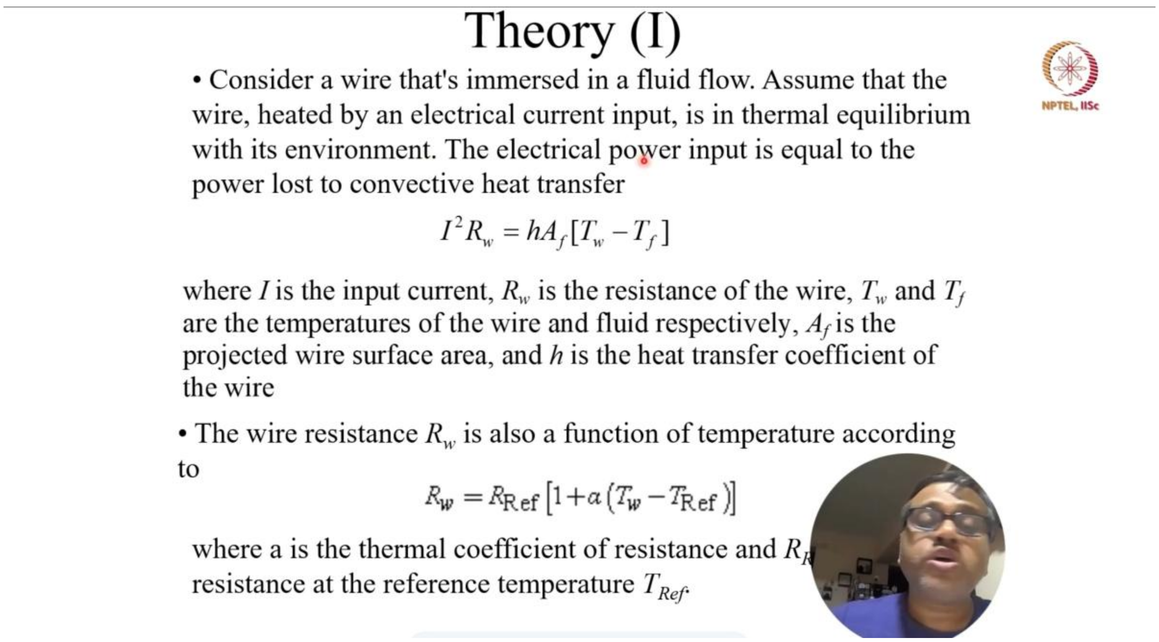
Then the electrical power input is equal to the power lost by the convective heat transfer. Remember, we neglected radiation; we neglected conduction; we neglected all that. So what happens is that this $I^2R_w = hA_f(T_w - T_f)$.

So this is what the expression means. This is already known to us. This is W equal to H. That is, the heat that is generated is taken out by convection.

All right. Now, the wire's resistance is also a function of the temperature. So according to this particular expression, So this Rw is dependent on some reference temperature, and it is given by $1 + \alpha$. This is α. This is not A; therefore, α is the thermal coefficient of resistance. And then there is a temperature differential, with some reference temperature taken from the temperature of the wire.

So this resistance of the wire is actually a function of the temperature at which the wire is at that particular moment. Therefore, this is needed when you want to evaluate this $I^2 R_w$. You are comparing it with the convective heat transfer, the power that is lost due to convective heat transfer. This is very important because the wire resistance is also a function of the temperature. We said this earlier, and we are saying it again that it is dependent on the temperature.

The resistance is captured at some reference temperature, and then there is a thermal coefficient of resistance.

