**Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer**
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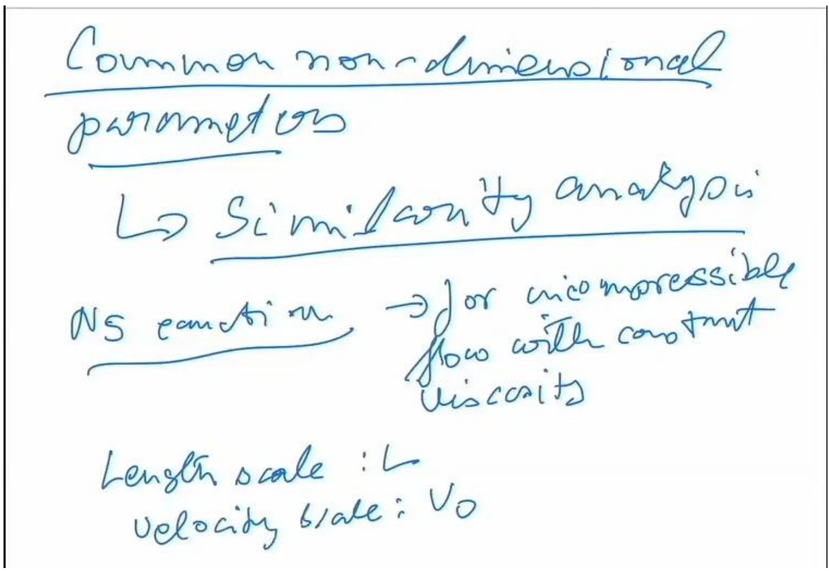
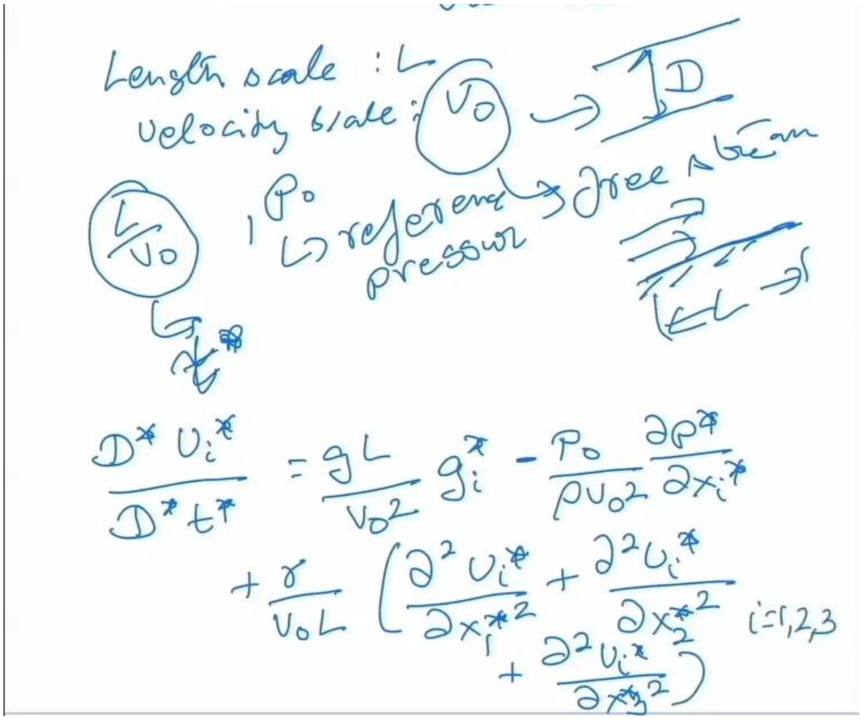
**Week – 01**
**Lecture - 03**
**Introduction to Fluids – 3**

So, as you can see, we talked about this. Now let us see, you know, or let's consider the Navier-Stokes equation. That is the equation that is most common, and most of you are actually familiar with the Navier-Stokes equation. Let's solve the Navier-Stokes equation for incompressible flow. with constant viscosity. So, in a gravitational field.




So, assume that there is a length scale. The length scale is equal to L, and then there is a velocity scale that is equal to V naught that can be identified. This is like the characteristic dimension of the boundary and the average free stream velocity. So something like the dimension of the system is a good length scale to start with.



