

Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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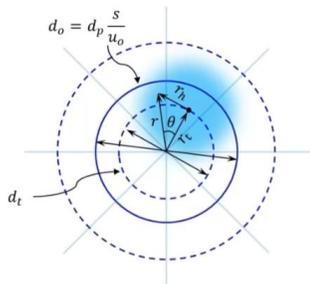
Week – 06

Lecture - 29

Depth from Defocus – 3

Hello, everyone, and welcome back. So in the last few lectures, we have covered what we mean by blurring in the first place. How can it be deduced from geometric optics? Like, how can we deduce the extent of blurring from simple geometric optics? How can we then define the blur circle and how light is redistributed within the blur circle using point spread functions? And then how point spread functions can be implemented in systems to emulate blurring in imaging systems by an operation that is known as convolution. So that is what we have done so far. So we ended the last lecture on the note of discussing this application to the blurring of a particle image. So, we will start from there in this lecture.

Convolution Integral Equation – Particle image



The blurred image is estimated by convolving the focused image of a particle of size d_o with a Gaussian blur kernel (shown as a *shaded circle*). The intensity (g_t) at each location (r_t) is evaluated by convoluting the focused image with the point spread function as:

$$g_t = i_f * h$$

Here $i_f(r)$ represents focused image and $h(r)$ is a Gaussian blur kernel

For a circular entity of diameter $2r_0$

$$i_f(r) = \begin{cases} 0 & r > r_0 \\ 1 & 0 < r < r_0 \end{cases}$$

In integral form

$$g_t = \iint i_f \cdot h \cdot dA = \int_0^{2\pi} \int_0^{\infty} i_f \cdot h \cdot r dr d\theta$$

Rao et. al, EXIF, 2024

So, with all the fundamentals that we have learned, we will now apply them to the problem of blurred image formation, especially related to dispersions or particles. So here we consider an axisymmetric, round particle that looks like a circle in the projected image, represented by the solid blue line here. So this is our focused image of the particle, and it is blurred using a Gaussian blur kernel. We operate this kernel in a convolution to generate the blurred image, which is represented by this equation.

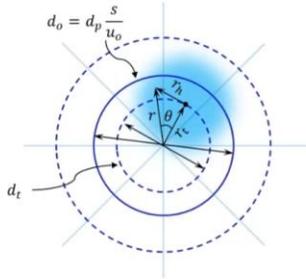
If you use your focused image, h is your blur kernel and g is your intensity map in the blurred image. We can define the focused image in a mathematical form like this. Since it is a circular particle, it is easier to choose a polar coordinate system. And in that system, when the radius is less than the radius of the particle size, which is R_0 , we assign a value of 1. And for the rest of the image, we assign a value of 0, which illustrates the background.

And as you can see, we are using 0s and 1s and all the other intensities; even in the blurred image, they will lie within it. So, we are normalizing the intensity in the range of 0 to 1, where 1 represents a shadow and 0 represents a background. And this convolution equation, when written in the form of, say, an integral equation, will look like this. So earlier when I showed you the convolution operation, I denoted it in a discrete fashion because usually the images that are captured in any system consist of π xels. So it is nothing but a matrix.

So every image is kind of a matrix, and here we are doing some analytical exercises to determine certain properties of the system, so we will stick to continuous modes. Here you can see we can now define the intensity of the blurred image in the form of this double integral, where if you use your focused image, h , it is the weight assigned. From the Gaussian blur kernel, this is integrated over an area that is reminiscent of a blur circle, or in this case, the region associated with the Gaussian blur kernel is locally spread out. That is why we use elemental area to integrate it, but we will do this for the whole space of the image because the Gaussian kernel is a smooth kernel without any sharp boundaries. So, it asymptotically approaches 0 as r tends to infinity.

So, here we have a bound of $0 \rightarrow$ infinity in the radial direction, and this is because we are considering the entire space in the angular direction, which will be from $0 \rightarrow 2\pi$. So this DA can be represented as DR into $RD\theta$, and you can just say this is just a standard way of doing it. So with this integral equation, I just want to emphasize a few things because you are looking at a lot of parameters like R , RH , RT , and it can be quite confusing. Uh, because we chose a polar coordinate system, it is better to have an $r-\theta$ kind of configuration fixed at the center of the particle, okay? Because it is easier to define a particle that way. But if you want to estimate the intensity in the blurred image at a location, say, r_t radial location r_t , then you have to perform the integral with this center, right? It's, uh.

Convolution Integral Equation – Particle image



$$g_t = \iint i_f \cdot h \cdot dA = \int_0^{2\pi} \int_0^{\infty} i_f \cdot h \cdot r dr d\theta$$

(r_t, g_t) : spatial distribution of intensity $g_t(r_t) = \int_0^{2\pi} \int_0^{\infty} i_f(r) \cdot h(r; r_t) \cdot r dr d\theta$
 (r, θ) : dummy variables for integration

The gaussian blur kernel

$$h(r_h) = \frac{1}{2\pi\sigma^2} e^{-\frac{r_h^2}{2\sigma^2}}, \quad r_h^2 = r^2 + r_t^2 - 2rr_t \cos\theta$$

As $i_f(r > d_o/2) = 0$, we can modify the integral limits

$$g_t = \int_0^{2\pi} \int_0^{\frac{d_o}{2}} \frac{1}{2\pi\sigma^2} e^{-\frac{(r^2+r_t^2-2rr_t \cos\theta)}{2\sigma^2}} r dr d\theta$$

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.. In the same way that we did with the matrices, wherever you want to evaluate the intensity in the final image, you place the matrix at that location, centered at that location, and do the weighted sum and weighted average to estimate the value at that location. So here, this blue blurred entity represents the Gaussian blur kernel, which is centered around this point where we want to estimate the intensity. But to integrate it, we have to do it in the local coordinate system with r_h as the radial coordinate and some θ . However, to do that, it becomes very tedious, so what we do is convert everything in terms of r and θ and use them as dummy variables for integration. So it may sound.

