Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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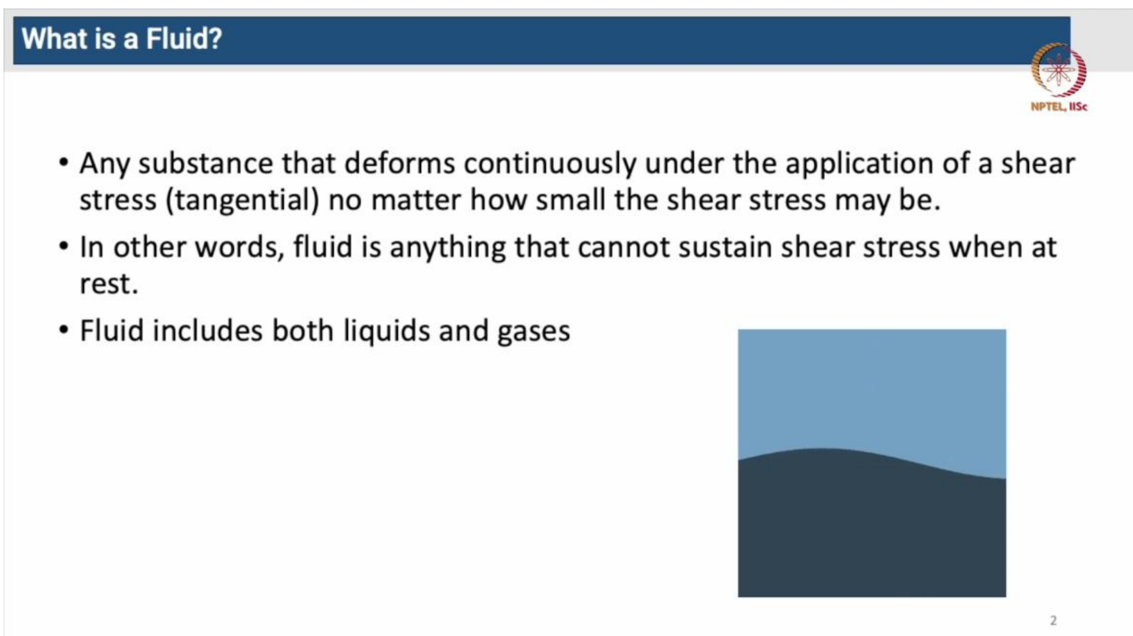
Week – 01

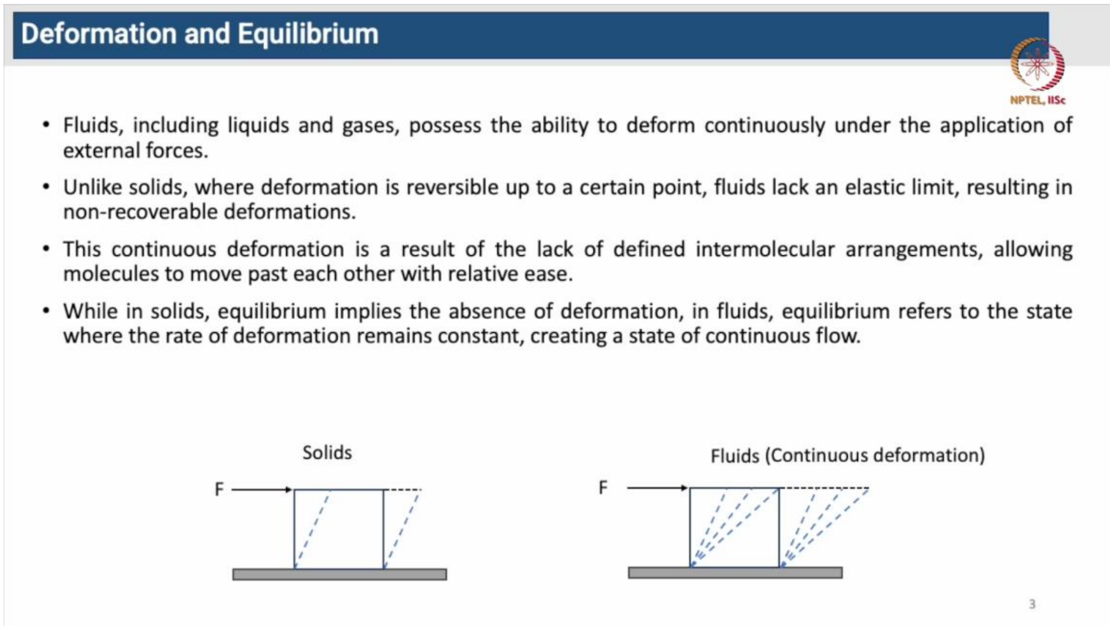
Lecture - 04

Recap and description of fluid flows

So, in this particular lecture, we will just do a recap of the fundamentals of fluid dynamics, which will be handy. So we have already taught, and we have gone through the mathematics; this is just an overview. So, what is a fluid? Anything that deforms continuously under the application of shear stress, including fluids, encompasses both liquids and gases. So, in the deformation and equilibrium part, fluids, including liquids and gases, possess the ability to deform continuously under the application of an external force. So, unlike solids, the deformation is reversible to a certain extent. The fluids therefore lack an elastic limit, so to speak.

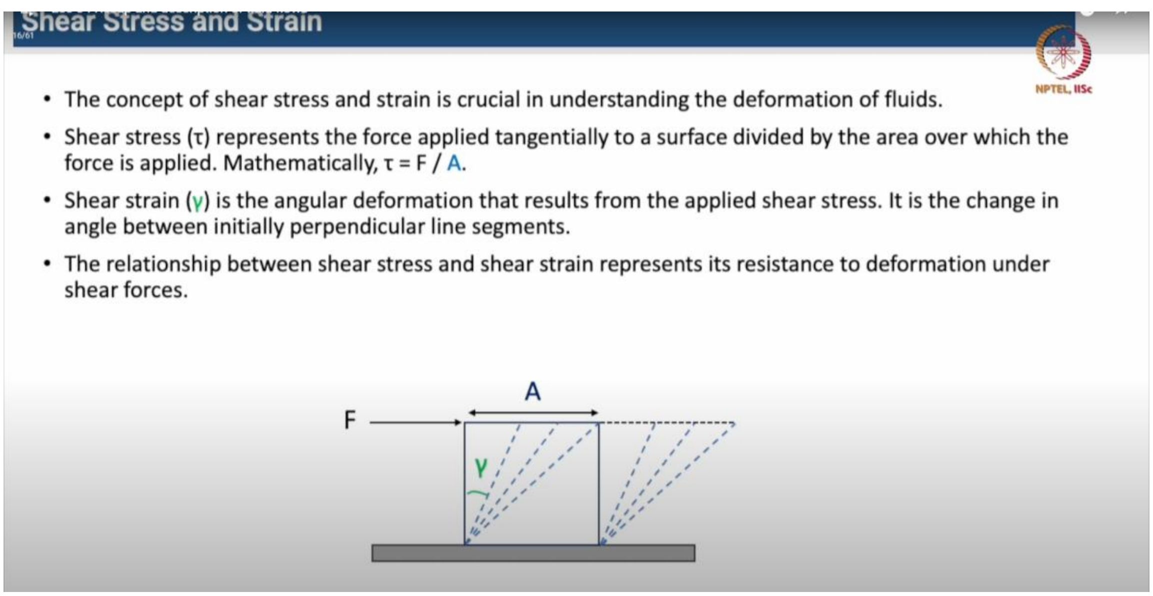


In solids, however, equilibrium implies the absence of deformation. In fluids, equilibrium refers to the state in which deformation remains constant.

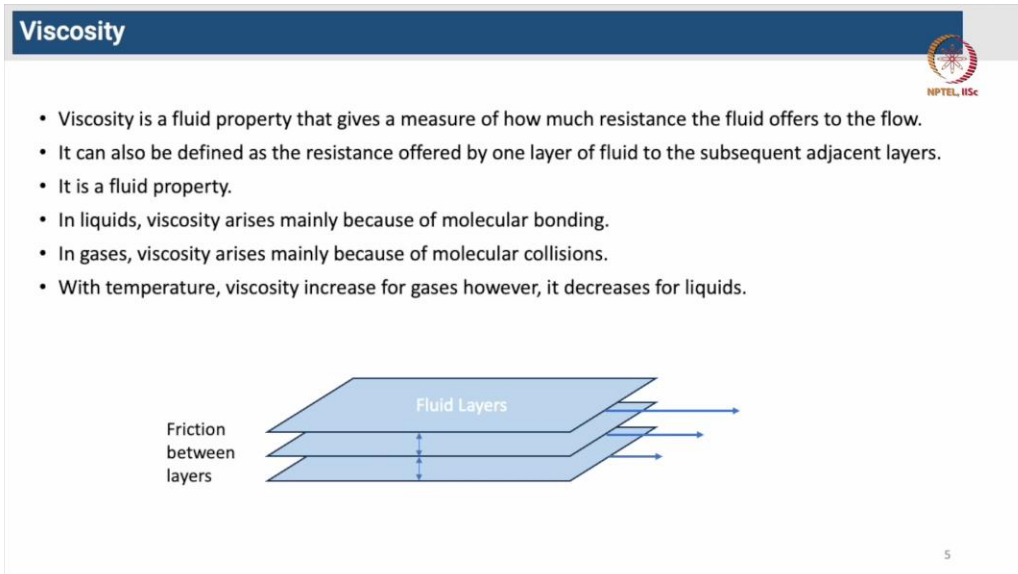


So, there is a concept of stress and strain. So, shear stress represents the force that is applied tangentially to the surface by the fluid.

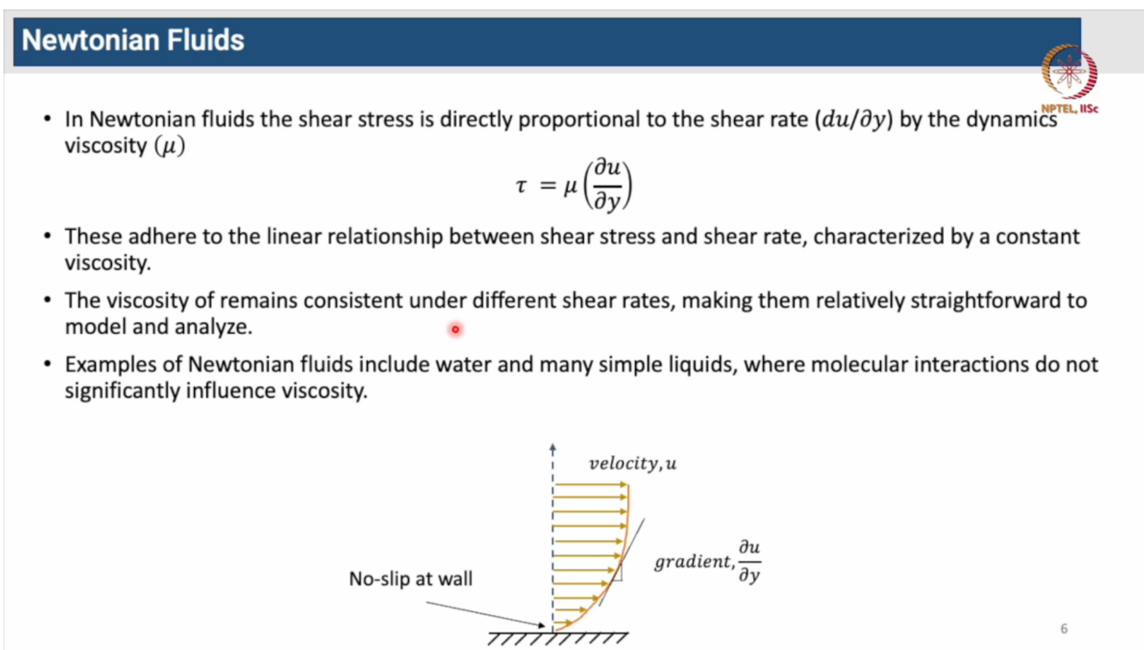
And divided by the area, the shear strain is the angular deformation that actually happens. So the relationship between shear stress and shear strain represents their resistance to deformation.



