Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
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Week – 08
Lecture - 40
Micro PIV – 3

So now we will talk about cedar particles, or the tracer particles, in depth. By now, you should appreciate the importance of cedar particles because, to visualize the fluid—whether it may be water or gases—it is really important to choose appropriate cedar particles, as those are the things, basically the entities, which are actually helπng you visualize the flow. So actually it should faithfully follow that flow and not behave on its own. So, yeah, I mean these pseudo particles are used to see how the flow is evolving in time and space, right? And there are a lot of issues related to that, but yeah, again it is one of the most important experimental decisions you should make because if you take a slightly larger particle or a smaller particle size. It can actually alter your results significantly, and this should be taken care of before starting your experiments. Another thing is that the quantity of particles plays an important role because at the micro scale, when you are dealing with channel sizes of micron scales like 100 microns or 50 microns, the particles should not clog the channel.

If they start clogging or if they start sticking to the channels, it can disrupt the flow, and as a result, you can get varying pressure drops across the channels, leading to erroneous results. Many x you are not even aware of these things happening inside your channel, so you should be very careful while doing the experiments and choose the quantity of particles very accurately. Another thing is, actually the most important thing, the cedar particle should respond immediately to any changes that are happening inside the fluid motion. A lag would actually result in some erroneous output, and because if you want to see a very small phenomenon that is occurring over a very small time scale, your particles should respond that fast so that you are able to capture that phenomenon at that small time scale.

Seeder particles

- The seeder particle is used to show us how the fluid is evolving.
- The choice of seeder/tracer particle is the most important experimental decision that can significantly alter the output.
- Along with that, the quantity of particles also play an important role. Since, at microscale, large number of particles can result in a clog and may disrupt flow.
- The seeder particle must response immediately to any change in fluid motion. A lag in the response will result erroneous output. Hence, a very small response time is desirable.
- Further, the size of the particle is important as at micron scales, the Brownian motion of the liquid molecules can result in some uncertainties.

Right, so it is very important that the particle should respond at a very small time scale because normally in microfluidics, the time scales are pretty small. Right, we are seeing things that are occurring over a very small period of time. We will see a bit more about it in the next slide: how to derive the particle's instantaneous velocity. Another important thing is that we are talking about smaller sizes, like the particle size should be small and those things, but again, if you start going to the very small side of the spectrum, the Brownian motion starts to come up and it can actually start causing some kind of uncertainty in your flow. So we should be very careful that the particle is not very small and it is not very large to get the desired output.

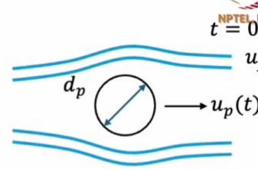
And this is just one image where you can see that these are the pseudo particles I am talking about. These are fluorescent particles kept inside a fluid channel. And these white particles, some of which are shining very brightly, some of which are not, are affected by the volume illumination, which we will talk about in later slides. Now, let us derive an equation for the particle velocity within a fluid. So here you can see a particle of diameter dp, and say you have at time t = 0 a fluid that is passing over it with a velocity of u_f.

We assume this uf to be constant for now. We want to see how long this particle takes from t_0 to 0, from a basically static position to reach the fluid velocity, right? So, let us consider this: the instantaneous velocity of the particle to be up. To model this, we will use a simple version of Newton's second law, where you can model the particle by saying that the rate of change of momentum, which is mass x the derivative of velocity with respect to time, is equal to the drag force the particle is experiencing. This drag force is due to the motion of the particle relative to the fluid velocity. If it is moving at some speed and the fluid is moving at a different speed, there will be a relative velocity between them, and because of that, there is a relative drag.

Now this drag force can be assumed to be the Stokes drag here, and we can write it as 3 x π x μ of the fluid x particle diameter x the relative velocity term $u_f - u_p$. On further solving this, you see that you can write the mass term as ρp, the density of the particle in this volume of the sphere, $\frac{4\pi R^3}{3}$ x $\frac{dUp}{dt}$, which is equal to the Stokes drag. On further solving this, you get something like this, where the rate of change of velocity is given by this term. Now you integrate both sides like this, and you get the log of the relative velocity term is equal to the negative of this term plus an integration constant. Now, to find the constant, you need a boundary condition, and as we already talked, we have a boundary condition that is at t = 0, the particle's velocity was 0.

