

Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

Prof. Saptarshi Basu

Department of Mechanical Engineering

Indian Institute of Science, Bengaluru

Week – 01

Lecture - 04

Recap and description of fluid flows

So, in this particular lecture, we will just do a recap of the fundamentals of fluid dynamics, which will be handy. So we have already taught, and we have gone through the mathematics; this is just an overview. So, what is a fluid? Anything that deforms continuously under the application of shear stress, including fluids, encompasses both liquids and gases. So, in the deformation and equilibrium part, fluids, including liquids and gases, possess the ability to deform continuously under the application of an external force. So, unlike solids, the deformation is reversible to a certain extent. The fluids therefore lack an elastic limit, so to speak.

What is a Fluid?



- Any substance that deforms continuously under the application of a shear stress (tangential) no matter how small the shear stress may be.
- In other words, fluid is anything that cannot sustain shear stress when at rest.
- Fluid includes both liquids and gases



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In solids, however, equilibrium implies the absence of deformation. In fluids, equilibrium refers to the state in which deformation remains constant.

Deformation and Equilibrium



- Fluids, including liquids and gases, possess the ability to deform continuously under the application of external forces.
- Unlike solids, where deformation is reversible up to a certain point, fluids lack an elastic limit, resulting in non-recoverable deformations.
- This continuous deformation is a result of the lack of defined intermolecular arrangements, allowing molecules to move past each other with relative ease.
- While in solids, equilibrium implies the absence of deformation, in fluids, equilibrium refers to the state where the rate of deformation remains constant, creating a state of continuous flow.



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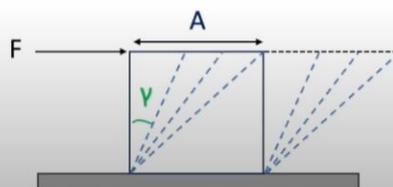
So, there is a concept of stress and strain. So, shear stress represents the force that is applied tangentially to the surface by the fluid.

And divided by the area, the shear strain is the angular deformation that actually happens. So the relationship between shear stress and shear strain represents their resistance to deformation.

Shear Stress and Strain



- The concept of shear stress and strain is crucial in understanding the deformation of fluids.
- Shear stress (τ) represents the force applied tangentially to a surface divided by the area over which the force is applied. Mathematically, $\tau = F / A$.
- Shear strain (γ) is the angular deformation that results from the applied shear stress. It is the change in angle between initially perpendicular line segments.
- The relationship between shear stress and shear strain represents its resistance to deformation under shear forces.

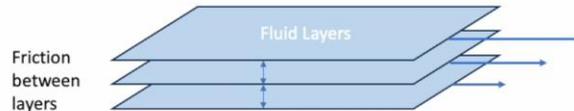


Then viscosity, also defined as a property that measures how much resistance the fluid offers to flow. It is also defined as the resistance offered by one layer of fluid to the subsequent adjacent layers. So with temperature, viscosity increases for gases; however, it decreases for liquids.

Viscosity



- Viscosity is a fluid property that gives a measure of how much resistance the fluid offers to the flow.
- It can also be defined as the resistance offered by one layer of fluid to the subsequent adjacent layers.
- It is a fluid property.
- In liquids, viscosity arises mainly because of molecular bonding.
- In gases, viscosity arises mainly because of molecular collisions.
- With temperature, viscosity increase for gases however, it decreases for liquids.



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So, if the fluid is called Newtonian, then we have the constitutive relationship that basically links the shear stress with the velocity gradient. This is the cross-velocity gradient, and μ is the coefficient of viscosity. This adheres to a linear relationship between shear stress and shear strain. So the viscosity remains constant under different shear rates, making it relatively straightforward to analyze. So, this is the velocity gradient, for example. This is the U velocity varying in the Y-direction.

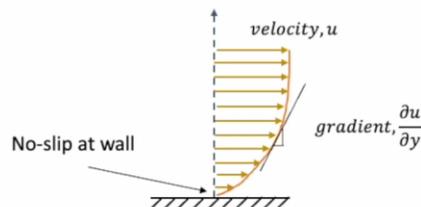
Newtonian Fluids



- In Newtonian fluids the shear stress is directly proportional to the shear rate ($du/\partial y$) by the dynamics viscosity (μ)

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right)$$

- These adhere to the linear relationship between shear stress and shear rate, characterized by a constant viscosity.
- The viscosity of remains consistent under different shear rates, making them relatively straightforward to model and analyze.
- Examples of Newtonian fluids include water and many simple liquids, where molecular interactions do not significantly influence viscosity.



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The examples of Newtonian fluids include water. These are all the things that we did. Then, if you talk about non-Newtonian fluids, they are fluids that exhibit complex behaviors; shear thinning is one example, and shear thickening is another example. This

behavior arises due to the response of the fluids to shear forces. It's not as simple as what we saw over there. Polymers can be an example of a complex fluid.

Non-Newtonian Behavior



- The deformation characteristics of fluids can deviate from linear relationships, leading to non-Newtonian behavior.
- Non-Newtonian fluids exhibit complex behaviors such as shear-thinning (viscosity decreases with increased shear rate) or shear-thickening (viscosity increases with shear rate).
- This behavior arises due to the varying responses of different types of fluids to shear forces.
- For example, polymers in solution can entangle and hinder flow under high shear rates, leading to shear-thickening.



Cornstarch Slurry
(shear-thickening)

Source: Google

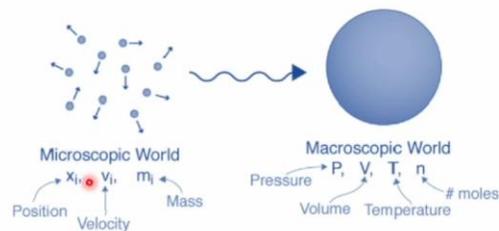
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Okay, so each of these things has a microscopic origin. So when you look at fluid dynamics in the macroscopic world, you have pressure, volume, temperature, etc. All of these occur in the microscopic world where the positions, velocity, and mass of each atom or molecule actually jitter. And that leads to the manifestation of the interaction at the microscopic level, leads to collective interactions, and leads to higher level organization, giving rise to the kinds of properties that we see in the macroscopic world.

Microscopic Origin of Macroscopic Scale Behavior



- The interaction of individual fluid particles on a microscopic scale, encompassing collisions and exchanges, leads to the manifestation of macroscopic phenomena.
- This emergence results from collective interactions, which, on a higher level of organization, give rise to new and unexpected outcomes.