Slightly complicated whatever I'm trying to say, but just to put it in simple terms: r and θ are the dummy variables of integration, and they will go away. r versus G_t will give you the spatial distribution of intensity, where r_t is your spatial radius and G_t is the intensity at that radial location. Okay, and because of axis symmetry, there is no θ dependence here, and to specify what parameters depend on what. As I have said, G_t is only a function of r_t , or intensity is only a function of radial position r_t . If the focused image is a function of r only because, in that coordinate system, it is the easiest to define, and H , your blur kernel, should also be defined in R because, as I told you, we have to convert it in terms of R so that we can integrate it.

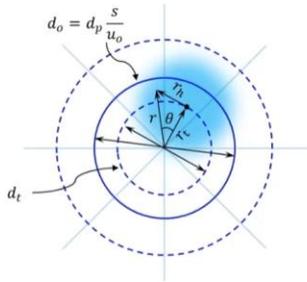
But at a given r_t , because that r_t dependent should stay here, right? Because then only both sides of the equations are functions of r_t , and this just depicts that r and θ are your integral parameters or just dummy variables for integration. Now, coming back to the blur kernel, we know this form of Gaussian blur kernel, but you can see here we have used r_h . To define this kernel correctly because the kernel is centered around this point where your local radial coordinate is r_h . So that's why r_h comes up here, and from simple

geometry, you can show that RH is dependent on R, r, and θ in this form. So it is fairly simple to solve; you can try that.

Also, one more parameter that we have is that we know what the functional form of this is. We know that when r is greater than the radius of the particle, this function goes to 0. So, this modifies the limit of the integral. Now, we do not need to solve the integral beyond $d_0/2$. So, we do not need to solve it from 0 \rightarrow infinity.

We just need to solve it from 0 \rightarrow d_0 in increments of 2. and substituting everything and also in that range this if is just one so if just disappears here because it is always one there and this is your blur kernel and this is your d_a and these are your new limits and this is now the integral equation which is of which represents the blurring of a circular particle this is what We use a specific technique called single camera depth from defocus. So, we will go there shortly, but we can just numerically solve this to estimate the blur distribution, and that is what we are going to do next to get some intuition about how the images look at different extents of blurring. Apart from this equation correlating with the various geometrical parameters, such as the dependence of intensity on the radial position, we have an intrinsic parameter, σ , which represents the extent of blur. We have already seen that this extent of blur, or deep focus, is dependent on depth, and we have used geometric optics to arrive at this equation that relates both of them.

Convolution Integral Equation – Particle image



$$g_t = \int_0^{2\pi} \int_0^{d_0/2} \frac{1}{2\pi\sigma^2} e^{-\frac{(r^2+r_0^2-2rr_0\cos\theta)}{2\sigma^2}} r dr d\theta$$

$$\sigma \propto \frac{C}{2} \Rightarrow \sigma = \frac{AC}{2}$$

The blur kernel size (or standard deviation) σ is then given as:

$$\sigma = \frac{ADM}{2f} |\Delta z|$$

- A = imaging constant
- D = imaging aperture diameter
- f = focal length
- M = Magnification

Correlating blur kernel (**Defocus**) with the **Depth** of the particle from plane of focus

Requisite for **Depth from Defocus**

So this big term that you see in front of it, if you don't change the aperture of your system or other parameters associated with your experimental system, will remain a constant, and it shows us that this σ is directly proportional to your mod Δz . And by $|\Delta z|$, it is also very critical to visualize or understand this mod sign. It tells us that blurring is symmetric across the object plane, so if you move away from the object plane, there

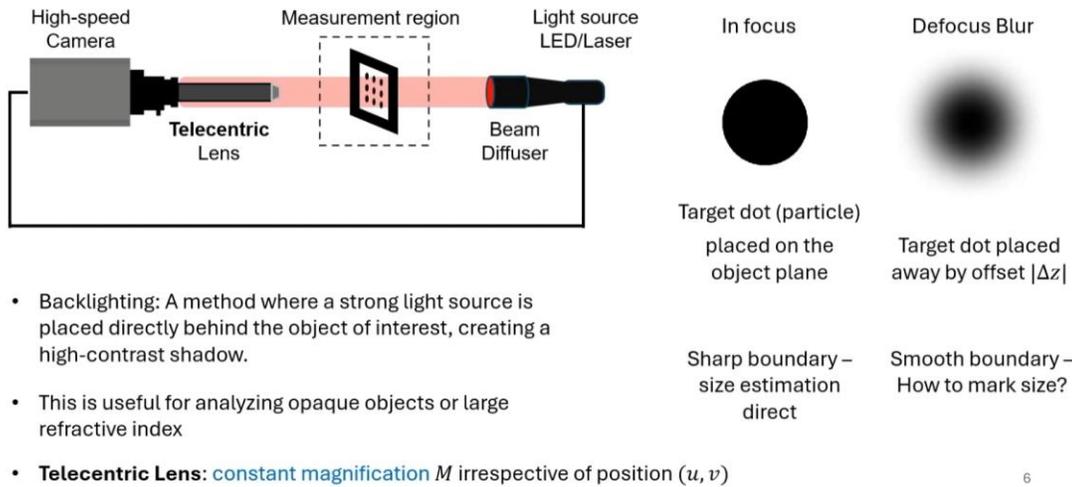
are two directions in which you can move: you can move either towards the camera or away from the camera. If you move by the same amount in either direction, the blurred image should look exactly the same under this assumption. The way we assume things to be, it comes out to be like that.

