Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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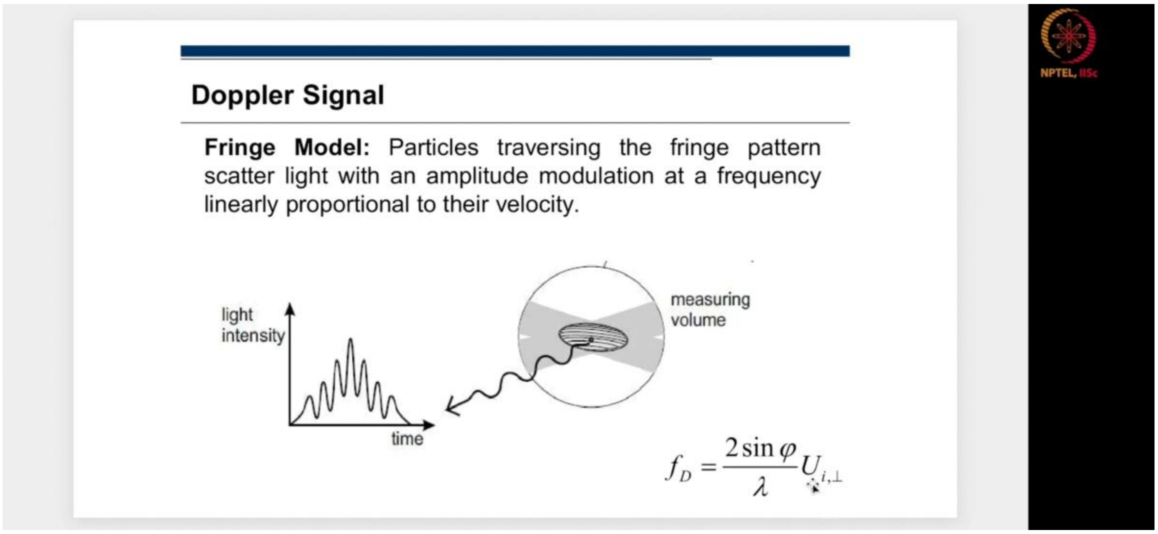
Week – 09
Lecture - 44
Laser Doppler Velocimetry – 3

So, coming back, you have gathered that in the case of LDV, two beams intersect, and you get an interference pattern that exhibits constructive and destructive interference. And the fringe spacing is given by this, where this Θ is basically the included angle or the angle between the two beams. As a result of that, the particle that is traversing the fringes will scatter alternatively high and low intensity values, and the frequency of the light intensity changes can hence be used to determine the velocity of the particle normal to the interference fringes. Remember, this is the velocity that is normal to the fringes. And this is the formula. So, this is very easy.

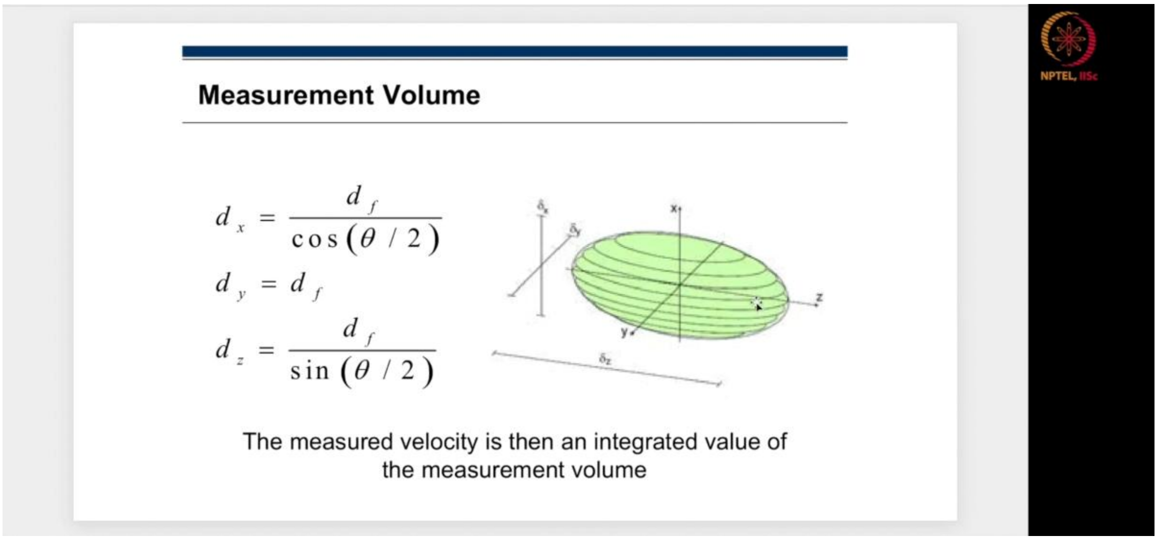
If you can measure this, it is on the order of 1 to 100 megahertz. The lambda is well-known, so it should be possible for us to calculate it. So now if you look at the fringe model, the Doppler signal, the particles that are traversing the fringe, if you look at that, the fringe and the particle just go through the fringe, through this measurement volume, so to say, it shows an amplitude modulation at a frequency that is linearly proportional to their velocity. So this is FD, and this is proportional to the velocity component that is normal to the fringes.