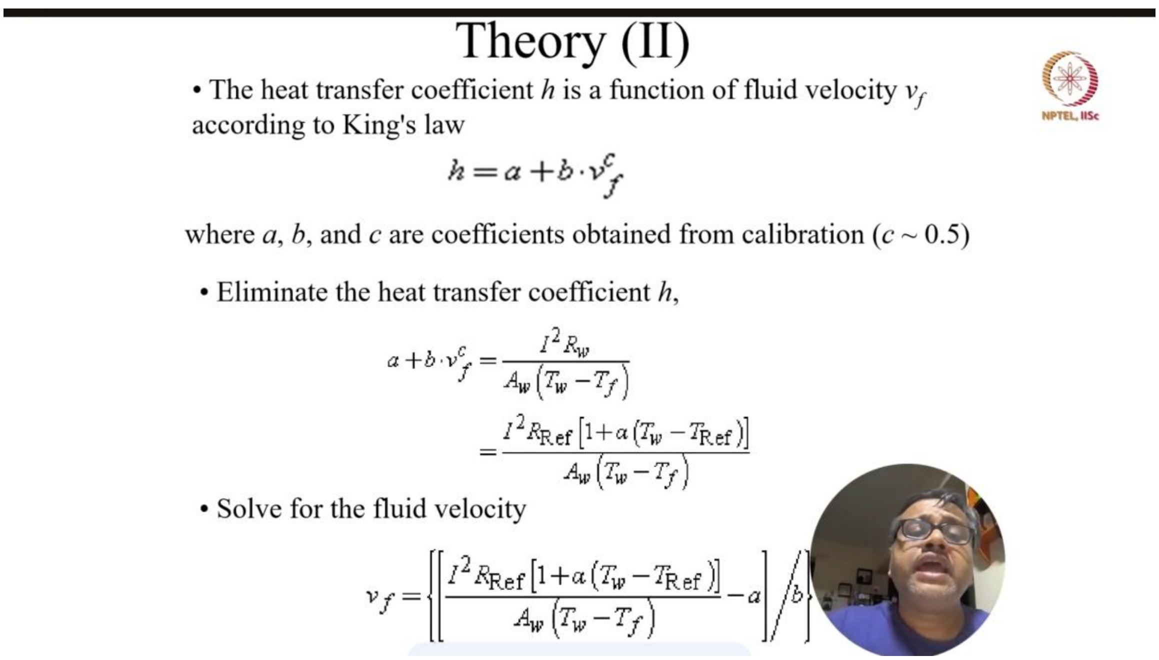


The next part of the theory is that the heat transfer coefficient h is a function of fluid velocity, as we already mentioned, and here we state that it is dependent on something called King's law. Not really a law; this is more of an empirical expression where it says that h is equal to a plus b multiplied by the velocity of the fluid raised to the power of c. So, a, b, and c are basically curve fit coefficients, and they are obtained for the calibration, and c is roughly of the order of about 0.

5; it's half. Because the Reynolds number is always of the order of half in the case of forced convection, ABC are coefficients that are determined for the calibration. Unlike LTV, where you did not need any calibration, here you do need a calibration a priori to know what these a, b, and c coefficients are all about. So, if you eliminate the heat transfer coefficient h now. By substituting this functional form, which is a function of the flow velocity, so that it is equal to this, and then you also substitute Rw, which is a function of the resistance calculated at some reference temperature, and then you deconvolute it to find out the fluid flow velocity, this gives you this kind of expression.

So it is nothing. It's just algebra. So you take a to the other side, and then b, you divide it out, and then you take the 1 by C-th root of the whole thing. So this is what the expression means that you get. This is all in terms of the native variables. The reference is already known.

It is calculated at a certain temperature, and alpha is also known. You know that A and B are obtained for the calibration, and therefore by measuring the current, you can actually measure what the flow velocity V_f is going to be. So, it is not a linear function, obviously, but once you measure it, you can actually now get a value of what your V_f is is going to be.



So now there is a constant-temperature hot-wire anemometer. For a hot wire anemometer that is powered by an adjustable current, what happens is that to maintain a constant temperature, Tw and Rw are therefore constants.

So, the fluid velocity is a function of the input current and the flow temperature. And the flow temperature is the fluid flow. So the temperature of the flow can be measured. The fluid velocity is therefore reduced to a function of the input current. Understood? So if it is maintained at a constant temperature, this is known.

T_f is is something that you already know a priori because we say that the flow is known, so you can just stick in a thermocouple and know the flow velocity, and the rest of the parameters are known, so your I is the only variable. You know how much current you are pumping in, and with that current, to maintain that equilibrium, W is equal to H. Therefore, once you know the current, you know what the fluid flow velocity is going to be. So that can be calculated. So this is a constant temperature hot wire anemometer (CTA).

All right, then there is a constant current hot wire anemometer.