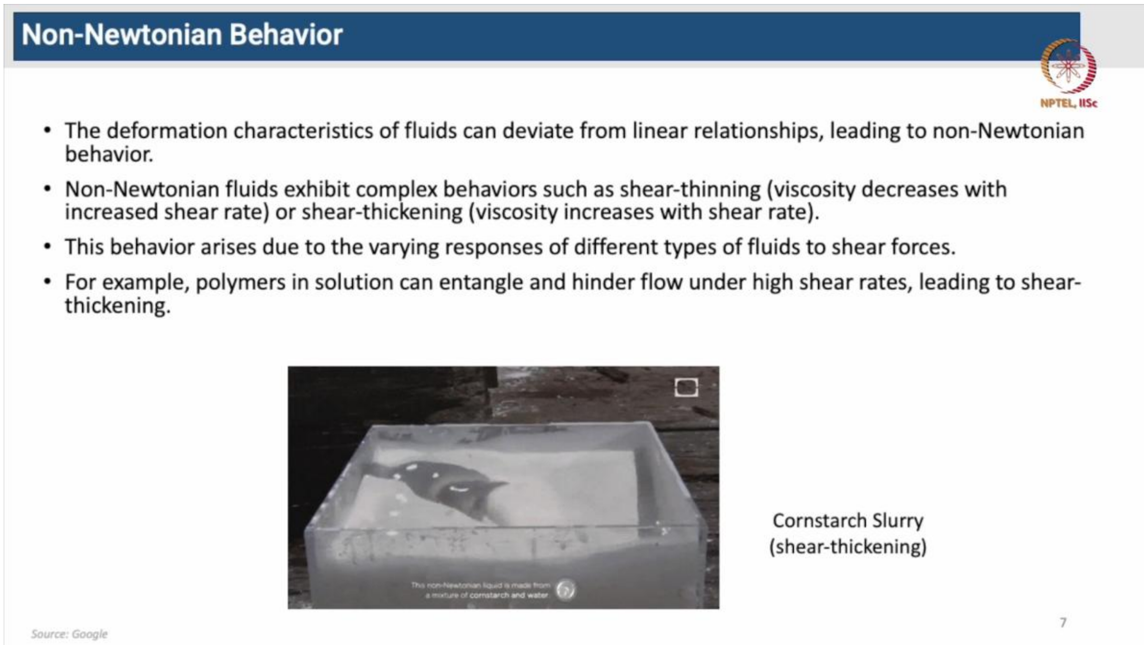
Then viscosity, also defined as a property that measures how much resistance the fluid offers to flow. It is also defined as the resistance offered by one layer of fluid to the subsequent adjacent layers. So with temperature, viscosity increases for gases; however, it decreases for liquids.



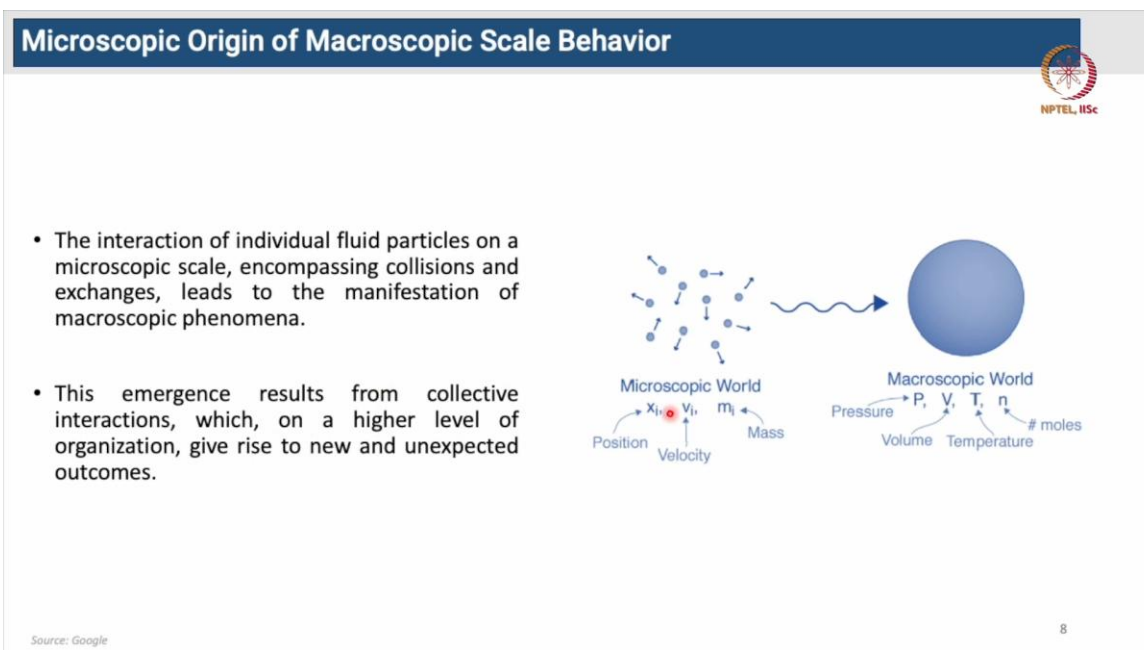
So, if the fluid is called Newtonian, then we have the constitutive relationship that basically links the shear stress with the velocity gradient. This is the cross-velocity gradient, and μ is the coefficient of viscosity. This adheres to a linear relationship between shear stress and shear strain. So the viscosity remains constant under different shear rates, making it relatively straightforward to analyze. So, this is the velocity gradient, for example. This is the U velocity varying in the Y-direction.



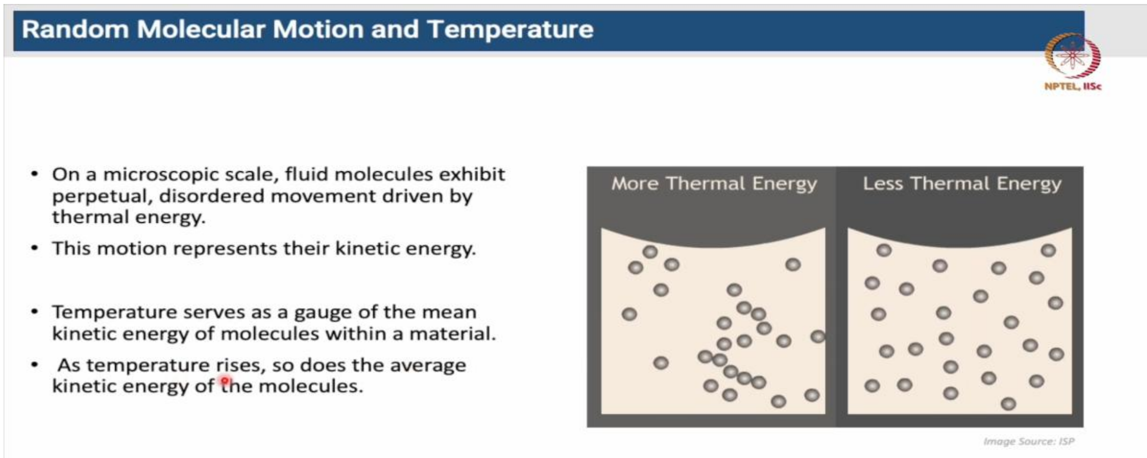
The examples of Newtonian fluids include water. These are all the things that we did. Then, if you talk about non-Newtonian fluids, they are fluids that exhibit complex behaviors; shear thinning is one example, and shear thickening is another example. This behavior arises due to the response of the fluids to shear forces.It's not as simple as what we saw over there. Polymers can be an example of a complex fluid.



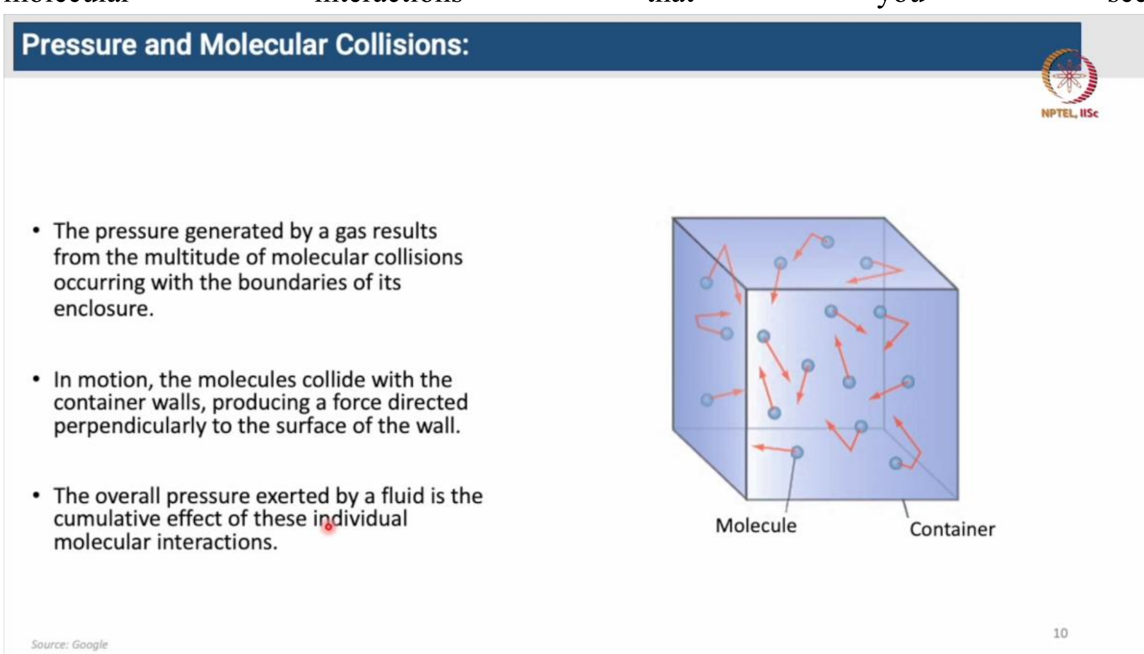
Okay, so each of these things has a microscopic origin. So when you look at fluid dynamics in the macroscopic world, you have pressure, volume, temperature, etc. All of these occur in the microscopic world where the positions, velocity, and mass of each atom or molecule actually jitter.And that leads to the manifestation of the interaction at the microscopic level, leads to collective interactions, and leads to higher level organization, giving rise to the kinds of properties that we see in the macroscopic world.



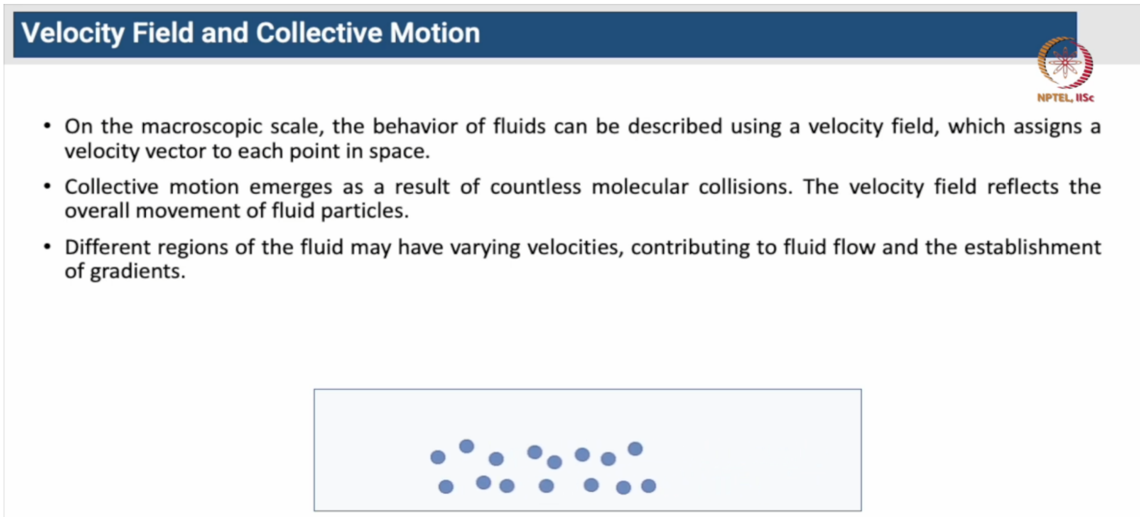
So each of them has an underlying microscopic theory. So, on a microscopic scale, for example, the fluid exhibits perpetual disordered motion. This is like its kinetic energy. They can also rotate.They can also have raw vibrational energies. So the temperature serves as a gauge of the mean kinetic energy of the fluid within the material for translational motion. For example, this has more thermal energy. This has less thermal energy. So, if the temperature rises, the average kinetic energy also goes up.



