Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
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Week – 09
Lecture - 43
Laser Doppler Velocimetry – 2

So in this particular lecture, which is basically on LTV, the second lecture on LTV, remember that in the last lecture we talked about the demands on the light source for LDV, and some of them were monochromatic, spatial and temporal coherence, linearly polarized, and well-focused. And we also say that the laser beams exhibit a Gaussian intensity profile, which is exactly the cross-section of the beam. So normally, you know, what we do is that if you try to focus a Gaussian beam, there is a low divergence angle. That means it is more or less collimated, which is what you see over here. There is a little bit of divergence. Now, what you do is bring the beam and then bring it to a focal point.

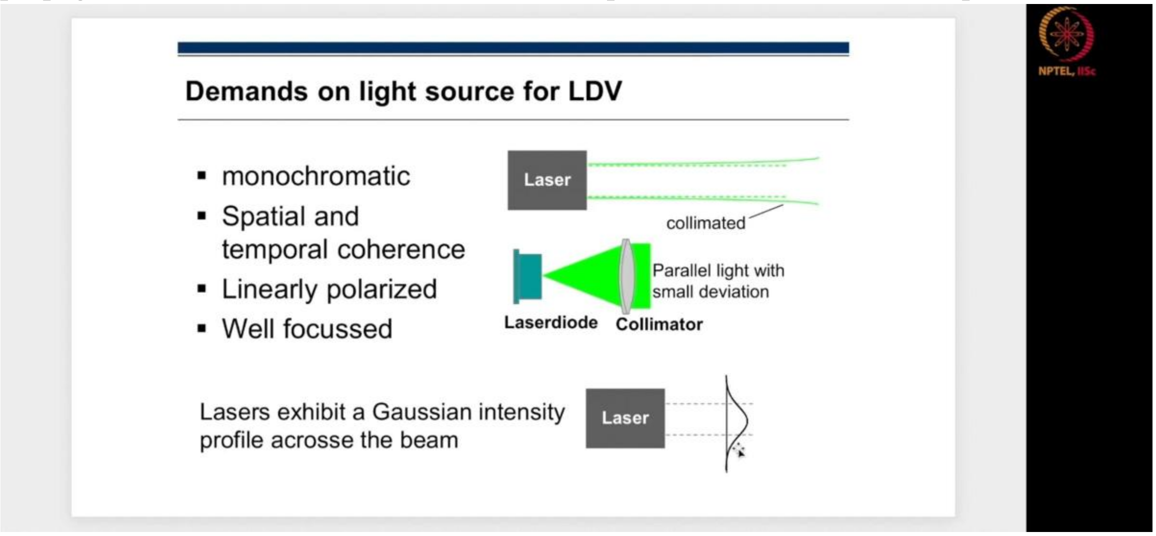
Length or a focal length L, and we will come across this a little later as well. So these are basically your spherical wave fronts, and this is the region where wave fronts are more or less planar, so to say, in this waist where the focusing point is, and then it starts to diverge once again. So the idea is basically that in this very small region in space where the beam is actually focused, the wave fronts are planar. And this will play a very significant role when we actually discuss LDV.

So focusing is not a trivial thing, therefore. It has a small divergence angle. The wavefronts are spherical in nature. So when you actually try to converge them or focus them, it is only in this small region that they are more or less in a planar fashion. So this is a beam, a Gaussian beam, that is actually focused.

Okay, so just keep that in mind when you actually do the stuff. So this is more about that. So this is a Gaussian intensity profile, as you can see over here. This is the focus as we go on. Forget about the math portion.

The math part is not important at this particular moment. So this is the beam waste, and there is a small region where you are supposed to get a more or less planar front. And then the.

.. The decay of the field starts to occur. Relative to the Z-axis. The Z axis is the propagation axis. And the beam axis is with respect to the X axis, for example. So this is



how a typical Gaussian beam actually looks.

So it is only a small region in space where you can actually expect some sort of planar wave front. And that is absolutely important for LDV purposes, as we will see. and not only the LDV, even for PIV purposes. When you try to form a laser sheet, for example, these things have to be kept in mind: this is what we are actually dealing with. Now, the fundamental point of LDV is essentially what we call the Doppler effect.

Now, this is kind of well known. OK, and perhaps you all studied it during your high school. What happens is that if you illustrate it in a very simple fashion, there is a conveyor belt, then there is a receiver, and there is a guy who is throwing cakes at you or placing cakes on the conveyor belt. So you receive the cakes at a certain frequency. He's placing the cakes at a certain frequency.

You're receiving the cakes at a certain frequency. Now what happens is that if this guy starts to move away or move in that particular opposite direction, you will get cakes at bigger or larger intervals. Now, if this guy starts to move towards you, okay, you will get the cakes at smaller intervals, or in other words, you will have what we call a frequency change. This is if this is the nominal frequency at which you would have received the cakes, uh, at a certain rate at which this guy is putting the cakes. If this guy maintains the same rate but is moving away at a certain relative velocity with respect to this conveyor belt, what will happen is that if he's moving away, it is quite clear that you will get the cakes at a larger interval.