Seeder particles

Q. How fast the particle start to move and reach the speed of the fluid?



From Newton's Second Law we have,

$$m\frac{du_p}{dt} = F_{drag}$$

$$m\frac{du_p}{dt} = 3\pi\mu d_p(u_f - u_p)$$

$$\rho_p\left(\frac{4}{3}\pi\left(\frac{d_p}{2}\right)^3\right)\frac{du_p}{dt} = 3\pi\mu d_p(u_f - u_p)$$

$$\frac{du_p}{dt} = \frac{18\mu}{\rho_p {d_p}^2}(u_f - u_p)$$

$$\int\frac{du_p}{(u_f - u_p)} = \frac{18\mu}{\rho_p {d_p}^2}\int tdt$$

$$ln(u_f - u_p) = -\frac{18\mu}{\rho_p {d_p}^2}t + C$$

At $t = 0, u_p = 0$

$$\boxed{ln(u_f) = C}$$

$$\frac{u_f - u_p}{u_f} = e^{\frac{18\mu}{\rho_p {d_p}^2}t} \qquad (1)$$

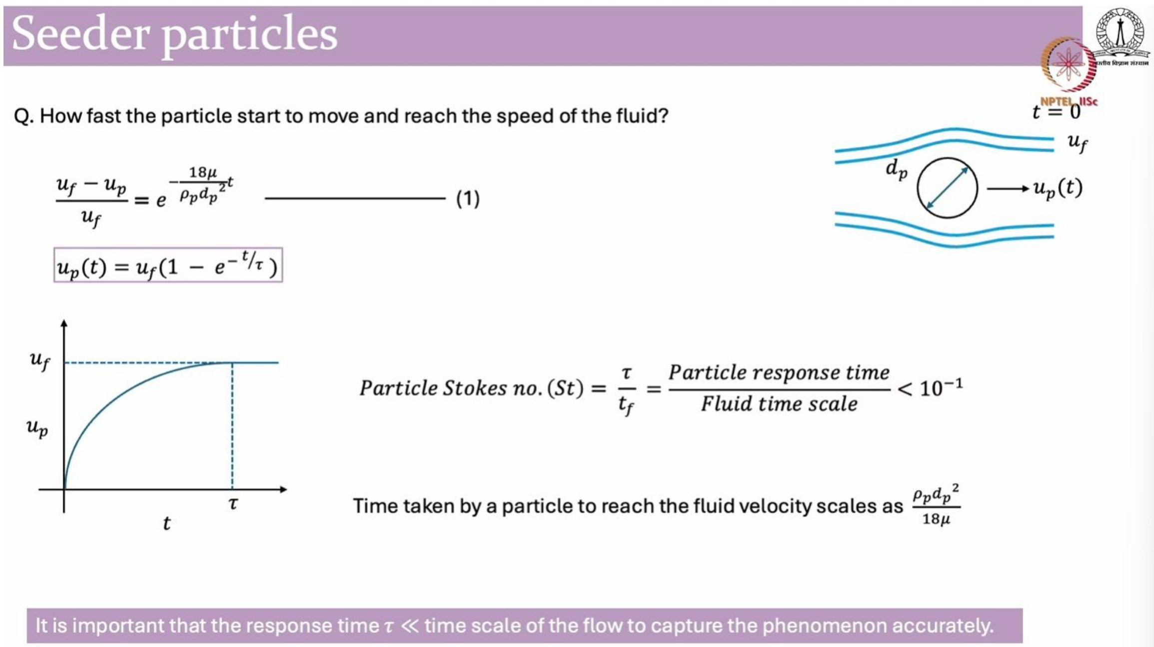
Now, the term $\frac{\rho_p {d_p}^2}{18\mu}$ corresponds to a time scale known as the relaxation time (τ)

Now, when you use this equation in this... You get the value of the constant as the logarithm of the fluid velocity, ln uf. You plug this back into the equation, and you get something like uf - up divided by u_f = e to the power of this term.