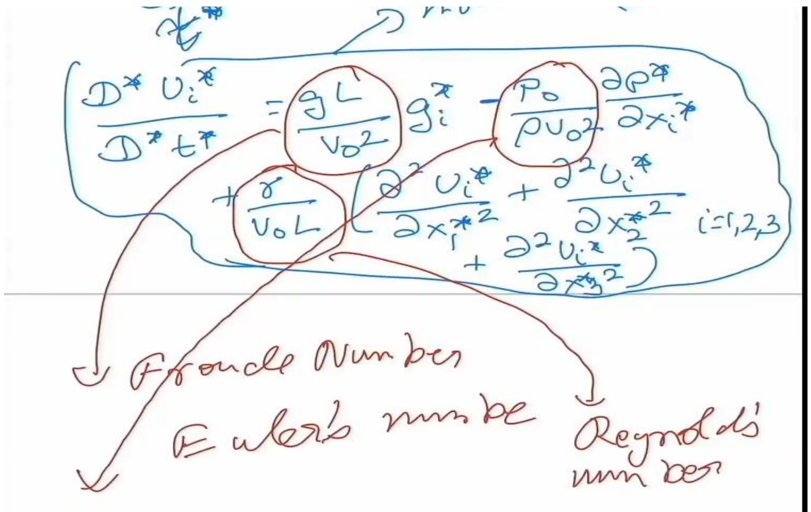

For example, if you have flow through a pipe, then this is the correct dimension of the velocity. This can be like the average velocity in the case of wall-bounded flow, or it can also be the free stream velocity in the case of, say, flow over a flat plate. In this case, for a flat plate, the dimension L is essentially the appropriate length scale. So you non-dimensionalize all distance and velocity components by their corresponding scales, and time is made non-dimensional by the reference, you know, by the time scale. So the time scale is, say, L divided by $V_0$.

And the pressure variation is normalized by the reference pressure value. The pressure is normalized by, say, $V_0$, which is the reference pressure. So this is like a time, a non-dimensional time, tau or T star, for example. So if you now denote the dimensionless variables using asterisks, one can express this Navier-Stokes equation like this:

$$\frac{D^* u_i}{D^* t} = \frac{g_L}{V_0^2} g_i^* + \frac{P_0}{\rho V_0^2} \frac{\partial P^*}{\partial x*_i} + \frac{v}{V_0 L} \nabla^2 u_i$$

Where I is equal to one, two, or three. Okay, so the dimensionless coefficients on the right-hand side have been identified as certain numbers. We will see what those numbers are. So this is like a dimensionless velocity Navier-Stokes equation. This is basically a momentum equation, right? So all these coefficients that you see on the three terms on the right-hand side, let me mark them. One is this; one is that, okay? And the other one is this: this particular term is called the fruit number; root number, this particular term that you see over here is called Euler's number. Euler's number, and this one is called the Reynolds number. Okay, so this is what you get after you non-dimensionalize everything; it pops out a few numbers. So, let us see the Reynolds number one by one. So the Reynolds number, look at the Reynolds number.

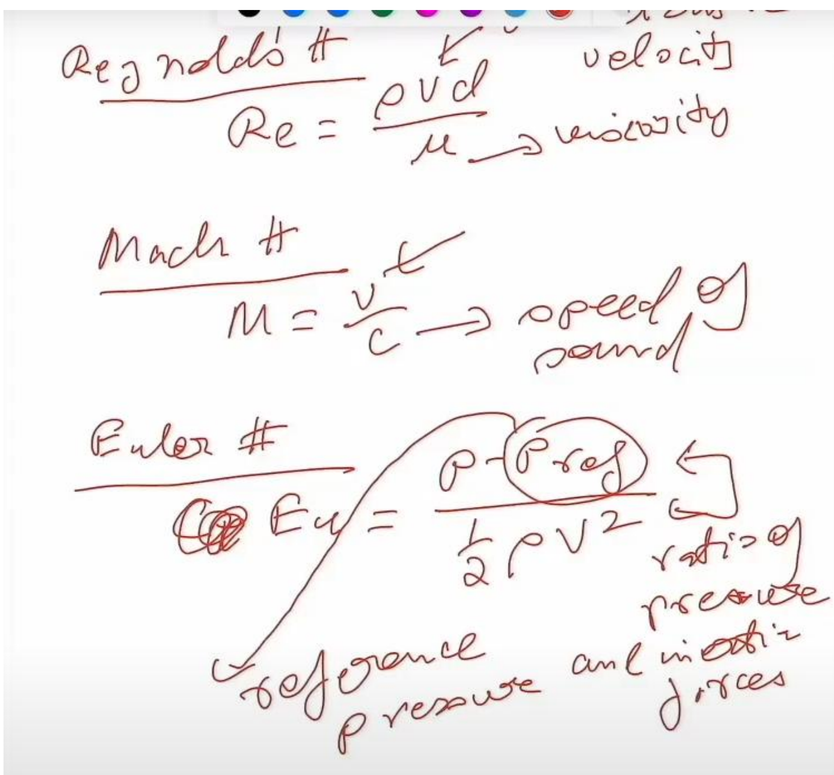
So the Reynolds number is basically ρVD/μ. V is the characteristic velocity, which in this case is $V_0$. You can define it in different ways. And mu is the viscosity. Yes, you all are not aware of this.