You can see that the interval has increased. If he moves toward you, you will get cakes at a much smaller interval. So there is a change in frequency depending on the velocity or the relative velocity of this transmitter. In this case, the transmitter is the guy who is throwing the cake, and you are the receiver. So this is all very good; this is something that you knew: the relative velocity changes a lot of stuff.

If you have to put it in a mathematical fashion, let's have the Doppler effect with a moving receiver. So you are the receiver, or in other words, you are this guy who is receiving the cake. Now, you may be moving at a certain relative velocity with respect to the direction of wave propagation. So the wave propagation direction is given by k_i, k_i being the unit vector in the i^{th} direction. And you are moving at a certain velocity with respect to this k^{th} vector and this direction of propagation.

So the transmitter, this guy is actually releasing a wave at a certain frequency, but you are moving away from that particular receiver at a certain relative velocity, okay? So if you had ideally been here, this would have been the frequency c_t by . Now that you are moving away, this is the inner product between U_i and k_i. It's nothing but the component of U_i in the k^{th} direction or k^{th} direction. So if you subtract the velocity, a new frequency comes into the picture. So now, if we take the new frequency, it is therefore the old frequency minus this particular factor.

Which is $\frac{U_i k_i}{c}$. Or in other words, the component of velocity in the k^{th} direction divided by c. So the moving receiver receives a different frequency, which is f'. So for him, the frequency has changed in the same way as this cake guy; it has changed. It has become something like an f'. And in this case, as you can see, the frequency is reduced.

So that is what happened. So this is a moving receiver. Now, if we have a moving transmitter and a stationary receiver, say, for example, you are standing still, maintaining your location to be fixed, and you have a.

.. The transmitter, which is moving at a certain velocity, once again, this is the velocity U_i, and it is giving out a wave in the ellioth direction. This is again the unit vector, and you are the stationary receiver who is actually receiving it. All right. The initial frequency, of course, is c_t/λ, as you can see over here. Now, it has obviously been reduced because C now is, you know, (c-u_i l_i), which is once again the inner product of the U^{th} velocity in the li^{th} direction.

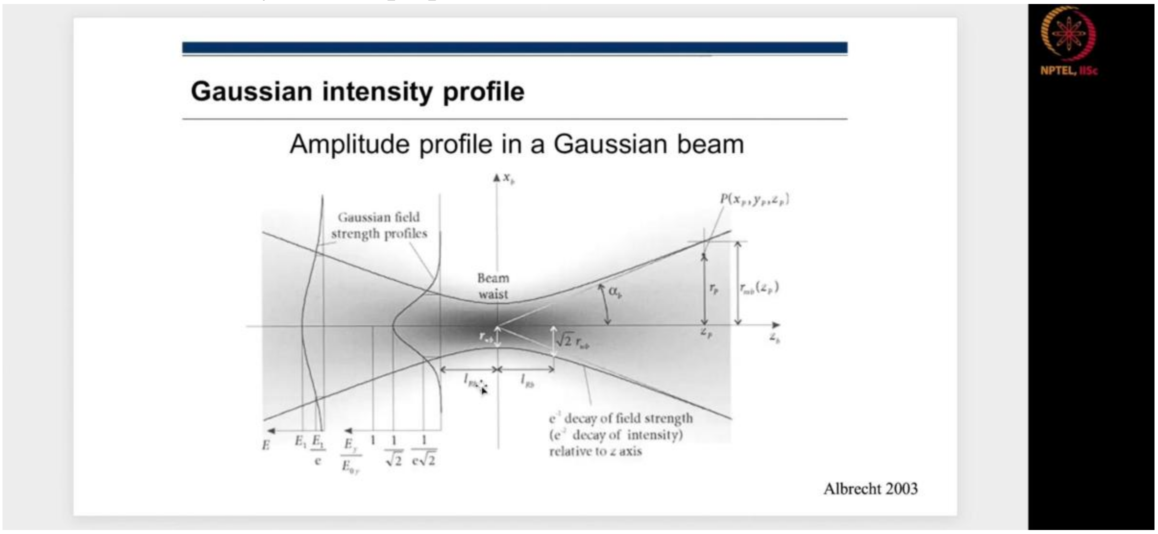
So the frequency once again now in this particular case will be also reduced. So the stationary receiver receives a different frequency of light. All right. So, if you basically look at this, if you have, you know, a receiver who is moving, you will see a different frequency. If you have a moving transmitter, you will also get a different frequency, similar to the picture over here.