Theory (III)

Constant-Temperature Hot-Wire Anemometers

For a hot-wire anemometer powered by an adjustable current to maintain a constant temperature, T_w and R_w are constants. The fluid velocity is a function of input current and flow temperature

$$a + b \cdot v_f^c = \frac{I^2 R_w}{A_w \left(T_w - T_f\right)} = f\left(I, T_f\right)$$

The temperature of the flow T_f can be measured. The fluid velocity is then reduced to a function of input current only

Theory (IV)

Constant-Current Hot-Wire Anemometers

For a hot-wire anemometer powered by a constant current I, the velocity of flow is a function of the temperatures of the wire and the fluid

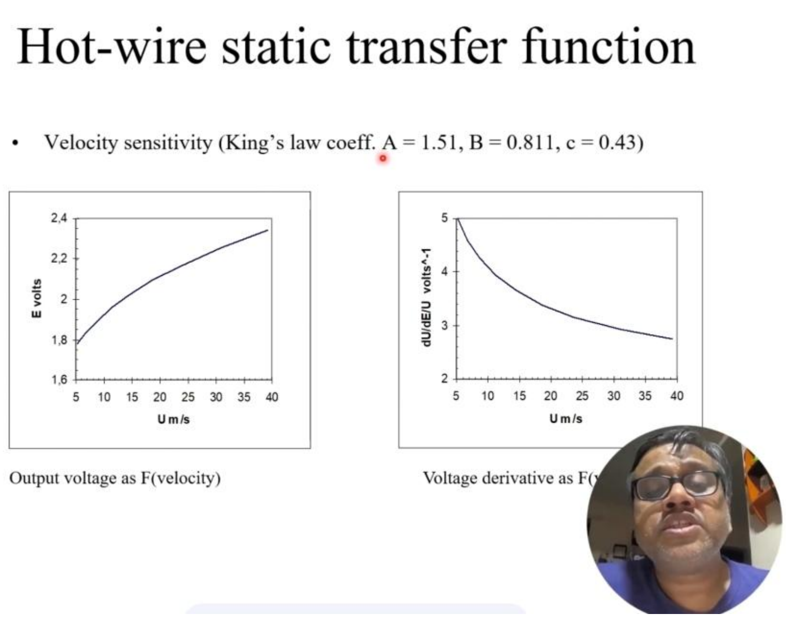
$$a + b \cdot v_f^c = \frac{I^2 R_{\text{Ref}} \left[1 + \alpha \left(T_w - T_{\text{Ref}}\right)\right]}{A_w \left(T_w - T_f\right)} = g\left(T_w, T_f\right)$$

If the flow temperature is measured independently, the fluid velocity can be reduced to a function of wire temperature T_w alone. In turn, the wire temperature is related to the measured wire resi... Therefore, the fluid velocity can be related to the wire r...

Now, for a hot wire anemometer, which is powered by a constant current, the velocity of the flow is a function of the temperature of the wire and the fluid. Alright, so it depends on the two temperatures, dw and tf. Now, the flow temperature is measured independently, so this is just plugged in. The fluid velocity can be reduced to a function of the wire temperature alone, therefore.

The wire temperature is related to the measured wire resistance. As a result, the fluid flow velocity can be related to the wire resistance. So this is a constant current hot wire anemometer. To understand the concept, we use a constant current; the previous one was variable current. The temperature actually changes to establish equilibrium, so the temperature goes high or low.