So the pressure is basically generated by, you know, the gas from the multitude of molecular collisions that occur at the boundary and at the enclosure. In motion, the molecules collide with the container walls, producing a force that is directed perpendicular to the walls. So the overall pressure exerted by a fluid is a cumulative effect of all these molecular interactions that you see.



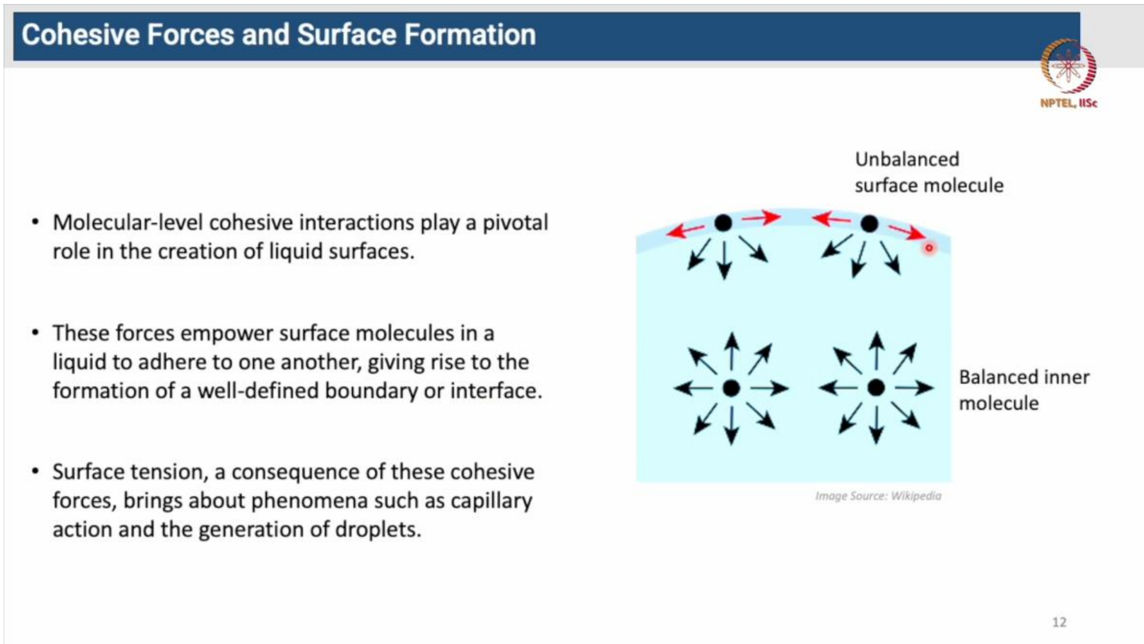
So, similarly to the macroscopic scale, the fluid velocity can be defined using a velocity field. It is typically the Eulerian representation, and there are vectors at each point in space that actually vary with time. So, the collective motion, however, emerges out of molecular collisions. The velocity field reflects the overall movement of these fluid particles. Different regions can have different velocities. So there can be spatial gradients of velocities, and there can be temporal and spatial variations of velocities in all four dimensions.



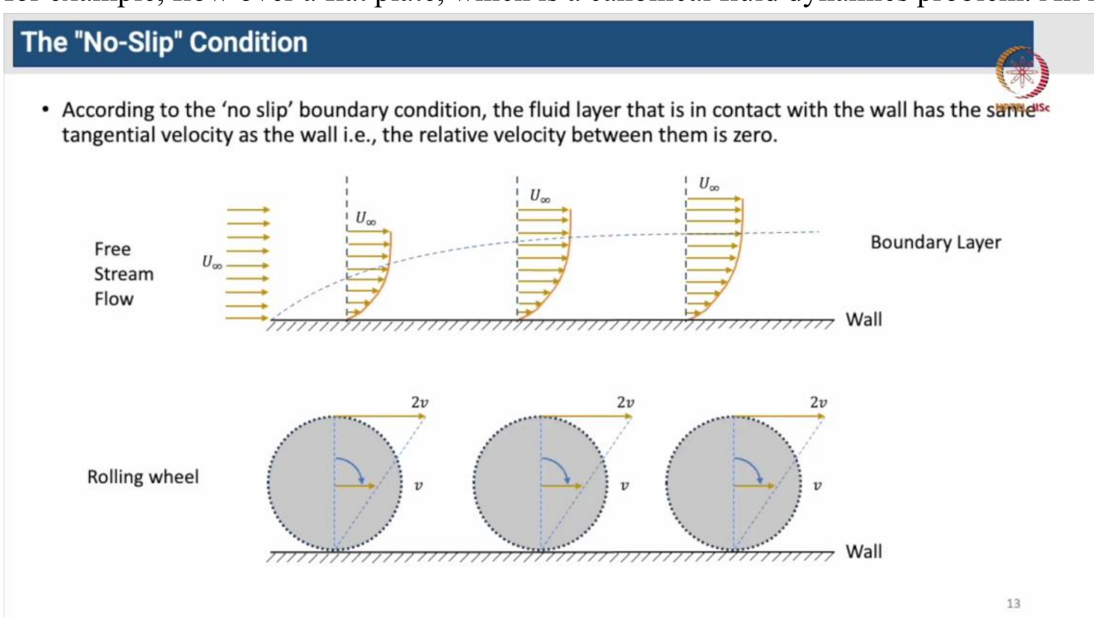
Then comes the concept of cohesive forces, which is particularly important because later on, when we do PDPA, LDV, and other things, we will use droplets, for example.

So you need to know a little bit about cohesive forces and surface formations. So basically, when you actually have, for example, a container containing some liquid. Now the molecules that are at the center actually have forces that are occurring in the same way in all directions. So they are in balanced condition. However, the molecules that are at the surface have an unbalanced component of the forces.

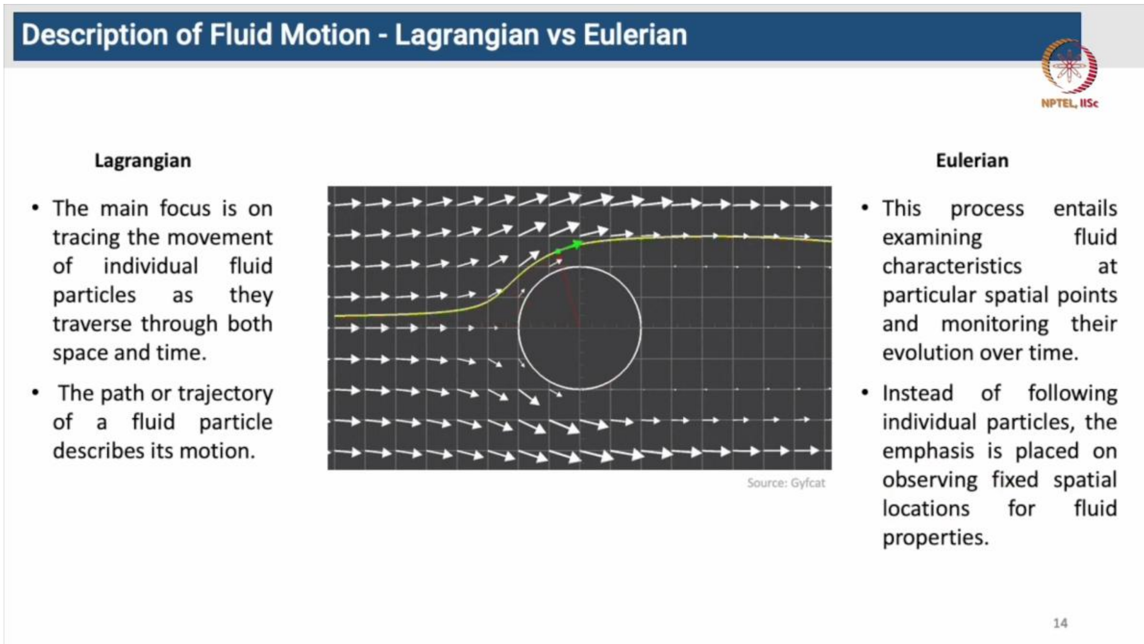
So, this leads to surface tension. It is a consequence of these cohesive forces, which actually brings about phenomena like the generation of droplets and capillary action. So, these forces empower the surface molecules of a liquid to adhere to one another, giving rise to the formation of this well-defined boundary or interface that you see over here.



Then, of course, the concept of no-slip boundary conditions is extensively used when you actually deal with wall-bounded flows and things like that. So, at a stationary wall, the relative velocity between the fluid and the wall is zero. Therefore, the wall has the same tangential velocity as the fluid. So they actually do not have relative velocity with each other. All right. So, this is the most important part that you should keep in mind. This is, for example, flow over a flat plate, which is a canonical fluid dynamics problem. All right.

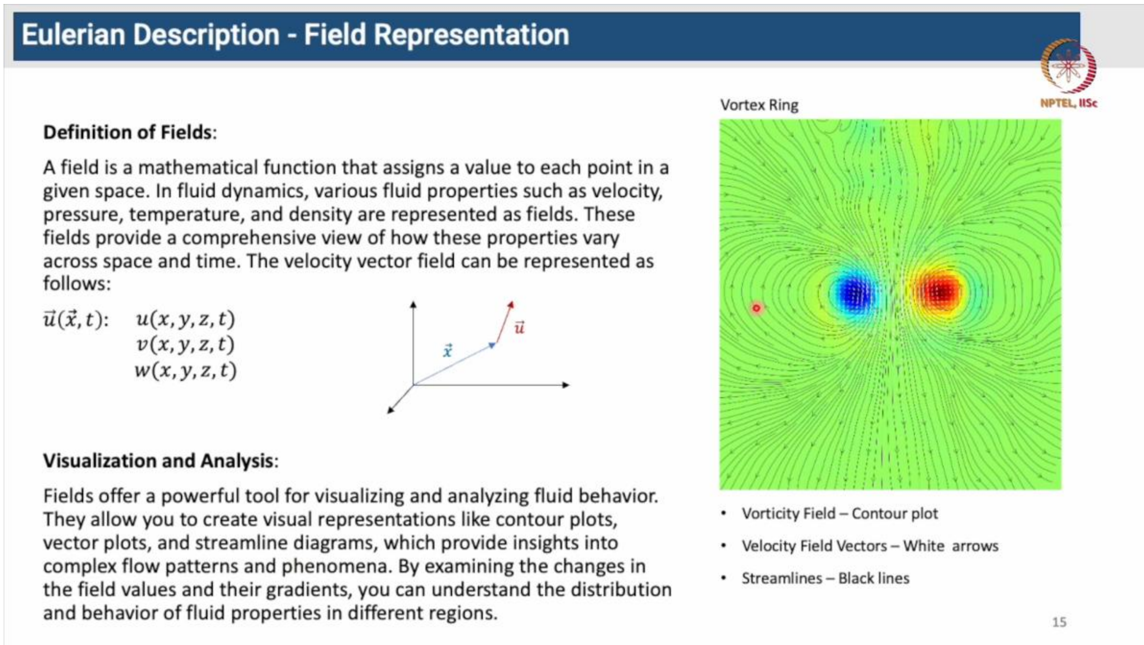


Then, of course, the concept of Lagrangian and Eulerian kinds of description. So, for example, in the Lagrangian, what you do, which we already covered in detail, is that you track individual particles, fluid particles as they traverse. But we said earlier that this is nearly impossible to do. But this is the path of the trajectory, for example, of a fluid particle that describes its motion. Eulerian is, however, that you monitor certain spatial points and observe the evolution of the fluid characteristics at those points.So instead of following individual particles, the emphasis is placed on observing fixed spatial locations for the fluid's properties. So that is Eulerian for you.