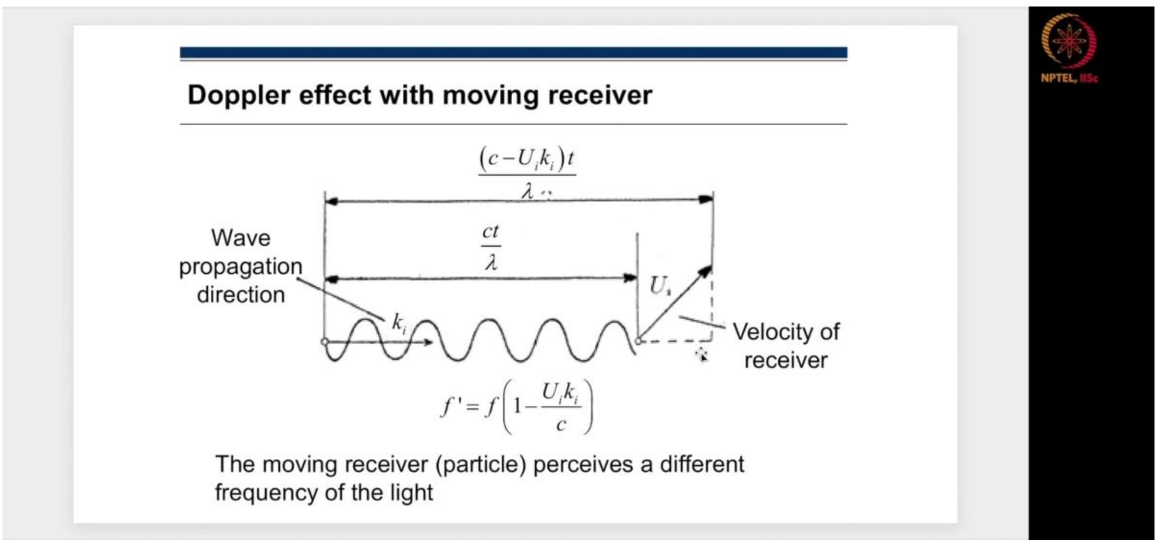
Here, of course, the transmitter was the person who was moving. In this case, we have two situations in which the receiver is moving and the other case where the transmitter is moving; in both cases, the stationary receiver perceives a different frequency, while in the case of the moving receiver, it perceives a different frequency of light. These are the two things that happen in these two cases, so if you imagine so. In both cases, because of the relative velocity of either the receiver or the transmitter, you are receiving different signals. So in one case, you are moving yourself, and that's why you see a different frequency.

In the other case, you are stationary, but the transmitter is moving. As a result of that, you also get a different kind of light that is coming to you. So this is the celebrated Doppler effect. Now, if you look at it, if you combine these two cases somehow, how do you combine these two cases? In this case, you have an illuminating wave that is coming from some transmitter, and it is in the k^{th} kind of direction. You have a particle that is scattering this wave.

Now, this particle is actually moving. At a velocity, some velocity U_i And then you have your receiver over here, which is now seeing a moving transmitter. In this case, you can see there is a stationary transmitter and a moving receiver. For the screen, which is stationed here and is going to collect the scattered wave, you have a moving transmitter and a stationary receiver.

So, you know, this part of the case basically corresponds to that. That means you have a stationary receiver, and this is basically the scattered light that is coming out of the particle, which is moving at a velocity u. On the other hand, this part, the left-hand side of the segment, is basically where there is stationary light. Transmitter, but a moving receiver. So basically, it's a superposition or a combination of these two cases, which

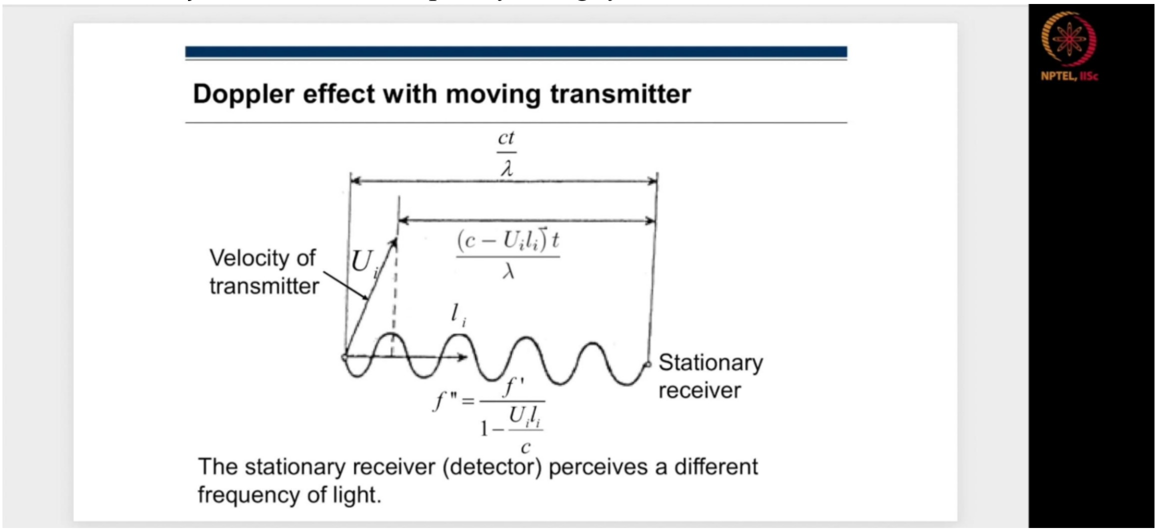




leads to what we are receiving on the screen.