Source: Google

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So each of them has an underlying microscopic theory. So, on a microscopic scale, for example, the fluid exhibits perpetual disordered motion. This is like its kinetic energy. They can also rotate. They can also have raw vibrational energies. So the temperature serves as a gauge of the mean kinetic energy of the fluid within the material for translational motion. For example, this has more thermal energy. This has less thermal energy. So, if the temperature rises, the average kinetic energy also goes up.

Random Molecular Motion and Temperature



- On a microscopic scale, fluid molecules exhibit perpetual, disordered movement driven by thermal energy.
- This motion represents their kinetic energy.
- Temperature serves as a gauge of the mean kinetic energy of molecules within a material.
- As temperature rises, so does the average kinetic energy of the molecules.

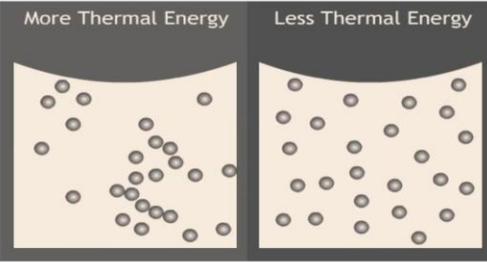


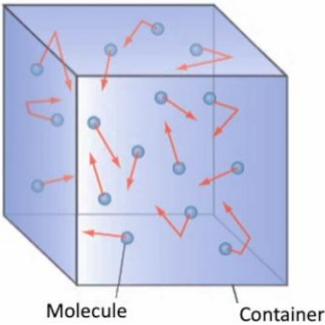
Image Source: ISP

So the pressure is basically generated by, you know, the gas from the multitude of molecular collisions that occur at the boundary and at the enclosure. In motion, the molecules collide with the container walls, producing a force that is directed perpendicular to the walls. So the overall pressure exerted by a fluid is a cumulative effect of all these molecular interactions that you see.

Pressure and Molecular Collisions:



- The pressure generated by a gas results from the multitude of molecular collisions occurring with the boundaries of its enclosure.
- In motion, the molecules collide with the container walls, producing a force directed perpendicularly to the surface of the wall.
- The overall pressure exerted by a fluid is the cumulative effect of these individual molecular interactions.



Source: Google

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So, similarly to the macroscopic scale, the fluid velocity can be defined using a velocity

field. It is typically the Eulerian representation, and there are vectors at each point in space that actually vary with time. So, the collective motion, however, emerges out of molecular collisions. The velocity field reflects the overall movement of these fluid particles. Different regions can have different velocities. So there can be spatial gradients of velocities, and there can be temporal and spatial variations of velocities in all four dimensions.

Velocity Field and Collective Motion



- On the macroscopic scale, the behavior of fluids can be described using a velocity field, which assigns a velocity vector to each point in space.
- Collective motion emerges as a result of countless molecular collisions. The velocity field reflects the overall movement of fluid particles.
- Different regions of the fluid may have varying velocities, contributing to fluid flow and the establishment of gradients.



Then comes the concept of cohesive forces, which is particularly important because later on, when we do PDPA, LDV, and other things, we will use droplets, for example.

So you need to know a little bit about cohesive forces and surface formations. So basically, when you actually have, for example, a container containing some liquid. Now the molecules that are at the center actually have forces that are occurring in the same way in all directions. So they are in balanced condition. However, the molecules that are at the surface have an unbalanced component of the forces.

So, this leads to surface tension. It is a consequence of these cohesive forces, which actually brings about phenomena like the generation of droplets and capillary action. So, these forces empower the surface molecules of a liquid to adhere to one another, giving rise to the formation of this well-defined boundary or interface that you see over here.

Cohesive Forces and Surface Formation



- Molecular-level cohesive interactions play a pivotal role in the creation of liquid surfaces.
- These forces empower surface molecules in a liquid to adhere to one another, giving rise to the formation of a well-defined boundary or interface.
- Surface tension, a consequence of these cohesive forces, brings about phenomena such as capillary action and the generation of droplets.

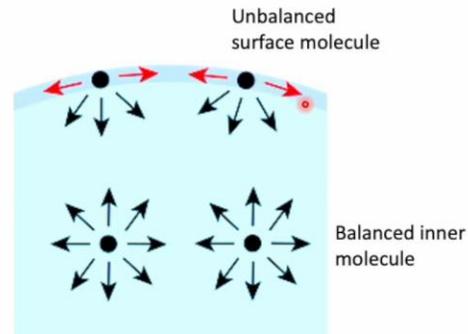


Image Source: Wikipedia

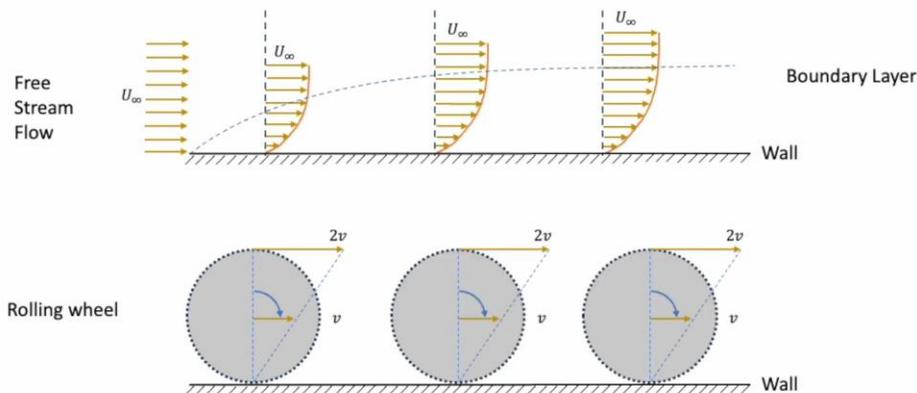
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Then, of course, the concept of no-slip boundary conditions is extensively used when you actually deal with wall-bounded flows and things like that. So, at a stationary wall, the relative velocity between the fluid and the wall is zero. Therefore, the wall has the same tangential velocity as the fluid. So they actually do not have relative velocity with each other. All right. So, this is the most important part that you should keep in mind. This is, for example, flow over a flat plate, which is a canonical fluid dynamics problem. All right.

The "No-Slip" Condition



- According to the 'no slip' boundary condition, the fluid layer that is in contact with the wall has the same tangential velocity as the wall i.e., the relative velocity between them is zero.