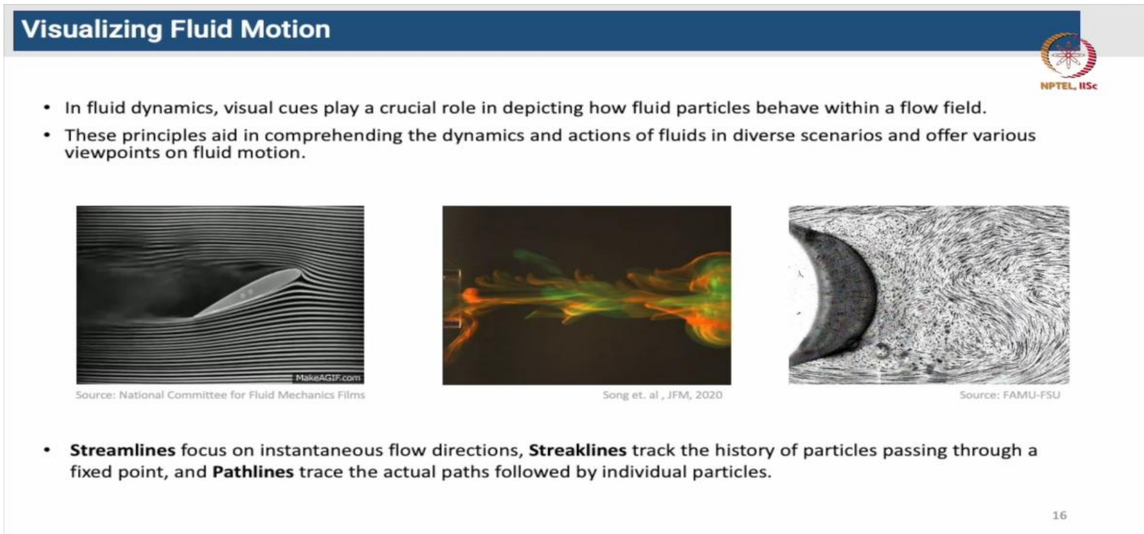
So if you, Eulerian is therefore a field representation, and it is extensively used in fluid mechanics, as we already know by this time. So this is, for example, a field equation for velocity. So the mathematical field is a mathematical function that assigns a value to each point in the space.

In fluid dynamics, various fluid properties such as velocity, temperature, pressure, and density are represented as fields. So it is very useful to visualize the flow field using this technique. For example, this shows vorticity, velocity, and streamlines all in one. So it allows you to create visual representations like contour plots, streamlines, which enables us to get insights into complex flow patterns. And PIV, as we will see, is exactly a field representation.



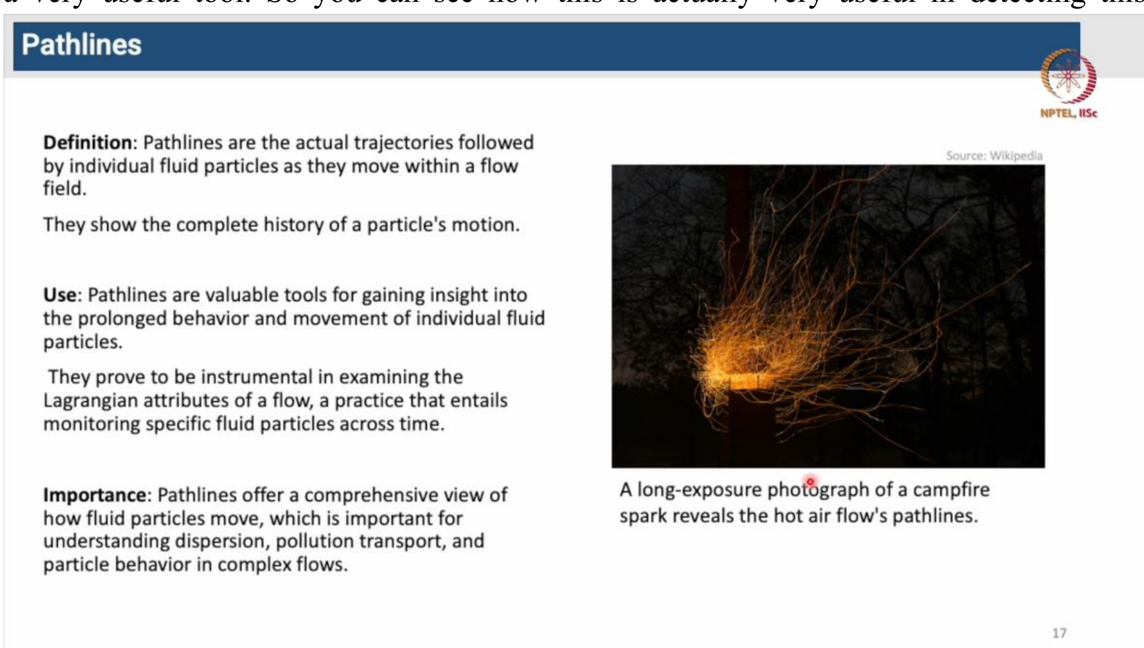
So this is, for example, a visualization. For example, this is behind a tilted aerofoil and how the flow separation actually happens. And this is like a vortex. And this is another shedding behind the body. So in all these cases, you see that there are visual cues that play a very strong role in how the fluid behaves.

Now there are, as we already talked about earlier, different ways to represent the flow field. We can have streamlines. which focuses on instantaneous flow characteristics. We can have streak lines that track the history of particles passing through a fixed point. So you are acting like a customs officer.And the path line traces the actual paths followed by the individual particles. So all of these are very useful ways of representation.

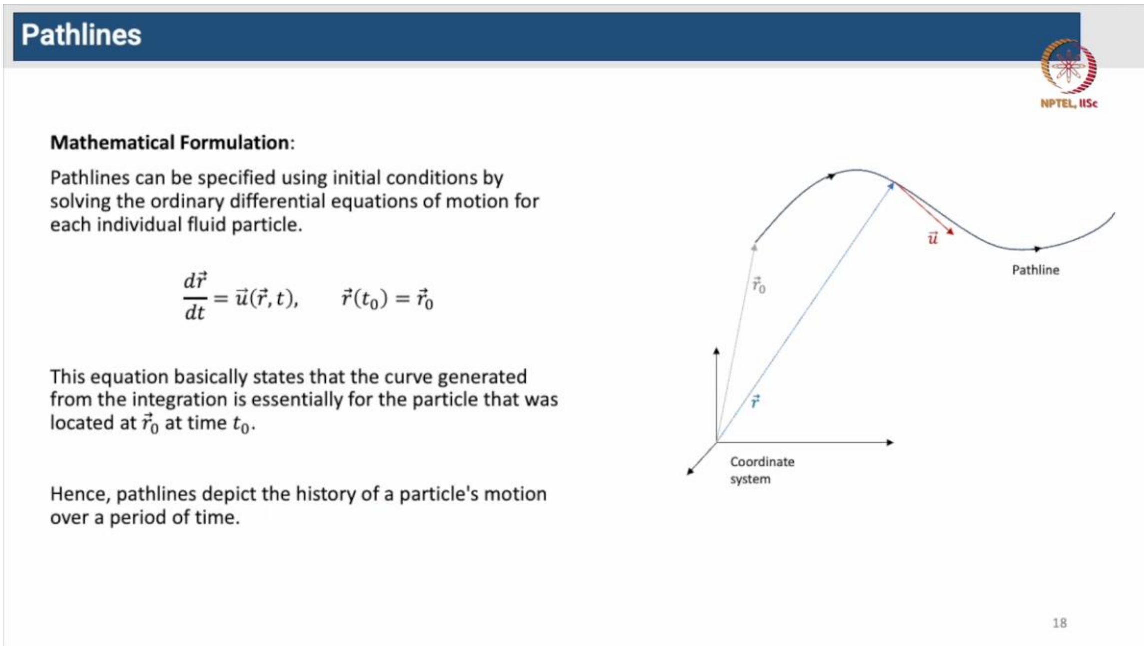


So these are, for example, pathlines. The pathlines are actual trajectories followed by individual fluid particles as they move. So this is a long exposure photograph of a campfire which shows the path lines.

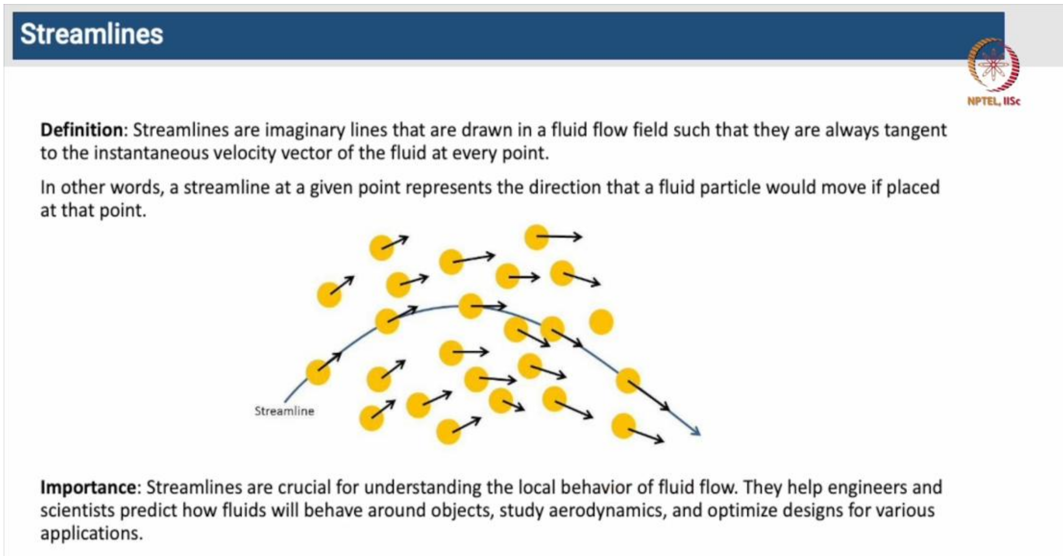
So path lines are valuable tools because they enable you to gain insight into the prolonged behavior and movement. They can also tell you how cohesive the path lines are. So understanding how the transport might have happened, especially in complex flows, this is a very useful tool. So you can see how this is actually very useful in detecting this.



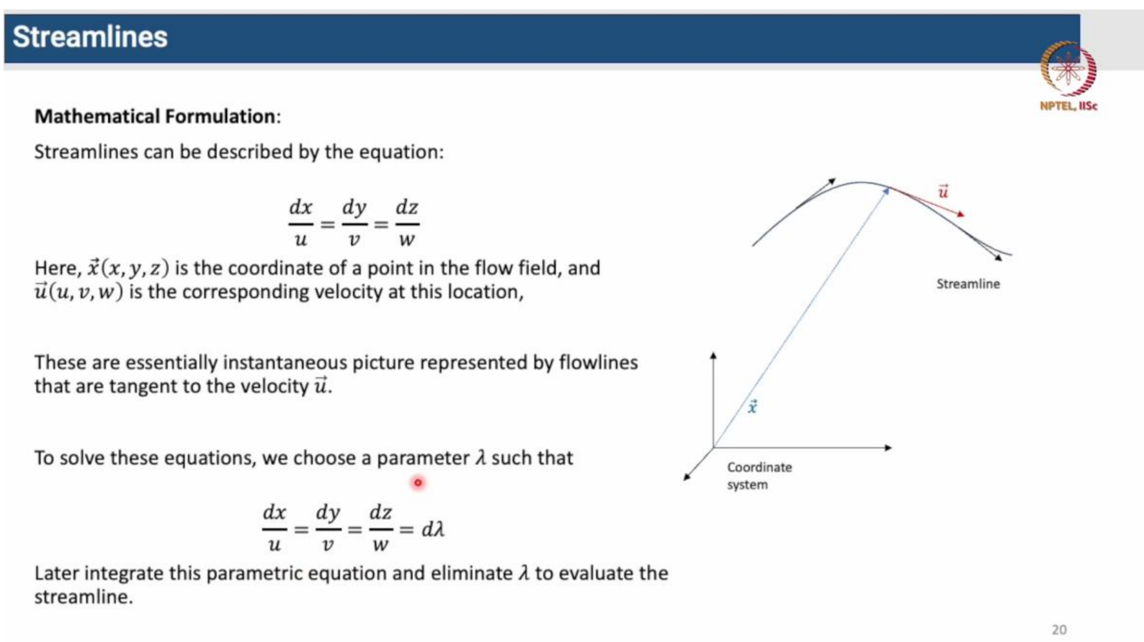
Then, of course, the path line has a mathematical formula.The path lines are basically specified by dr/dt, which is u. This is like a Lagrangian description. That's how the position vector is changing with time, and that is the pathline. And of course, you have an initial position, which is r0, and then how it evolves over time. So this is how this vector actually changes. The position vector changes with time. So pathlines depict the history of the particle's motion over a period of time.