This screen is a stationary receiver. But because there is a particle in between, we are not directly looking at this wave; we are looking at the scattered wave that is coming out of this particle, which is moving at a velocity U_i in a particular direction, and you are monitoring it over here. So where you are monitoring depends because we saw the scattering. The scattering scatters in all directions. So it depends on where your scattering screen is; your frequency will be according to that.

So let us look at this particular item. The frequency shift now is f''. Why is it f''? So there is a frequency shift that this particle experiences. And then there is an additional frequency shift that the receiver here actually sees because of this moving particle. So, therefore, this f'' is whatever frequency this guy, or the receiver at the end, or the



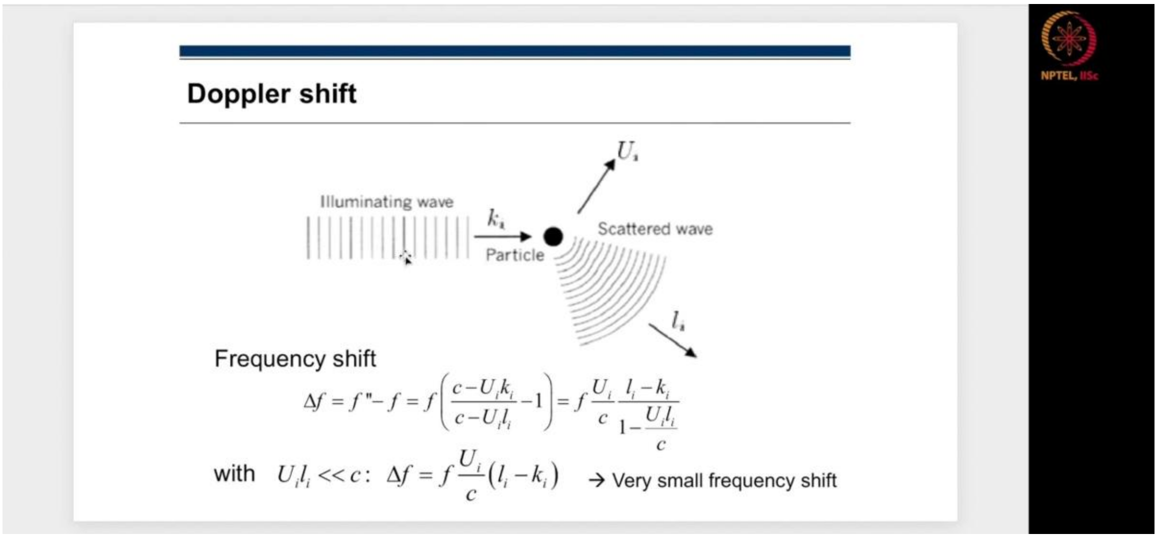
photodetector actually looks at.

So this is $f'' - f$, which is the frequency of the illuminating wave. This $f'' - f$ is given by this relationship. Now, you should also realize that $U_i l_i$ is a term that represents the component of u in the direction of l_i, in whichever scattering direction you are actually measuring it. It is much smaller than c. Or in other words, your frequency shift is basically the original frequency, and this is the unit vector's direction.

So one is the k-1 direction, and the other one is the direction in which you are actually monitoring the scattering. So this depends. So if you are monitoring in the same direction, which is like forward scattering, this l_i and k_i is in the same direction. If you are scattering it in the other direction, which is like backward scattering, it will be the opposite of that.

So this is the premise. That means these are unit vectors in the forward scattering direction or the scattering direction in which you are measuring. And l_i is basically the direction in which the light with which the scattering is measured, and k_i is the direction of the illuminating wave that is falling on the particle. And the particle has a velocity which is U_i. So this can be any arbitrary direction in which the particle might be moving.

And it's a combination of two cases. The left-hand side is a stationary emitter and a moving receiver. And this right-hand side is basically a case of a moving transmitter and a stationary receiver. So these two are the cases that we already discussed in these two slides: the Doppler effect with a moving receiver and the Doppler effect with a moving transmitter. Both of these things were given. These are combined, and this is the final expression you get with the caveat that U_i is actually much smaller than c, which is the velocity of light.