Fringe lines. And it is related to it by a purely geometric factor and the wavelength of the laser. So the light intensity, if you look at it, fluctuates like this. If you see the intensity of the light, it fluctuates like that. So there are high, low, high, low, high, low.



And this varies over time because of the particle transit. And this amplitude modulation happens at a frequency that is linearly proportional to the velocity, so this is how a detector should actually see it, because here at the dark fringe it will scatter less light, and at the bright fringe it will scatter more light, so the light intensity therefore changes quite a bit. Now, if you look at the measurement volume now. You will find that this is rather interesting. So dx and dy are more or less of the same order because the Θ angle is not that significant.

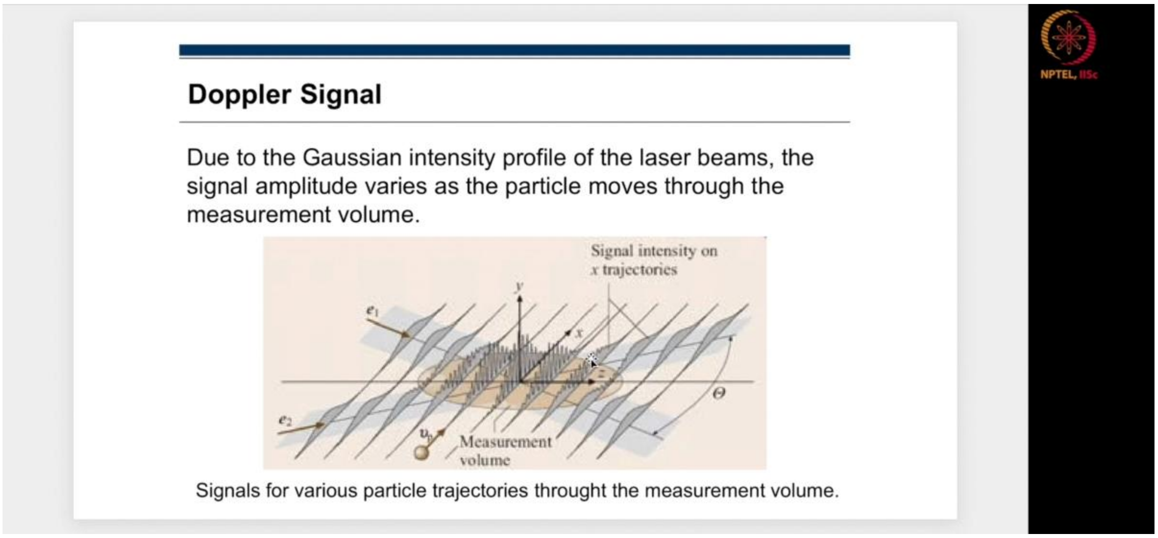
So they are of the same order. So dx and dy are kind of very similar. You can calculate this. And but the dz, uh, because it has a $sin\theta$ by two component, okay, so this shows a little bit of an enlargement along the direction, uh, along the optical axis direction, so it more resembles a cigar type of volume in which the z is actually extended, while x and y are of very similar orders. So the measured velocity that you obtain is an integrated value over this measurement volume.



Now, this measurement volume, therefore, is not a very symmetrical volume. It has lower dimensions on the x and y sides and higher dimensions along the z. And this is how the fringe lines also appear. So, the measured velocity is an integrated value of the measurement volume. This is the important part to know about this.

Now, if we look at the Doppler signal, due to the Gaussian intensity profile of the beam, as you can see, this is the Gaussian profile of the first beam, E1. This is the Gaussian distribution profile of the beam, E^2. So these beams now start to interact in this particular area. But this zone, if you see, is not very homogeneous, so to speak. So, for example, a particle that is going through the middle will see at the edge and will start to see the burst.

OK, and then as it passes through the middle where the two beams have kind of intersected, it will see more bursts. I mean, this modulation kind of taps off at the tail. It will show kind of. So a particle that is passing at the edge, for example, sees only a single beam. Then it starts to see that it comes to the edge of the beam, and then it starts to see the second beam.