And of course, the other non-dimensional term will be the Mach number. So the Mach number is basically m, which is equal to v divided by c, where c is the speed of sound. What you see over here is actually the fluid velocity. It is the ratio of the inertia versus the elastic forces. Then you have the Euler number, which you can also call c_e.

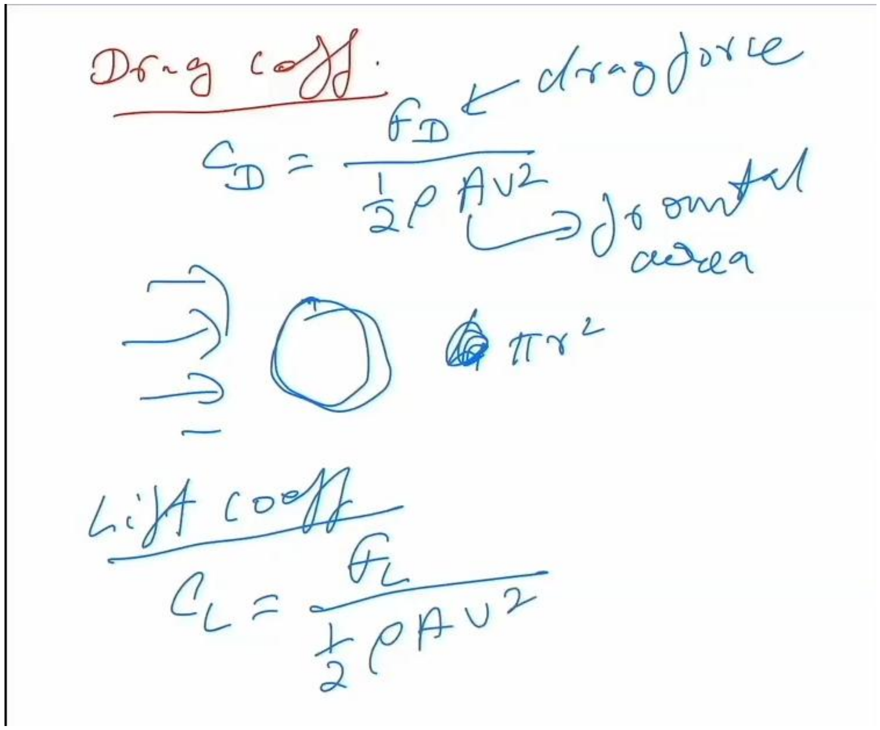
So, it is also, can you can call it basically let's call it Euler p minus p_ref. Divided by $\frac{1}{2}\rho V_0^2$ . P ref is basically a reference pressure. It represents the ratio of pressure to inertia forces. Ratio of pressure to inertia forces.

And P is basically a reference pressure. Okay, so for some of you, it can also be defined as half of the pressure coefficient. So you can also call it the pressure coefficient.




Then, of course, there is the drag coefficient. Okay, the drag coefficient. The drag coefficient, written as $C_D$, is effectively equal to $F_D$ divided by $\frac{1}{2}\rho A V_0^2$. So FD is the drag force. Okay, and A is the frontal area. So, for example, when you actually have a body and then there is a flow around it , it is that frontal area. So it basically sees if it's a sphere; it sees basically $\pi R^2$ of the sphere as a frontal area.

So that is the area that it actually sees. So you have these flows and then, of course, you have the lift coefficient. The lift coefficient, denoted as $C_L$, is equal to $F_L$, which is the lift force divided by $\frac{1}{2}\rho A V_0^2$ ; this represents the ratio of the lift force to the inertia force. So, this is the lift coefficient.

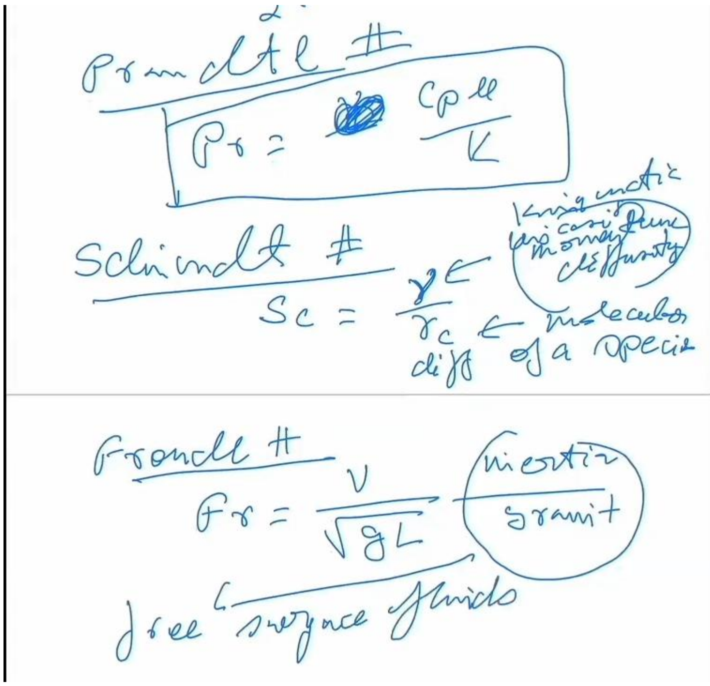



Then you have the Prandtl number, which surprisingly is not a flow property anymore; it is the ratio of two properties and is basically given as $\frac{\rho \nu C_p}{k}$ .