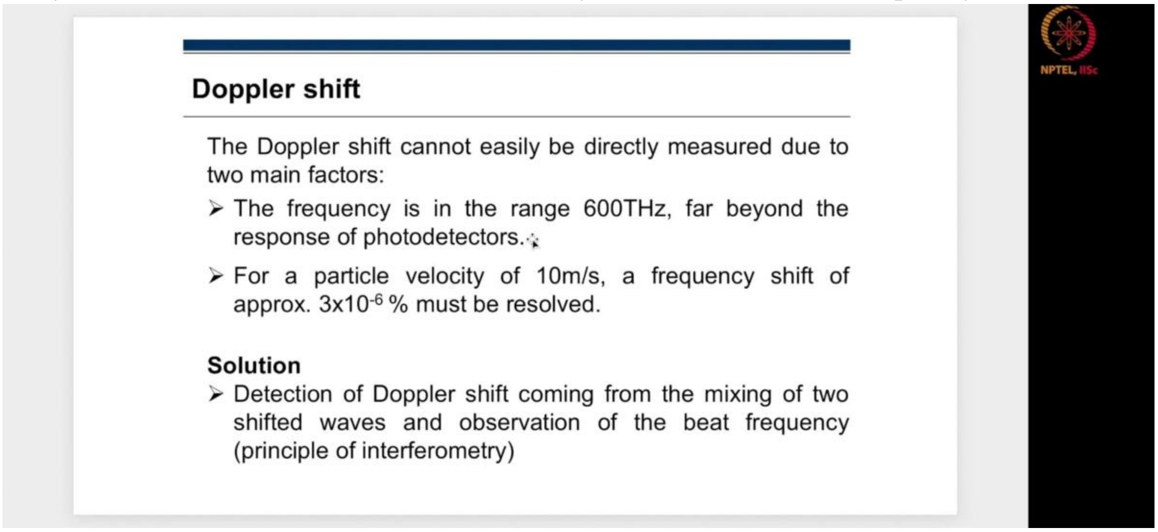


As a result of that, you get a frequency shift that is very, very small. You see, this frequency is multiplied by U_i and c. U_i is a small number, so therefore this becomes a very, very small, uh, you know, you are dividing it by a very large number. As a result of that, your frequency should go up quite a bit because U_i/c is a number that is much, much smaller than one. As I say, it is in the fraction, the point zero zero zero zero something.

Okay, as a result of that, this frequency therefore blows up. So if your parent frequency is whatever it is, say it is like 10 Hz , let's assume for the time being that if there is a frequency associated with it, that frequency is 10 Hz, okay? So this gets scaled, and if this is, say, one meter, and this light's velocity is 10^8, it gets multiplied; it's like divided by 10^8. So that is the kind of frequency shift that you are going to get, all right? So the Doppler shift cannot be easily measured due to these two common factors that I mentioned. The frequency is in the range of 600 teraHz, which is beyond the response of any photodetectors that you will have. All right, so if we have a particle that has a velocity of 10 meters per second, the frequency shift calculated from this expression, even assuming the maximum frequency shift that you can have, is approximately $3 * 10^{-6}$ %.

So three parts per million type of percentage resolution must be resolved. So one is that the base frequency is very high, and the frequency shift is therefore very, very small. So this is onerous because you don't have photodetectors. You don't have that kind of resolution. The other way of doing this is to detect the Doppler shift, which comes from the mixing of two shifted waves.

So you mix two waves from two lasers, and you observe the beat frequency, which is



basically the principle of interferometry. So you observe the beat frequency and use it to measure what the velocity is going to be. Because you can see this expression is nice because you measure the velocity, if we had been able to measure this frequency shift, your velocity would be automatic. You know the direction in which you are measuring the scattering. You can easily measure the velocity because this is the velocity in which you are interested.

Remember, these are the tracer particles in the flow. So we are trying to measure the velocity of these tracer particles because once you measure the velocity of these tracer particles, you can imagine that, because they are small in size and all those things, they will represent the flow field. So, in other words, this tracer particle velocity U_i is the velocity of the fluid at that particular location and at that particular time. Right? So ideally, if we had been able to measure this Δf, then U_i would have been pretty much a linear function. So you can actually then back-calculate U_i in no time. However, you have these problems because of the 600 teraHz and the frequency shift of approximately three parts in a million resolution problem.

What you do is mix two frequencies coming from two different shifted waves, and then you observe the beat frequency. And then you use the beat frequency to calculate the velocity because your ultimate aim is to calculate the velocity. You're not interested in the optics, but you want the optics to be such that you can measure the velocity. So, that is what it is. How do you do this? So if you do this, you have Doppler shift with two illuminating beams.

So say there is a beam illuminating a wave from one laser; this is the wave coming from another laser. It can also come from the same laser, with a little bit of a, you know, but they have the same phase essentially. So there is a half angle, which is phi, theta, whatever you call it. We have used different nomenclatures; this time it is Φ.

This is half the angle. So what you have here is now two directions. One is $k_{i,1}$ and the other is $k_{i,2}$. So these are coming from two different directions at you. And this is the particle in question. And then there is the photodetector, which now measures the scattered wave that is emitted by this particular particle.