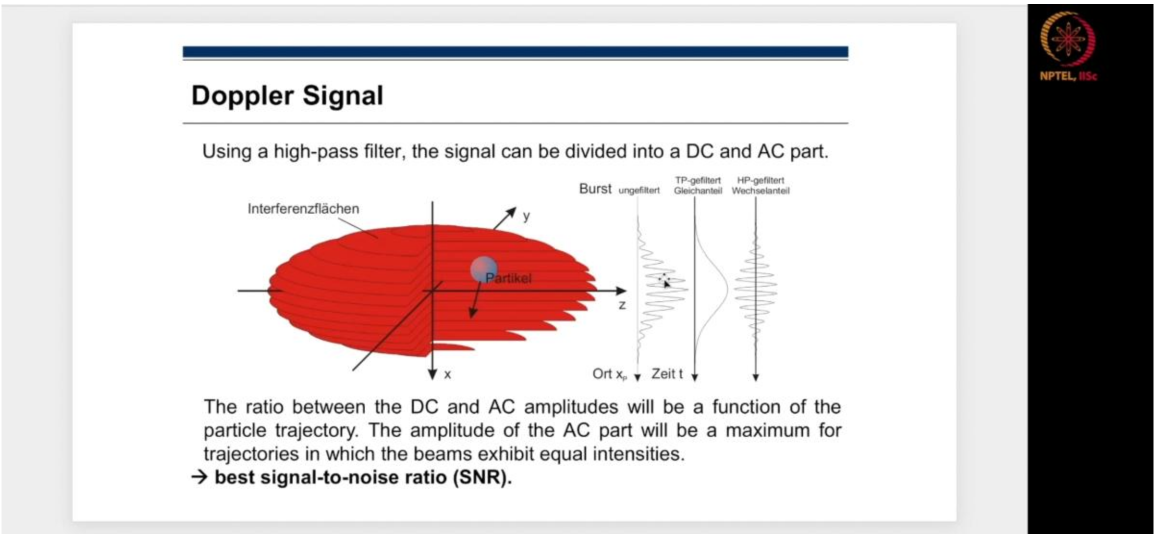


So this amplitude modulation or intensity modulation for a beam or a particle passing at the edge of this measurement volume will be very different from a particle passing through the center of the measurement volume. So, due to the Gaussian intensity profile, the amplitude varies; the signal amplitude varies as the particle moves through the measurement volume, and the signals for different particle trajectories are actually different throughout the measurement volume. So you see the edge; it is something different because here it sees one beam, then it sees the other beam. There's a very small degree of overlap that may actually be there. In some cases, it may be just seeing one beam and then the other.

So all these things have to be taken into consideration when we actually show this. So, this is rather interesting. If you look at it that way, it is quite interesting. And this is what actually happens. So, more on this.

If you consider this to be the kind of interference fringes, again, there is a little bit of German here. This is from Professor Tropia's lecture notes. If you look at these, they are kind of like slices, three-dimensional interference slices. So to say, what happens is that as the particle—so this is X and Y, and then this is Z—as it passes through this measurement volume, this is typically what we call a Doppler burst. So you can see that there are, depending on how the particle is moving, amplitude modulation.

When the modulation is off, you know, if this goes to zero, then of course you have a modulation that is kind of one. So this is the kind of typical signal that you receive. So if you look at it carefully, this signal has a varying mean or an envelope function, something like this, and then there is modulation that sits on top of it. It is almost like the ratio between a DC amplitude and an AC amplitude, and this will be a function of the particle trajectory. The amplitude of the AC part will be maximum for trajectories in which the beams exhibit equal intensity.



So this is required for the best signal-to-noise ratio. We have an enveloping function, and then you have a fluctuating function of it, much like a DC and an AC component. Right, so if you now look at it, the ratio between the DC and the AC amplitude is known as the modulation depth. Now, when you examine the modulation depth carefully, it is $\frac{2(a_1)(a_2)}{a_1^2+a_2^2}$. So when you actually have your a_1 and a_2 to be the same, Okay, this modulation depth becomes equal to one.

Or in other words, if you look at it, when this becomes, when this goes absolutely to the end, okay, so then this amplitude actually becomes equal to one. Other than that, if that is not the case, then you get something like this. So if you look at this, this is the amplitude for this, and this is the amplitude for the AC part of the signal, all right? So if you now kind of classify this, you will see that there is the total signal, and this is the mean, some kind of offset basically, and then this is the fluctuating signal. So, for waves of equal amplitude, the modulation achieves a maximum value of one. The signal is then low-pass filtered to improve the signal-to-noise ratio.

So this is what happens when the particle passes through these Doppler fringes. So how this happens is that you have a Doppler burst, and your photo detector sees something like this. Then you pass it through an amplifier and then you pass it through an adjustable bandpass filter. What the bandpass filter does is that there's a high-pass filter, which removes the low frequencies arising from the Gaussian intensity profiles of the beam.

So, that is the envelope. And then there is a low pass filter which removes the high frequency electronic noise that is kind of embedded into the system. So you pass it through these kinds of filters to basically generate the final output. OK, so you can see that there is a DC component, and then there is an AC component. I want to emphasize once again that this is the total signal.