This is called the Prandtl Number. It is a ratio of the momentum diffusivity divided by the thermal diffusivity. But then you have something similar to the Prandtl number. Again, this is a number that is a ratio of two properties. It's called the Schmidt number.

The Schmidt number is a ratio, written as Sc, of this by Prandtl. C, which you see, is the molecular diffusivity of the species. And this one, the one that you see on top, is basically the momentum diffuser. This is the momentum and the kinematic viscosity. So that is what you get over there.

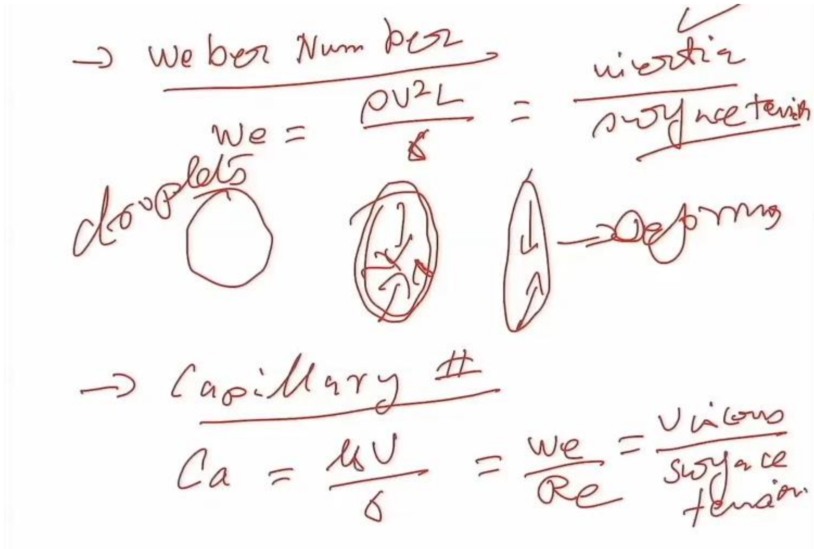
So this is the kinematic diffusivity or kinematic velocity. So, that is the Schmidt number. You have the Froude number. Froude Number. The food number is basically for you can write it in whatever way you want; it represents the ratio of inertia to gravitational forces, the inertia by gravitational forces.




Okay, so it basically is applied to free surface fluids. Then, of course, you have something which is called the Weber number. This is also used very routinely in droplet breakup, for example. The Weber number is $\rho V^2 L/\sigma$. So this is the ratio of inertia to surface tension.

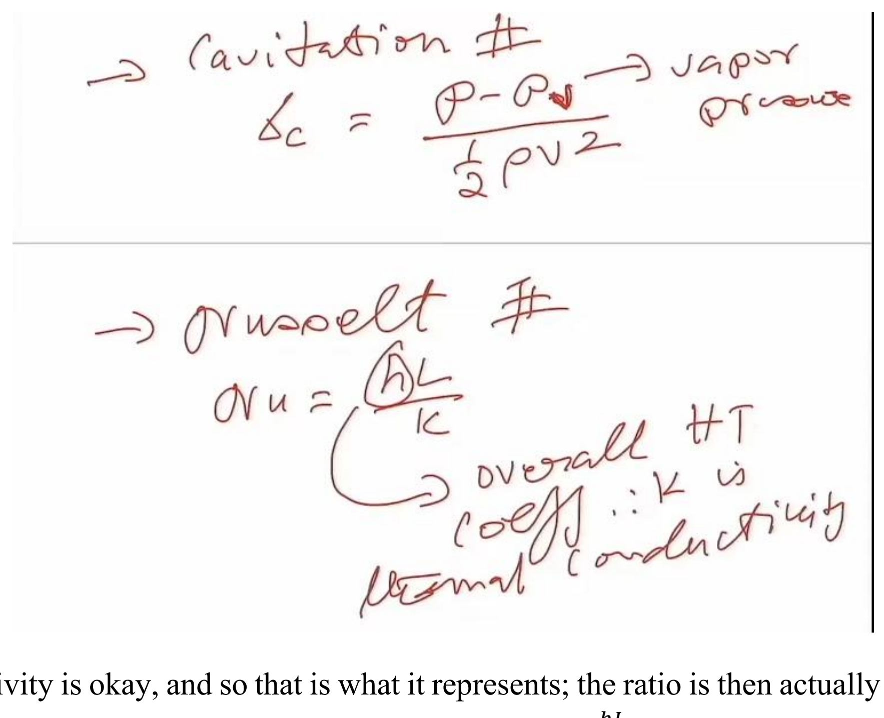
Surface tension. This actually tells you how the droplet, for example, a droplet like this, can become deformed, if it deforms. So that would mean that its inertia is high compared to that of surface tension. Surface tension tries to bring things back together. Inertia tries to deform it. So the Weber number is basically that.