In a certain specific experimental setup, we can achieve this, and that is what makes this system very interesting. Although it is still an idealized system, in practical systems it is not always true. But yeah, this mod sign tells you that your blurring is symmetric in the z direction if you go either towards the camera or away from the camera. You will get the same amount of blurring if you shift by the same amount in the depth direction, and this equation is the one that relates depth with defocus and is the motivation for the development of the technique depth from defocus. Now, coming to this defocus blurring.

Like the model that we have used, we will use it to generate some synthetic images. So, we just took a circle like this; as I have told you, we assign an intensity of 1 to the particle and an intensity of 0 to the background, and this is the focused image where $\sigma = 0$. And if we apply the equations that we solved earlier at each pixel location and numerically solve them using certain discretization, we can get these blurred images for different extents of blurring. And this was done, as we have seen, using a Gaussian blur kernel. So here we write it in terms of the fraction of particle diameter because even if we change the particle size, it will have that geometrical similarity and it will look the same.

So if we blur it using 10% of the particle diameter, it will look something like this, and if you blur it using 50% of the particle diameter, it will look extremely blurred, right? And you see that even the intensity goes to a lower value because this is still kind of a moving average operation with certain weights assigned to it. And one more interesting or important thing to note is that here you have a sharp interface, so you can clearly demarcate the boundary, but here there is no boundary. Visually, you can still say, "Okay, the particle is somewhere here," but to estimate it quantitatively, it's still a challenge. So the images that I've shown you in the last slides actually correlate pretty well with the experiments. When I say experiment, what kind of imaging system am I talking about? For dispersion size measurement using imaging, shadowgraphy or shadow imaging is the most suitable candidate, where what you do is use backlighting.

Shadow Imaging – Defocus Blur



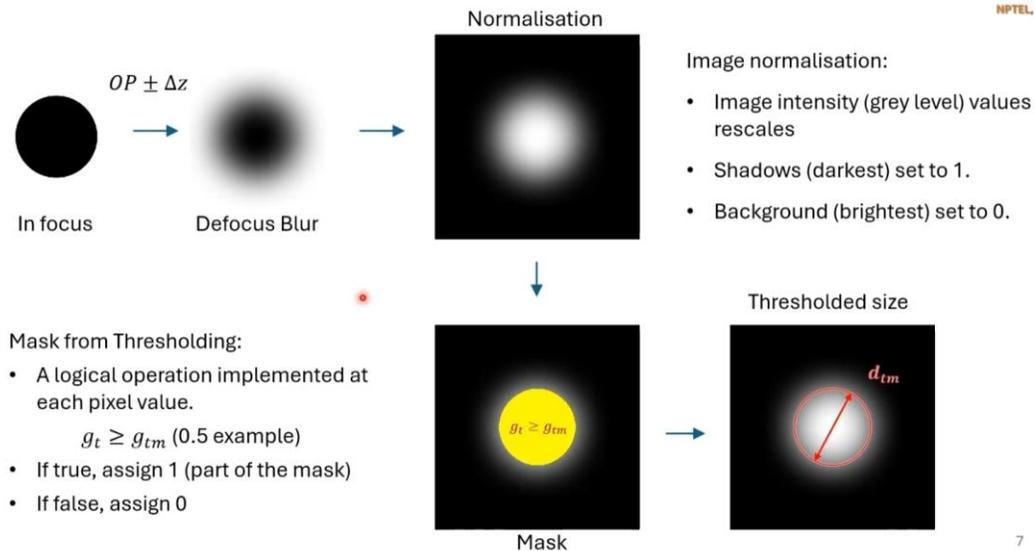
So, what do we mean by backlighting? It means a strong light source placed directly behind your object of interest. Which creates extremely strong shadows, so it is represented here in this diagram: you have a light source, you have the object, it is in line with the camera, and the light source is placed behind the object, so it forms a shadow that is captured by your high-speed camera. Your background is usually bright and has small features as shadows, so if you have a droplet, it will look something like this if it is in focus, where the background is bright and the droplet is visualized as a shadow. But if it is defocused or if the particle is away from the object plane, it looks blurred; so here you have sharp boundaries, and it is very easy to estimate size because you can directly take the diameter, right? Or you can estimate the area and use the formula $\frac{\sqrt{A}}{\pi}$, something like that, to get an equivalent radius. But here you don't have any of the boundaries to detect the size, right? So the boundary is extremely smooth.

How will you mark size here? That becomes a challenge. Also, you may have a question: if we have droplets, say of water, water is transparent, right? So how will the shadows look? Then these systems are useful if there is a large refractive index contrast between the particle's matter and the surrounding matter, which is mostly air, or it can be anything else as well. There can be a lot of different kinds of dispersions. So, if your refractive index contrast is pretty high, then that still leads to the formation of these sharp shadows, and yeah, it is because of something that is known as the ball lens effect and things like that. You can look it up, but it will look like a shadow, that is for sure.