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Then, of course, the concept of Lagrangian and Eulerian kinds of description. So, for example, in the Lagrangian, what you do, which we already covered in detail, is that you track individual particles, fluid particles as they traverse. But we said earlier that this is nearly impossible to do. But this is the path of the trajectory, for example, of a fluid particle

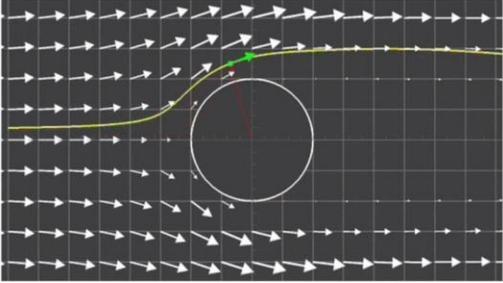
that describes its motion. Eulerian is, however, that you monitor certain spatial points and observe the evolution of the fluid characteristics at those points. So instead of following individual particles, the emphasis is placed on observing fixed spatial locations for the fluid's properties. So that is Eulerian for you.

Description of Fluid Motion - Lagrangian vs Eulerian



Lagrangian

- The main focus is on tracing the movement of individual fluid particles as they traverse through both space and time.
- The path or trajectory of a fluid particle describes its motion.



Eulerian

- This process entails examining fluid characteristics at particular spatial points and monitoring their evolution over time.
- Instead of following individual particles, the emphasis is placed on observing fixed spatial locations for fluid properties.

Source: Gyfcat

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So if you, Eulerian is therefore a field representation, and it is extensively used in fluid mechanics, as we already know by this time. So this is, for example, a field equation for velocity. So the mathematical field is a mathematical function that assigns a value to each point in the space.

In fluid dynamics, various fluid properties such as velocity, temperature, pressure, and density are represented as fields. So it is very useful to visualize the flow field using this technique. For example, this shows vorticity, velocity, and streamlines all in one. So it allows you to create visual representations like contour plots, streamlines, which enables us to get insights into complex flow patterns. And PIV, as we will see, is exactly a field representation.

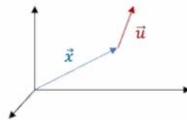
Eulerian Description - Field Representation



Definition of Fields:

A field is a mathematical function that assigns a value to each point in a given space. In fluid dynamics, various fluid properties such as velocity, pressure, temperature, and density are represented as fields. These fields provide a comprehensive view of how these properties vary across space and time. The velocity vector field can be represented as follows:

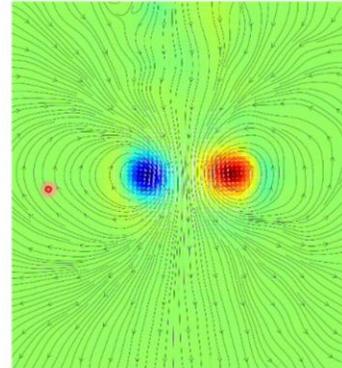
$$\vec{u}(\vec{x}, t): \begin{aligned} &u(x, y, z, t) \\ &v(x, y, z, t) \\ &w(x, y, z, t) \end{aligned}$$



Visualization and Analysis:

Fields offer a powerful tool for visualizing and analyzing fluid behavior. They allow you to create visual representations like contour plots, vector plots, and streamline diagrams, which provide insights into complex flow patterns and phenomena. By examining the changes in the field values and their gradients, you can understand the distribution and behavior of fluid properties in different regions.

Vortex Ring



- Vorticity Field – Contour plot
- Velocity Field Vectors – White arrows
- Streamlines – Black lines

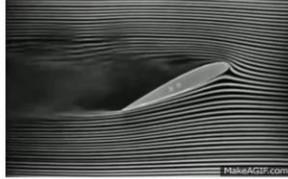
So this is, for example, a visualization. For example, this is behind a tilted aerofoil and how the flow separation actually happens. And this is like a vortex. And this is another shedding behind the body. So in all these cases, you see that there are visual cues that play a very strong role in how the fluid behaves.

Now there are, as we already talked about earlier, different ways to represent the flow field. We can have streamlines, which focuses on instantaneous flow characteristics. We can have streak lines that track the history of particles passing through a fixed point. So you are acting like a customs officer. And the path line traces the actual paths followed by the individual particles. So all of these are very useful ways of representation.

Visualizing Fluid Motion



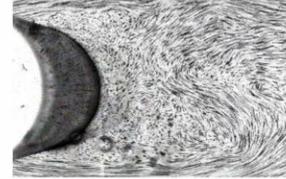
- In fluid dynamics, visual cues play a crucial role in depicting how fluid particles behave within a flow field.
- These principles aid in comprehending the dynamics and actions of fluids in diverse scenarios and offer various viewpoints on fluid motion.



Source: National Committee for Fluid Mechanics Films



Song et. al, JFM, 2020



Source: FAMU-FSU

- **Streamlines** focus on instantaneous flow directions, **Streaklines** track the history of particles passing through a fixed point, and **Pathlines** trace the actual paths followed by individual particles.

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So these are, for example, pathlines. The pathlines are actual trajectories followed by individual fluid particles as they move. So this is a long exposure photograph of a campfire which shows the path lines.

So path lines are valuable tools because they enable you to gain insight into the prolonged behavior and movement. They can also tell you how cohesive the path lines are. So understanding how the transport might have happened, especially in complex flows, this is a very useful tool. So you can see how this is actually very useful in detecting this.

Pathlines



Definition: Pathlines are the actual trajectories followed by individual fluid particles as they move within a flow field.

They show the complete history of a particle's motion.

Use: Pathlines are valuable tools for gaining insight into the prolonged behavior and movement of individual fluid particles.

They prove to be instrumental in examining the Lagrangian attributes of a flow, a practice that entails monitoring specific fluid particles across time.

Importance: Pathlines offer a comprehensive view of how fluid particles move, which is important for understanding dispersion, pollution transport, and particle behavior in complex flows.

Source: Wikipedia



A long-exposure photograph of a campfire spark reveals the hot air flow's pathlines.

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Then, of course, the path line has a mathematical formula. The path lines are basically specified by $\frac{dr}{dt}$, which is u . This is like a Lagrangian description. That's how the position vector is changing with time, and that is the pathline. And of course, you have an initial position, which is r_0 , and then how it evolves over time. So this is how this vector actually

changes. The position vector changes with time. So pathlines depict the history of the particle's motion over a period of time.

Pathlines

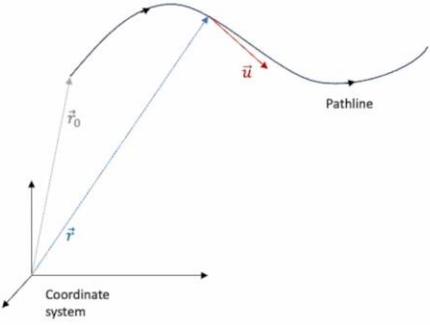
Mathematical Formulation:

Pathlines can be specified using initial conditions by solving the ordinary differential equations of motion for each individual fluid particle.

$$\frac{d\vec{r}}{dt} = \vec{u}(\vec{r}, t), \quad \vec{r}(t_0) = \vec{r}_0$$

This equation basically states that the curve generated from the integration is essentially for the particle that was located at \vec{r}_0 at time t_0 .