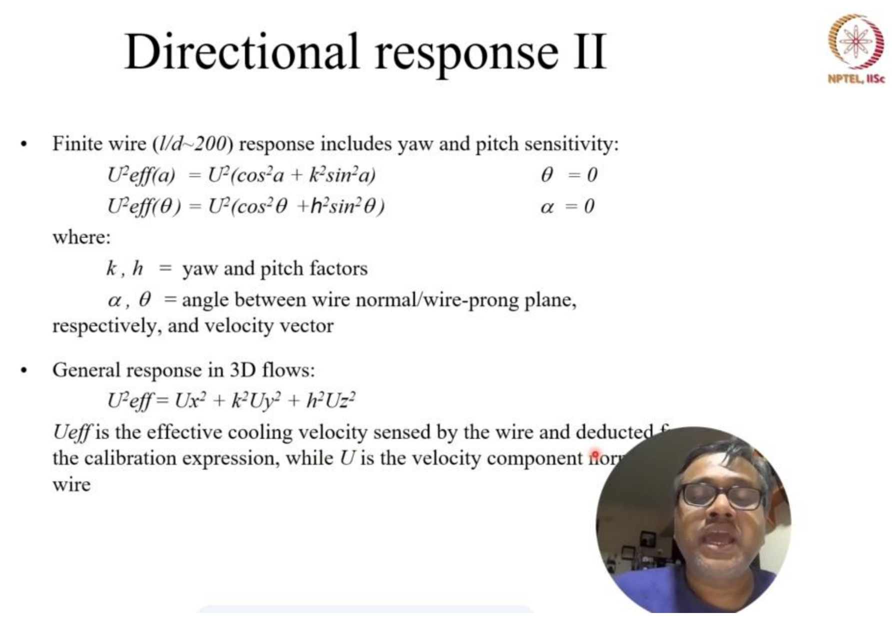
What happens is that you know the fluid flow temperature because that is calculated independently. This temperature, Tw, therefore, means that the fluid flow velocity is a function of Tw alone. So how do you know the wire temperature? It is related to the wire resistance. Because you know that what resistance will give you that exact equilibrium, the fluid flow velocity can now be related to the wire resistance. These are the indirect ways by which you can calculate what your fluid flow velocity will be.



All these things require calibration. Okay, so the simplified static analysis is, you know, W is equal to H, equal to I squared times R. This is the expression. This is the ambiance. You can write it as TA or TF, right? So this is the expression.

H is the film coefficient. This is the transfer area. K_f is is the thermal conductivity of the fluid. This is the dimensionless heat transfer coefficient. So in a forced convection region, the Nusselt number is given by this, which is basically nothing but King's law. As we said, this is how it is done; this I^2R_w is also measured as the voltage drop, so this can also be used to measure the velocity.

This is a way by which you can do all these measurements. Basically, the concept is pretty simple: there is Joule heating, and therefore there is convection heat transfer. You relate the two, essentially, and this expression for the Nusselt number is very common, and therefore King's law is also something that we are kind of very much familiar with. So this is the static function; if the velocity changes and you know the voltage derivative, this is the output voltage as a function of the velocity, and this is the voltage derivative. So this is how the wire remembers that any transducer or measurement system has to respond to the physical quantity that you are trying to measure.

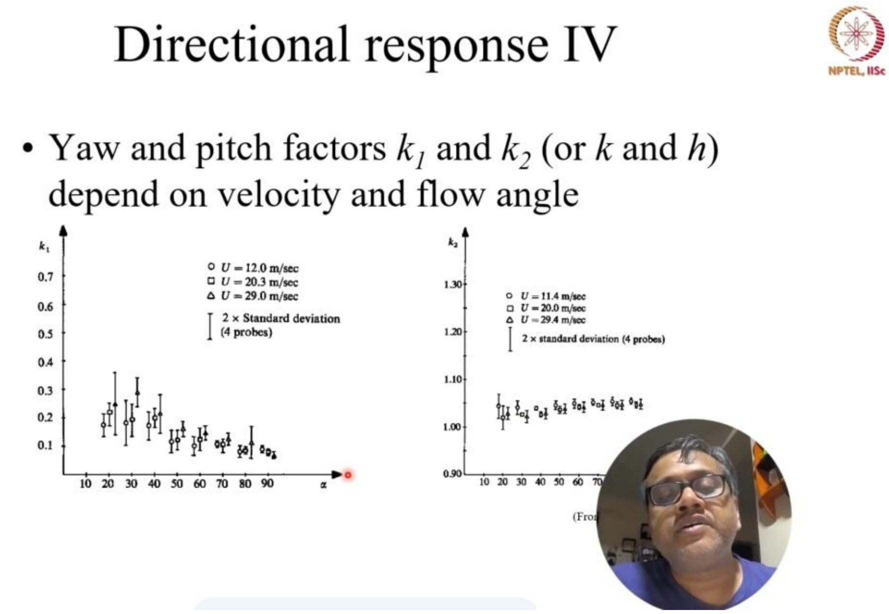


Okay, so that is the most important part. If this changes by a very small amount, then your measurement sensitivity is going to be a problem. So, the velocity sensitivity is the King's law coefficient, which is given by A, B, and C essentially. Okay, and so that is an important part of this exercise. So now you have this, you know the directional response now. The probe coordinate system is very important because, as we said, the velocity should be perpendicular to that wire.