Now that we have seen how shadow imaging is done and how particles look in those images. Say we have a particle in focus here, and we shift the particle from the object plane by a certain distance, Δz , and it becomes defocused. We have seen that there is no

boundary associated with it to determine its size. So now we will develop an approach known as the thresholding approach, where we can determine an effective size for the particle that might not be the true size of the particle. So, before going to that procedure, we normalize the image, as I have discussed earlier as well.

Thresholding approach



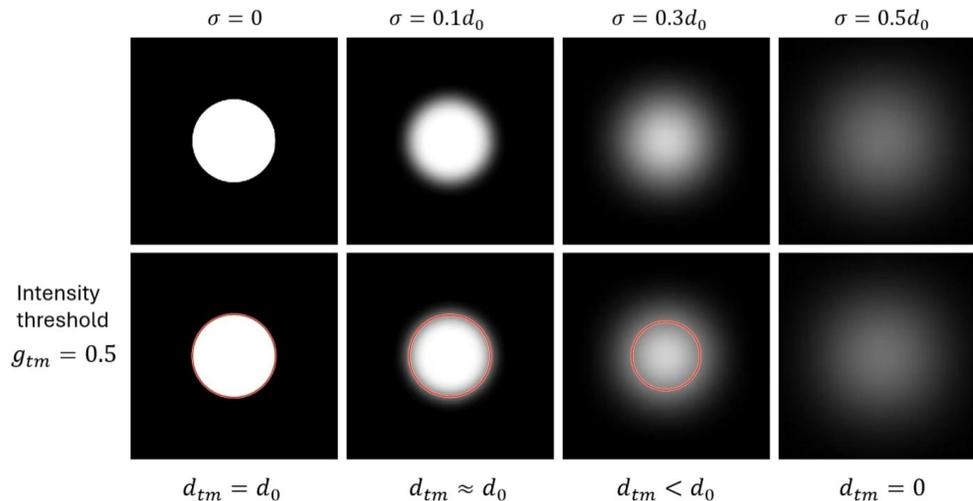
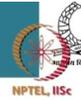
We rescale the image from 0 -> 1, where 0 values are assigned to the background. In the shadow imaging system, the background is usually bright. So the brightest points are set to zero. In shadow imaging, the darkest points are associated with the particle. So, we will assign a value of 1 there.

So, this is the operation that we will do for normalization, and these image intensity values are also known as gray level intensity because these shadows are usually not wanted; we only want the intensity values. So, we collect only a monochromatic signal, and that's why we call it gray level values, because it's just black and white or grayscale. The next step is thresholding. What do we mean by that? It is a logical operation where we assess if the logic is true or false. So we define a logic in a way that you choose a threshold value, say 0.5, and at each pixel location, you determine whether the intensity at that location is greater than 0.5 or not. If yes, assign a value of 1. If not, you assign a value of 0.

All the ones together form a mass. So that is the idea: here in this normalized image, at each pixel location, we see if this logic is valid or not, and all the collective points where this logic is valid look like this yellow blob. In this yellow blob, the intensity was greater than the threshold value, and that is something that is known as a mask. Okay, and we can now. See that this mask has a distinct boundary so we can determine its size; here we can determine its size, and that is something known as a thresholded size. Clearly, it will

be close to the particle size but not the same.

Thresholding approach – Different Blur



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We will see how. So here we have taken particles with different extents of blurring, as I showed you earlier, and we chose a threshold value of 0.5 to see what is being detected. So, for the same particle, these are the images for the same particle but with a different extent of blurring. So the first one where the blurring is 0, the thresholded size is the same as the particle size, right? Because there is no blurring. But in the case where there is some blurring, say 10% of the particle size, by the extent of 10% of the particle size, we see that you can still see the particle image is not that blurred and that this thresholded size is roughly the same as the particle size.

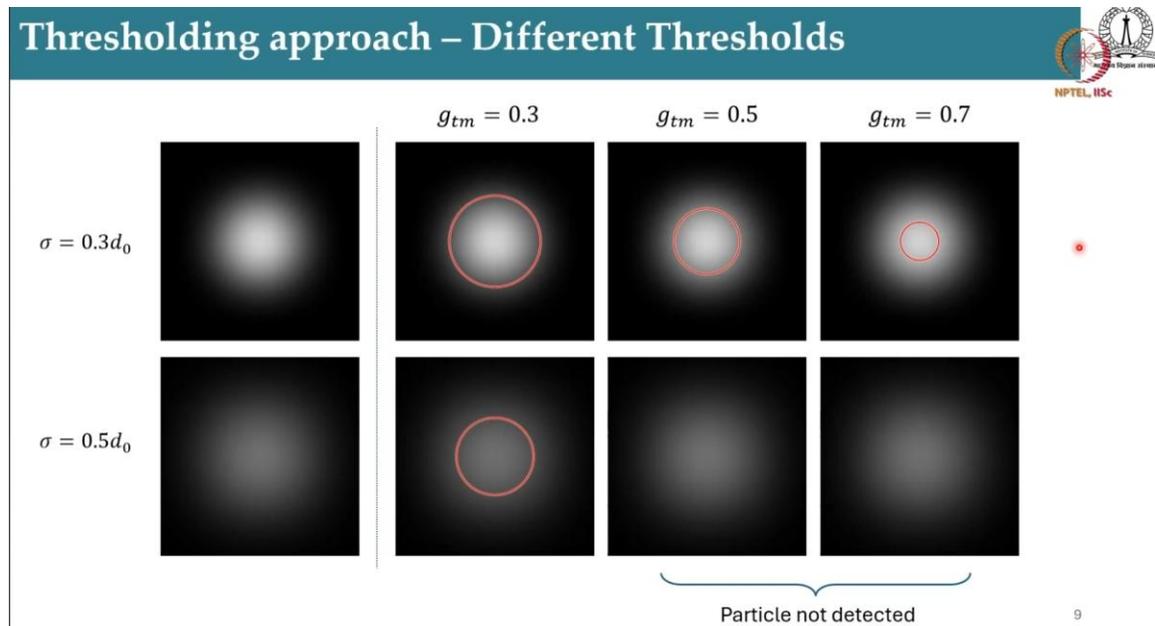
But now, in this case, where the particle is slightly more blurred, you can see it does not resemble the actual particle. Here we can see that the thresholded size is smaller. It's now smaller than the actual particle sizes. Why is this happening? Because the convolution using a Gaussian blur kernel is similar to a weighted moving average and it smooths out features. So, at each and every point the intensity goes to much lower value than the initial one.