Hence, pathlines depict the history of a particle's motion over a period of time.



NPTEL, IISc

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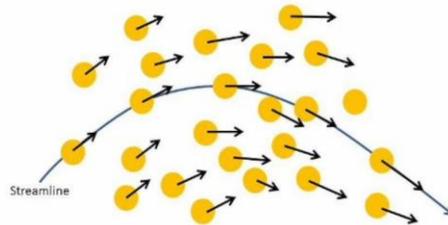
And then, of course, there is the streamline which you already know. The streamlines are constructed in such a way that, everywhere along the streamline, the flow velocity is tangential at that particular point. So these are imaginary lines that you can see. The fluid flow field is always tangential to the instantaneous velocity vector at every point. So in other words, it gives a representation of a fluid parcel that would move if it were placed at this particular point. So they are crucial for understanding the local behavior of the fluid flow. This enables us, scientists and engineers, to understand how the fluid will behave around objects, aerodynamics, and other things. So this is also useful for the representation of, you know, when you want to represent PIV. So this would be one of the key features of it.

Streamlines



Definition: Streamlines are imaginary lines that are drawn in a fluid flow field such that they are always tangent to the instantaneous velocity vector of the fluid at every point.

In other words, a streamline at a given point represents the direction that a fluid particle would move if placed at that point.



Importance: Streamlines are crucial for understanding the local behavior of fluid flow. They help engineers and scientists predict how fluids will behave around objects, study aerodynamics, and optimize designs for various applications.

So streamlines have this mathematical representation, where x , y , and z are the coordinates. So $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$. So here x and u , y and v , z and w are the corresponding coordinate system, and u , v , and w are the corresponding velocity components. So this is the instantaneous picture that is actually represented. So, to solve this equation, you need to choose a parameter and then use it later to integrate this parametric equation.

Streamlines



Mathematical Formulation:

Streamlines can be described by the equation:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

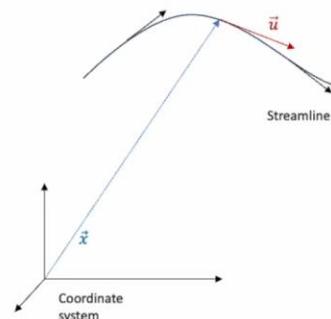
Here, $\vec{x}(x, y, z)$ is the coordinate of a point in the flow field, and $\vec{u}(u, v, w)$ is the corresponding velocity at this location,

These are essentially instantaneous picture represented by flowlines that are tangent to the velocity \vec{u} .

To solve these equations, we choose a parameter λ such that

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = d\lambda$$

Later integrate this parametric equation and eliminate λ to evaluate the streamline.



So streak lines are paths traced by a fluid particle passing through a specific point. So if you are monitoring at a particular point and you are seeing all the particles, the paths that are traced out by them provide information about the temporal evolution of the fluid field and are very useful for unsteady flows. So, for example, this is laminar flow. These are the

flows with eddies. This is a highly turbulent flow. They have injected some dye into the water. You can also see at a specified location how the smoke injection, for example, happens. So each of them has their own advantage.

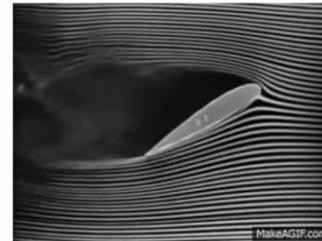
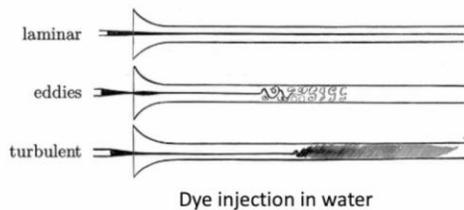
Streaklines



Definition: Streaklines are the paths traced out by particles that have passed through a specific point in a fluid at earlier times.

They can be formed by continuously releasing dye or other markers into the flow at a fixed location and monitored how they move in time.

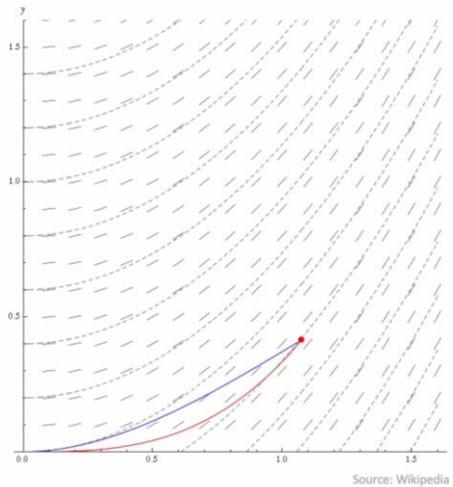
Importance: Streaklines provide information about the temporal evolution of fluid motion and are useful for analyzing unsteady flows.



So if you look at the flow visualization situation, you will see pathlines, streaklines, and streamlines. Streamlines are obviously those dotted lines, and the red and blue are the path lines and streak lines, respectively. So you can see, the red particle moves; the red particle moves in a fluid following the flow exactly. The trace is actually a pathline. The endpoint of the trail, which is the trail of the blue path that is initially released from the starting point, adheres to the moving particles; however, in contrast to the unchanging path line, the ink released afterward interacts with an alternate flow field. So the dashed lines represent the velocity contours and illustrate the entire field's movement at any given point.

So these lines are streamlines, and the velocity at any point is tangential to the streamlines. So this is just a demonstration of how the pathlines, streamlines, and streaklines actually vary from one another.

Flow field visualisation



- Pathline
- Streakline
- - - Streamlines

- The red particle moves in a fluid following the flow exactly. The trace is the pathline.
- The endpoint of the trail of blue ink, which is initially released from the starting point, adheres to the moving particle. However, in contrast to the unchanging pathline, the ink released afterward interacts with an alternate flow field. Consequently, the collection of ink particles creates a trace that deviates from the original pathline, and this evolving trace is known as a streakline.
- The dashed lines depict the velocity field's contours, illustrating the entire field's movement at a given moment. These lines are streamlines, and the velocity vector at any point aligns tangentially with these lines.

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Now we have already covered the concept of the material derivative, so if we consider an Eulerian quantity Q that is fixed at a specific position in time, then for a fluid element that is at time t at that location, the value of that quantity is $Q(\vec{r}, t)$, the Eulerian velocity at that point is $\vec{V}(\vec{r}, t)$, and the position of the fluid element is this; therefore, the rate of change of Q for the fluid element. Which is an Eulerian quantity, is denoted by capital DQ over DT , which is given by this. So, if you use a Taylor series expansion to the first order, you get something like that. So the Taylor series expansion with respect to a scalar gives you something like this.