And then, of course, there is the streamline which you already know. The streamlines are constructed in such a way that, everywhere along the streamline, the flow velocity is tangential at that particular point. So these are imaginary lines that you can see. The fluid flow field is always tangent to the instantaneous velocity vector at every point. So in other words, it gives a representation of a fluid parcel that would move if it were placed at this particular point. So they are crucial for understanding the local behavior of the fluid flow. This enables us, scientists and engineers, to understand how the fluid will behave around objects, aerodynamics, and other things. So this is also useful for the representation of, you know, when you want to represent PIV. So this would be one of the key features of it.

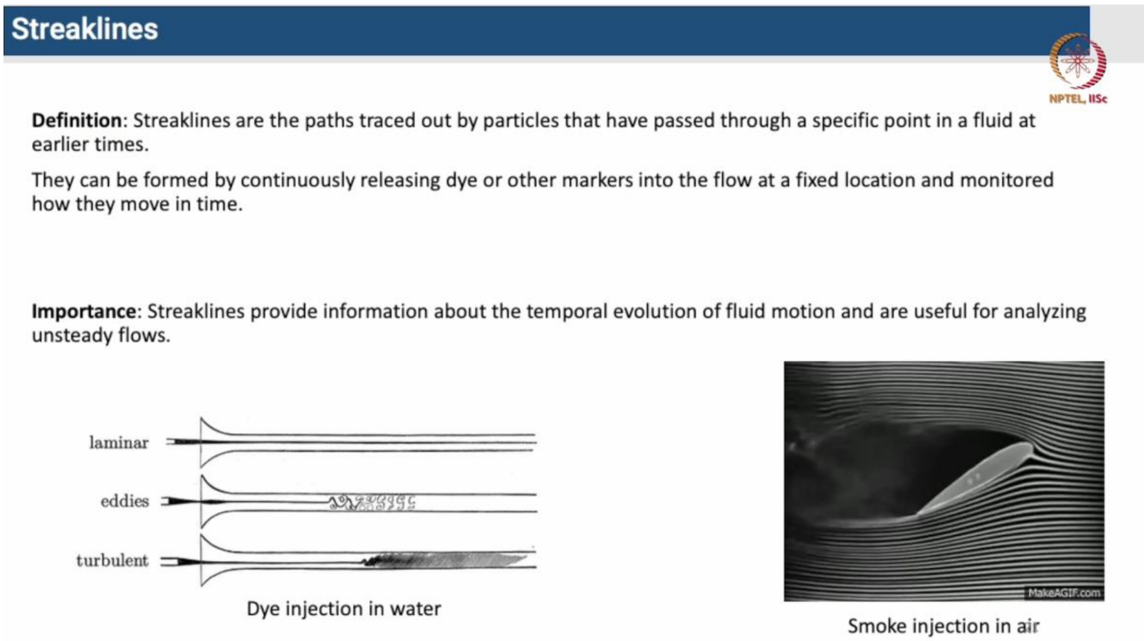


So streamlines have this mathematical representation, where x, y, and z are the coordinates. So $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$. So here x and u, x is the corresponding coordinate system, and u is the corresponding velocity. So this is the instantaneous picture that is actually represented. So, to solve this equation, you need to choose a parameter and then use it later to integrate this parametric equation.



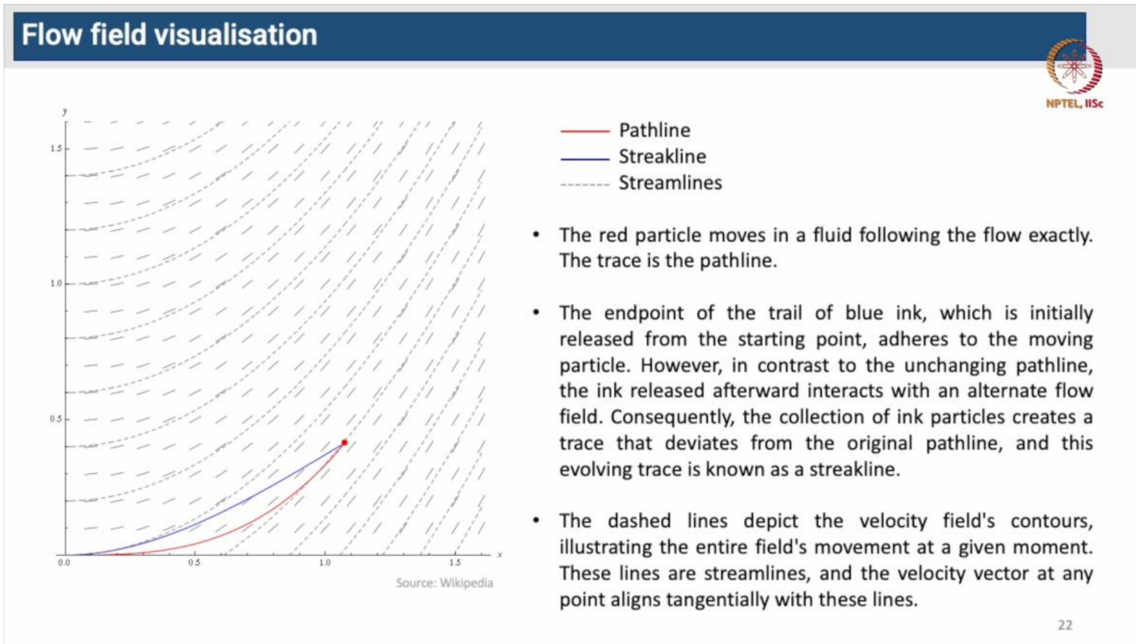
So streak lines are paths traced by a fluid particle passing through a specific point. So if you are monitoring at a particular point and you are seeing all the particles, the paths that are traced out by them provide information about the temporal evolution of the fluid field and are very useful for unsteady flows. So, for example, this is laminar flow. These are the

flows with eddies.
This is a highly turbulent flow. They have injected some dye into the water. You can also see at a specified location how the smoke injection, for example, happens. So each of them has their own advantage.

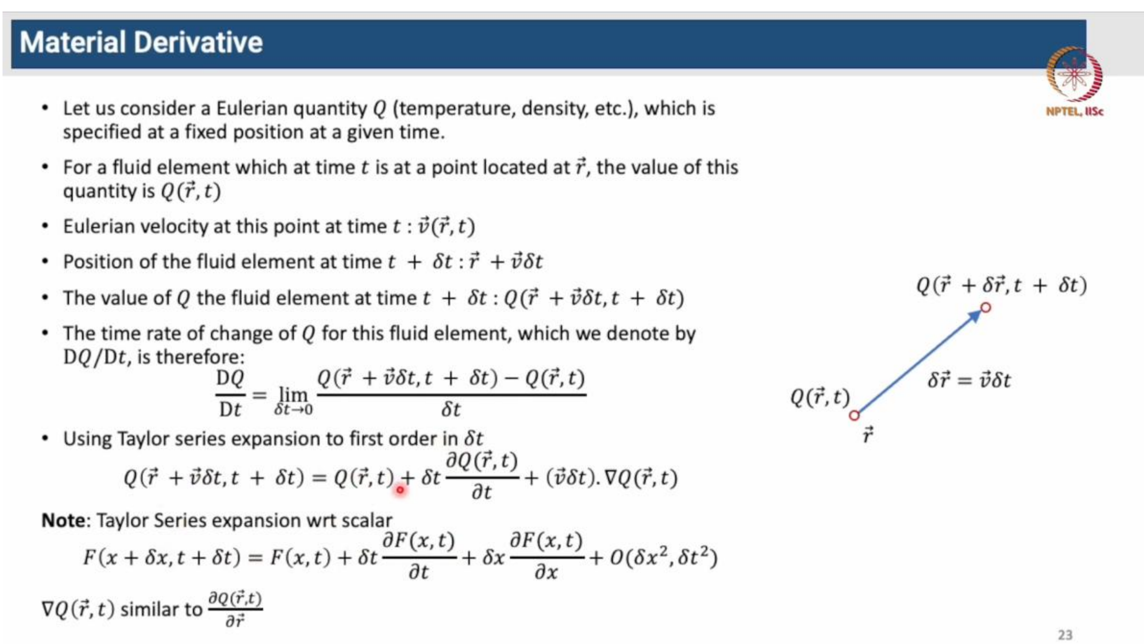


So if you look at the flow visualization situation, you will see pathlines, streaklines, and streamlines. Streamlines are obviously those dotted lines, and the red and blue are the path lines and streak lines, respectively.So you can see, the red particle moves; the red particle moves in a fluid following the flow exactly. The trace is actually a pathline. The endpoint of the trail, which is the trail of the blue path that is initially released from the starting point, adheres to the moving particles; however, in contrast to the unchanging path line, the ink released afterward interacts with an alternate flow field. So the dashed lines represent the velocity contours and illustrate the entire field's movement at any given point.

So these lines are streamlines, and the velocity at any point is tangential to the streamlines. So this is just a demonstration of how the pathlines, streamlines, and streaklines actually vary from one another.



Now we have already covered the concept of the material derivative, so if we consider an Eulerian quantity Q that is fixed at a specific position in time, then for a fluid element that is at time t at that location, the value of that quantity is $Q(\vec{r}, t)$, the Eulerian velocity at that point is $\vec{V}(\vec{r}, t)$, , and the position of the fluid element is this; therefore, the rate of change of Q for the fluid element. Which is an Eulerian quantity, is denoted by capital DQ over DT, which is given by this. So, if you use a Taylor series expansion to the first order, you get something like that. So the Taylor series expansion with respect to a scalar gives you something like this.



We have already covered. So, if you substitute everything, you will get this. So the material derivative is the Lagrangian rate of change, the substantial derivative, or the total derivative. It is basically the unsteady term, plus there is a convective term associated with it or an advective term. So the time derivative follows this kind of formulation. So this is the local rate of change due to time variations of Q at a particular fixed point. Then this is the convective rate of change because of the fluid element being transported to a different position along the gradient of Q. So instead of the scalar Q, it can also be vectors like position, velocity, and momentum. It can be anything. So this is what the material derivative is all about.

Material Derivative

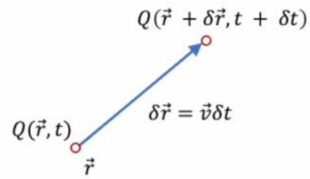
NPTEL, IISc

- Substituting we get:

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{v}.\nabla Q$$

- The Lagrangian rate of change or Material derivative or Substantial derivative or Total Derivative.
- The time derivative following the motion of a fluid element is then described by the following operator:

$$\frac{D()}{Dt} = \frac{\partial ()}{\partial t} + \vec{v}.\nabla()$$



$\frac{\partial Q}{\partial t}$: **Local** rate of change due to time variations of Q at a fixed point

$\vec{v}.\nabla Q$: **Convective** rate of change due to the fluid element being transported to a different position along the gradient of Q

Instead of a scalar Q, it can also be a vector (position, velocity, momentum etc.)

