Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

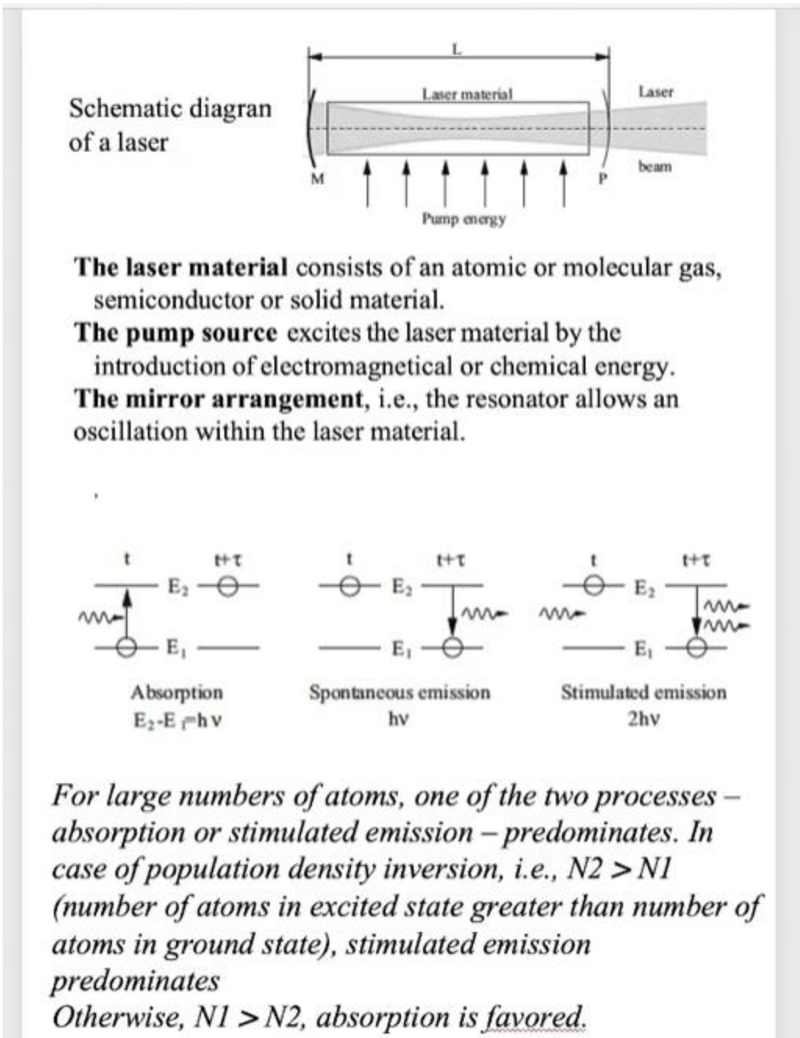
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru

Week – 07

Lecture - 33

Particle Image Velocimetry – 3

Okay, so we are going to proceed with our work on the PIV. Okay, so let us look at it. So in the PIV, as we know we have talked about the seeding and all other things, now let us look at the lasers, okay? So now that we have it in the PIV, now let us look at how the lasers can actually, how would they affect the PIV. So what you see here is a schematic of the laser. Okay, so what a laser actually does is that it consists of atomic or molecular gas, or semiconductor, or solid material. Okay, and then you have a pump source.