This is the envelope. And then you have the other pedestal functions. And this is basically the envelope. Modulation and the modulation depth are equal to one when a1 and a2 are basically the same, and also remember that the particles passing through different areas or different trajectories across the measurement volume produce those different types of signals, so to say. So this is how the photodetector assembly actually happens. So the measurement volume is defined by the amplitude of the modulation and the region in which the velocity measurement is performed.

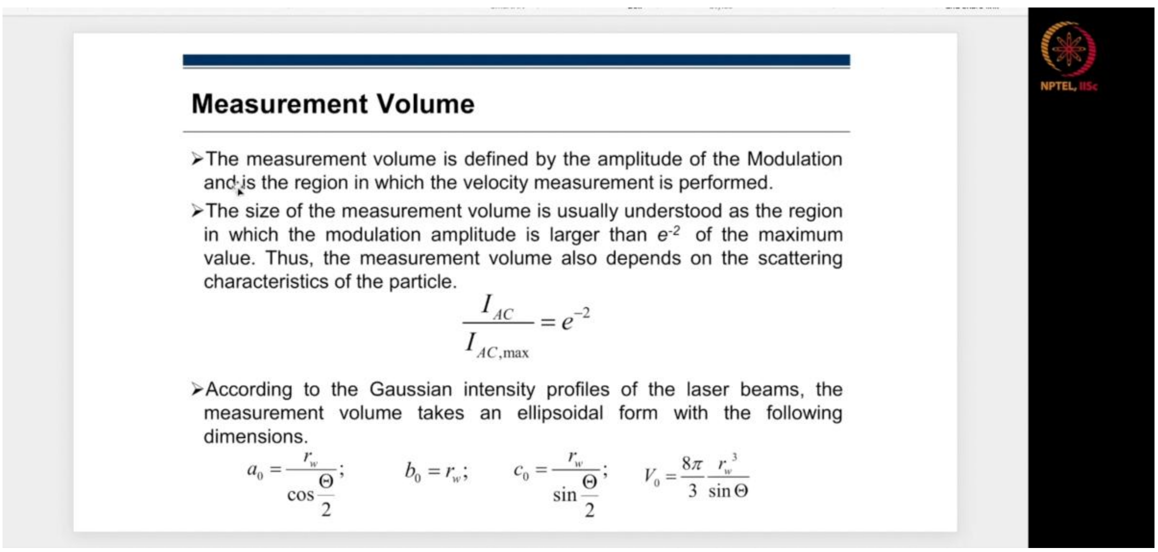
Now, the size of the measurement volume is usually understood as a region in which the modulation amplitude is larger than e to the power of minus two times the maximum value. As the measurement volume also depends on the scattering characteristics of the particles. So this is important, right? Because you know that we are basically putting certain cutoffs. So normally, in the intersection of the two beams, that particular volume can have many types of scattering particles. But then if you place certain filters in place, which basically reject low intensity versus high intensity, then you are pre-selecting only a part of the measurement volume.

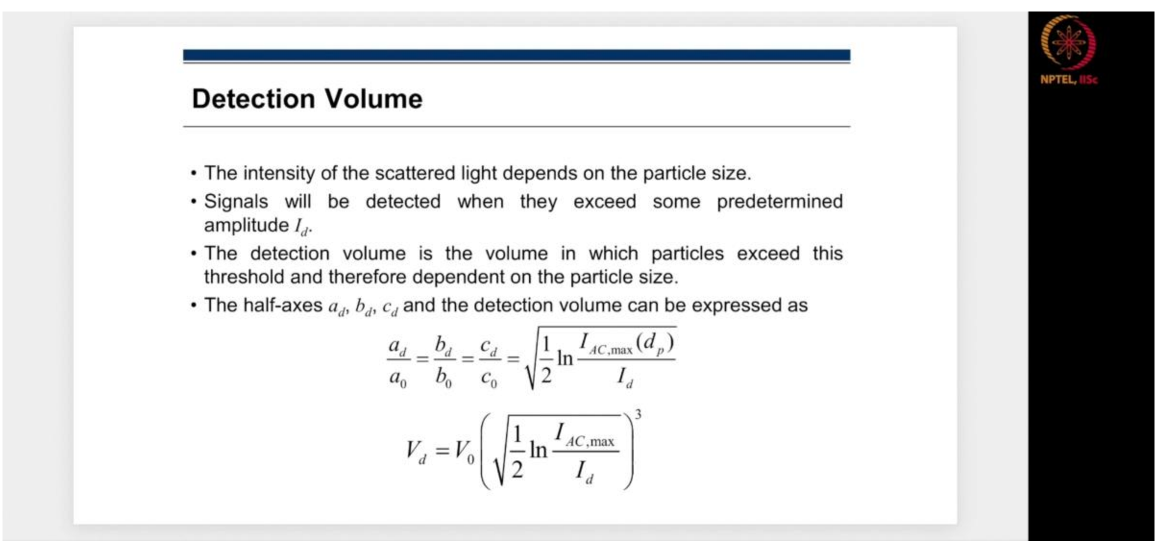
That means the signals from all other parts of the measurement volume will be basically ignored, or you are throwing them out. So in effect, your measurement volume or the actual acceptance level of the measurement volume may be actually smaller than the actual measurement volume that is defined by the crossing over of the two beams. So the measurement volume, as you can see, is $\frac{I_{ac}}{I_{ac,max}}$ which is e^{-2}. So according to the Gaussian intensity profiles of the beam, the measurement volume takes an elliptical form with the following dimensions that you see over here. Okay, so these are the dimensions of the measurement volume.