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So there are other approaches that use multivariate calculus.We can also show the total derivative, and these are different ways of showing how the material derivative works.

Material Derivative – Another approach

NPTEL, IISc

- We have

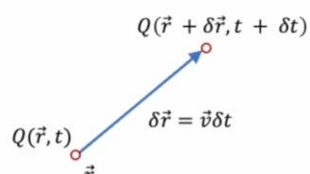
$$\vec{r} = \vec{r}(x, y, z)$$
$$Q = Q(\vec{r}, t) = Q(x, y, z, t)$$

- From multivariable calculus, mathematically we have for total derivative

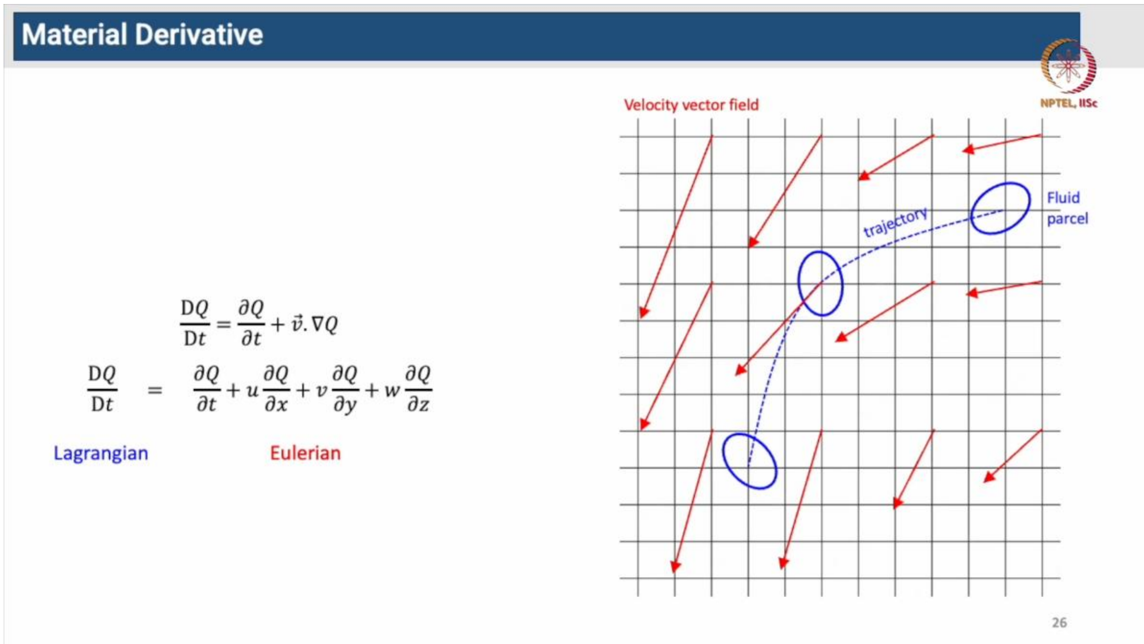
$$dQ = \frac{\partial Q}{\partial t}dt + \frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy + \frac{\partial Q}{\partial z}dz$$
$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x}\frac{dx}{dt} + \frac{\partial Q}{\partial y}\frac{dy}{dt} + \frac{\partial Q}{\partial z}\frac{dz}{dt}$$

- Using : $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$ and $w = \frac{dz}{dt}$

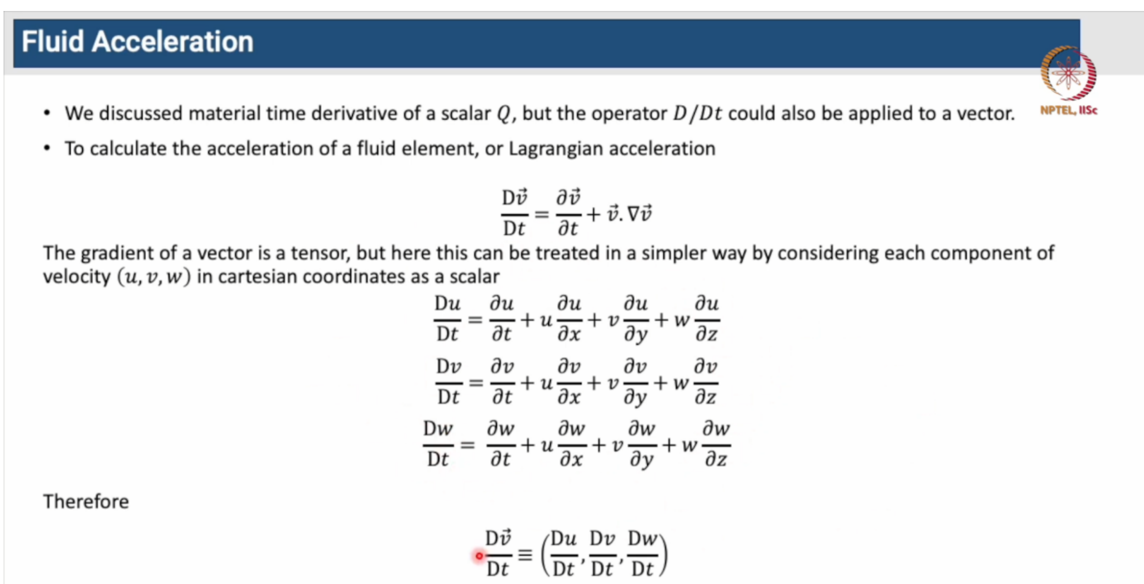
$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + u\frac{\partial Q}{\partial x} + v\frac{\partial Q}{\partial y} + w\frac{\partial Q}{\partial z} = \frac{\partial Q}{\partial t} + (u, v, w).\left(\frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial y}, \frac{\partial Q}{\partial z}\right)$$
$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{v}.\nabla Q$$



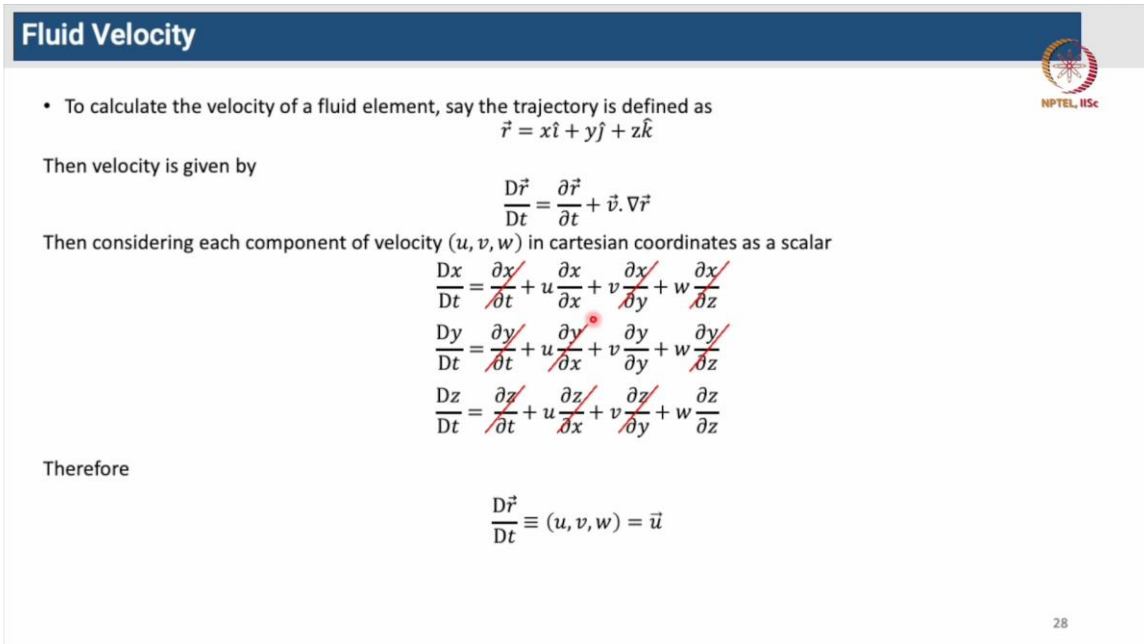
So this part is Lagrangian, and this part is therefore Eulerian. So this is the Lagrangian fluid parcel, and this is the corresponding Eulerian field representation. This is how to also visualize the material derivative.



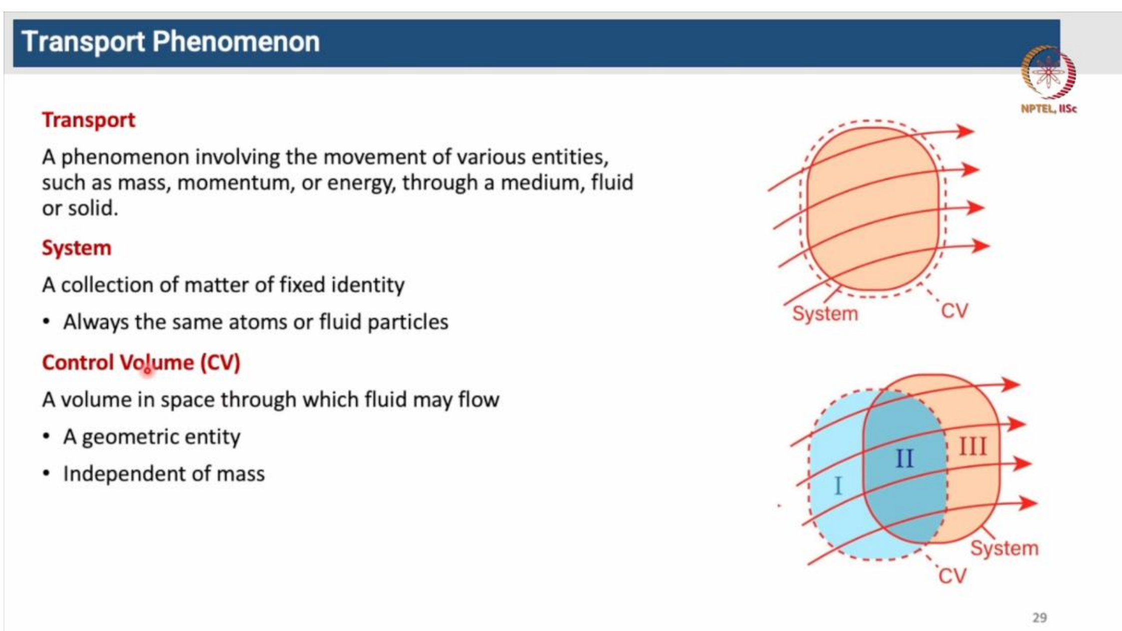
Then the fluid acceleration, we always did it with a scalar, but if it is applied to a vector, for example, that vector is v or the velocity, then this is the quantity that you get.So, the gradient of a velocity is a tensor, but here it can be treated in a simpler way. So if you look at this du by dt, or the first, or the x direction, for example, that is u, so you get this particular term, right? Okay, so then you have the y and the z, so all of these show this is the family of, you know, the family of equations that you see.



Okay, so in order to calculate the velocity of a fluid element, what do you do? For example, if $r = xi + yj + zk$, so if you take the material derivative of that, you get this. So, in other words, considering each component in the Cartesian system as a scalar, you get this particular form. Okay, so in other words, if you look at it, the convective derivative is actually the acceleration term. Basically, what we are talking about.