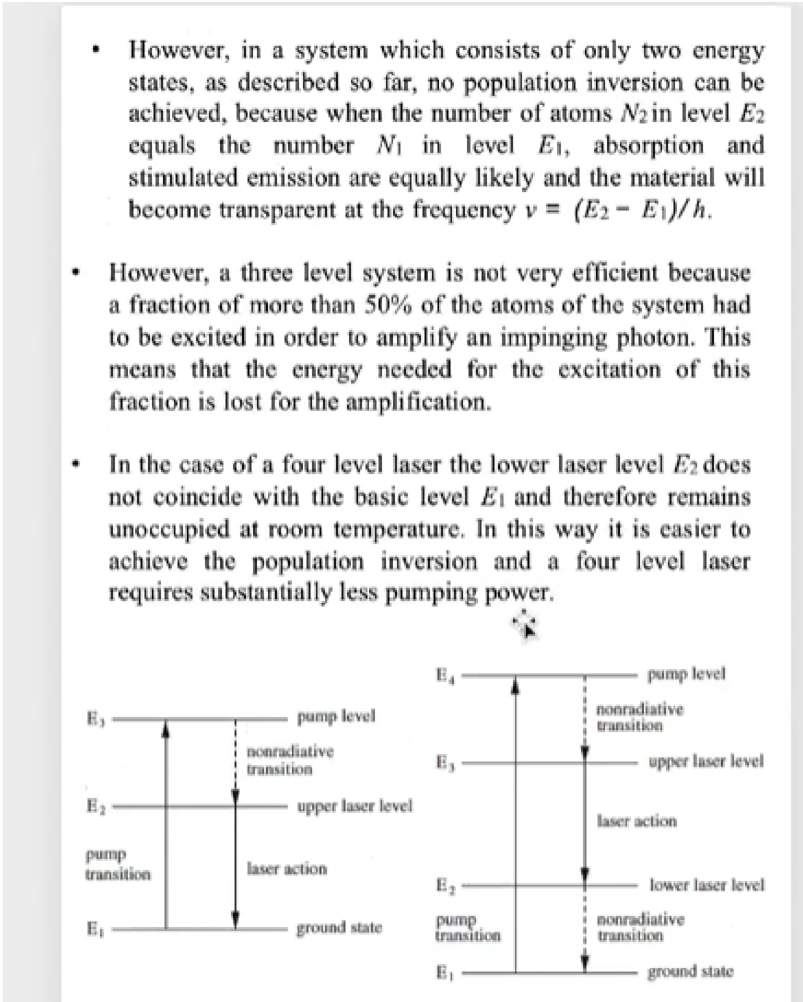


We have already talked a little bit about the laser; it excites the laser material by the introduction of electromagnetic or chemical energy. Now there is a mirror arrangement that basically is a resonator, which allows oscillation within the laser material and therefore helps us to populate a particular mode. So this is the schematic of the laser that we are already familiar with. So remember that it consists of gas, semiconductor, and solid material as well.

Now, let us look at how lasing happens. We already talked about it in our laser presentation. Let me just quickly go through it one more time. So this is the typical Einstein two-level diagram in which there is a lower energy level and a higher energy level. So when an incident photon of a particular wavelength or color impinges on this energy level, what it can do is that there are a large number of interactions, so what happens is that this photon now interacts with this electron, and this electron is now promoted.

It absorbs the energy and gets promoted to a higher energy level after some time, which is τ. So this is called absorption, where the difference in energy actually matches the color of the photon. Now there can be spontaneous emission also. This electron will go down to ground level and emit a photon after some time. And then there is stimulated emission.

So that means there is an electron that is already in the excited state. You basically bombard it with photons. And you basically get two photons coming out of it, which actually have coherence; they're phase locked. For a large number of atoms, like one of the two processes, absorption or stimulated emission predominates. In case of population inversion, N_2 is greater than N_1. Normally, your $N_1 > N_2$ under thermal equilibrium. So, as we can see, you know, just a second, let me lower this thing. So you can see that, however, in a system where there are only two energy states, as we described, the population inversion cannot be achieved because the number of atoms in N_2 in level E_2 equals the number N_1 in level E_1. Therefore, what will happen is that absorption and stimulated emissions will be equally likely, and the material will become transparent at the frequency which is $(E_2 – E_1)/ H$. However, if we introduce a three-level system, which is right over here, this is also not very efficient.



What a three-level system does is that there is a metastable state, which is... E_2 is over here. So E_3 is the upper energy level where the electrons are pumped, and then you have a metastable upper laser level where the electrons from E_3 undergo non-radiative transition and come to this state, and then they actually come back to the ground state, and this is where the laser action happens.

So, laser action happens between E_2 and E_1. So what happens here is that a fraction of more than 50% of the atoms has to be excited in order to amplify an impinging photon. This means that the energy needed for this excitation is lost for the amplification. So in this case, what people do is go to a four-level laser system in which the lowest level, which is E_2, does not coincide with this. So you see, the lasing actually happens between E_3 and E_2.

Okay, so in this way, it is easier to achieve a population inversion, and a four-level laser system requires substantially less pumping power. So, therefore, you pump it to E_4; then there is a first non-radiative transition, followed by a lasing transition, and then there is another non-radiative transition that actually happens. So this is how this entire thing actually works. This requires substantially less power to begin with. Alright, so this is good.

So, mostly for PIV you will use solid state lasers. So solid-state laser materials are generally pumped by electromagnetic radiation. Semiconductor lasers are operated by electric current, while gas lasers are operated by the collision of atoms or molecules with electrons and ions. So this is the way these lasers are pumped. Because you need pumping energy to make the electrons go to a higher energy level to create that inversion.

- Solid laser materials are generally pumped by electromagnetic radiation, semiconductor lasers by electronic current, and gas lasers by collision of the atoms or molecules with electrons and ions.
- As a consequence of population inversion through energy transfer by the pump mechanism spontaneous emission occurs in all directions which causes excitation of further neighboring atoms. This initiates a rapid increase of stimulated emission and therefore of radiation in a chain reaction.
- In the case of a cylindrical shape of the laser material, the rapid increase of radiation occurs in a defined direction because the amplification increases with increasing length of the laser medium. Within an optical resonator (mirror arrangement) the laser material can be arranged to form an oscillator. The simplest way to achieve this is to place the material between two exactly aligned mirrors. In this case, a photon which impinges randomly on one of the mirror surfaces is reflected and amplified in the laser material again. This process will be repeated and generates an avalanche of light which increases exponentially with the number of reflections, finally resulting in a stationary process. In other words, standing waves are produced for a resonator length corresponding to the condition

$$L = \frac{m\lambda}{2n} \tag{2.11}$$

where n is the refractive index, m an integer number, and L the resonator length. Since the frequency ν according to the transition $\nu h = E_2 - E_1$ does not correspond to exactly one wavelength, but rather to a spectrum of a certain band width $\Delta\nu$ depending on the transition time τ of the process, these conditions can be fulfilled by different wavelengths λ or frequencies ν and the resonator can oscillate in many axial modes with distinct frequencies.