Material Derivative



- Let us consider a Eulerian quantity Q (temperature, density, etc.), which is specified at a fixed position at a given time.
- For a fluid element which at time t is at a point located at \vec{r} , the value of this quantity is $Q(\vec{r}, t)$
- Eulerian velocity at this point at time t : $\vec{v}(\vec{r}, t)$
- Position of the fluid element at time $t + \delta t$: $\vec{r} + \vec{v}\delta t$
- The value of Q the fluid element at time $t + \delta t$: $Q(\vec{r} + \vec{v}\delta t, t + \delta t)$
- The time rate of change of Q for this fluid element, which we denote by DQ/Dt , is therefore:

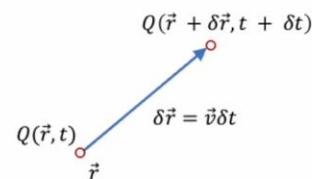
$$\frac{DQ}{Dt} = \lim_{\delta t \rightarrow 0} \frac{Q(\vec{r} + \vec{v}\delta t, t + \delta t) - Q(\vec{r}, t)}{\delta t}$$

- Using Taylor series expansion to first order in δt
- $$Q(\vec{r} + \vec{v}\delta t, t + \delta t) = Q(\vec{r}, t) + \delta t \frac{\partial Q(\vec{r}, t)}{\partial t} + (\vec{v}\delta t) \cdot \nabla Q(\vec{r}, t)$$

Note: Taylor Series expansion wrt scalar

$$F(x + \delta x, t + \delta t) = F(x, t) + \delta t \frac{\partial F(x, t)}{\partial t} + \delta x \frac{\partial F(x, t)}{\partial x} + O(\delta x^2, \delta t^2)$$

$$\nabla Q(\vec{r}, t) \text{ similar to } \frac{\partial Q(\vec{r}, t)}{\partial \vec{r}}$$



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We have already covered. So, if you substitute everything, you will get this. So the material

derivative is the Lagrangian rate of change, the substantial derivative, or the total derivative. It is basically the unsteady term, plus there is a convective term associated with it or an advective term. So the time derivative follows this kind of formulation. So this is the local rate of change due to time variations of Q at a particular fixed point. Then this is the convective rate of change because of the fluid element being transported to a different position along the gradient of Q . So instead of the scalar Q , it can also be vectors like position, velocity, and momentum. It can be anything. So this is what the material derivative is all about.

Material Derivative



- Substituting we get:

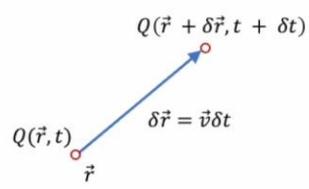
$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{v} \cdot \nabla Q$$
- The Lagrangian rate of change or Material derivative or Substantial derivative or Total Derivative.
- The time derivative following the motion of a fluid element is then described by the following operator:

$$\frac{D(\cdot)}{Dt} = \frac{\partial(\cdot)}{\partial t} + \vec{v} \cdot \nabla(\cdot)$$

$\frac{\partial Q}{\partial t}$: **Local** rate of change due to time variations of Q at a fixed point

$\vec{v} \cdot \nabla Q$: **Convective** rate of change due to the fluid element being transported to a different position along the gradient of Q

Instead of a scalar Q , it can also be a vector (position, velocity, momentum etc.)



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So there are other approaches that use multivariate calculus. We can also show the total derivative, and these are different ways of showing how the material derivative works.

Material Derivative – Another approach



- We have

$$\vec{r} = \vec{r}(x, y, z)$$

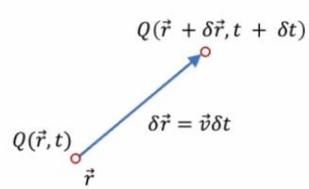
$$Q = Q(\vec{r}, t) = Q(x, y, z, t)$$
- From multivariable calculus, mathematically we have for total derivative

$$dQ = \frac{\partial Q}{\partial t} dt + \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial y} dy + \frac{\partial Q}{\partial z} dz$$

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} \frac{dx}{dt} + \frac{\partial Q}{\partial y} \frac{dy}{dt} + \frac{\partial Q}{\partial z} \frac{dz}{dt}$$
- Using : $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$ and $w = \frac{dz}{dt}$

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + w \frac{\partial Q}{\partial z} = \frac{\partial Q}{\partial t} + (u, v, w) \cdot \left(\frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial y}, \frac{\partial Q}{\partial z} \right)$$

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{v} \cdot \nabla Q$$



So this part is Lagrangian, and this part is therefore Eulerian. So this is the Lagrangian fluid parcel, and this is the corresponding Eulerian field representation. This is how to also visualize the material derivative.

Material Derivative

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{v} \cdot \nabla Q$$

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + w \frac{\partial Q}{\partial z}$$

Lagrangian Eulerian

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Then the fluid acceleration, we always did it with a scalar, but if it is applied to a vector, for example, that vector is v or the velocity, then this is the quantity that you get. So, the gradient of a velocity is a tensor, but here it can be treated in a simpler way. So if you look at this du by dt , or the first, or the x direction, for example, that is u , so you get this particular term, right? Okay, so then you have the y and the z , so all of these show this is the family of, you know, the family of equations that you see.

Fluid Acceleration

- We discussed material time derivative of a scalar Q , but the operator D/Dt could also be applied to a vector.
- To calculate the acceleration of a fluid element, or Lagrangian acceleration

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$

The gradient of a vector is a tensor, but here this can be treated in a simpler way by considering each component of velocity (u, v, w) in cartesian coordinates as a scalar

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

Therefore

$$\frac{D\vec{v}}{Dt} \equiv \left(\frac{Du}{Dt}, \frac{Dv}{Dt}, \frac{Dw}{Dt} \right)$$

Okay, so in order to calculate the velocity of a fluid element, what do you do? For example,

if $r = xi + yj + zk$, so if you take the material derivative of that, you get this. So, in other words, considering each component in the Cartesian system as a scalar, you get this particular form. Okay, so in other words, if you look at it, the convective derivative is actually the acceleration term. Basically, what we are talking about.