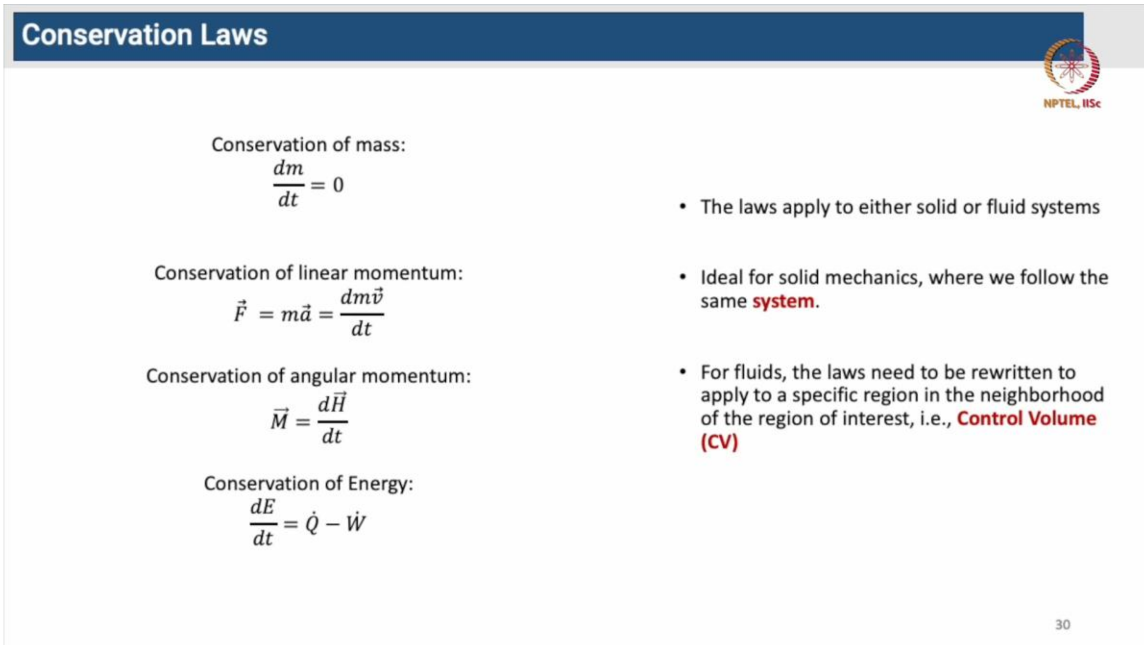


Then, of course, is the transport phenomena. The concept of the system is the control volume, represented by the dotted line. The system is a collection of matter with a fixed identity, always the same atoms or fluid particles. The control volume is a geometric entity, and it is independent of mass.



So, the conservation laws are like this: So, this is the conservation of mass, momentum, angular momentum, and energy.

So, the laws apply to either solid or fluid systems ideal for solid mechanics, but in fluids, you need to bring the control volume approach for a specific reason because you want to apply it to a specific region of the fluid of interest.



So then we have the Reynolds transport theorem, which basically establishes the link between a system and a control volume. So for moving or deforming a control volume, any rate of change of any property B, which is an extensive property, is given. This beta is an intensive version of the same properties. So it is the rate of change of that particular thing within the control volume and whatever is coming out: the flux.

That is coming out of the control volume or going into the control volume. Here, this Vr is a relative velocity, and this is a dot product with respect to the area. So, this is basically a scalar dot product. So basically, you have the flux, which is normal, or a component of the flux that is normal to the area. So there are special cases; there can be non-deforming, moving control volumes, etc.

There can be a fixed control volume, and there can be a steady state control volume. For the steady state, of course, this will be equal to zero. So, for a steady-state control volume. So, if you look at these equations, these are the equations for when you have a fixed control volume; this is the equation. So the flux term is basically nothing but whatever is going out minus whatever is going in.

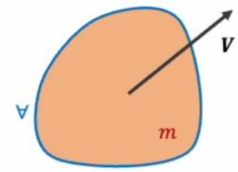
So, that is all there is to it. So remember, this is across the control volumes. It's a volume integral. This is a surface integral.

Reynold's Transport Theorem (RTT)

An analytical tool to shift from describing the laws governing fluid motion using the system concept to the CV concept.

General RTT (for moving and deforming CV)

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\left(\int_{CV} \beta\rho d\forall\right) + \int_{CS} \beta\rho(\boldsymbol{V_r}.\hat{\boldsymbol{n}})dA$$



Special Cases:

- Non-deforming, moving CV

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\beta\rho)d\forall + \int_{CS} \beta\rho(\boldsymbol{V_r}.\hat{\boldsymbol{n}})dA$$

- Fixed CV

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\beta\rho)d\forall + \int_{CS} \beta\rho(\boldsymbol{V}.\hat{\boldsymbol{n}})dA$$

- Steady flow:

$$\frac{d}{dt}() = 0$$

B = Extensive property
β = Intensive property
$$\beta = \frac{B}{m}$$

Flux terms for uniform flow across discrete CS's (steady or unsteady)

$$\int_{CS} \beta\rho(\boldsymbol{V}.\hat{\boldsymbol{n}})dA = \sum(\beta\dot{m})_{out} - \sum(\beta\dot{m})_{in}$$

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Okay, so now you know mass, when you have the corresponding beta of 1, when you have momentum, it is just the velocity, the beta, which is the intensive variable, and when it is energy, it is E, right? So that is the nomenclature; those are the nomenclatures.

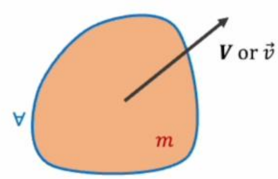
Conservation Laws

For uniform distribution of B

$$\beta = \frac{B}{m}, \qquad B = \beta.m$$

For non-uniform distribution of B

$$\beta = \frac{dB}{dm}, \qquad B = \oiiint \beta.dm = \oiiint \beta\,\rho d\forall$$



Property	B	$\beta = B/m$	Governing Law
Mass	m	1	Conservation of mass
Momentum	$\vec{p} = m\vec{v}$	$\vec{v}$	Conservation of linear momentum
Energy	E	e	Conservation of Energy

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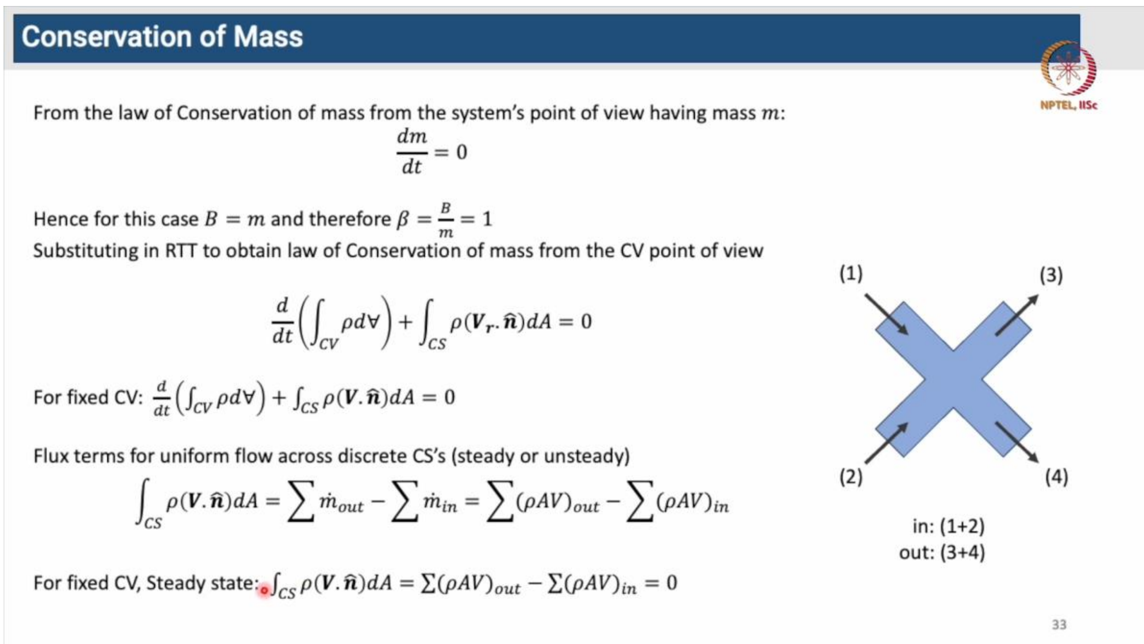
So you can use the conservation principles, such as the conservation of mass, for a system like this, where there are a couple of inlets and two outlets; it's like a pipe.

You can think of it like a pipe. So $\frac{dm}{dt} = 0$. For this case, m is equal to 1. So beta is equal to 1. So, if you apply the Reynolds transport theorem, you will see that this is equal to 0. So, for a fixed control volume, this is equal to 0.

So, now if you do this, there are two inlets and two outlets. So, it just becomes ρAv. So, ρ times A times velocity minus whatever is going out plus whatever is going in. So, for a fixed control volume in steady state, this is the equation that you have that is actually occurring for a fixed control volume.



Similarly, you have a conservation of mass and conservation of momentum. In this case, the intensive variable beta is v, and the extensive variable is mv.

Again, you apply the Reynolds Transport Theorem. You have an external force that is equal to ma. So this is basically the acceleration term again; it's the same thing that you see over here. This is v, and remember this is the relative velocity. For a fixed control volume, this is what you get. Okay, this V becomes capital V now, and so here in this case you sum total all this stuff.

Okay, and you know the external force will be the rate of change of momentum or the change of momentum, so that is what you get: whatever is out minus whatever is in will give rise to the force over here. If it is a steady state, so that unsteady terms actually drop out.

Conservation of Linear Momentum

From the system's point of view having mass m, momentum is given as $m\boldsymbol{V}$, rate of change of which depiction the net forces on the system.

Hence for this case $B = m\boldsymbol{V}$ and therefore $\beta = \frac{B}{m} = \boldsymbol{V}$

Substituting in RTT to obtain law of Conservation of momentum from the CV point of view

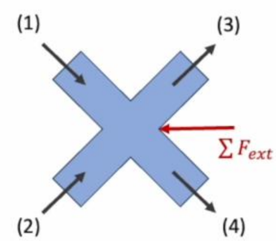
$$\frac{d(m\boldsymbol{V})_{sys}}{dt} = \sum F_{ext} = \frac{d}{dt}\left(\int_{CV} \rho \boldsymbol{V} d\forall\right) + \int_{CS} \boldsymbol{V}(\rho \boldsymbol{V_r}.\hat{\boldsymbol{n}})dA$$

For fixed CV: $\sum F_{ext} = \frac{d}{dt}\left(\int_{CV} \rho \boldsymbol{V} d\forall\right) + \int_{CS} \boldsymbol{V}(\rho \boldsymbol{V}.\hat{\boldsymbol{n}})dA$

Flux terms for uniform flow across discrete CS's (steady or unsteady)

$$\int_{CS} \boldsymbol{V}(\rho \boldsymbol{V}.\hat{\boldsymbol{n}})dA = \sum(\dot{m}\boldsymbol{V})_{out} - \sum(\dot{m}\boldsymbol{V})_{in} = \sum(\rho AV.\boldsymbol{V})_{out} - \sum(\rho AV.\boldsymbol{V})_{in}$$

For fixed CV, Steady state: $\sum F_{ext} = \int_{CS} \boldsymbol{V}(\rho \boldsymbol{V}.\hat{\boldsymbol{n}})dA = \sum(\rho AV.\boldsymbol{V})_{out} - \sum(\rho AV.\boldsymbol{V})_{in}$



in: (1+2)
out: (3+4)

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Similarly, you can have conservation of energy as well. So it is like, you know, Q is added to the system, and W is the work done by the system.

So, this is the first law of thermodynamics, as we all know. Similarly, you can write this expression. I'm not going to go into the details of all this.

Conservation of Energy

From the system's point of view, 1st law of Thermodynamics states:

$$dE = \delta Q - \delta W$$

Q = heat added to the system,
W = work done by the system
E = total internal energy of the material.
Here, δ indicates the differential element for path-dependent quantities (inexact differential), and d indicates exact differential.
We can also write

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Hence for this case the property we are interested in is the total internal energy $B = E$ and therefore the total specific internal energy is defined as $\beta = \frac{B}{m} = e$

$$e = e_{int} + e_{kinetic} + e_{potential} + e_{other}$$

$$e = u + \frac{v^2}{2} + gz + e_{other}$$

Where u = internal energy per unit mass due to microscopic interaction potential and microscopic kinetic energy.