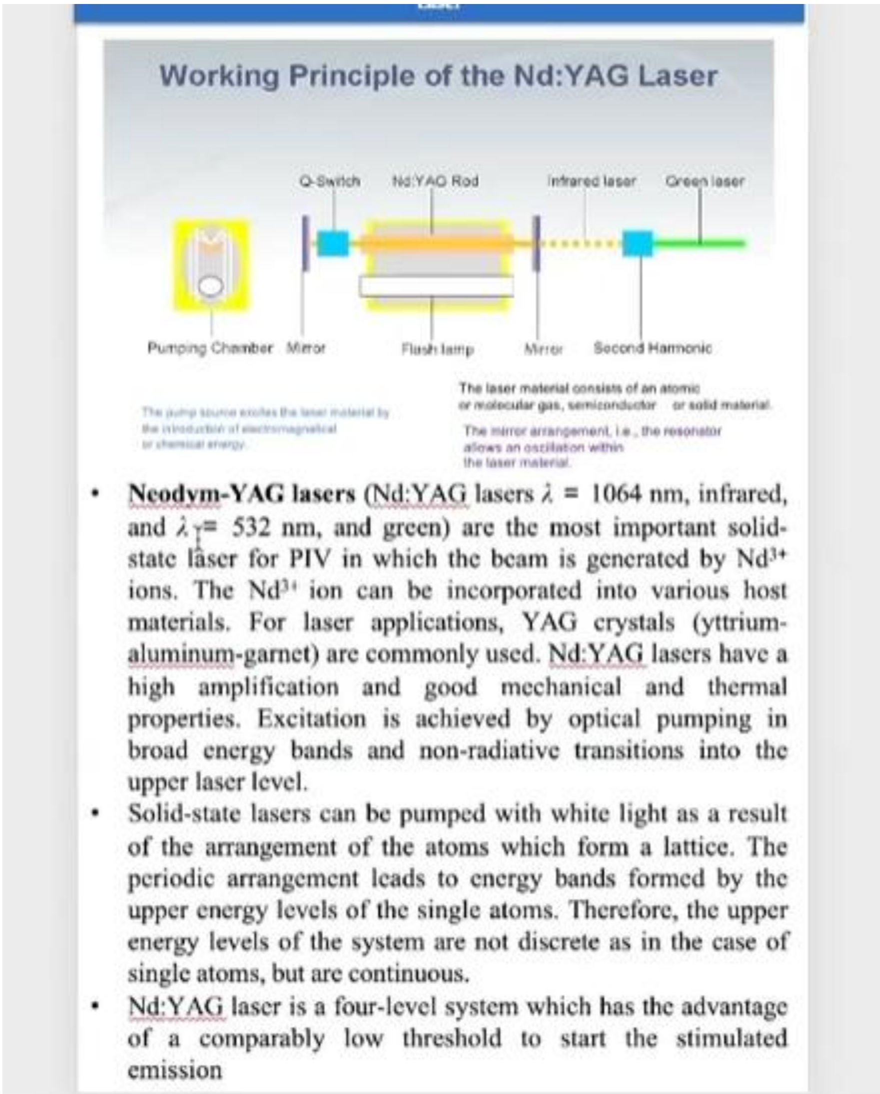
So, as a consequence of this population inversion, what happens is that the pump mechanism and spontaneous emission occur in all directions, which basically causes excitation of the neighboring atoms. So this initiates a rapid increase in stimulated emission and therefore radiation in a chain reaction. So this is how this actually happens. So in case you look at a laser material, it is of cylindrical shape. So, the rapid increase in radiation occurs in a defined direction.

Okay, because the amplification increases with the length of the laser material, you have an optical resonator, which is a mirror arrangement for the laser material to form an oscillator. The simplest way to do this is to have two exactly aligned mirrors. A photon impinges on one of the mirrors, is reflected, and then is amplified in the laser material. This process is repeated. So that you see more and more photons, it basically comes out.

It increases as we continue increasing the number of reflections because more and more photons will be knocked off the material, from the laser material, and therefore it will generate what we call a stationary process. At the stationary process, in other words, what we will do along the resonator length is form these standing waves, which are basically as you can see here: l is equal to $m\lambda /2n$, where n is the refractive index, m is an integer number, and l is the length of the resonator. So the frequency according to the transition does not correspond to exactly one wavelength but rather to a spectrum of a certain band. This we already covered during the lecture. Because this also depends on the short transition time that you have for these two processes.

In addition to this, the lasers will have different cross-sections. These are the different transverse modes of the laser. So, this is a cross-section of the laser beam that can be divided into several ranges. This is TM00. And then you can go higher and higher.

Okay, so these are the cross sections of the laser beam. That happens because of out-of-phase oscillations. That's actually why this happens. Okay, so the working principles of an Nd:YAG laser, you will see that there is a flash lamp and this is the Nd:YAG rod. And these are mirrors; you can have a second harmonic generator, and this is the pumping chamber.



Okay, and this is a cue switch. So, the laser material basically consists of, this is an infrared laser. As we know, it is 1064 nanometers, and the second harmonic is at 532 nanometers; it is green. So these are the most important solid-state lasers.

This is an ND YAG. So it is a neodymium YAG laser. So its ND3+ ions basically generate the beam. So, these ions are incorporated into various host materials, and the YAG crystals in these cases are commonly used. So these have very high amplification. That's why they make good candidates, as they have good thermal and mechanical properties.

So how do you do that? The excitation is achieved by optical pumping in the broad energy bands and through non-radiative transitions into the upper energy level. It's a four-level laser system. So solid-state lasers, as we know, can be pumped with white light because of the arrangement of the atoms in the lattice. So, this is a long story. We need not consider all aspects of it, but one can say that the upper energy levels of this system are not very discrete; instead, they are more continuous.