Okay, so here you see that this is x, y, and z, and the velocity can be coming from a very arbitrary direction. So the velocity has, if you vector decompose it, actually got three components. So the velocity vector is composed of three components: u_x the tangential velocity, and u_z, the binormal velocity. So this is the wire when the flow velocity is coming at a very strange angle with respect to the wire because this alignment may not always be very easy to achieve. Okay, so the finite wire dimensions—if you have a finite wire where the l/d is about 200—so the response includes yaw and pitch sensitivity.

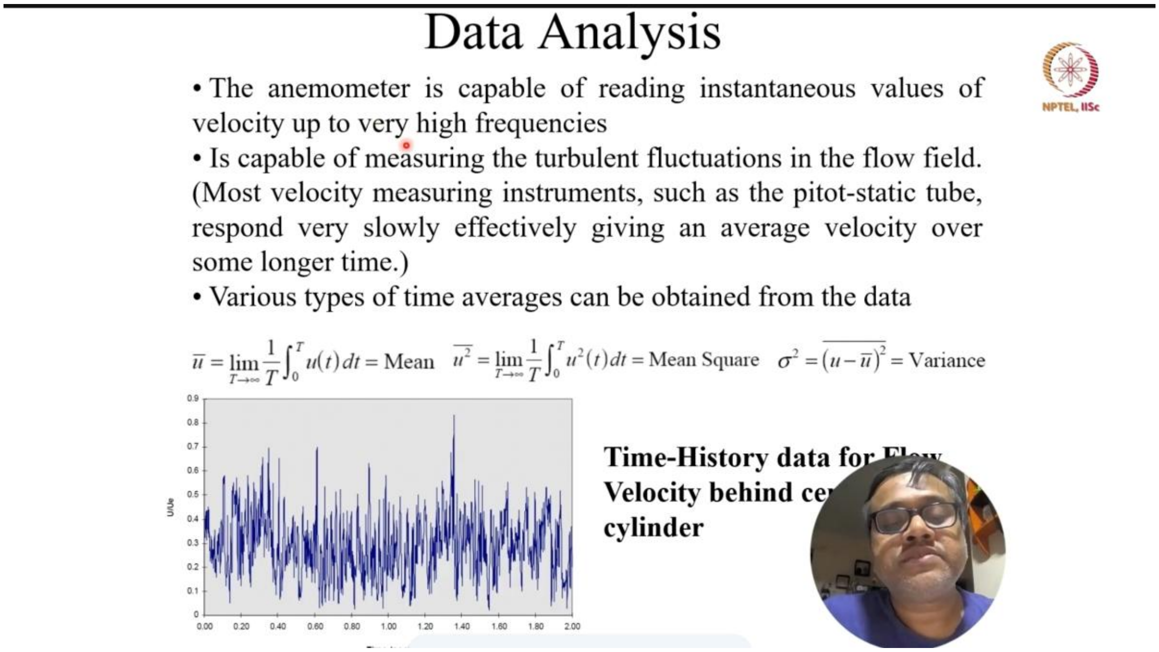
Thus, the u effective is basically given by this. And u is effective for alpha equal to zero, and this is where k and h are the u and the pitch factors, essentially over here. α and θ are basically the angles between the wire normal and the velocity vector. Okay, so this is the effective velocity that it is kind of actually measuring. So, the general response in a 3D flow is something like this.

So this is a U_x response, and then there are, you know, contributions from U_y and U_z as well. So the U_f is an effective cooling velocity that is sensed by the wire and deducted from the calibration equation. Well, U is the velocity component normal to the wire. The U component is a velocity that is normal. The typical directional response of a hot wire probe is pretty complicated; it requires a lot of a priori calibration and response function testing.



Okay, so these are the yaw and pitch factors k1 and k2, or k and h, depending on the velocity and the flow angle. So you can see how. You know, for different velocity values, uh, okay, so this is a two standard deviation, and these are four probes kind of method, so you can see how much, uh, you know, this varies with alpha, you know how the k1 and k2 actually vary. Okay, so this is how it is dependent on the flow velocity and the flow angle. So the flow angle is that alpha, which indicates how it is attacking the thing.