Then, of course, you have the capillary number. So this is routinely used in the case of droplets, for example. The capillary number Ca is basically equal to $\frac{\rho v v}{\sigma}$, which is a ratio of the Weber number to the Reynolds number. This is the ratio of viscous forces to surface tension. So there is no inertial effect in that way.




Then you have the cavitation number. Okay, so what is the cavitation number? Let's call it sigma C. Sigma C is basically $\frac{P - P_v}{\frac{1}{2}\rho V_0^2}$. So $P_v$; sorry. This is basically the vapor pressure, okay? So, lastly, you have the Nusselt number, or the last few, basically.

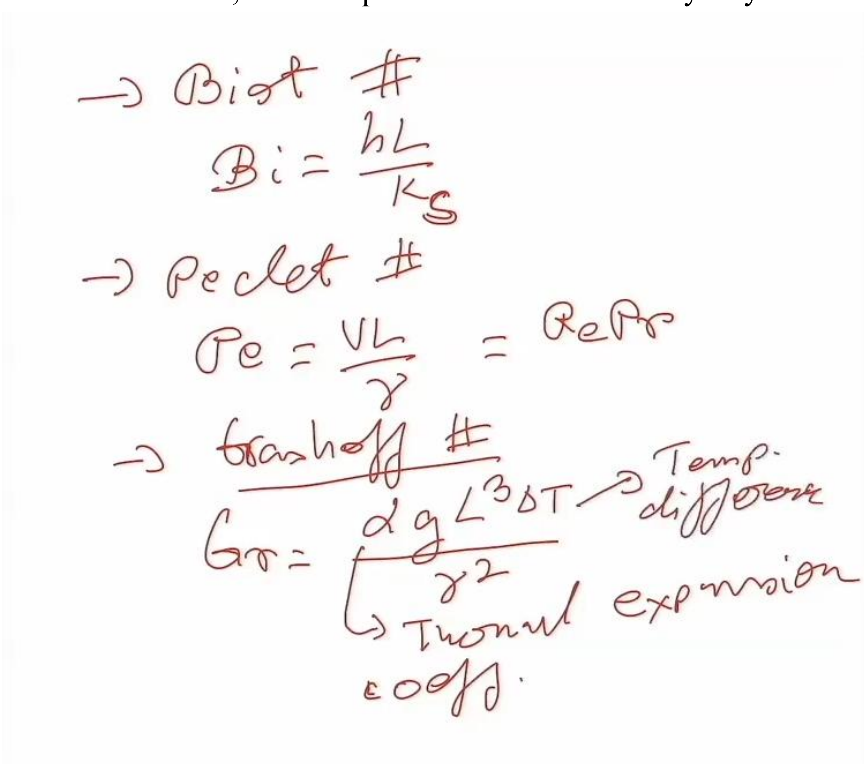
The Nusselt number is related to heat transfer. As you know, it is $\frac{hL}{k}$, where H is the overall heat transfer coefficient. K is the thermal conductivity.




Activity is okay, and so that is what it represents; the ratio is then actually the Biot number. The Biot number, which is given as Bi, is equal to $\frac{hL}{k}$.

The Biot number is h, which is the overall heat transfer coefficient from the solid surface to a fluid, and k is the thermal conductivity of the solid, so maybe we should put it as s. So it represents the ratio of heat transfer rates between the surrounding fluid and the solid interior. And whereas the Nusselt number basically represents the ratio of the total heat transfer rate to the conductive heat transfer rate in a fluid. These are very different things. Then you have something called a Péclet number.

You see that the large gamut of numbers that you see, the Peclet number is basically VL divided by gamma, which is the kinematic viscosity, which is the Reynolds number multiplied by the Prandtl number. It represents the ratio of heat convection to conduction. Then you have the Grashof number. Grashof number. The Grashof number is represented as Gr, which is $\frac{\alpha G\, l^3 \Delta T}{\gamma^2}$; so this $\alpha$ is basically the thermal expansion coefficient, $\Delta T$ is the temperature difference, and it represents the ratio of buoyancy forces to viscous forces.