If your blur kernel strength and size are very large, that is what is happening; there is now a smaller region where this logic is valid. If you go to even higher blurrings, like 0.5 times the particle diameter, which is an extremely high blurring, you can barely see the particle; here it looks like just a rough patch in the image. Here, there are no pixel values where your intensity is 0.

5. So no particle is being detected here; you will detect nothing using this threshold. This tells us something very important: that thresholding does not give you the exact particle

size. Depending on the extent of blurring, it will deviate a lot, and it will always be less than your original particle size. Secondly, there is a particular extent of blurring beyond which the thresholding will give you nothing.

So there is no particle detected. So it basically tells you that σ is related to depth. So if your particle goes beyond a certain depth, it will not be detected, and that marks your detection depth. So we will discuss that more later. The second point is the choice of the threshold.



Say you have this particle that is blurred by 0.3 times the particle diameter. If you choose a threshold of 0.3, it looks like this. If you choose a higher value, it will look smaller. And if you choose an even higher value, it looks even smaller.

Because high intensities are still preserved at the center, it smooths out as we go out and decreases smoothly as we go radially outwards, right? So here, you can see that the choice of threshold also plays a critical role. So, the effective size definitely cannot be directly predicted using a threshold. Particularly in this case, where the blurring is very high, you can see that, as we have seen earlier, it was not getting detected for 0.5. It is still being detected for 0.3. But if you choose a higher threshold value, it will not be detected. So the event detection depth will be dependent on the threshold you choose. So both the apparent size and detection depth, as well as all these things, are not universal and cannot be relied on. Although they will give you some idea associated with the size, it will not tell you the exact size. Now, talking about depth from defocus in the context of particles, it is a two-parameter system.

Because the particle itself can be defined using two parameters: the size of the particle

and the position of the particle, which is something we want to determine, and all we have is a blurred image of the particle, so in that blurred image we have to somehow quantify the defocus and then correlate that defocus with the depth. And that's how the depth from defocus technique can be applied to this two-parameter system. Before going into that, let us understand how the blurring increases for particles. We have seen that if the particle is moved away from the plane of focus or the object plane, as depicted here, you will see that it will become more and more blurred.

Depth from Defocus (DFD) - Two parameter system

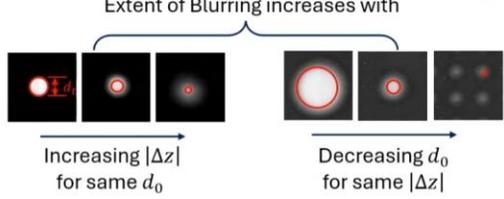


- Estimates depth by analyzing the amount of defocus blur in the image.
- Relates depth to blur radius and camera parameters.
- For any particle under consideration, it can be defined using

Size: d_0
 Position: $|\Delta z|$
- We can only estimate apparent size d_t from the image using a threshold.
- for fixed intensity threshold

$d_t = \text{fn}(|\Delta z|, d_0)$

Extent of Blurring increases with



Increasing $|\Delta z|$ for same d_0

Decreasing d_0 for same $|\Delta z|$

- Equation (1)
- Knowns (only 1):
 - d_t from the experimental image
- Unknowns (3):
 - Size: d_0
 - Position: $|\Delta z|$
 - Functional form fn
- Insufficient information to solve the problem

Right? So the extent of blurring increases. With the increase in the extent of blurring, the thresholded diameter decreases, which is also something we have seen in the last few slides. Apart from that, the size of the particle is also important. So, if you have all the particles tagged in the same plane, they are all on the same plane, which is away from your object plane. So basically all the particles are moved away by the same amount. But you will see that the larger particles will be blurred less, and the smaller particles will be blurred more.

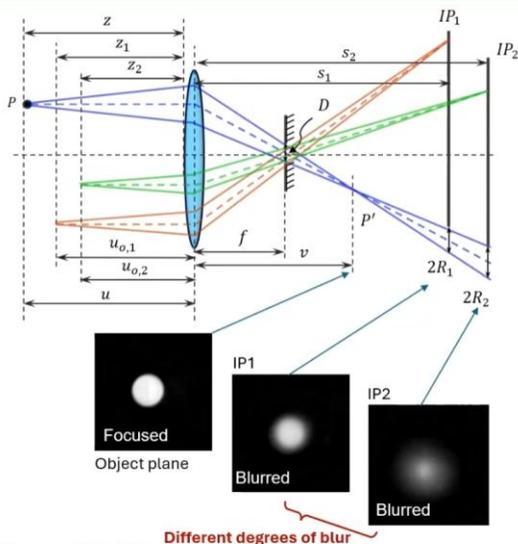
So, for the same Δz , decreasing d_0 also increases the extent of blurring. So there are two kinds of independent self-contradicting parameters that show that blurring can be controlled either by depth or by the actual size of the particle. So, that is why these two parameters are extremely important in the system, not just because we want to know it, but also because they affect the amount of blurring. So, in this way, for a fixed intensity threshold, say 0.5, this thresholded size is a function of depth and particle diameter.