Now, if you actually see this term, it is basically nothing; if you write the units of this term, you get something like kg/ms. This is the dynamic viscosity multiplied by a ρ term, so that is nothing but kg/m^3, and you have this diameter squared. If you actually cancel the terms, you get a time inverse term. If you name this equation as 1, then this term standing here is basically nothing but 1/t, and this is the relaxation time we are talking about. Now, this term corresponds to a time scale known as the relaxation time, and we actually name it τ.

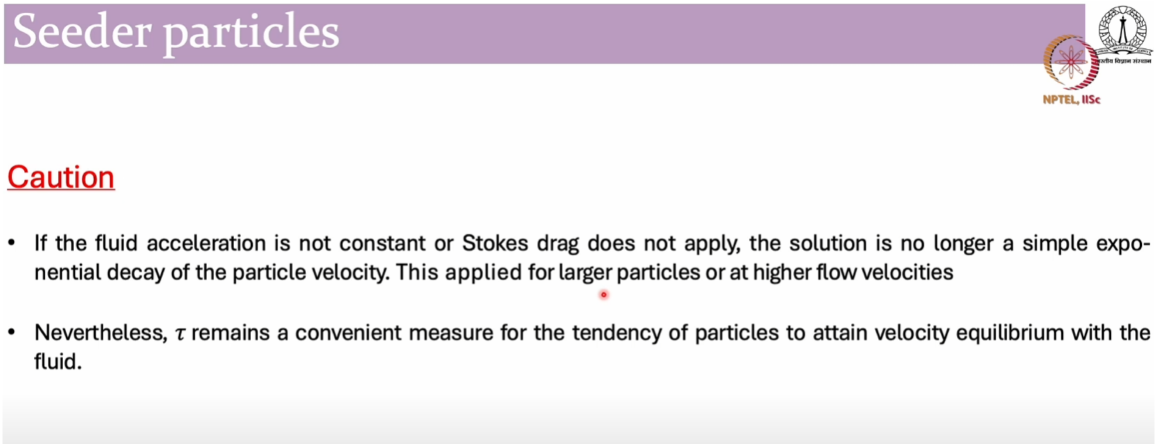
Again, I have written this equation here as you can see, and now, on further solving this, you get a relation which looks like this: the instantaneous velocity is equal to u_f

multiplied by (1 - exponential to the power of - t /τ). When you plot this, you get something like it. It is very easy to plot, and it is a routine activity. You can see that the velocity of the particle starts to increase and then stagnates at a point where it reaches the value of uf, which is the fluid velocity, and the time taken to do that is τ. Now you can understand that the particle takes τ time to reach from its initial position to an initial velocity of 0 to the fluid velocity, and this is the relaxation time.



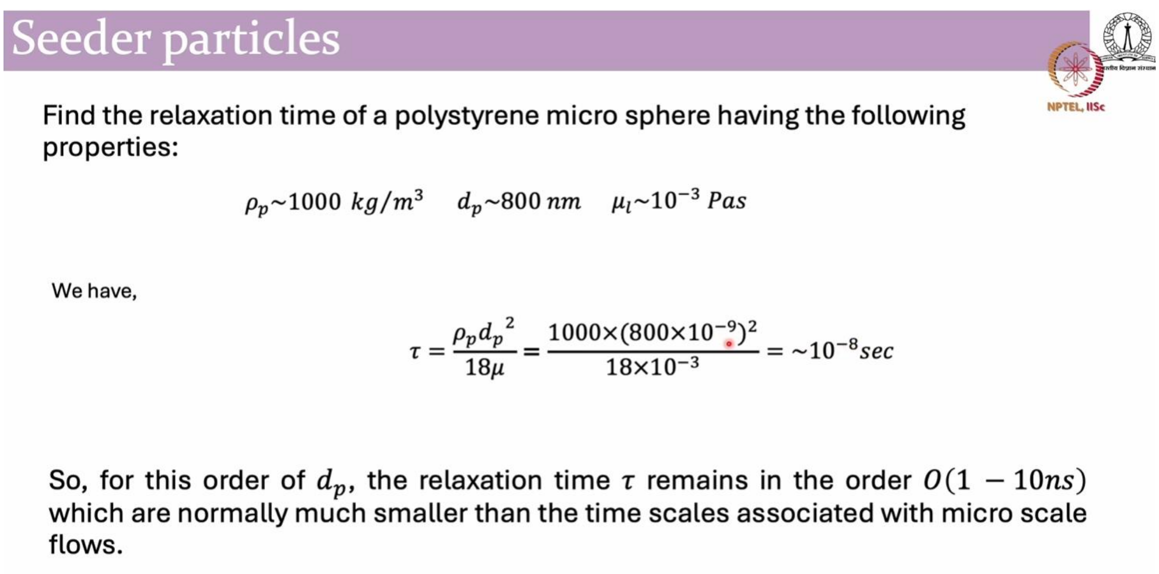
Now the time taken by a particle to reach the fluid velocity scales with this relaxation time, as we saw previously. Now you know what this relaxation time is. So, how do you use it? How do you use this information to basically say that this size and relaxation time of the particle are convenient for your flow? So, for that, you define our non-dimensional number, which is the Stokes number or the particle Stokes number, and that is nothing but the ratio of the relaxation time of the particle to the time scale of your flow. So, what is the time scale of your flow—basically, the fluid time scale? So, for example, as I mentioned earlier, if you want to see something that is happening over a period of one second, you cannot take a particle whose response time is much longer than that. You won't be able to capture that phenomenon.