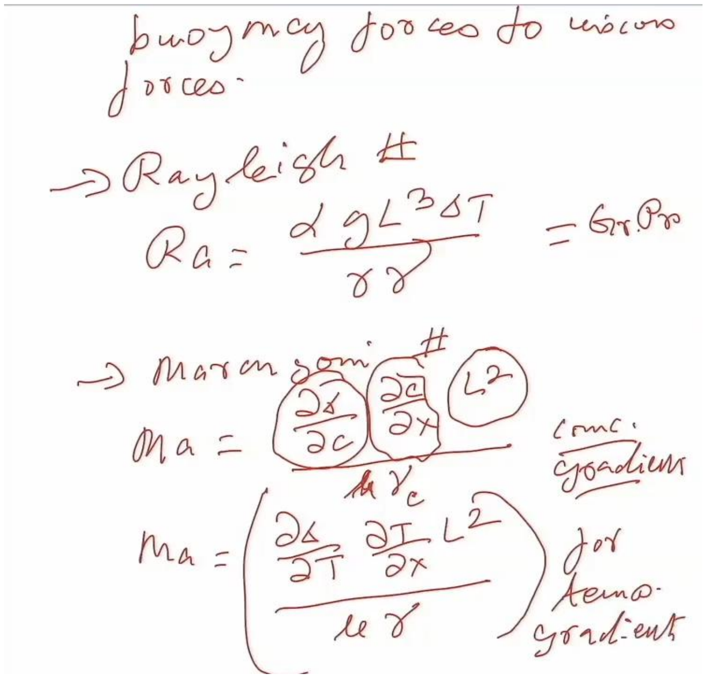



So it's a ratio of buoyancy forces to viscous forces. All right, so then you have the Rayleigh number also. Relay number, which basically looks pretty similar to a Grashof number, is l cubed into delta t by gamma, and the thermal, so it is gamma by that; it is basically Grashof into Prandtl. Then you have the Marangoni number. So the Marangoni number is two Marangoni numbers.

If you write it as d sigma by dc, you can see it is the, and then dc by dx into L squared.

And then you divide it by μ. So this is for concentration gradients. Can see what it is; it is the rate of change of surface tension with composition and the composition gradient across the length, and this is multiplied by the characteristic length scale. Okay, so this is for concentration gradients, and then you have the Marangoni number, which is very important, for example, in droplets and in other flows.

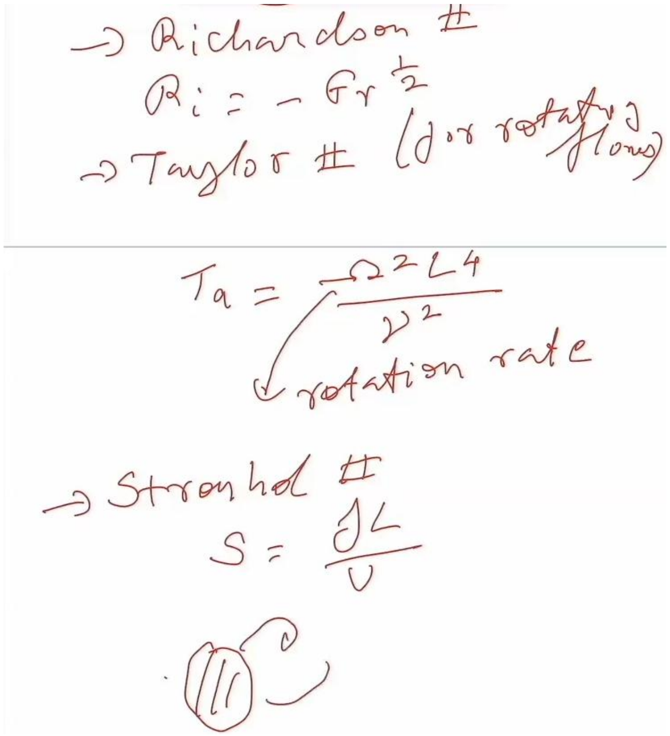
Also, this is, for example, with respect to temperature. mu gamma. So this is now divided by the thermal diffusivity.



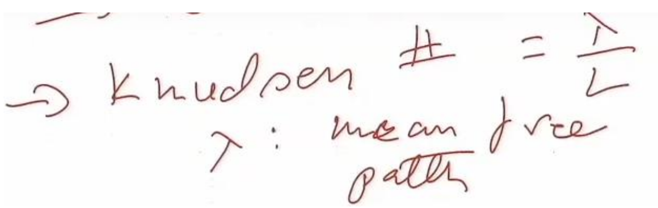

This is for temperature gradients. All right. So then you actually have something that is called the Richardson number. The Richardson number, which is basically Ri, is equal to minus half of the fluid number. It represents the ratio of potential energy associated with gravity to kinetic energy. So, that is the Richardson number.

Then you have the Taylor number. Taylor numbers for rotating flows. And this is also very common: the rotating flows, because fluid dynamics is something like that. So this is the Taylor number, which is given as gamma squared, omega squared, rather, the square of that. So, this is basically what is called the rotation rate.