Some implicit function can be linear or non-linear; we do not know for now. And if we want to see this as a system of equations that we are trying to solve, we only know there is this one equation or functional equation that relates these three parameters. But for

now, we don't know the functional form of it. Apart from that, we also know the particle size and depth.

All we have is DT or thresholded size. So we have only one known parameter and one equation that we know exists, but we don't know its functional form. The things that we want to determine from it, or the unknowns, are the particle size and the depth, so we have insufficient information or constraints to solve this problem at this stage. The next obvious step is to generate more equations and collect more information; in that way, you can determine these two parameters: the depth and the size. One approach to doing this is using two cameras. And we will still apply the thresholding approach, but we will capture two simultaneous images of the same particle with a different extent of blurring.

Two Camera DFD – Thresholding approach



- We use two cameras (1 & 2) with different path lengths after the lens (s) to simultaneously capture the same particle
- We can estimate two apparent size d_{t1} and d_{t2} from two images.

$$d_{t1} = fn_1(\Delta z_1, d_0)$$

$$d_{t2} = fn_2(\Delta z_2, d_0)$$

- Equation (2)
- Knowns (2):
 - d_t from the two simultaneous images
- Unknowns (4):
 - Size: d_0
 - Position: z
 - Functional form fn_1 and fn_2
- Still insufficient information to solve the problem

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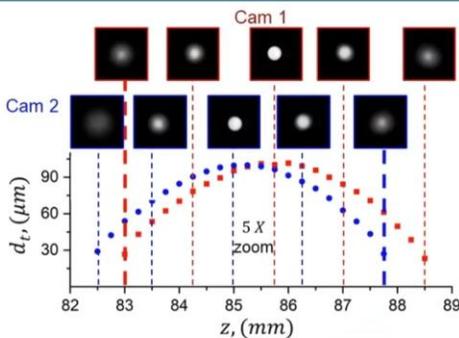
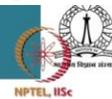
How do we do that? For that, we choose the imaging system in the same way that we have shown earlier, the only difference being that there are two cameras. So we put the two cameras in a way that the imaging planes are offset by some Δ value so they are not aligned; for the same particle, two different images will be formed. This is the particle p on the imaging plane one; you see it is blurred by some extent to r_1 , and in the imaging plane two, it is blurred even more by some extent to r_2 . So it looks something like this; these are actual experimental images you can see. When the focus of the particle looks like this in the imaging plane 1, it looks like this, and in the imaging plane 2, it looks like this, and there is a different degree of blur captured.

So now, if we have two simultaneous images with a different degree of blur for the same particle, we can use them to determine the actual particle. For camera 1, we will have this functional form that we were discussing earlier; for camera 2, we will have this functional form. As the extent of blurring is different, the threshold sizes will be

different. And the Δz will also be determined based on its respective object planes. So now we have two equations; we know there are two equations, but we still don't know the functional form.

And we have two knowns now. So we have these thresholded sizes from the two images. But we now have the same two unknowns, d_0 and the position z , but we have two functions whose functional forms are not known, so it still seems like we don't have enough information to solve the problem. However, it looks like we are going in the right direction. So, the next step is to determine the functional form because if we have that, then we have the proper system where there are two calculated known parameters, two functions, and using that system of equations, we generate the two outputs or unknowns: the size and depth of the image or the particle.

Two Camera DFD – Calibration



- Target dot images captured by two cameras at known depth locations (different degree of defocus).
- This generates thres. size $d_{t,i}$ vs depth z_i map.
- A curve fitting enables calibration curve estimation.

- Calibration using the blurred images to evaluate functions k_i in red and blue for the two cameras. d_t is the image diameter at a threshold grey level of $g_t = 0.6$

$$\frac{d_{t,i}}{d_0} = k_i \left(\frac{z_i}{d_0} \right)$$

- Equation (2)
- Knowns (4):
 - d_t from the two simultaneous images
 - Functional form k_1 and k_2
- Unknowns (2):
 - Size: d_0
 - Position: z
- Now the problem can be solved

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How do we do that? We perform calibration. We use a calibration operation to generate those two functions. And how do we do that? First of all, this shows the operation that we do. We take target dots of known size and place them in the plane of focus first, so it will look something like this. Say in camera one; I'll talk about camera one first.

So we will now move it to a different depth location, say this point. And we will get a blurred image. We know the exact location of this depth. So we track this depth's location. So we will move it another step to a different depth location and capture the image again. And we'll do that in one direction, and we'll do the same in the other direction as well.