So this allows us to have a white-light excitation source. You know, this is different from the energy bands that are formed by single atoms. The NDAG is a four-level laser system, and it has a very low threshold to start the stimulated emission. So you can get all those things. The basic arrangement is still the same.

There's a flashlight. There are two mirror assemblies, and there is an optical resonator. Therefore, there is a second harmonic, and you get a green light out of it. One uses the 1064 as well, but that is invisible to the naked eye. So the pumping source excites the laser material by the introduction of electromagnetic, electro-magnetic, or chemical energy. So that is how this lasing actually occurs.

If you now look at the different aspects of the laser, this is, for example, a double oscillator system because in PIV you need two pulses, remember, which are well separated by a certain separation distance. So these are more about the complexities of the whole thing. This is a double-oscillator laser with critical resonators. And then there is the beam alignment that happens behind the frequency doubling, which basically converts it to 532; okay, so this is the arrangement that is actually shown. This is the output beam, this is the 532 nanometer beam shutter, so if you open up a laser, you will see all these things; okay, so there is the crystal doubler, which basically creates the.

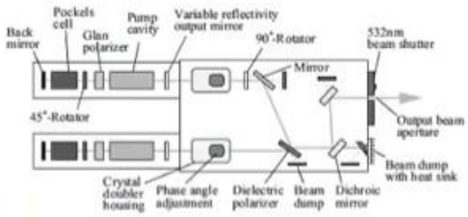


Fig. 2.35 Double oscillator laser system with critical resonators. The beam alignment behind the frequency doubling leads to two polarization directions of the laser beams. (typically one laser beam horizontally and the other beam vertically polarized)

Table 2.5 Properties and specifications of modern Nd:YAG PIV-laser systems

Repetition rate[a]	10 Hz
Pulse energy for each of two pulses	320 mJ
Roundness at 8 m from laser output[b]	75%
Roundness at 0.5 m from laser output[c]	75%
Spatial intensity distribution at 8 m from laser output[d]	<0.2
Spatial intensity distribution at 0.5 m from laser output[d]	<0.2
Linewidth	1.4 cm^{-1}
Power drift over 8 hours[e]	<5%
Energy stability[f]	<5%
Beam pointing stability[g]	100 μrad
Deviation from collinearity of laser beams	<0.1 mm/m
Beam diameter at laser output	9 mm
Divergence[h]	0.5 mrad
Jitter between two following laser pulses	2 ns
Delay between two laser pulses	0–10 ms
Resolution	5 ps
Working temperatures	15–35°C
Cooling water[i]	10–25°C
Power requirements	220–240 V, 50 Hz

[a]And integral fractions of 10 Hz, eg. 5, 2.5 Hz etc
[b]Ratio between two perpendicular axis (major and minor axis)
[c]If laser beam is elliptical, major axis of both oscillators must be parallel
[d]$|(I_{max} - I_{min})|/|(I_{max} + I_{min})|$, with I being the peak intensity in the spatial distribution limited by the diameter at half maximum for both oscillators
[e]Without readjustment of phase-matching for ambient temperatures of $18°C < T < 35°C$
[f]Shot to shot, peak to peak, 100% of shots
[g]RMS, on 200 alternating pulses at the focal plane of a 2 m lens
[h]Full angle on 200 pulses at e^{-2} of the peak, 85% of total energy
[i]Secondary circuit, 10 l/min pressure, 1.5–3 bar

.. and there is a phase angle adjustment, which you will see shortly, what that actually does. So this is the pump cavity; basically, these are the Pockels cells. And this is the variable reflectivity mirror, okay? And then you have the crystal doubler, and then you have the polarizer, and then you have the dichroic mirrors, and then you have an output beam. So some of the properties of the modern Nd:YAG systems include rep rates. I mean, these are low-speed PIV lasers, which can go up to 10 hertz.

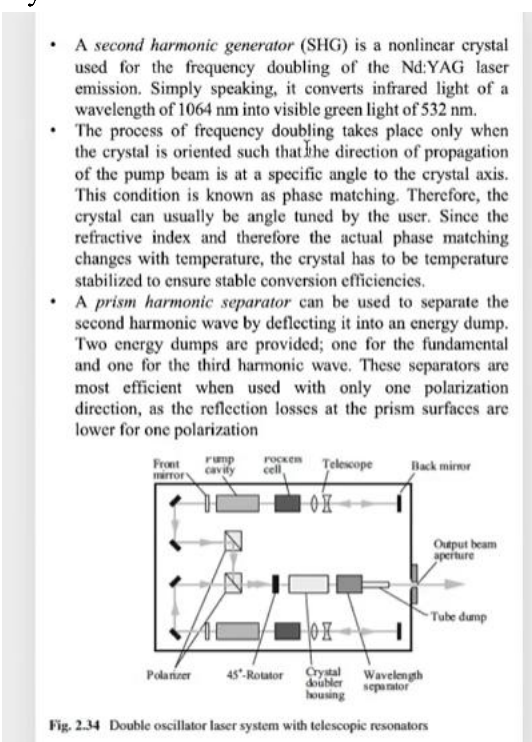
Nd:YLF lasers actually go up to, you know, uh... Kilohertz order, okay, so then there is a roundness of the beam, uh, eight meters from the laser output and 0.