Similarly, you have other numbers. For example, let's talk about a true whole number. Stuhl number is like the shedding of vortices behind bluff bodies. So, like this, if you have a bluff body here, behind the, as the vortices are shed, this frequency of shedding is what comes here, and this is the characteristic length, this L, and this is the corresponding velocity, V. So this is the Stuhl number.




You can also have something called a Knudsen number. The maximum number, which is basically represented as $\frac{\lambda}{L}$, where lambda is basically the mean free path, and L is again the characteristic length. So, you have a lot of numbers like this. When you actually use these numbers for non-dimensionalization, they are accepted numbers, so to speak. So this has its own utility.

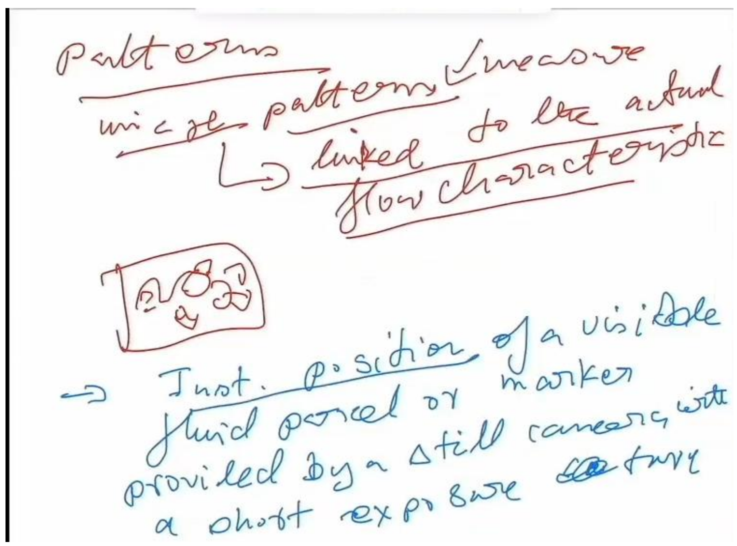



So apart from this, the fluid has many patterns as well.

Okay, flow visualization and measurement methods provide images or records of the fluid or the transported mixtures, and they give you information on the flow. To correctly interpret the observed patterns, it is necessary to understand their relationship with the actual flow characteristics. For example, we imagine patterns, and we imagine patterns. How these patterns are linked to the flow characteristics is important because we measure patterns. You can see, for example, that you get all kinds of flow velocities and vector fields around it.

How is it linked to the actual flow characteristics? There are ways, for example, to take an instantaneous position of a fluid particle or marker provided by a still camera with a short exposure time. So you can determine the instantaneous position of the visible. This can be introduced by dye or tracers, the fluid parcel, or marker. Marker provided by a still camera.

A camera with a short exposure time. What is the exposure time? We will learn later. Exposure time, but assume that this is a short exposure time. You know what you are trying to do. And then, so this is the instantaneous position, for example.

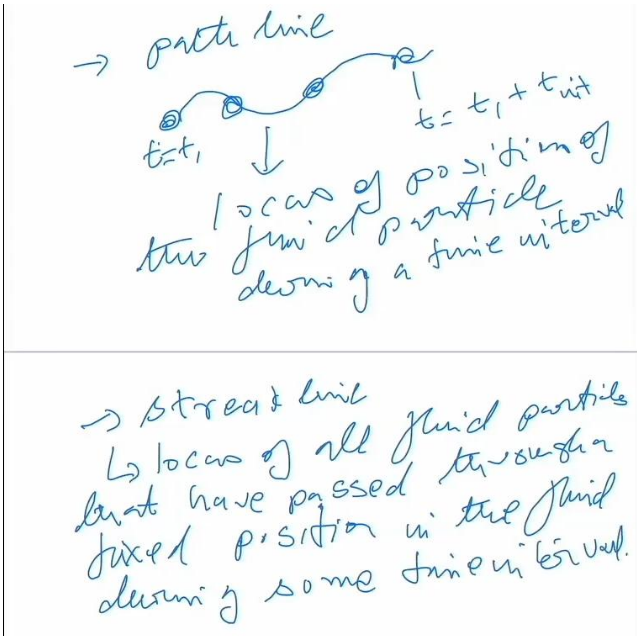



Similarly, you can determine something like a pathline, which will be the locus of the positions of the fluid particles during a time interval.

This is a fluid particle that is moving around. So this is at t equal to $t_1$. This is at t equal to t1 plus some interval t. So this is the locus of a fluid, the locus of the positions of the fluid particles. Locus of the positions of the fluid particles during a time interval.