Fluid Velocity



- To calculate the velocity of a fluid element, say the trajectory is defined as

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Then velocity is given by

$$\frac{D\vec{r}}{Dt} = \frac{\partial \vec{r}}{\partial t} + \vec{v} \cdot \nabla \vec{r}$$

Then considering each component of velocity (u, v, w) in cartesian coordinates as a scalar

$$\frac{Dx}{Dt} = \frac{\partial x}{\partial t} + u \frac{\partial x}{\partial x} + v \frac{\partial x}{\partial y} + w \frac{\partial x}{\partial z}$$

$$\frac{Dy}{Dt} = \frac{\partial y}{\partial t} + u \frac{\partial y}{\partial x} + v \frac{\partial y}{\partial y} + w \frac{\partial y}{\partial z}$$

$$\frac{Dz}{Dt} = \frac{\partial z}{\partial t} + u \frac{\partial z}{\partial x} + v \frac{\partial z}{\partial y} + w \frac{\partial z}{\partial z}$$

Therefore

$$\frac{D\vec{r}}{Dt} \equiv (u, v, w) = \vec{u}$$

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Then, of course, is the transport phenomena. The concept of the system is the control volume, represented by the dotted line. The system is a collection of matter with a fixed identity, always the same atoms or fluid particles. The control volume is a geometric entity, and it is independent of mass.

Transport Phenomenon



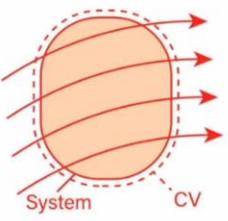
Transport
A phenomenon involving the movement of various entities, such as mass, momentum, or energy, through a medium, fluid or solid.

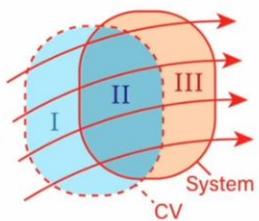
System
A collection of matter of fixed identity

- Always the same atoms or fluid particles

Control Volume (CV)
A volume in space through which fluid may flow

- A geometric entity
- Independent of mass





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So, the conservation laws are like this: So, this is the conservation of mass, momentum, angular momentum, and energy.

So, the laws apply to either solid or fluid systems ideal for solid mechanics, but in fluids, you need to bring the control volume approach for a specific reason because you want to apply it to a specific region of the fluid of interest.

Conservation Laws



Conservation of mass:

$$\frac{dm}{dt} = 0$$

Conservation of linear momentum:

$$\vec{F} = m\vec{a} = \frac{dm\vec{v}}{dt}$$

Conservation of angular momentum:

$$\vec{M} = \frac{d\vec{H}}{dt}$$

Conservation of Energy:

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

- The laws apply to either solid or fluid systems
- Ideal for solid mechanics, where we follow the same **system**.
- For fluids, the laws need to be rewritten to apply to a specific region in the neighborhood of the region of interest, i.e., **Control Volume (CV)**

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So then we have the Reynolds transport theorem, which basically establishes the link between a system and a control volume. So for moving or deforming a control volume, any rate of change of any property B, which is an extensive property, is given. This beta is an intensive version of the same properties. So it is the rate of change of that particular thing within the control volume and whatever is coming out: the flux.

That is coming out of the control volume or going into the control volume. Here, this V_r is a relative velocity, and this is a dot product with respect to the area. So, this is basically a scalar dot product. So basically, you have the flux, which is normal, or a component of the flux that is normal to the area. So there are special cases; there can be non-deforming, moving control volumes, etc.

There can be a fixed control volume, and there can be a steady state control volume. For the steady state, of course, this will be equal to zero. So, for a steady-state control volume. So, if you look at these equations, these are the equations for when you have a fixed control volume; this is the equation. So the flux term is basically nothing but whatever is going out minus whatever is going in.

So, that is all there is to it. So remember, this is across the control volumes. It's a volume integral. This is a surface integral.

Reynold's Transport Theorem (RTT)

NPTEL, IISc

An analytical tool to shift from describing the laws governing fluid motion using the system concept to the CV concept.

General RTT (for moving and deforming CV)

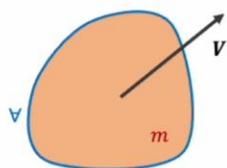
$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \left(\int_{CV} \beta \rho dV \right) + \int_{CS} \beta \rho (\mathbf{V}_r \cdot \hat{\mathbf{n}}) dA$$

Special Cases:

- Non-deforming, moving CV
- Fixed CV
- Steady flow:

$$\frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t} (\beta \rho) dV + \int_{CS} \beta \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA$$

$$\frac{d}{dt} (\) = 0$$



$B =$ Extensive property
 $\beta =$ Intensive property
 $\beta = \frac{B}{m}$

Flux terms for uniform flow across discrete CS's (steady or unsteady)

$$\int_{CS} \beta \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA = \sum (\beta \dot{m})_{out} - \sum (\beta \dot{m})_{in}$$

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Okay, so now you know mass, when you have the corresponding beta of 1, when you have momentum, it is just the velocity, the beta, which is the intensive variable, and when it is energy, it is E, right? So that is the nomenclature; those are the nomenclatures.

Conservation Laws

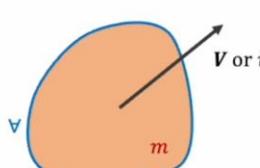
NPTEL, IISc

For uniform distribution of B

$$\beta = \frac{B}{m}, \quad B = \beta \cdot m$$

For non-uniform distribution of B

$$\beta = \frac{dB}{dm}, \quad B = \iiint \beta \cdot dm = \iiint \beta \rho dV$$



Property	B	$\beta = B/m$	Governing Law
Mass	m	1	Conservation of mass
Momentum	$\vec{p} = m\vec{v}$	\vec{v}	Conservation of linear momentum
Energy	E	e	Conservation of Energy

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So you can use the conservation principles, such as the conservation of mass, for a system like this, where there are a couple of inlets and two outlets; it's like a pipe.

You can think of it like a pipe. So $\frac{dm}{dt} = 0$. For this case, m is equal to 1. So β is equal to 1. So, if you apply the Reynolds transport theorem, you will see that this is equal to 0. So, for a fixed control volume, this is equal to 0.

So, now if you do this, there are two inlets and two outlets. So, it just becomes ρAv . So, ρ times A times velocity minus whatever is going out plus whatever is going in. So, for a fixed control volume in steady state, this is the equation that you have that is actually occurring for a fixed control volume.