And this is, these dimensions are basically shown here. This is A naught and B naught; you know, the Δt is basically the time. And these are the signal characteristics as the particle passes through it. And you can see that $\frac{I_{ac}}{I_{ac,max}}$ which is e^{-2}. And so all these characteristics are actually marked here.

So you pre-choose, define what is important for you, and then can down-select only a portion of the measurement volume. So the detection volume, therefore, since the intensity of the scattered light depends on the particle size, signals will be detected when they exceed some predetermined amplitude, ID, for example. So the detection volume is the volume in which the particles exceed this threshold, and therefore it depends on the particle size. So the half-axes are AD, BD, and CD, and the detection volume can be expressed like this. So this is the max, and this is the ID.

So this is a predetermined value, a predetermined amplitude. And so, the detection volume is the total volume multiplied by this particular term over here. All right. So, this is the expression for the detection volume. In other words, the relationship between the measurement and the detection volume is therefore like this.

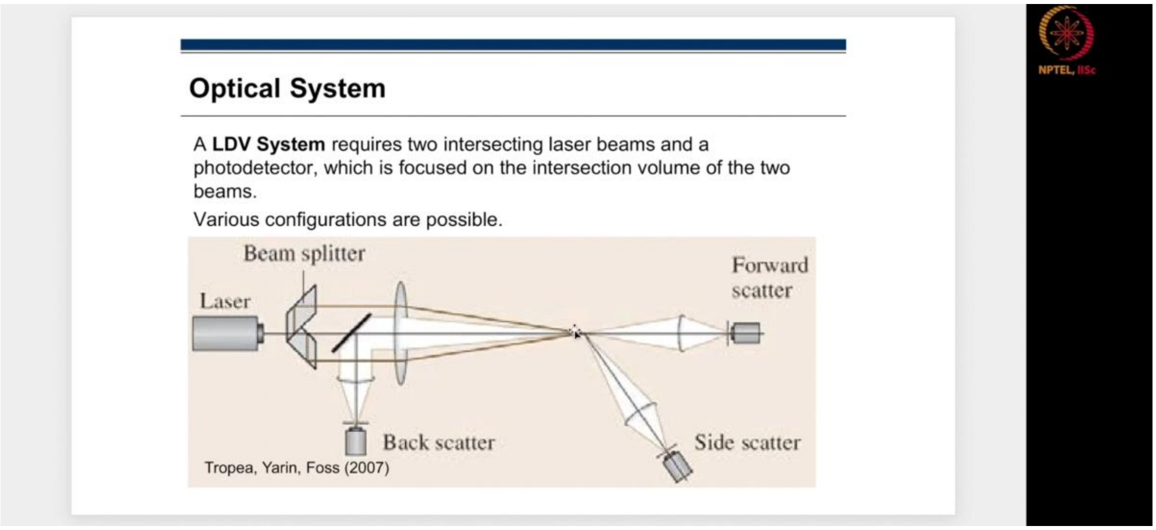




You can see that the actual detection area is somewhere here, whereas this is the total measurement area. And these are some of the relationships. OK, this you can always work out, and we can go through the paper as well. So this is only a small portion. Of the measurement volume depending on what you have chosen to be your ID.

And this is a predetermined value that you are basically supplying to your system. So this is important. So what we have learned is that what a DC component is and what an AC component is, and how the detection volume is different from the measurement volume. And we have also seen how you can use filters basically to throw out the high-frequency electronic noise versus the low frequency, the shape profile, the shape, or the pedestal function. So the measurement volume is therefore, as you can see, defined by the amplitude in which the velocity is measured.

So this is the detection volume, and this is how it is calculated. So, it depends on the scattering. The actual detection volume is actually smaller than this, just because of the way that we have chosen a predetermined threshold. Okay, now if we go to the optical system that actually measures, which is actually used for measurements, what we do is have a laser, and then we split the laser light into two using a beam splitter. You preserve the coherence of the phase, then you pass through the lens and focus it on a particular measurement volume that we already know.