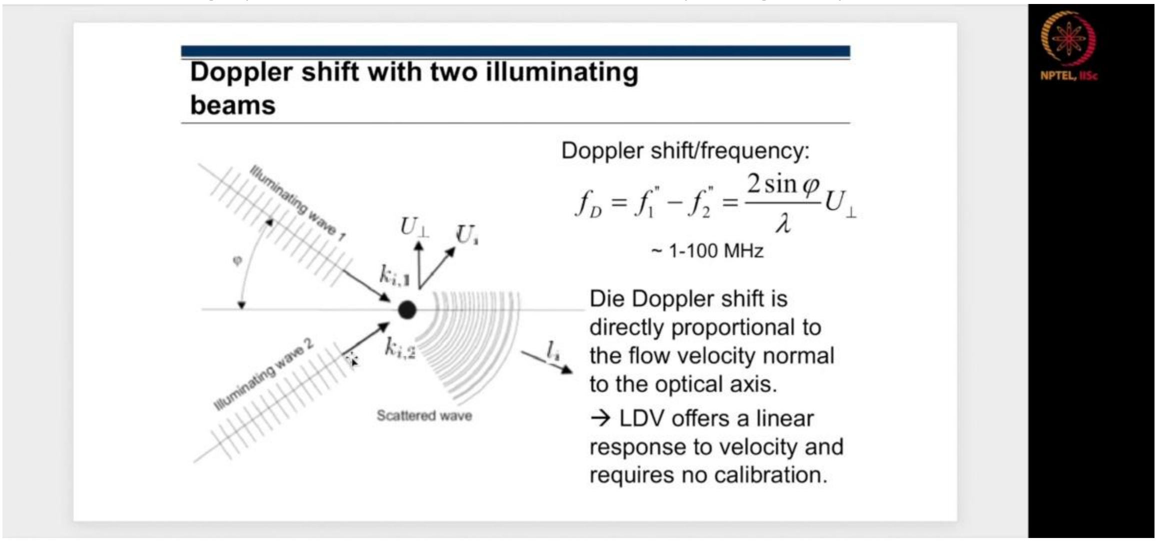
So the particle is moving with a velocity U_i, so everything else remains the same. This is the component of the velocity of the particle; this is $U_\perp$, which is actually in the direction that is perpendicular to the z-axis. If this is the axis, then this is what it is. And l_i is once again the direction at which your photodetector is placed to monitor the scattered waves. So what happens here? If you consider one beam, you have a frequency shift of f_1''.

If you have the second beam, you have a frequency shift of f_2'', so the Doppler shift of frequency is basically the difference in the shift of the two waves, which is nothing but, you can show this, this is actually $2sin\Phi/\lambda$, λ being the wavelength of the incident beams, multiplied by the velocity in the perpendicular direction, perpendicular to this particular axis. We understand that f_{11} is essentially nothing but the shift that you calculated here. This is one of the shift. So this is the other shift that is coming from the second beam. So the difference between these two shifts is a change in the Doppler shift.

So Doppler shift itself means a shift in frequency. What we are talking about here is the difference between the shifts of two frequencies, right? And that is equal to the perpendicular component of the velocity perpendicular to the z-axis multiplied by $2\ (sin\Phi)\lambda$, where Φ is basically the half angle and λ is the wavelength of the incident beam. Here you can see that if this frequency comes out to be in the order of 1 to 100 megaHz, there are photo detectors that can easily measure this shift in frequency. Now, if we can measure this shift in frequency and we know the half-angle between the two incident beams, we can calculate the perpendicular component of the velocity. The velocity component in the direction that is perpendicular to this optical axis, remember, is not ui that you are measuring; it is the u perpendicular component. So, the Doppler shift is therefore proportional to the flow velocity normal to the optical axis.

Remember, the velocity of the particle is the flow velocity. Because that is what our assumption is. And it offers a linear response to velocity and, therefore, requires no calibration. So it is a linear response, as you can see. You know this. You know the half-angle, you know the λ, because these are the experimental setups.

There is nothing dynamic about it. The FD is the only thing that you need to measure



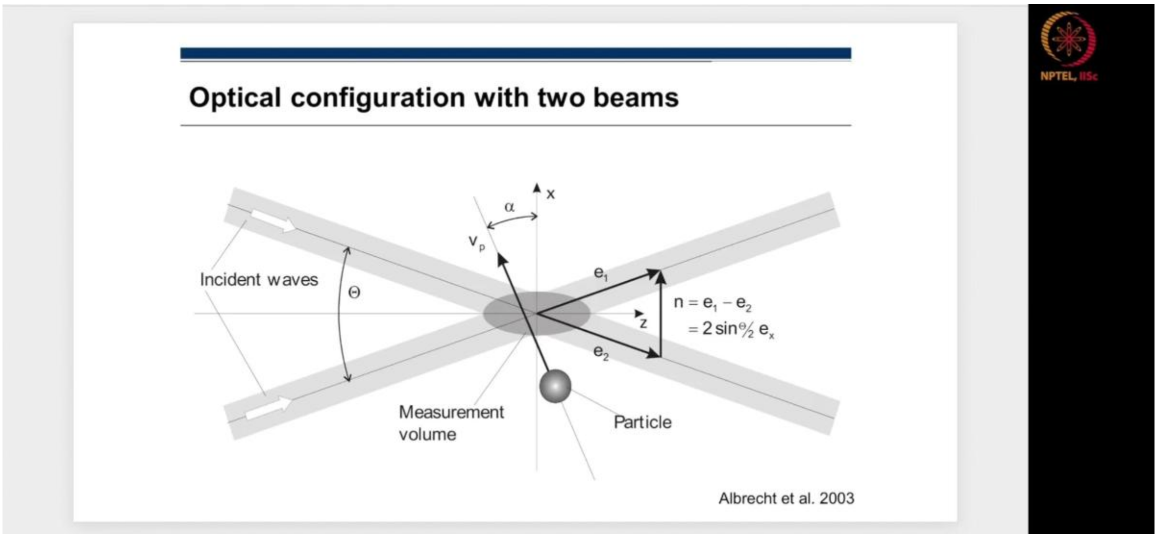
with your photodetector. And you measure, then you can recalculate what the perpendicular component of the velocity is going to be. Nice and easy. And it comes from the Doppler shift of the two beams and the difference between them.