5 meters from the laser output. Then there is a spatial intensity; the line width, the line width. Remember, this is not one frequency, contrary to popular belief, but it is still very narrow, the laser line width. Then there is also a beam pointing stability. Then there is a divergence because the light actually diverges a little. Then there is jitter between the two laser pulses.

Then there is a delay. The delay can be anywhere between zero and 10 milliseconds. We can adjust the delays. The lasing duration is usually on the order of nanoseconds. The working temperature is generally within room temperature and not in a very hot environment. Okay, so this is kind of, you know, the kind of laser, so all the beams you can see come through the same hole.

So, in other words, there are not two separate holes. So you use the same beam alignment optics to do this double oscillator laser system. Okay. So what is a second harmonic oscillator and a second harmonic generator? That is basically, you know, a nonlinear crystal. It's a nonlinear crystal used for frequency doubling of the Nd:YAG laser emission.

So what it does is convert the infrared light into visible green. So how does it happen? The process of frequency doubling takes place when the crystal is oriented in the direction of the propagation of the pump beam. So that the direction of the propagation of the pump beam is at a particular angle with respect to the crystal axis. Okay, so because this is a nonlinear crystal and this condition is called phase matching, the crystal can usually be angle tuned by the user, so you can actually do the crystal tuning yourself. If you had opened it, then you would have seen those crystals; the refractive index, and therefore the actual phase matching, varies a little bit with temperature because the crystal has to be temperature stabilized.



- A *second harmonic generator* (SHG) is a nonlinear crystal used for the frequency doubling of the Nd:YAG laser emission. Simply speaking, it converts infrared light of a wavelength of 1064 nm into visible green light of 532 nm.
- The process of frequency doubling takes place only when the crystal is oriented such that the direction of propagation of the pump beam is at a specific angle to the crystal axis. This condition is known as phase matching. Therefore, the crystal can usually be angle tuned by the user. Since the refractive index and therefore the actual phase matching changes with temperature, the crystal has to be temperature stabilized to ensure stable conversion efficiencies.
- A *prism harmonic separator* can be used to separate the second harmonic wave by deflecting it into an energy dump. Two energy dumps are provided; one for the fundamental and one for the third harmonic wave. These separators are most efficient when used with only one polarization direction, as the reflection losses at the prism surfaces are lower for one polarization

Fig. 2.34 Double oscillator laser system with telescopic resonators

The prism harmonic separator can be used to separate the second harmonic by deflecting it into an energy dump. The two energy dumps are therefore provided. Two energy dumps are actually provided: one for the fundamental and one for the third harmonic. So these separators are, basically, to separate out the second harmonic because you don't want the 1064, and you definitely do not want the third harmonic. The third harmonic is also useful for, you know, acetone pliff and stuff like that.

But you know, the separators—these separators—are essential when you are actually dealing with the lasers, okay? So this is an important statement that we are actually making here. So now let us look at when you actually use light sheet optics. So we can also have white light sources and light delivery, but light-sheet optics is what we want to do. So the essential element for doing PIV is that generation of a light sheet. So the essential element for this is a cylindrical lens.

2.3.4 *White Light Sources*

Even though most PIV investigations are performed using laser light sheets, white light sources might also be used. Due to the finite extension of these sources and since white light cannot be collimated as well as monochromatic light, they clearly have some disadvantages. On the other hand, the spectral output of sources like Xenon lamps is well suited for use with CCD cameras because of their similar spectral sensitivity. Systems are commercially available which can easily be triggered and offer a repetition rate that matches the video rate. Two flash lamps can be linked by optical fiber bundles in order to achieve short pulse separation times. If the outputs of the fibers are arranged in line, the generation of a light sheet is considerably simplified.

An attractive alternative to discharge flash lamps is provided by white-light LEDs that are increasingly finding widespread uses in general lighting applications replacing less efficient incandescent light sources. Regarding its electrical properties the white-light LED is no different to its monochromatic counterparts and can be operated in a pulsed mode in the same manner as described before. The emission spectrum of commonly available white LEDs is plotted in Fig. 2.39 and shows two distinct maxima, one sharp peak in the blue and a broader peak with a maximum in the yellow. The reason lies in the fact that the "white-light" LEDs actually are phosphor coated blue LEDs. The phosphor down-converts the blue light to a yellowish light such that, in combination with the blue light, it is perceived as white light. In some cases this bi-modal emission spectrum may not be desired, for instance for liquid crystal thermometry [12]. In such cases, the combination of several different-colored LEDs should be considered, in particular, if matched to the color filters of a color camera.

The main advantage of these white light sources is – aside from reduced cost – that their application is not hampered by laser safety rules.

4. Light Delivery

1. *Light Sheet Optics*

This section treats the optics for the illumination of the particles by a thin light sheet. Therefore, we describe three different lens configurations which are frequently used. For a more detailed analysis or the design of complex optical systems ray tracing programs can be used, but this is beyond the scope of the book. Rules for the calculation of the light sheet intensity distribution are not given herein. The reason for this is that geometric optics rules are already sufficient for a general layout of the chosen lens configuration. They do not require a special description and can readily be found in any book on optics [23]. On the other hand, more sophisticated calculations based on Gaussian optics usually require some assumptions, which are

So when using lasers with a sufficiently small beam diameter and divergence, like argon-ion lasers, one cylindrical lens can be sufficient to generate a light sheet. But for NDAG lasers, you need a combination of different lenses. At least one additional lens must be used. So, if you look at the configuration over here, there is a third cylindrical lens as well. So there is also a third cylindrical lens that you can see to generate a light sheet of constant height.