Now you can measure whatever, after that whatever happens, you are not concerned about, but then you are focusing your photo detector, and you are either measuring this as a typical side scatter, a forward scatter, or a back scatter. So you are measuring this kind of, I mean, this side scatter can be at any particular angle; the forward and the backward scatter are like this, so various configurations are therefore possible to actually do this. The interesting part is that because there is a linearity. With the frequency shift and the velocity, this becomes a very easy tool to use. Now, in the two-beam configuration, if you look, the forward scatter is a very simple problem.

So you have the laser, you divide it using a beam splitter, you preserve the coherence of the phase, and then you use a thin lens basically to focus it at a particular measurement volume with all the particles going through that measurement volume. Right, and then you have a receiving end, which is a detector that also focuses on that particular measurement volume. This is what you collect: the forward scatter. The forward scatter, as you know, can be quite high—hundreds of times higher than the backward scatter. The backward scatter is done with the detector and the laser all in one unit, so it is backscattered and fed by another optical system, where it goes into.

The advantage of forward scatter is that it is very simple. The backward scatter requires a little bit of engineering, but at the same time, it has the advantage that, you know, it can be integrated within the optical system and therefore you don't require external alignment and maneuvering, which is a big deal because in the forward scatter, or any side scatter as a matter of fact, you need to position the receiving unit so that it is perfectly focused and can come from any particular direction, etc., so this needs to be a very comprehensive unit, and it requires alignment, but this is like pre-built. by the manufacturer, and you cannot really, you don't have to do anything.

It is already pre-adjusted and pre-aligned. So, therefore, though the magnitude of the scattered intensity is lower here, you can basically live with it if your experimental setup becomes a much easier proposition, particularly this, all right? So now we go to the case of the transmission optics. Now, in the transmitting optics, what you see over here is something interesting. So this is how you now transmit the transmitting optics. So, the size of the measurement volume is usually an important parameter. It is determined by the laser beam diameter, which is given by one over E of the maximum intensity.

So, the minimum beam diameter is known as the beam waist. Let's just look at this. So you have an incoming beam that has a beam diameter or beam waist of $2rw_1$. This expands, and then there is a lens that actually focuses it to another beam diameter, which is $2rw_2$.

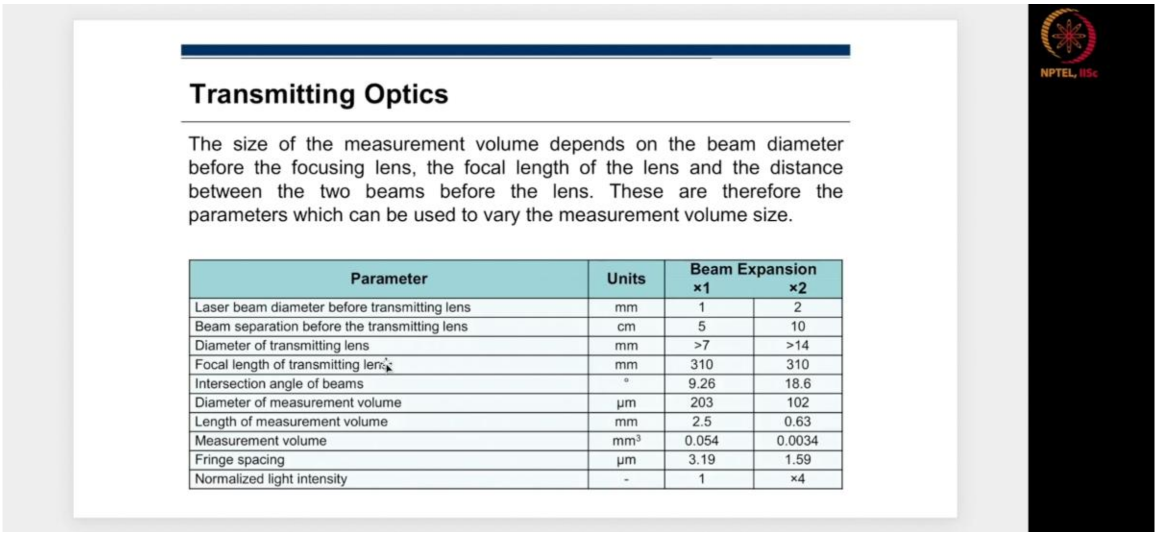
This is actually two. And there are Z1 and Z2, and then there are the focal points. So, if you look at the second beam diameter over here, this is dw_1, and this is dw_2.

So dw_2 is given as... 4πfλ is the focal length, λ is the wavelength, divided by dw_1. So this is dw_1. So, in other words, what you can see is that dw_1 is in the denominator. So smaller the value of this is, the more the value of this will be.