Conservation of Mass



From the law of Conservation of mass from the system's point of view having mass m :

$$\frac{dm}{dt} = 0$$

Hence for this case $B = m$ and therefore $\beta = \frac{B}{m} = 1$

Substituting in RTT to obtain law of Conservation of mass from the CV point of view

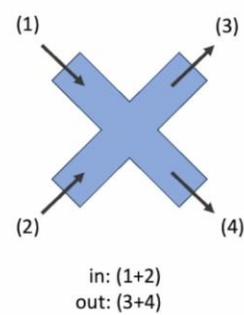
$$\frac{d}{dt} \left(\int_{CV} \rho dV \right) + \int_{CS} \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA = 0$$

For fixed CV: $\frac{d}{dt} \left(\int_{CV} \rho dV \right) + \int_{CS} \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA = 0$

Flux terms for uniform flow across discrete CS's (steady or unsteady)

$$\int_{CS} \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA = \sum \dot{m}_{out} - \sum \dot{m}_{in} = \sum (\rho AV)_{out} - \sum (\rho AV)_{in}$$

For fixed CV, Steady state: $\int_{CS} \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA = \sum (\rho AV)_{out} - \sum (\rho AV)_{in} = 0$



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Similarly, you have a conservation of mass and conservation of momentum. In this case, the intensive variable β is v , and the extensive variable is mv .

Again, you apply the Reynolds Transport Theorem. You have an external force that is equal to ma . So this is basically the acceleration term again; it's the same thing that you see over here. This is v , and remember this is the relative velocity. For a fixed control volume, this is what you get. Okay, this V becomes capital V now, and so here in this case you sum total all this stuff.

Okay, and you know the external force will be the rate of change of momentum or the change of momentum, so that is what you get: whatever is out minus whatever is in will give rise to the force over here. If it is a steady state, so that unsteady terms actually drop out.

Conservation of Linear Momentum



From the system's point of view having mass m , momentum is given as $m\mathbf{V}$, rate of change of which depicts the net forces on the system.

Hence for this case $B = m\mathbf{V}$ and therefore $\beta = \frac{B}{m} = \mathbf{V}$

Substituting in RTT to obtain law of Conservation of momentum from the CV point of view

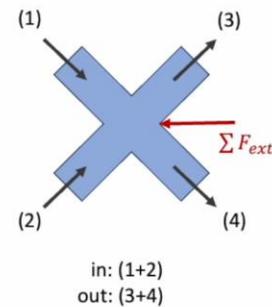
$$\frac{d(m\mathbf{V})_{sys}}{dt} = \sum F_{ext} = \frac{d}{dt} \left(\int_{CV} \rho \mathbf{V} d\mathcal{V} \right) + \int_{CS} \mathbf{V} (\rho \mathbf{V}_r \cdot \hat{\mathbf{n}}) dA$$

For fixed CV: $\sum F_{ext} = \frac{d}{dt} \left(\int_{CV} \rho \mathbf{V} d\mathcal{V} \right) + \int_{CS} \mathbf{V} (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA$

Flux terms for uniform flow across discrete CS's (steady or unsteady)

$$\int_{CS} \mathbf{V} (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA = \sum (\dot{m}\mathbf{V})_{out} - \sum (\dot{m}\mathbf{V})_{in} = \sum (\rho AV \cdot \mathbf{V})_{out} - \sum (\rho AV \cdot \mathbf{V})_{in}$$

For fixed CV, Steady state: $\sum F_{ext} = \int_{CS} \mathbf{V} (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA = \sum (\rho AV \cdot \mathbf{V})_{out} - \sum (\rho AV \cdot \mathbf{V})_{in}$



34

Similarly, you can have conservation of energy as well. So it is like, you know, Q is added to the system, and W is the work done by the system.

So, this is the first law of thermodynamics, as we all know. Similarly, you can write this expression. I'm not going to go into the details of all this.

Conservation of Energy



From the system's point of view, 1st law of Thermodynamics states:

$$dE = \delta Q - \delta W$$

Q = heat added to the system,

W = work done by the system

E = total internal energy of the material.

Here, δ indicates the differential element for path-dependent quantities (inexact differential), and d indicates exact differential.

We can also write

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Hence for this case the property we are interested in is the total internal energy $B = E$ and therefore the total specific internal energy is defined as $\beta = \frac{B}{m} = e$

$$e = e_{int} + e_{kinetic} + e_{potential} + e_{other}$$

$$e = u + \frac{v^2}{2} + gz + e_{other}$$

Where u = internal energy per unit mass due to microscopic interaction potential and microscopic kinetic energy.

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You can use the Reynolds transport theorem once again for this. Okay. But this time it is, you know, the rate of change of energy is equal to $\frac{dQ}{dt} - \frac{dW}{dt}$, which is equal to all these energy terms.

So $\frac{dQ}{dt}$ is the heat input. This is the work done, and then you can write this equation. You can see this in any heat transfer book.

Conservation of Energy



Substituting in RTT to obtain law of Conservation of momentum from the point of view of a fixed CV

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt} \left(\int_{CV} e \rho dV \right) + \int_{CS} e (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA$$

where

$\frac{dQ}{dt} = \dot{Q}$ = rate of heat input, can be due to Conduction, Radiation, Convection

$\frac{dW}{dt} = \dot{W}$ = rate of workdone or power, $\dot{W} = \dot{W}_{shaft} + \dot{W}_{pressure} + \dot{W}_{viscous}$

Specifically the pressure work is given as (similar to $dW = p dV$)

$$d\dot{W}_{pressure} = (-p) dA (-\mathbf{V} \cdot \hat{\mathbf{n}})$$

$$\dot{W}_{pressure} = \int_{CS} p (\mathbf{V} \cdot \hat{\mathbf{n}}) dA$$

Substituting and also using $h = u + \frac{p}{\rho}$

$$\dot{Q} - \dot{W}_{shaft} - \dot{W}_{viscous} = \frac{d}{dt} \left(\int_{CV} e \rho dV \right) + \int_{CS} \left(h + \frac{v^2}{2} + gz \right) (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA$$

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So ultimately, at the end of the day, you get something like this. For a steady-state kind of system, you already know that the heat transfer minus the work minus the viscous dissipation is something like this. If \dot{m}_1 is equal to \dot{m}_2 , whatever mass is entering is actually exiting; you have something like this.

So, this is the steady-flow version of the SFE. In general, there are multiple inlets, so you have to do the summations.

Conservation of Energy



$$\dot{Q} - \dot{W}_{shaft} - \dot{W}_{viscous} = \frac{d}{dt} \left(\int_{CV} e \rho dV \right) + \int_{CS} \left(h + \frac{v^2}{2} + gz \right) (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA$$

For steady state 1D problem

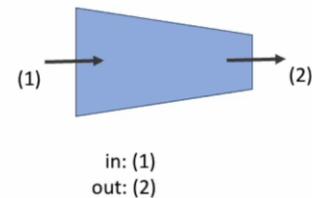
$$\dot{Q} - \dot{W}_{shaft} - \dot{W}_{viscous} = \dot{m}_2 \left(h_2 + \frac{v_2^2}{2} + gz_2 \right) - \dot{m}_1 \left(h_1 + \frac{v_1^2}{2} + gz_1 \right)$$

From mass conservation $\dot{m}_1 = \dot{m}_2$

$$h_1 + \frac{v_1^2}{2} + gz_1 + \dot{q} = h_2 + \frac{v_2^2}{2} + gz_2 + \dot{w}$$

This is the steady flow energy equation or SFEE. In general, with multiple inlet and exit ports

$$\left(h + \frac{v^2}{2} + gz \right)_{inlet} + \dot{q}_{net} = \left(h + \frac{v^2}{2} + gz \right)_{exit} + \dot{w}_{net}$$



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So, these are the conservation laws in integral form. This is what you need to know. So, this is basically $F = Ma$.