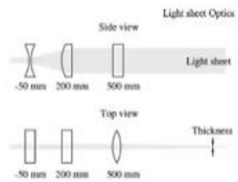


Fig. 2.46 Light sheet optics using three cylindrical lenses (one of them with negative focal length)

- The essential element for the generation of a light sheet is a cylindrical lens. When using lasers with a sufficiently small beam diameter and divergence – like for example argon-ion lasers – one cylindrical lens can be sufficient to generate a light sheet of appropriate shape. For other light sources – like for example Nd:YAG lasers – a combination of different lenses is usually required in order to generate thin light sheets of high intensity. At least one additional lens has then to be used for focusing the light to an appropriate thickness. Such a configuration is shown in Fig 2.46 where also a third cylindrical lens has been added in order to generate a light sheet of constant height.
- The combination of a cylindrical lens together with two spherical lenses actingas a telescope makes the system more versatile. This is shown in Fig. 2.47 where spherical lenses have been used, because they are in general easier to manufacture, especially if short focal length lenses are required. The height of the light sheet shown in Fig. 2.47 is mainly given by the focal length of the cylindrical lens in the middle. A diverging lens – negative focal length – could also be used. The adaptation of the light sheet height has to be done by replacing the cylindrical lens with a cylindrical lens

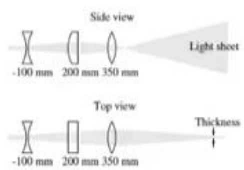
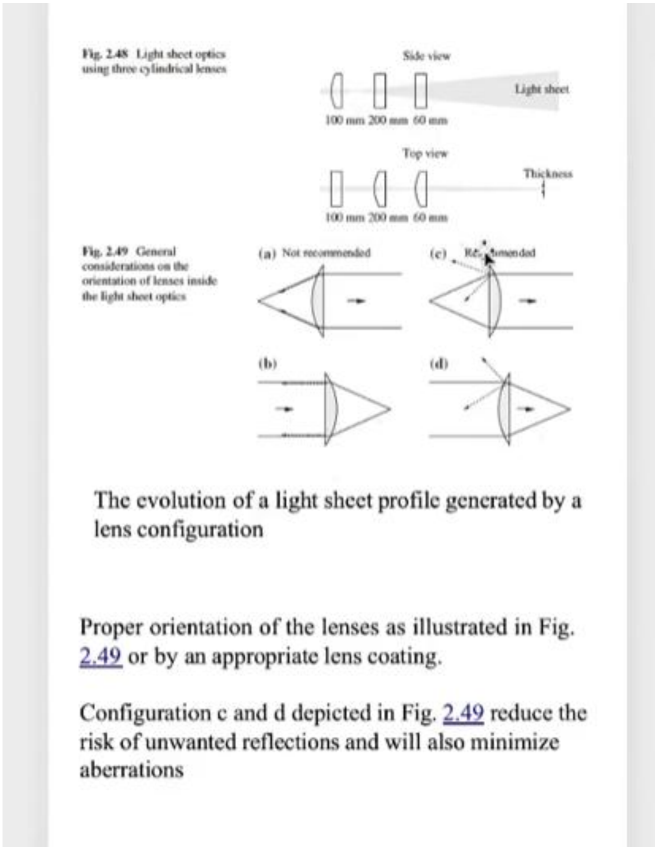


Fig. 2.47 Light sheet optics using two spherical lenses (one of them with negative focal length) and one cylindrical lens

So now, the combination of a cylindrical lens together with two spherical lenses acts as a telescope that makes the system very versatile. So this is what is shown in this particular figure. Remember, this is from Marcus Raphael's book. So here you basically use two spherical lenses to make the system more versatile. So the height of the sheet is shown over here, and it is mainly given by the focal length of the cylindrical lens in the middle.

A diverging lens that has a negative focal length could also be used in these cases to open up, and this adaptation of the light sheet height has to be done by replacing the cylindrical lens with another cylindrical lens. So this is how these things operate. You can also see the evolution of the light sheet profile with different optics. This is, for example, light-sheet optics using cylindrical lenses.

And these are some of the recommended ones. These are the non-recommended ones. For example, the proper orientation of the lenses is shown here; in this case, this configuration and this are recommended because they reduce unwanted reflections and also curtail spherical aberrations. This is what happens. Now we take a little bit of a look at the imaging of small particles. You know that we have already done it in our diffraction lectures.



So what happens is that when you actually have light waves that are hitting an opaque screen containing a small circular aperture, which can also be a particle, they generate a far-field diffraction pattern on a distant observing screen. This we have already seen about the ARRI disks. So what happens is that the image it casts on the screen can be captured by a camera. So the image of a distant point source, for example, a small scattering particle inside a light sheet, does not appear as a point. It appears that it forms a diffraction pattern even if the lenses are completely aberration-free, so ideal optics also gives rise to a diffraction pattern; therefore, the central peak looks something like this.