So you ideally want this to be large. So that means you need a large beam diameter. To have a very small spot size, a very small dw_2 is of vital importance because in many cases you want to focus your beam to a very small location. Why this is important is because you know the small-scale location, for example, you want to. You know, place it in a flow where you want to have your spot size or the measurement volume, so to speak, of the order of, say, tens of microns. In order to achieve this, what you need is a very large D because all the other things are kind of given, a very large D.

So this large D can be attained by using a beam expander. which is used as a collimator. So it actually expands the beam. And therefore, you can kind of focus them on a much, much smaller location. So this is quite interesting. So, what do you do? Normally, you have this coming from the lasers.

Then you use a concave lens assembly to expand this out. Then you use a collimator to make it parallel, and then you use a thin lens to focus it at a particular point. So you are concerned with this large D, which is what can actually give you a very tight spot. So these are some of the interesting features, because you sometimes need that very small measurement volume, so to speak, depending on the type of experiments that you are doing. Because, remember, it is volume-averaged over that particular measurement volume. So you need it to be very small to get the kind of fidelity that you want because, remember, as your particle passes through it, if it has different velocities— for example, if it passes through, then if it has different velocities— it is all averaged out in that particular, you know, measurement volume.



Transmitting Optics

The size of the measurement volume depends on the beam diameter before the focusing lens, the focal length of the lens and the distance between the two beams before the lens. These are therefore the parameters which can be used to vary the measurement volume size.

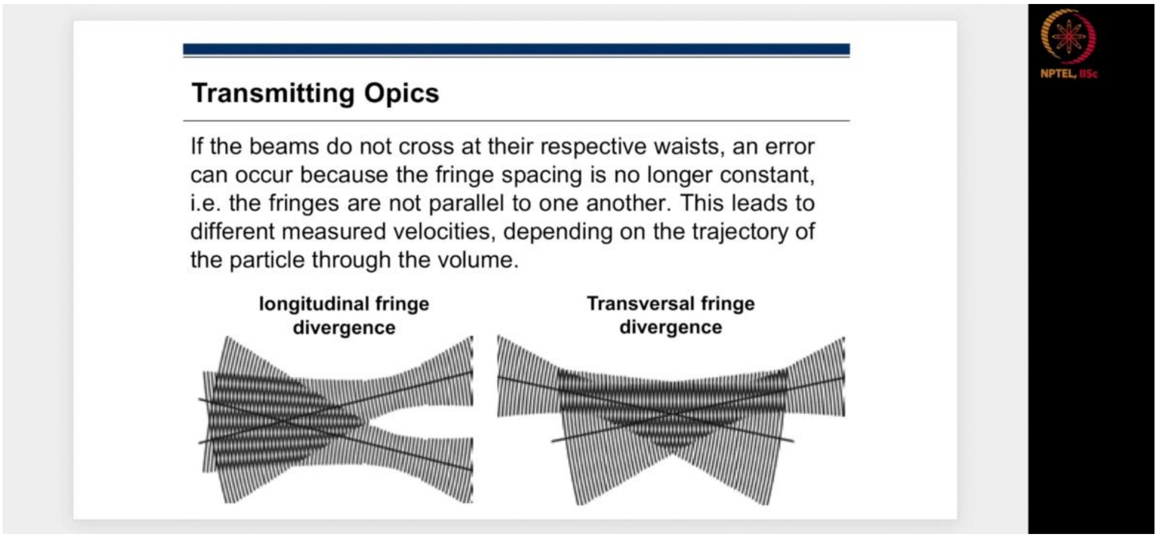
Parameter	Units	Beam Expansion ×1	Beam Expansion ×2
Laser beam diameter before transmitting lens	mm	1	2
Beam separation before the transmitting lens	cm	5	10
Diameter of transmitting lens	mm	>7	>14
Focal length of transmitting lens	mm	310	310
Intersection angle of beams	°	9.26	18.6
Diameter of measurement volume	µm	203	102
Length of measurement volume	mm	2.5	0.63
Measurement volume	mm^3	0.054	0.0034
Fringe spacing	µm	3.19	1.59
Normalized light intensity	-	1	×4

So you need to have large intensity there, and you need to have the particles go through that measurement volume. So that you can assume that it is small enough that spatially, in that particular region of interest, the particle's velocity does not change by much. So this is the interesting part, and this depends on what the incoming diameter of this is. So the transmitting optics, for example, the size of the measurement volume, therefore depends on the beam diameter before the focusing lens. This is important: the size of the measurement volume, as this particular expression shows.

So it depends on the beam diameter before the focusing length, the focal length of the lens, and the distance between the two beams before the lens. These are therefore the parameters that can be used to change the measurement volume, but to get a tighter spot, you need a beam diameter that is large, and that is where the beam expander comes into the picture. Then, of course, use the collimators and other things, and thin lens formulas. So, for example, take a look at this table. So the laser beam, before it goes to the transmitting lens, say 1x means there is no expansion, is one millimeter.