So if you know the fluid time scale, if you know the approximate time scales, then you can choose a particle response time that is much, much lower than your fluid time scale, right? And in general, if it is less than 0.1, then that is actually acceptable. So a few words of caution here: basically, when we derived the previous equation for the exponential decay of the particle velocity, we actually assumed that the fluid acceleration is constant and that the stroke drag is applied. Now, if that is not the situation, then the solution may no longer be a simple exponential decay. This should be taken into consideration if your flow has a large velocity or if the particle size is much larger; but in general cases, if I generalize it, the derivation we have done is a very convenient one to measure the tendency of the particle to attain an equilibrium velocity and can be used easily.



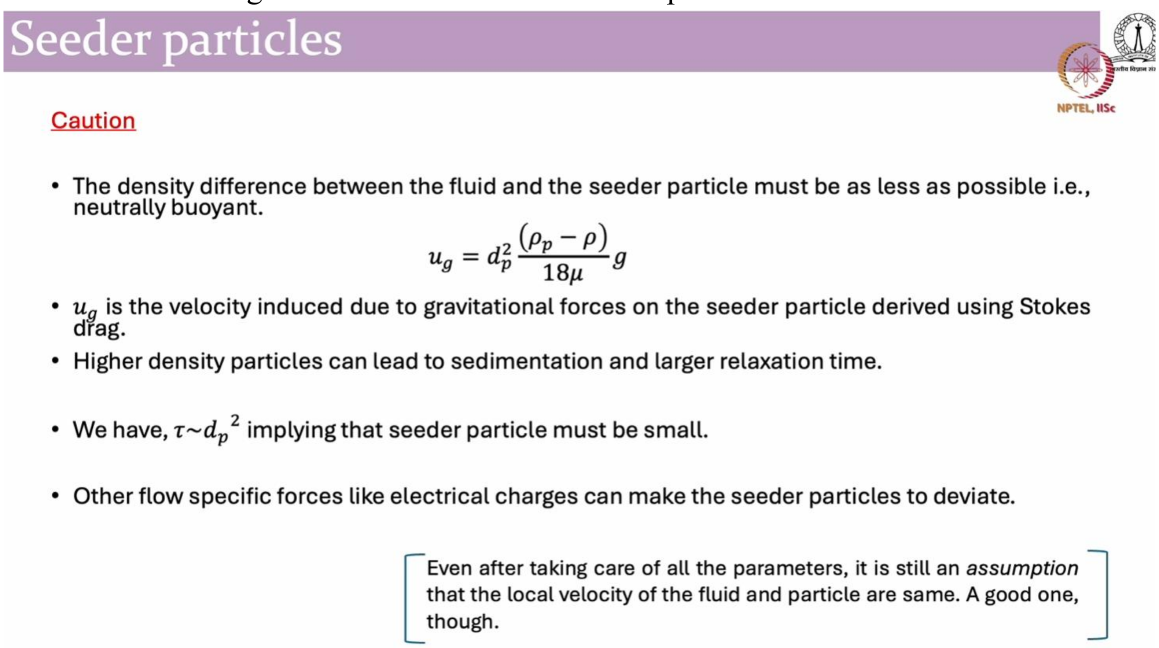
So now we will solve a simple mathematical problem to find the relaxation time of a polystyrene microsphere with the following properties. This exercise is just to give you an intuition of how things vary at this scale. So, given that the particle density is 1000, which is very close to that of water. The particle diameter is approximately 800 nanometers. The viscosity of the liquid, which is water, is 10^{-3} Pa/s.