Imaging of Small Particles

Diffraction Limited Imaging

- If plane light waves impinge on an opaque screen containing a circular aperture they generate a far-field diffraction pattern on a distant observing screen. By using a lens – for example an objective in a camera – the far field pattern can be imaged on an image sensor. However, the image of a distant point source (e.g. a small scattering particle inside the light sheet) does not appear as a point in the image plane but forms a diffraction pattern even if it is imaged by a perfectly aberration-free lens. The central peak of the intensity distribution is called Airy disk,and the rings around the maximum are called Airy rings.
- Small aperture diam- eters correspond to large Airy disks and large apertures to small disks as can be seen in Fig. 2.54.
- The Airy function represents the impulse response – the so-called point spreadfunction of an aberration-free lens. It is equivalent to the square of the first order Bessel function

The value of the radius of the ring and therefore of the Airy disk for a given aperture diameter D_a and wavelength λ is:

$$\frac{I(x)}{I_{max}} = 0 \Rightarrow \frac{d_{diff}}{2x_0} = 1.22$$

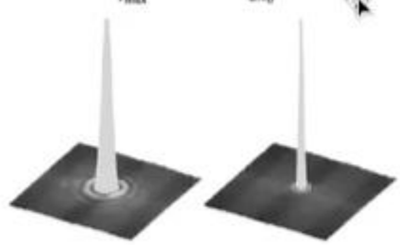


Fig. 2.54 Airy patterns for both a small (50 μm, left hand side) and large (100 μm, right hand side) aperture diameter. A helium-neon laser was used to create the pinhole intensity profile. The intensity is normalized by the maximum intensity for both the contour levels and the z-axes

For example, this is a 50-micron, and this is a 100-micron, so as you can see. 50 microns is left, 100 microns is right. So a smaller particle gives rise to a wider area of disks. So the small aperture diameter, or in this case small particles, for example, corresponds to large area disks.

whereas the bigger particles correspond to smaller arydiscs. So the central peak of the distribution is called the Airy disc, and the rings that are around it are called the Airy rings, okay? So the airy function represents an impulse response. This is a spread function of an aberration-free lens. And if you do a little bit of math, you will find that this can be represented by a first-order basal function. So the value of the radius of the ring and therefore the Airy disc for a given aperture diameter DA and wavelength λ is given by this. So the rule of thumb is that the smaller the particle, the larger the size of the iridescence.

Okay, so we already know this from our previous experience. Now, if we imagine that we are imaging objects in air, the same medium on both sides of the lens, etc. etc. This is geometric optics; this is the object plane; this is the image plane that you can see. Z naught is the distance from the image plane to the lens, and Z_o is the distance between the lens and the object plane. So you already know the magnification, which is given as z_o / Z_o.

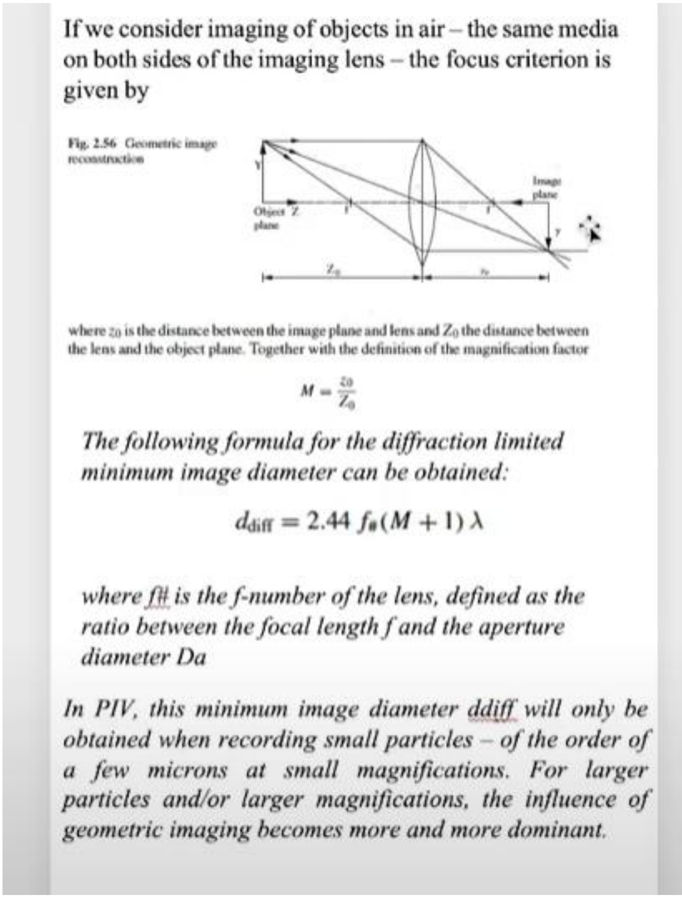

If we consider imaging of objects in air – the same media on both sides of the imaging lens – the focus criterion is given by

Fig. 2.56 Geometric image reconstruction

where z_0 is the distance between the image plane and lens and Z_0 the distance between the lens and the object plane. Together with the definition of the magnification factor

$$M = \frac{z_0}{Z_0}$$

The following formula for the diffraction limited minimum image diameter can be obtained:

$$d_{diff} = 2.44\, f_{\#}(M + 1)\,\lambda$$

where f# is the f-number of the lens, defined as the ratio between the focal length f and the aperture diameter Da

In PIV, this minimum image diameter ddiff will only be obtained when recording small particles – of the order of a few microns at small magnifications. For larger particles and/or larger magnifications, the influence of geometric imaging becomes more and more dominant.