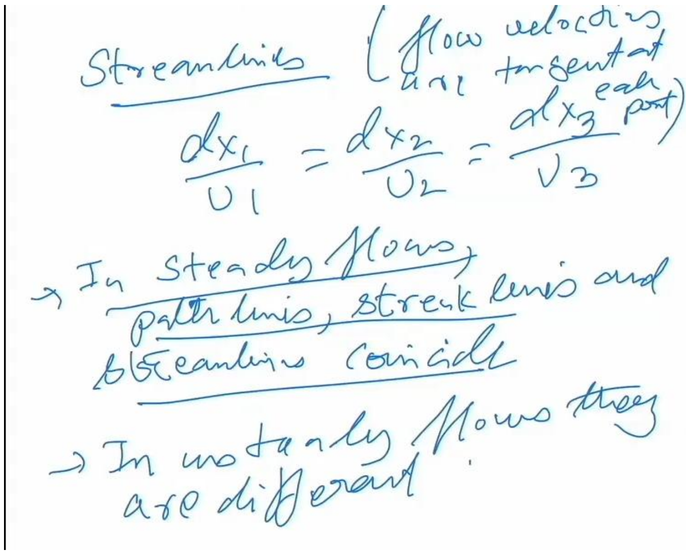
During a time interval. Then you have something called a streakline. What is a streakline? The streak line is the locus of all fluid particles that have passed through a fixed position in the fluid during some time interval. So it is basically the locus of all fluid particles, fluid particles that have passed through a fixed position in the fluid during some time interval. So all these are processes that you can measure.

So this is called a streak line. And then you, of course, have a timeline, namely the locus at a given point of all the fluid particles. By combining the preceding patterns, the time streak lines, or photographic exposures, you can determine many things. Then it is also important to understand that all the preceding patterns are distinct.



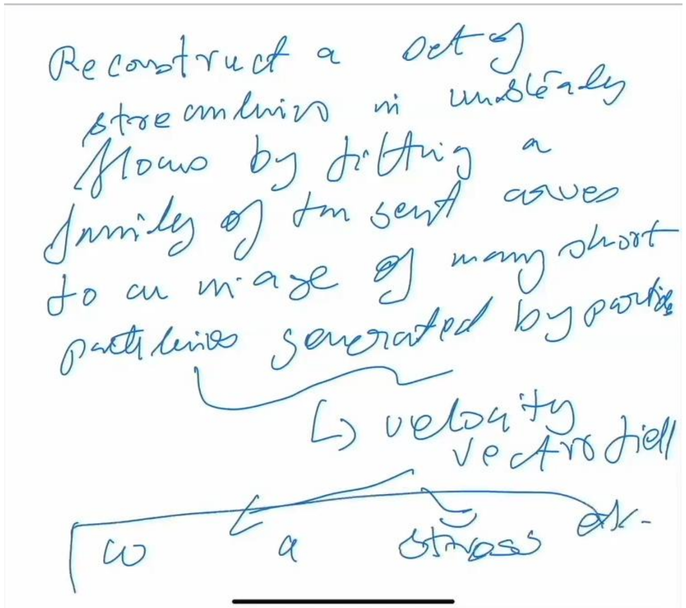

From the instantaneous streamlines, namely lines in the fluid that are tangent at all points to their local velocity vector, if you know that the streamlines are given by something like this one: $\frac{dx_1}{u_1} = \frac{dx_2}{u_2} = \frac{dx_3}{u_3}$, please consult your fluids notes.

In steady flows, in steady flows, steady flows. You know, pathlines, streaklines, and streamlines coincide. This is also something that you probably already know. But in unsteady flow, they may be different. Flows are different. The streamlines where the flow velocities are tangent at each point, okay, so you see, in stationary or turbulent flows of relatively low intensity, these lines may approximately coincide on average, but their instantaneous features are different.




So one may reconstruct a set of streamlines in unsteady flows by fitting a family of tangent curves to an image of many short pathlines generated by adjacent particles and thus representing the vector velocity field. So, this is interesting. So you can say you can reconstruct the streamlines; to reconstruct a set of streamlines in unsteady flows or steady flows, you can fit a family of tangent curves to an image of many short pathlines generated by particles, and therefore you can generate the velocity vector field. Because our aim is to form the velocity vector field, this is what will then be used for calculating vorticity, acceleration, stress, etc. So this is the most important thing: you need the velocity vector field, and you can obtain it from the images in a variety of ways.

Varieties of ways you can generate these kinds of images, and that is the most important part. So this is what a very short description of fluid mechanics is all about, where we have considered all kinds of flows, the key features of the flow, how to non-dimensionalize a particular quantity, and how to non-dimensionalize an equation. And come up with the predominant scales. And how, if you get an image and measured data, can you generate these non-dimensional numbers and gain more insights into the flow field? If you are talking about velocity, that is essential for a whole lot of derivative quantities.




Similarly, you want a temperature that can now be utilized for a variety of things, including calculating the rate of heat transfer. Okay, so we will end the lecture here.