After expansion, it is like two mm thick. So the beam separation before the transmitting lens is five centimeters and ten centimeters, for example. The diameter of the transmitting lens is of that order. The focal length is 310 mm. The intersection of the beams is about nine degrees, and this is about 18 degrees here.

The length of the measurement volume is 2.5 mm. And 0.63 mm, you proceed to the beam expansions. You can see how much reduction we had because if you, okay, it gives you the normalized light intensity, which goes up by four. This is because if you reduce the diameter of the laser beam and the measurement volume, then what will happen is



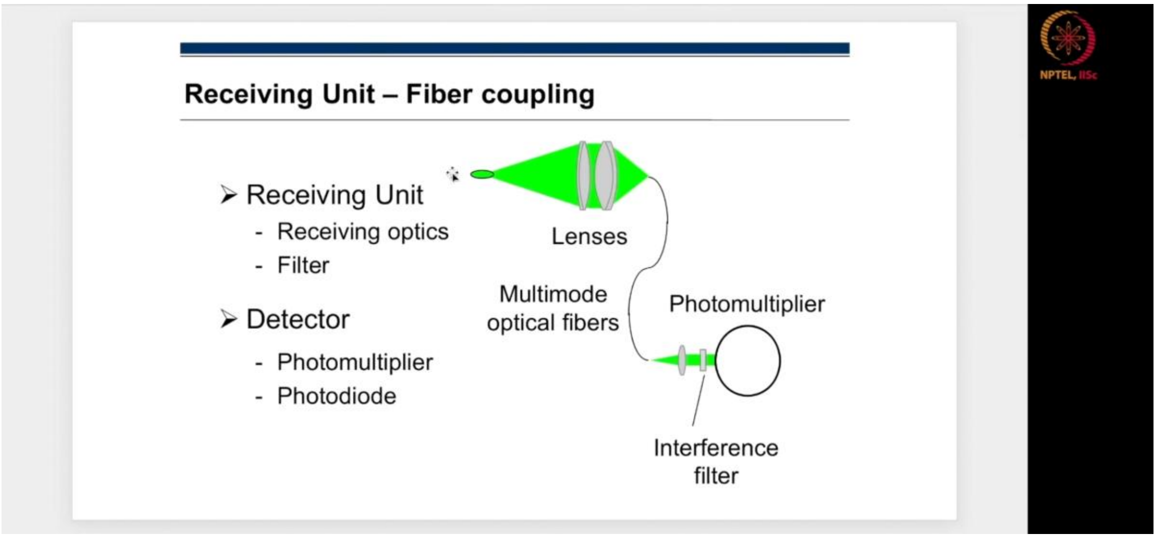
that the same intensity is now captured within a lower volume or a lower surface area; in that particular case, this has a D square dependence. Therefore, the light intensity in that particular spot goes up by four times if you can reduce it to half.

If you can reduce it by half, that is what is going to happen. So if you see the measurement volumes, these are off by a factor of four. So you can see the normalized light intensity, why it goes up when you actually have a beam expansion, and the measurement volume also becomes much tighter. But for that, you need the beam to be expanded. That means the laser beam with beam expansion will be quite large.

So, this is the caveat. So when you are trying to focus this Gaussian beam optics, you need to take this into consideration. This is all because of the Gaussian beams and the intensity profiles that they carry. Once again, if you take a look, this is the intensity profile that the Gaussian beam has. So, naturally, the particle sees different intensities at different points in time. If these had been of the same intensity, then it would not have that envelope function anymore.

The envelope function would actually be redundant if these all had the same intensity, but it is not. So that is the most crucial part of the argument. So we have talked a lot about the transmitting optics and how the beam expansion actually increases the laser beam diameter. As a result of that, you can focus it on a much tighter position. So if the beams do not cross at their respective waists, an error can occur because the fringe spacing is no longer constant and the fringes are not parallel to each other.

So this leads to different measured velocities depending on the trajectory of the particles through the volume. So this is called longitudinal fringe divergence, and this is the transverse beam fringe divergence. So if this happens when the beams do not cross at



their respective waists. So what are the waste over here? The respective wastes will be shown in this particular spot.

So these are the respective wastes that we are concerned about. So now we will, I mean, in the next class, we will talk a little bit about coupling. That means how the receiving unit actually uses optics, fiber optics, and other things to collect the signal. So you can see, you can roughly get the idea that there is a linear one-to-one correspondence between the frequency and the velocity. The velocities are perpendicular to the optical axis; this you should keep in mind. Gaussian beams require a special kind of focusing, and the larger the beam diameter, the tighter the spot you can actually focus.

As a result, you can get a very tight volume. I mean, you can get a very tight volume, and here we are essentially looking at the beat. For the interference so that we can calculate what the velocity of the particles will be, and the particles, remember, they follow the flow field, so we stop here.