So now, if we choose a threshold, say 0.6, and plot the threshold diameter with the depth positions, we get a curve something like this. These red points represent the curve for

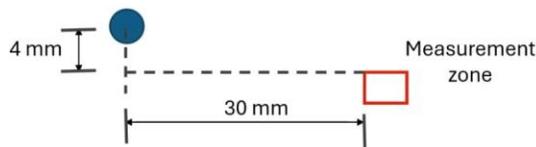
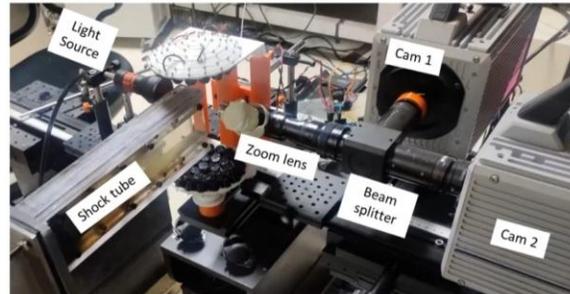
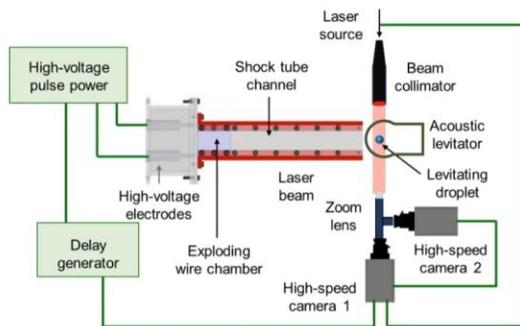
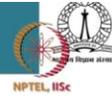
camera 1. Now for camera 2, the same curve will be replicated, only offset by a certain Δz . That Δz is related to the offset in the imaging plane because the imaging plates are shifted, so you will get the focused image slightly away from camera 1, which is somewhere here. So this is the plane of focus for camera 1; this is the plane of focus for camera 2.

But you will get the same curve, just shifted, because all the rest of your optical parameters remain the same. So this curve is the calibration function that correlates dt with z , and we get a functional form something like these two functions from this, and in this map, we can fit a curve to get the calibration function. This gives us the two equations, the functional form of two equations, which looks like an inverted parabola, and actually, we usually fit a sixth-order polynomial in this. And now we have known parameters: two thresholded diameters.

And now there are two unknowns: the size and depth. And it looks like a sufficient condition now for the problem to be solved. And that is true. So this, as these two are kind of not linear and are shifted, but they are sixth-order polynomials. We have to use some numerical juggling to solve these two equations with two nodes.

But we can do that and estimate the actual size and depth of the particles. And to do that experimentally, how do we place two imaging sensor planes with a fixed or predetermined offset? That is done using a splitter plate. So we have this backlight imaging system. So this is the light source. It comes through the measurement volume and over the lens. After the lens, we place a splitter plate that divides the light beam into two parts, and we can now take and place the cameras in both of these paths; by changing the position of the camera, we change the path length or the value of S .

Two Camera DFD – Setup



Experimental parameters:

Initial drop size ~ 2 mm
 Weber number ~ 900, 2000, 4000
 Droplet Breakup mode = SIE

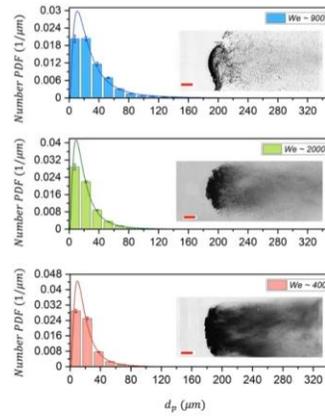
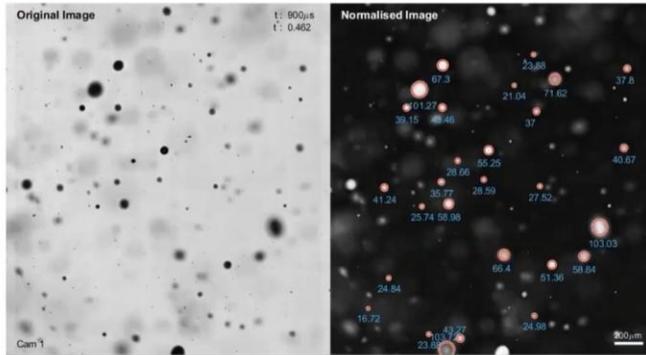
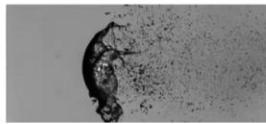
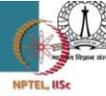
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So, in this image, you can see this box consists of the splitter plate. This is the lens. This is the light source. This is camera 2, and this is camera 1. So the splitter plate moves one beam here and another beam there. Here, what you see is an orange band or ring that is the spacer, which is adding an extra value of S in this direction or extra path length in this direction.

By that, we get two planes of imaging with different path lengths. And this problem, uh, is particularly related to shock-induced droplet atomization, and there we implemented this two-camera DFD to estimate the droplet sizes. So here are the results that we obtained. So the top video actually shows you the droplet disintegration and formation of many daughter droplets, and the bottom one shows a zoomed-in view of the actual ROI where we did the measurements. So here you can see a wide array of droplet sizes and the extent of blurring. So they are volumetrically distributed, and they also have different extents of blurring depending on their size and position.

Two Camera DFD – Shock droplet atomization



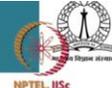
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So we normalize the images. Identify these particles or droplets, and then apply our technique to get the effective sizes, like the thresholded sizes from the two cameras. We generate a calibration curve before doing these experiments, and then feed these two values to determine the actual particle diameter and position. For this particular problem at hand, the position is not that important. We are more interested in the size distributions. So that can be plotted in terms of probability density functions, which are nothing but normalized histograms.