So, for a very large flow angle, for example, K2 or H is approximately 1. This actually has a variation of about 0.2 to about 1. So this is the directional response bar. So what we do in terms of data analysis is that the anemometer is capable of reading instantaneous values of the velocity up to very high frequencies.

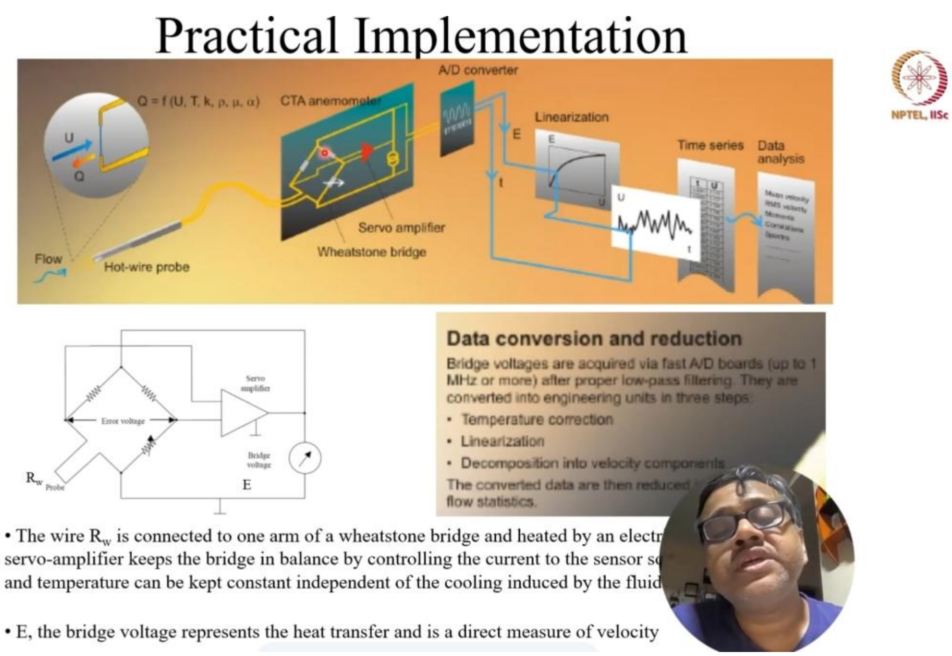


So it is capable of measuring turbulent fluctuations in a flow field, for example. So most other probe-based measurement techniques, which have a physical probe like the p-dot static tube, respond very slowly, giving an average velocity, so you won't have the temporal resolution that you need. So you can basically see this as a time history data for a flow behind a cylinder. And you can see that these are the flow fluctuations as a function of time.

Remember, it's almost an analog response. So you can now use this to calculate the mean over a certain time period, the standard deviation, or the variance, and things like that. You can actually take the time history data and determine all the statistics that are essential when dealing with high-speed flows, so to speak, or flows with turbulent fluctuations, allowing you to find out all the turbulent statistics, including turbulent kinetic energy and whatnot. By using this thing, remember that the active element is very small, so it has a very low thermal mass; as a result, it can respond very quickly. This is taken from Dantec Dynamics, so you have the implementation.

You have a flow and the heat loss. Now, this is the hot wire probe; this is effectively connected to a servo amplifier and a Wheatstone bridge. Then you pass it through an analog-to-digital converter. Then you kind of linearize it, generate the time series data, and perform the corresponding data analysis. So what happens is that this RW of the probe is connected to one arm of the Wheatstone bridge.

So the Wheatstone bridge has four resistors. So one arm, this is the resistance of the wire; this particular wire is now connected to one arm of the Wheatstone bridge and heated by an electrical element.



The servo amplifier, what does it do? It keeps the bridge in a balanced condition by controlling the current to the sensor section. And the temperature can be kept constant, independent of the cooling introduced by the fluid. So the E, which is the bridge voltage, represents the heat transfer and therefore is a direct measure of the velocity.

We have already seen this. So the servo amplifier basically keeps the Wheatstone bridge in a balanced condition. And this is actually incorporated into one of the arms of the Wheatstone bridge. The actual probe, that is how these things are done.