Now, we have the relaxation time as ρ_p $\Delta p^2/18\mu$. If you substitute the values of each of the terms, you will get 1000 x 800 x 10 to the power of – 9^2. You have to keep in mind that you are doing everything in SI units divided by 18 x 10^{-3}. This gives you around 10^{-8} seconds. You can see that this time is already in nanoseconds, right? So, for this order of the particle diameter, say 200 to 2000 nanometers, the relaxation time remains in the order of 1 to 10 nanoseconds.



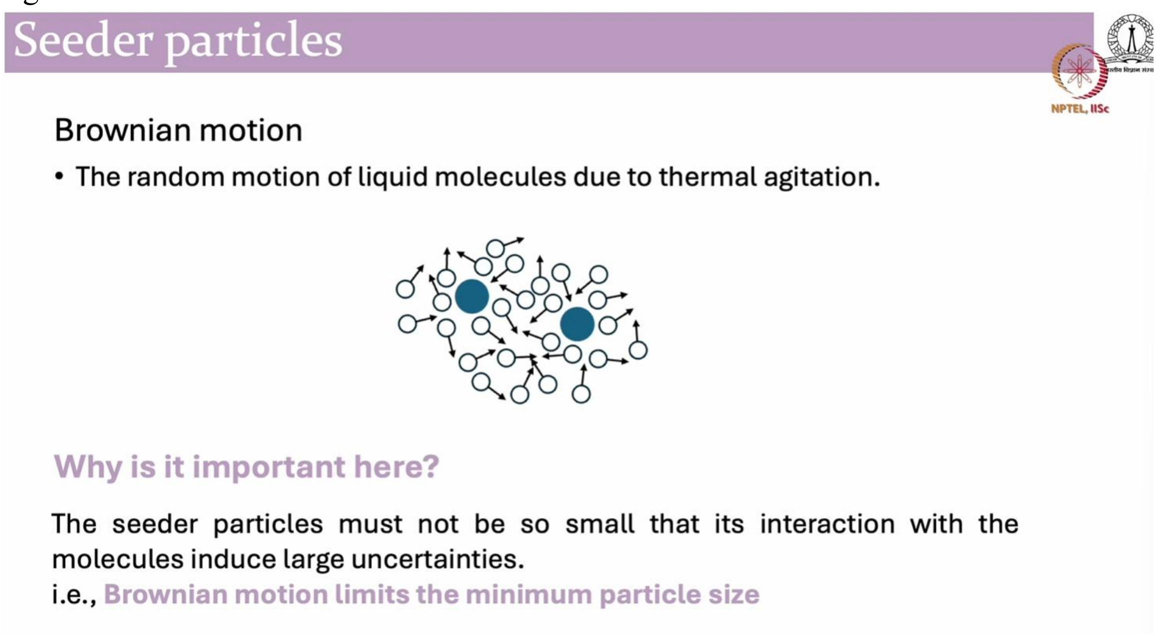
which are much smaller than the time scale that is associated with microscale flows. So, normally the microscale time scale should be much larger than this. And if you find the Stokes number, it will be much less than 1. So, you can do an exercise on your own to see how varying this dp actually changes the relaxation time. And you will see there is a dependence on the square term.

So, if you decrease this dp, your τ should decrease, right? So, taking smaller particles is favorable when you are doing experiments. Now, again, a word of caution regarding the density difference between the particle and the fluid. Now, it is very crucial to select the cedar particle that has a density very near to that of your working fluid. We can say that the particles are neutrally buoyant and are not tending towards any kind of buoyant force, which might not be the force that you want in your flow field. So, using the Stokes drag and the forces on the particle, you can actually derive a u_g term that is the velocity induced due to the gravitational forces on the cedar particles.