So, all of them share that general form, which involves beta. So now there is a specialized form which we also discussed that Bernoulli's equation. So Bernoulli's equation is valid along the streamline for frictionless or inviscid flow and incompressible or constant density. So if it's an unsteady flow, this is called Euler's equation, and for a steady flow, the integral part of this is called Bernoulli's equation. Whatever is in the brackets is basically Euler's equation. So if you flow along the streamline, you can apply Bernoulli's at two different points in the flow field.

This is routinely used where you have a balance between pressure head, velocity head, and potential head.

Bernoulli's Equation



Assumptions

- Along the streamline
- Frictionless or inviscid flow
- Incompressible or Constant density

For Unsteady flow

$$\int_1^2 \left(\frac{\partial V_s}{\partial t} ds + V_s dV_s + \frac{dp}{\rho} + g dz \right) = 0$$

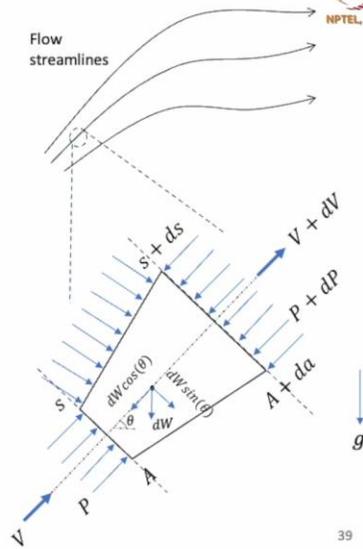
where

s = coordinate along the streamline

V_s = velocity magnitude at the coordinate s

For Steady flow

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \text{constant}$$



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So you can also have the differential forms of these quantities, and you can derive them from the integral version by using the divergence theorem, and then you can shrink them to a point. So this is what you get. This is the conservation equation, and if it is incompressible, the volumetric dilatation is actually equal to zero.

Conservation Laws – Differential Forms



Conservation of Mass

$$\begin{aligned} \frac{d}{dt} \left(\int_{CV} \rho dV \right) + \int_{CS} \rho (\mathbf{V} \cdot \hat{\mathbf{n}}) dA &= 0 \\ \Rightarrow \int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CS} (\rho \mathbf{V} \cdot \hat{\mathbf{n}}) dA &= 0 \end{aligned}$$

Using Gauss divergence theorem $\int_{CS} (\mathbf{F} \cdot \hat{\mathbf{n}}) dA = \int_{CV} (\nabla \cdot \mathbf{F}) dV$

$$\Rightarrow \int_{CV} \frac{\partial \rho}{\partial t} dV + \int_{CV} \nabla \cdot (\rho \mathbf{V}) dV = 0 \Rightarrow \int_{CV} \left\{ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) \right\} dV = 0$$

For this to be uniformly valid within the CV

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

For incompressible flow $\rho = \text{const.}$

$$\nabla \cdot \mathbf{V} = 0$$

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Similarly, you do the same thing with the momentum equations. Okay, and you ultimately get this. This is the stress term; this is the tensor. Okay, and so τ_{ij} is written this way. τ_{xx} and τ_{yy} are the normal stresses, while the other ones, τ_{xy} , are the shear stresses. So, the moment you substitute all of them, you get something like this, which is the Navier-Stokes

equation.

This is the viscous term. This is a substantial derivative, which is the acceleration term. This is the pressure, and this is the body force. So, if it is inviscid, you will get the Euler term because the viscosity will actually drop off.

Conservation Laws – Differential Forms



Conservation of Linear Momentum

$$\frac{d(mV)}{dt} = \sum F_{ext} = F_{body} + F_{surface}$$

$$f_{body} = \rho g$$

$$f_{surface} = \nabla \cdot \tau_{ij}$$

(f : per unit volume). Where the stress tensor

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

Newtonian fluid: Stress \propto Strain ($\tau_{ij} \propto \epsilon_{ij}$)

$$\rho \frac{dV}{dt} = \rho g + \nabla \cdot \tau_{ij}$$

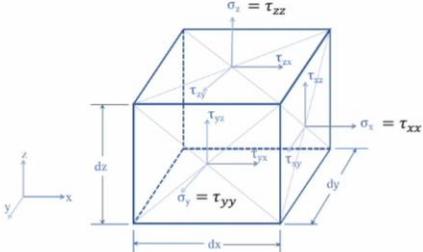
On simplification with Newtonian fluid assumption for incompressible flow

$$\rho \frac{dV}{dt} = -\nabla p + \rho g - \mu \nabla^2 V \quad \text{Navier Stokes Equation}$$

Additionally, if inviscid

$$\rho \frac{dV}{dt} = -\nabla p + \rho g \quad \text{Euler Equation}$$

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Then the conservation can also be written in a form like this. H is the enthalpy. ϕ is the viscous dissipation. It can be neglected in certain cases because it depends on the square of the velocity gradient, but not on the cross velocity gradient.

Conservation Laws – Differential Forms



Conservation of Energy

$$\rho \frac{dh}{dt} = \frac{dP}{dt} + \nabla \cdot (k\nabla T) + \Phi$$

Where Dissipation function $\Phi = \tau'_{ij} \frac{\partial u_i}{\partial x_j}$

For Newtonian fluid

$$\Phi = \mu \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + 2 \left(\frac{\partial w}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right] + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2$$

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So these are the equations in differential form. You know, the conservation of mass, the conservation of linear momentum, and the conservation of energy.

Conservation Laws – Differential Forms



Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Conservation of Linear Momentum

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \rho \mathbf{g} - \mu \nabla^2 \mathbf{V}$$

Conservation of Energy

$$\rho \frac{dh}{dt} = \frac{dP}{dt} + \nabla \cdot (k\nabla T) + \Phi$$

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Okay. So we will end our lecture here, where we have actually covered a lot in terms of what these quantities are. And we have already covered, you know, that we have shown mathematically how these can be best illustrated. But you know, this is just a summary. A very quick summary: in measurement, you are supposed to measure all these things, and you should have an idea of what we are trying to measure. Either it's a field measurement or it's a Lagrangian type of measurement. Mostly, ours are field measurements.