So the following formula, which you have also seen, for diffraction-limited minimum image diameter is this. So the f number of the lens, which is the f-stop, is defined as the ratio of the focal length of the lens and the aperture, which is DA. So, aperture; this we have already covered in quite a bit of detail, and M is the magnification, λ is the wavelength. So the diffraction, you know, the limited minimum image diameter comes out to be something like this. So in this case, in PIV, this will be obtained by recording small particles on the order of a few microns at small magnifications.

Okay, for larger particles and larger magnifications, the geometric imaging becomes more and more dominant, but this is the minimum image diameter that you are going to get. So, this poses that if we do not have any lens aberrations, for example, and the point spread function can be approximated by the Airy function, then the following, you know, A formula can be used, which is $d\tau = (Mdp)^2 + d_{diff}^2$, when the size of the particle's geometric image, which is Mdp in the subscript, is considerably small. So, if this is considerably smaller than df, then the expression is dominated by diffraction. So if the particle is very small, then it undergoes diffraction.

If lens aberrations can be neglected and the point spread function can be approximated by the Airy function, the following formula can be used for an estimate of the particle image diameter

$$d_\tau = \sqrt{(Md_p)^2 + d_{diff}^2}$$

Whenthe size of the particle's geometric image Mdp is considerably smaller than ddiff, this expression is dominated by diffraction effects and reaches a constant value of ddiff.

It is dominated by the geometric image size for geometric image sizes considerably larger than ddiff where dτ ≈ Mdp

First, an analysis of PIV evaluation shows that the error in velocity measurements strongly depends on the particle image diameter (see e.g. Sect. 6.2.2). For most practical situations, the error is minimized by minimizing both the particle image diameter d_τ and the uncertainty in locating the image centroid or correlation peak centroid respectively.

Second, sharp and small particle images are particularly essential in order to obtain a high particle image intensity I_{max}, since at constant light energy scattered by the tracer particle the light energy per sensor area increases quadratically with decreasing image area

So your diameter simply does not change. You get this unique size. And it reaches a constant; it is constant, virtually constant. But if it is dominated by the geometric image size, then where the particle size images are considerably larger, this becomes equal to MDP, okay? That means larger particles are favored. The smaller particles can be below the diffraction limits. So you can see that the diffraction limit is very, very important because beyond a certain size you cannot really, you know, recognize the particle any longer.

So if you go and look at it, you will see that it is very, very hard. It is obviously a function of the wavelength. The smaller the wavelength, the less the diffraction diameter. So you can essentially go to smaller particle sizes. This is what it means. So first, your PIV evaluation should show an error in velocity measurements because of the particle diameter.

Okay, this is from the book; you can read it. For most practical situations, the error is minimized by minimizing both the particle diameter and the uncertainty in locating the image centroid. Second, sharp and small particles are particularly essential. Okay to obtain a high particle image density, but then this carries the issue that you need to reduce scatter, and this also leads to imaging problems. Smaller particles are required for PIV because they act as good tracers, but at the same time, they bring the additional baggage of low scattering, which is low intensity, and this diffraction limit. So the particle diameter obtained earlier can now be used to find the depth of field for typical macroscopic PIV, and the formula is this.

Particle image diameter obtained earlier equation can be used to estimate the depth of field for typical macroscopic PIV recordings δZ using the following approximation

$$\delta_Z = 2 f_{\#} d_{\text{diff}} (M + 1)/M^2$$

Table 2.7 Theoretical values of f-number, image diameter and depth of focus for diffraction limited imaging of small particles ($\lambda = 532$ nm, $M = 1/4$, $d_p = 1$ µm)

$f_{\#} = f/D_a$	d_τ [µm]	δ_Z [mm]
2.8	4.7	0.5
4.0	6.6	1.1
5.6	9.1	2.0
8.0	13.0	4.2
11	17.8	7.8
16	26.0	16.6
22	35.7	31.4

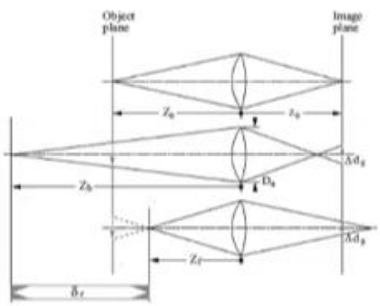


Fig. 2.57 Depth of focus for an acceptable diameter of the geometric image sketch for both extremal positions of out-of-focus imaging

It can be seen that a large aperture diameter (small f-number) is needed to collect sufficient light from each individual particle within the light sheet, and to get sharp particle images

Unfortunately, a large aperture diameter yields a small focal depth which is a significant problem when imaging small tracer particles

So this is the depth of field that you have. It is given by this. This is diffraction, and this is magnification. So you can see that a large aperture, that is, a small f-number, is needed to collect sufficient light. All right, a small F-number is needed. That's the large aperture that is actually needed to collect sufficient light from each individual particle within the light sheet and to get those sharp images. Unfortunately, a large aperture also leads to a small depth of field because it's a small F-number.

Large aperture leads to a small focal depth. We have already seen this in our previous lectures. This is a significant problem when you try to image small tracer particles that may go out of the depth of field quite easily. So these are the problems. These are problems, and this is what you need to address. Therefore, these are the problems that one needs to figure out before choosing your tracer, choosing your light source, and determining what kind of depth of field you should be using, and so forth.

So in the next class, we will also look a little bit at perspective projections and see how that actually affects the PIV.