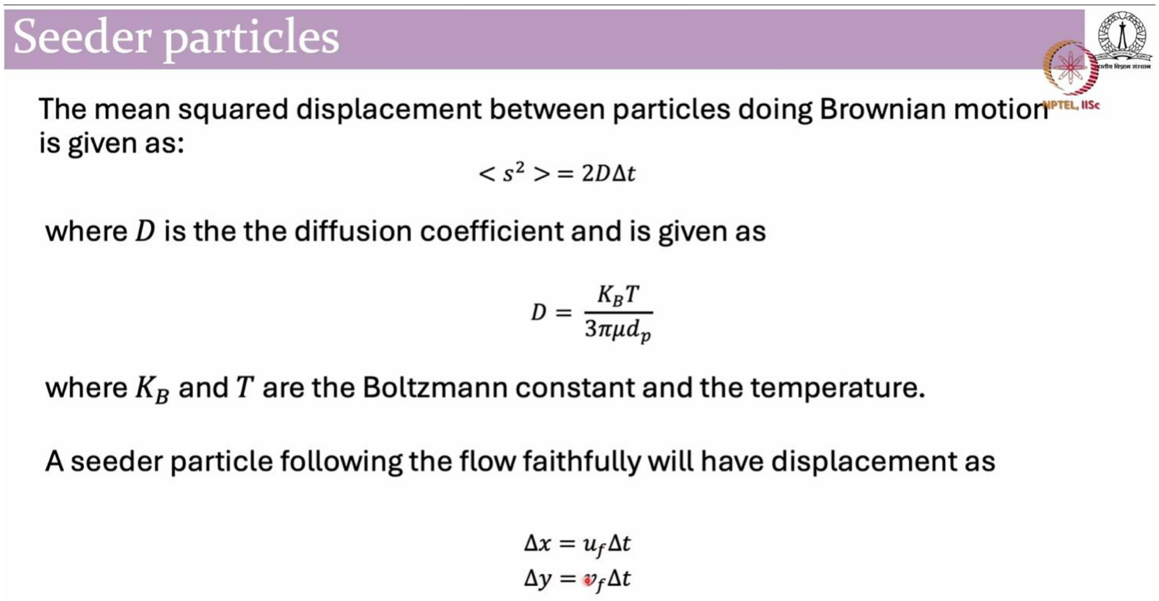
And now our aim is to basically minimize this so that this becomes very small and it does not affect your flow. To minimize this, if you minimize this difference, you can achieve a very low value of ug, and this should be your aim before starting the experiment. Another thing is that higher density would definitely lead to sedimentation, which again is what we were talking about, and the relaxation time will also increase, so we don't want that to happen. And now again, previously we saw that τ is directly proportional to the square of the particle diameter, which implies that your particle diameter must be very small, right? But again, if it is very, very small, then the Brownian motion can come into the picture, and that's why we have to be very careful while selecting the diameter of the particle. And there can be other forces, like the electrical charges, if there is a flow that has some kind of electrical charges in it that can actually act on the particles, and it can make the cedar particle deviate.

There can be other things, like if you are dealing with some kind of active fluid or things like that; those can actually play a role while your cedar particles are moving into the flow. So one has to be very careful in choosing these particles, the cedar particles, when you are not dealing with traditional flow fields, and another very important thing of which we should be aware is that even after taking care of all the parameters like the relaxation time, the particle diameter, and the density difference, it is still an assumption that the local velocities of the fluid and particle are the same because Again, there will be some kind of relaxation time that the particle will have, although very small, but it will have, and there will be some error; however, this assumption is still a good one to get the flow field to a certain reasonable accuracy. Now we will talk a bit about Brownian motion, of which probably everyone is aware, and it is basically the random motion of liquid molecules due to thermal agitation. So, I will just give you a pictorial depiction of what I am talking about: suppose these blue particles represent the particle diameter, and these are your cedar particles, with many large molecules around them. Now, if your particle size starts to become so small that it begins to reach the order of these molecular particles, then the interaction of these molecules with your particles can become very significant.