So, this is the optical configuration. So here, of course, we have used θ. So this is the incident wave. This is the incident wave. This is where they actually meet. So this is the measurement volume. As you can see, if you just do that y, the perpendicular component, you can see the n.

This is n, which basically indicates that there are two directionalities. So, the n is basically e_1-e_2. That is given as $2\sin\left(\frac{\theta}{2}\right)e_x$, which is the component in the X direction. And that is how you get this. You get this relationship because $U_\perp$ is nothing but U_x, which is the beam in the perpendicular direction.

And N is nothing but e_1-e_2. Okay, this is just vector subtraction. And that is what you get. This is the included angle. So this angle and this angle are the same as C. So this is how you get $2\,sin\frac{\theta}{2}$.

In this case, it is $\frac{\theta}{2}$. In the previous slide, this was half of the angle. So, it's basically the same. Okay, so it is basically half the angle into two sine of half the angle multiplied by two into e_x, where e_x is the unit vector in the X direction. And that is the reason why it becomes a velocity in the X direction that we are actually measuring over here. Okay, so what happens is that if the two coherent beams intersect with each other, they create an interference fringe pattern, okay? So, these fringe patterns exhibit maxima and minima.



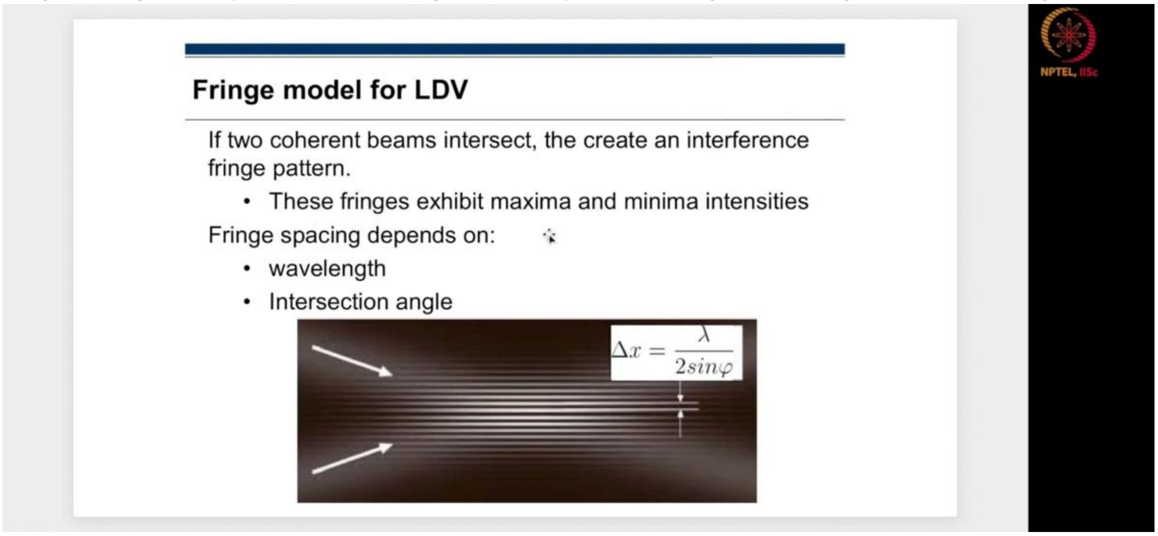
So you have bright spots, bright lines, and dark lines. So, the fringe spacing depends on the wavelength and the intersection angle. So, as you can see, this difference between the two bright fringes is given by $\lambda\ /2sin\Phi$, where Φ is the included angle of the intersection; basically this. So, these are the two coherent beams. They actually intersect with one another.

They create an interference pattern, a fringe pattern. This fringe pattern has maximum and minimum intensity. It is bright and dark. And this fringe spacing depends on the wavelength and the intersection angle. And this separation of the fringe spacing is dependent on whatever laser beam you are using. As you can see, the smaller the wavelength of the beam, the smaller the fringe spacing will be.