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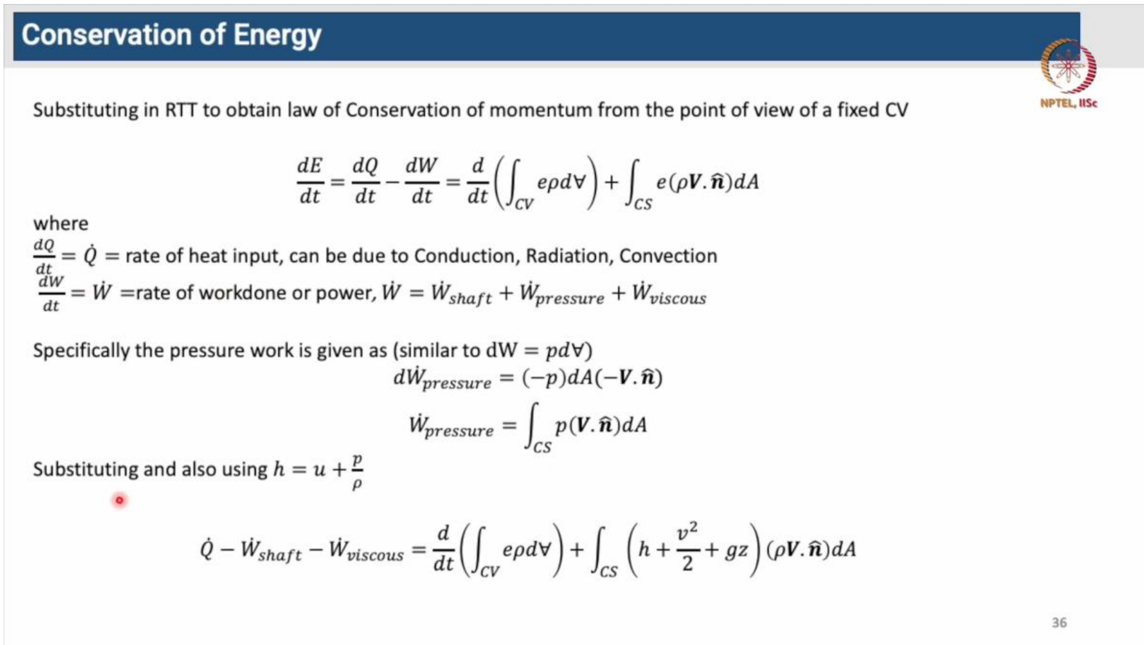
You can use the Reynolds transport theorem once again for this. Okay. But this time it is, you know, the rate of change of energy is equal to $\frac{dQ}{dt} - \frac{dW}{dt}$, which is equal to all these energy terms.

So $\frac{dQ}{dt}$ is the heat input. This is the work done, and then you can write this equation. You can see this in any heat transfer book.



So ultimately, at the end of the day, you get something like this. For a steady-state kind of system, you already know that the heat transfer minus the work minus the viscous dissipation is something like this. If $\dot{m}_1$ is equal to $\dot{m}_2$, whatever mass is entering is actually exiting; you have something like this.

So, this is the steady-flow version of the SFE. In general, there are multiple inlets, so you have to do the summations.

Conservation of Energy

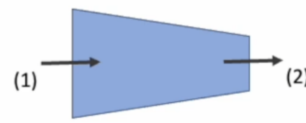
$$\dot{Q} - \dot{W}_{shaft} - \dot{W}_{viscous} = \frac{d}{dt}\left(\int_{CV} e\rho d\forall\right) + \int_{CS}\left(h + \frac{v^2}{2} + gz\right)(\rho \boldsymbol{V}.\hat{\boldsymbol{n}})dA$$

For steady state 1D problem

$$\dot{Q} - \dot{W}_{shaft} - \dot{W}_{viscous} = \dot{m}_2\left(h_2 + \frac{v_2^2}{2} + gz_2\right) - \dot{m}_1\left(h_1 + \frac{v_1^2}{2} + gz_1\right)$$

From mass conservation $\dot{m}_1 = \dot{m}_2$

$$h_1 + \frac{v_1^2}{2} + gz_1 + \dot{q} = h_2 + \frac{v_2^2}{2} + gz_2 + \dot{w}$$



in: (1)
out: (2)

This is the steady flow energy equation or SFEE. In general, with multiple inlet and exit ports

$$\left(h + \frac{v^2}{2} + gz\right)_{inlet} + \dot{q}_{net} = \left(h + \frac{v^2}{2} + gz\right)_{exit} + \dot{w}_{net}$$

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So, these are the conservation laws in integral form. This is what you need to know. So, this is basically $F = Ma$.

So, all of them share that general form, which involves beta. So now there is a specialized form which we also discussed that Bernoulli's equation. So Bernoulli's equation is valid along the streamline for frictionless or inviscid flow and incompressible or constant density. So if it's an unsteady flow, this is called Euler's equation, and for a steady flow, the integral part of this is called Bernoulli's equation. Whatever is in the brackets is basically Euler's equation. So if you flow along the streamline, you can apply Bernoulli's at two different points in the flow field.

This is routinely used where you have a balance between pressure head, velocity head, and potential head.

Bernoulli's Equation

Assumptions
- Along the streamline
- Frictionless or inviscid flow
- Incompressible or Constant density

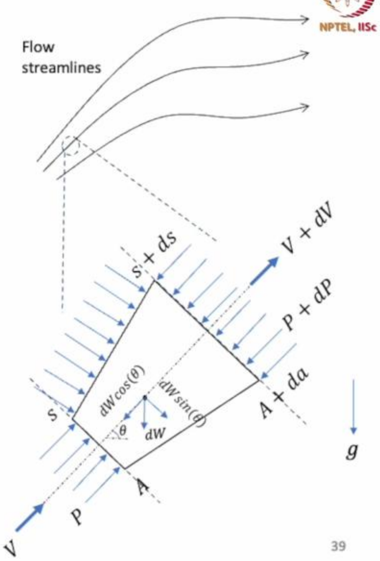
For **Unsteady flow**

$$\int_1^2 \left(\frac{\partial V_s}{\partial t} ds + V_s dV_s + \frac{dp}{\rho} + g dz\right) = 0$$

where
s = coordinate along the streamline
V_s = velocity magnitude at the coordinate s

For **Steady flow**

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = constant$$



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So you can also have the differential forms of these quantities, and you can derive them from the integral version by using the divergence theorem, and then you can shrink them to a point. So this is what you get. This is the conservation equation, and if it is incompressible, the volumetric dilatation is actually equal to zero.

Conservation Laws – Differential Forms

Conservation of Mass

$$\frac{d}{dt}\left(\int_{CV} \rho d\forall\right) + \int_{CS} \rho(\boldsymbol{V}.\hat{\boldsymbol{n}}) dA = 0$$

$$\Rightarrow \int_{CV} \frac{\partial \rho}{\partial t} d\forall + \int_{CS} (\rho \boldsymbol{V}.\hat{\boldsymbol{n}}) dA = 0$$

Using Gauss divergence theorem $\int_{CS} (\boldsymbol{F}.\hat{\boldsymbol{n}}) dA = \int_{CV} (\boldsymbol{\nabla}.\boldsymbol{F}) d\forall$

$$\Rightarrow \int_{CV} \frac{\partial \rho}{\partial t} d\forall + \int_{CV} \boldsymbol{\nabla}.(\rho \boldsymbol{V}) d\forall = 0 \Rightarrow \int_{CV} \left\{\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{V})\right\} d\forall = 0$$

For this to be uniformly valid within the CV

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{V}) = 0$$

For incompressible flow $\rho = const.$

$$\boldsymbol{\nabla}.\boldsymbol{V} = 0$$

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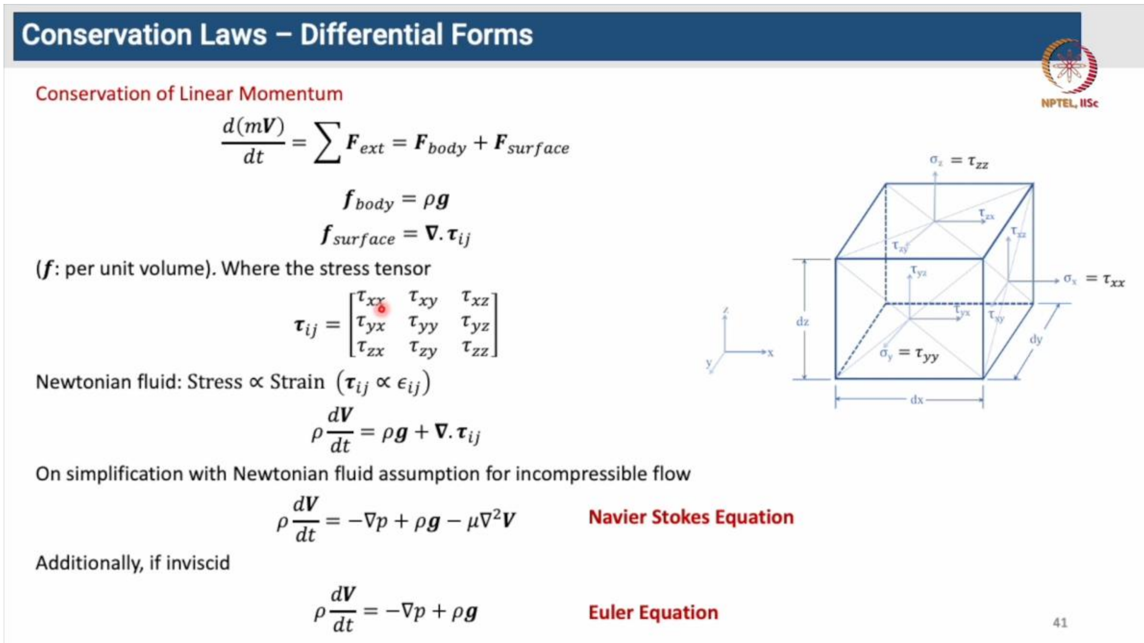
Similarly, you do the same thing with the momentum equations. Okay, and you ultimately get this. This is the stress term; this is the tensor. Okay, and so τ_{ij} is written this way. τ_{xx} and yy are the normal stresses, while the other ones, xy, are the shear stresses. So, the moment you substitute all of them, you get something like this, which is the Navier-Stokes equation.

This is the viscous term. This is a substantial derivative, which is the acceleration term. This is the pressure, and this is the body force. So, if it is inviscid, you will get the Euler term because the viscosity will actually drop off.



Then the conservation can also be written in a form like this. H is the enthalpy. ϕ is the viscous dissipation. It can be neglected in certain cases because it depends on the square of the velocity gradient, but not on the cross velocity gradient.

Conservation Laws – Differential Forms

NPTEL, IISc

Conservation of Energy

$$\rho \frac{dh}{dt} = \frac{dP}{dt} + \boldsymbol{\nabla}.(k\boldsymbol{\nabla}T) + \Phi$$

Where Dissipation function $\Phi = \tau'_{ij}\frac{\partial u_i}{\partial x_j}$

For Newtonian fluid

$$\Phi = \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2\right] + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2$$

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So these are the equations in differential form. You know, the conservation of mass, the conservation of linear momentum, and the conservation of energy.

Conservation Laws – Differential Forms

NPTEL, IISc

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{V}) = 0$$

Conservation of Linear Momentum

$$\rho \frac{d\boldsymbol{V}}{dt} = -\nabla p + \rho \boldsymbol{g} - \mu \nabla^2 \boldsymbol{V}$$

Conservation of Energy

$$\rho \frac{dh}{dt} = \frac{dP}{dt} + \boldsymbol{\nabla}.(k\boldsymbol{\nabla}T) + \Phi$$

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Okay. So we will end our lecture here, where we have actually covered a lot in terms of what these quantities are. And we have already covered, you know, that we have shown mathematically how these can be best illustrated. But you know, this is just a summary. A very quick summary: in measurement, you are supposed to measure all these things, and you should have an idea of what we are trying to measure. Either it's a field measurement or it's a Lagrangian type of measurement. Mostly, ours are field measurements.