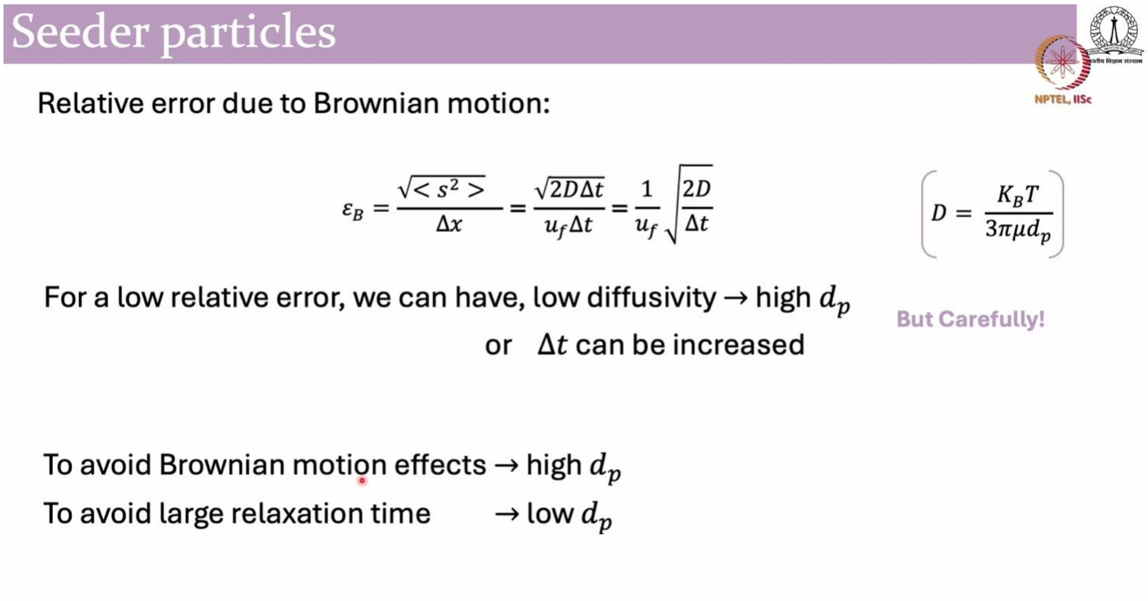
And that can actually cause some kind of uncertainty in your flow, and that can actually deviate your particles, and this can be an additional force that actually starts acting on your particle. So, we should be very careful about that. Now, why is it important? Again, it is important that the pseudo particle not be very small so that its interaction with the molecules does not induce large uncertainties. And so we can say that Brownian motion actually limits the minimum particle size of the cedar particles that we should consider while doing micro PIV. So now let us see how we can actually find the mean square displacement between particles that actually perform Brownian motion.

So, if you find the relation, it actually looks like this: where you have the displacement S and the mean square displacement is given as 2 x capital D x Δ T, where D is the diffusion coefficient and that is given as the ratio of K B to T. This is the Boltzmann constant, T is the temperature in Kelvin, and this is divided by 3 π μ x the particle diameter. And this Δ T is the time scale; basically, you can say it is the time between which you are actually capturing your images. So, if we say that the cedar particles are following the flow faithfully, It will have displacement given as Δ x is equal to the velocity of the fluid x Δ t, and this velocity is basically uf, which is in the x direction, and again in Δ y you will have v_f, which is in the y direction, x Δ t. Now, this should hold good when we have confirmed that the cedar particle is following the flow faithfully.



So then, if we talk about the relative error that is caused by the Brownian motion in the flow field, we can write it as this ε b, just a second. This ε b, which is the relative error, can be expressed as this term: the Brownian displacement, basically the square root of the mean square displacement given by Δ x, and I am just solving it in one direction. So, this is the normal displacement of the particle, and if you solve it, you will get something like this, which is the normal formula we derived in the previous slide. Which gives us a relation that is $(1/u_f)\sqrt{(2d/\Delta t)}$. So, from this relation, we can assume that for a low relative error, that is, for a lower velocity of ε b, we should have a low diffusivity of the particle, and if you have a low diffusivity, your particle should be higher in dp; like it should have a higher.

Particle diameter, which is again not desirable, can lead to a larger Δ t, which is very subjective because normally this Δ t depends on your flow. It is a very subjective matter, and it cannot be randomly increased or decreased. So, again, there is a tradeoff that one has to consider. In case you want to keep this relative error very small, that's what I have shown here: this D is inversely proportional to D_p. So, if you want to decrease this D, the diffusion coefficient, you have to increase the D_p, which is normally undesirable.