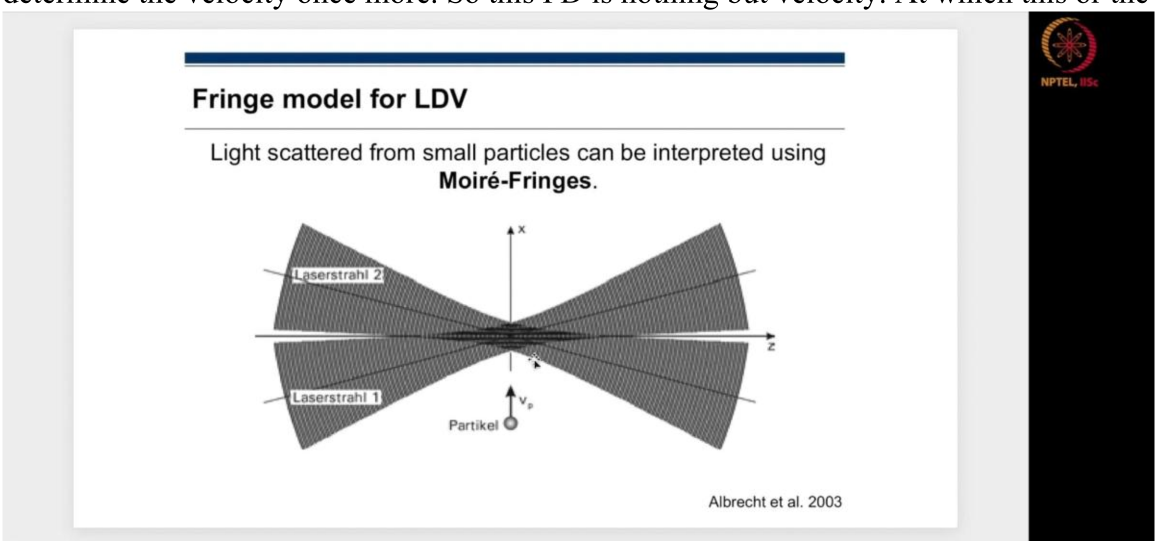
And then it is also dependent on the geometric angle at which you have placed the two beams together. So the laser light that is scattered by small particles can be interpreted using the Moyer fringes. So, these are the two laser lights. It is assumed that they are planar wave fronts, and this is the particle with a velocity v or the effective velocity V_p that is actually traversing that fringe.

You can see these are the fringes that are present. And this is the z-axis, and the detector is somewhere there. All right. The interference fringe model is that in the area where these two beams intersect, an interference pattern can be imagined exhibiting constructive and destructive interference. The fringe spacing is given by Δ x, which is $\lambda\ /2\sin\left(\frac{\Phi}{2}\right)$.

In this case, this is the whole angle; this is the half angle. Particles traversing the fringes will scatter alternatively high and low-intensity light because, as they pass through the bright fringes, they will scatter high intensity. When it goes through the dark fringes, it



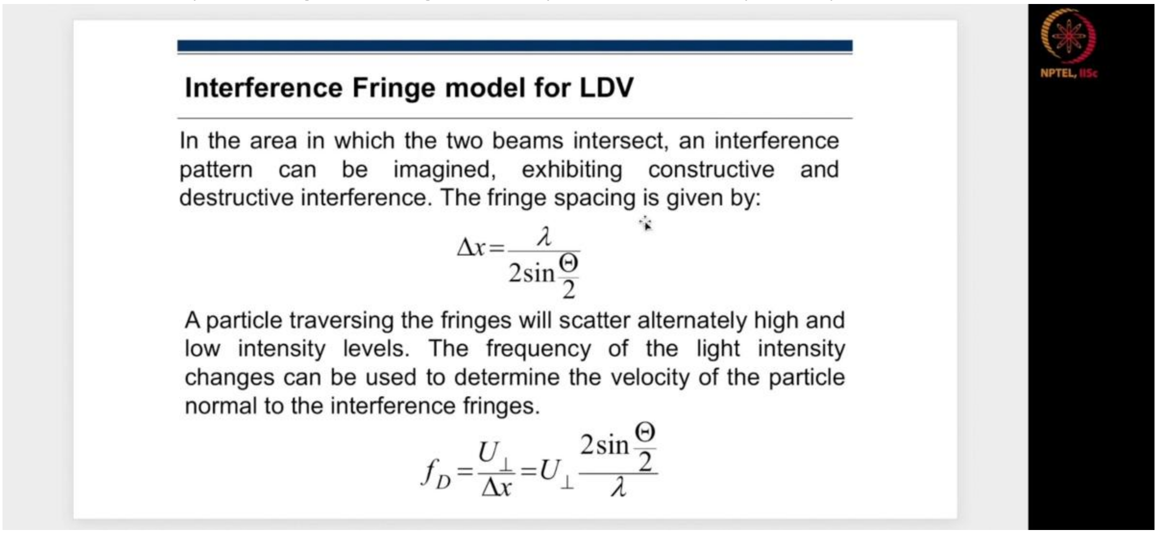
will scatter low intensity light. The frequency of the light intensity changes can be used to determine the velocity once more. So this FD is nothing but velocity. At which this or the



perpendicular velocity is divided by the Δ x between the fringes, which is basically this.

So, the frequency of the light intensity changes. It is what is actually used to determine the velocity of the particle in this particular case. And this is given by this simple relationship. This is the U, and this is the difference in the fringes. As you can see, if you make λ smaller, these fringes will be smaller.

So, therefore, you can get more granularity on the velocity. Okay. And so this is how it is



actually achieved. So in the next class, we'll see; we'll move further on this.