But yeah, there is a range for that, and you can selectively choose that value of D_p which is giving you. Relatively lower error, so this is what I mean: to sum up, to avoid Brownian motion effects, you should have a high D_p (the particle diameter), and to avoid larger relaxation time, your D_p should be low. So, it is again a trade-off, and this is something a person who is doing the experiment should calculate very carefully and then select their particle diameter. Now, we will just solve a simple problem that will give a better intuition of the relative error that is caused because of Brownian motion. So, we have a problem that states we need to find the time interval between two images, Δ t, which must be considered to maintain a relative error.

E_b value of 5% and the fluid velocity in the x direction is given by 100 micrometers per second, and the particle diameter is 500 nanometers. So, let us start solving this, and let us see where we end up. So, we need to find the Δ T. First, we will see what the diffusion coefficient is, which is given as the Boltzmann constant x temperature divided by 3 π μ dp, and if you put the values of the Boltzmann constant and the temperature in Kelvin, you end up getting a value of 8.

7 x 10 to the power of - 13. square meter per second. This is the diffusion coefficient. Then the error term, which is the ε b, that we found out was like this. It comes out from this that what you have is the error you want, which is 5%; you put it here, and you get the Δ t to be 70 ms. So basically, this is the time you should actually select if you want your error to be within 5% due to the Brownian motion, and if you have these fluid velocity and particle diameter scales.

However, I mean this relative error can again be reduced, and it should be reduced because this is what the whole exercise is for. A single particle in normal conditions has multiple particles, and so if you have several particles, intuitively the relative error will be reduced. If you do some ensemble averaging over several realizations, I mean if you take multiple runs. In that situation, you can reduce the error by a huge margin.

So, I will just show you how we can do it. Suppose you have 3 images: this is 1, this is 2, and this is 3, and in each image, you have 4 particles. So, basically, this m is the number of images and n is the number of particles. Particles, so if you multiply these two, you get a term that is n. Okay? So the diffusive uncertainty decreases as 1 divided by the square root of n. So if you have a higher M, basically you have a higher root n; if you have a higher root n, the diffusive uncertainty actually decreases more, so basically if you have a large number of images or realizations, you can have a lower error.

Seeder particles

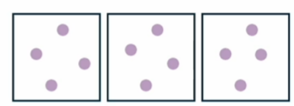
Find the time interval between two images (Δt) that must be considered to maintain a relative error (ε_B) value of 5%. The fluid velocity (u_f) is 100 $\mu m/s$ and the particle diameter is 500 nm.

$$D = \frac{K_B T}{3\pi\mu d_p} = \frac{1.38\times10^{-23}\times298}{3\pi\times10^{-3}\times500\times10^{-9}} = 8.7\times10^{-13}\ m^2/_s$$

Then,

$$\varepsilon_B = \frac{1}{u_f}\sqrt{\frac{2D}{\Delta t}}$$

$$\Rightarrow \Delta t = 0.0696 \text{ sec} \sim 70\ ms$$



$$N = M \times n$$

Then, the diffusive uncertaintiy decreases as $^1/_{\sqrt{N}}$

So, if number of images $M = 6$ and $n = 4$, then

The uncertainty reduces to $\frac{\varepsilon_B}{\sqrt{N}} = \frac{0.05}{\sqrt{24}} \sim 1\%$

The relative error can be reduced by both averaging over several particles in a single interrogation spot and by ensemble averaging over several realizations.

So, if the number of changes, if the number of images m = 6, say, and the number of particles shown here is 4, then the uncertainty reduces to $\varepsilon_b/\sqrt{n}$, which is nothing but your ε n for a single particle; ε b for a single particle was 0.05 /√6 x 4, which is 24. This gives you a relative error of around 1%. So, see, I mean, if you do a lot of experiments, you can get a lot of images, and you can actually reduce the uncertainty error. However, again, doing a lot of experiments and performing ensemble averaging over larger relations becomes very challenging over time.

And yeah, so this is basically how you can deal with the relative error that arises because of Brownian motion.