So you define bins, and in each bin, you count the number of droplets that appeared, and you normalize the whole thing in a way that the area under this curve is 1. So you will see that as we increase the Weber numbers or the shock strengths, the smaller droplets are produced; the only limitation set to this system is the resolution in space and time, so this DFD technique can be applied to a very high-speed phenomenon if you have a high-speed camera. Apart from that, the limitation is not there because of, say, Like a lot of droplets distributed in a volume, which is typically the case in droplet measurements where only point measurements can be done, here a volumetric measurement is being done; so that is an advantage that we have. And with this advantage of volumetric measurement, we can quantitatively estimate as well, which is even more important. And how is that? First of all, what do we mean by detection depth? As I told you earlier, there are, say, larger particles that are moved.

Two Camera DFD – Detection depth

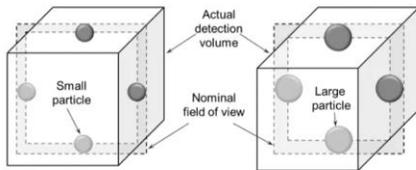


- Correction for measurement volume size
- Linear relation between particle size and detection depth

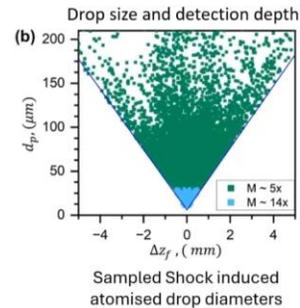
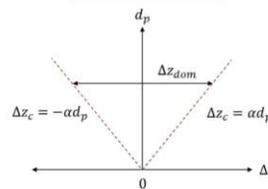
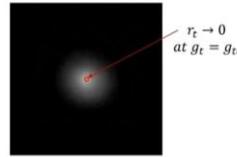
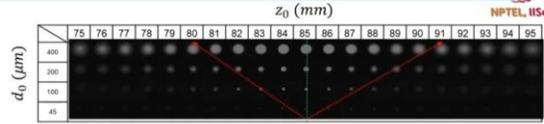
$$\Delta z_{dom} = 2\alpha(d_p - d_{p,0})$$

- Actual FOV: $a \times b$
- Effective FOV: $(a - d_p) \times (b - d_p)$
- Detection volume given as:

$$V_d = \Delta z_{dom}(a - d_p)(b - d_p)$$



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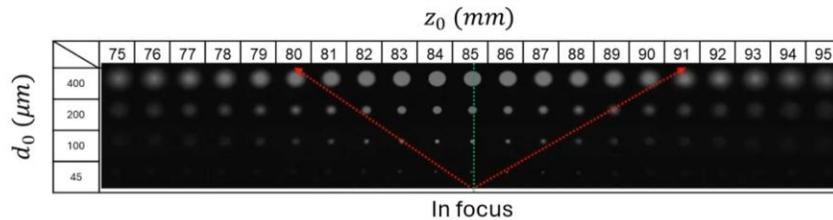


Away from the plane of focus, you see they get blurred, and when the smaller particles are moved away, you see they get blurred much faster and disappear almost entirely. So you see that the reason they can be detected is bound by this dotted line. Even in the experiments, we see that in the shock experiment that I've shown you earlier, if I plot all the points and their estimated positions along with the particle sizes, we see they are inside this envelope. The boundary is just a line, so this line can be represented in the form of some parameter alpha, and this is the line of detection depth.

If we take it in both directions, it marks the measurement volume. With this, we can exactly know what the measurement volume is; if we know alpha, we can know exactly the measurement volume, and that is done in this way. You are capturing particles in the region of interest, which is this, and in the depth direction, that is this, and the particles on the edges are neglected. We usually take the effective frame of view as the actual a by b of the FOV. Minus the particle sizes, this is the effective frame of view.

If you multiply the detection volume or detection depth, then we get the detection volume. We know that for this particle size and that particular particle diameter, the detection volume behaves this way. If we change the particle size, the detection volume changes, so we have an exact correlation between the detection volume and the particle size, which is very crucial. As we have seen, larger particles are detected at greater depths compared to smaller particles. So the measurement volume associated with larger particles is larger. So we are seeing them more often than smaller particles because of this bias, and this oversampling of larger particles must be corrected.

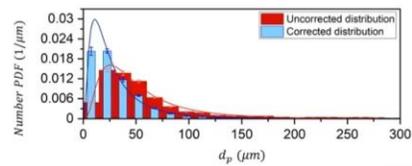
Two Camera DFD – Size dependent bias



- Larger particles can be detected and sized over a larger depth than small particles.
- This leads to a biased estimates and the statistics must be corrected.
- The correction for this volumetric measurement bias is done by taking number (or frequency) per unit volume and pdf is also modified accordingly

$$n_i(d_i) = \frac{N_i(d_i)}{V_d(d_i)}, \quad f_n(d_i) = \frac{n_i(d_i)}{\sum n_i(d_i) \cdot \Delta d_i}$$

where
 d_i = i^{th} bin position – diameter
 N_i = Number of particles in this bin
 n_i = Number of particles per unit volume in this bin



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And how it is done that instead of counting the number of particles in each bin, in the histogram we can count the number of particles appearing per unit volume. So, if we regularize it using per unit volume, then irrespective of the size of the volume, we are averaging it in a kind of way, and in that way, we can correct it. So, the undersampled small droplets would be corrected to appear as per unit volume, and that is also reflected in this plot. You can see that after corrections, the number of smaller droplets went up and how we achieved that; you can see that here in the equation. Earlier, we were only counting the number of particles in the bin d_i , and now we have normalized that frequency with the volume measurement associated with that bin d_i , which leads to this number concentration of particles in that bin.

And when that is substituted into the probability distribution function, we get this corrected function. So, in this way, this knowledge is extremely important and is useful not just for bias correction, but also to get a number concentration of particles in your system. So with this, our two-camera DFD measurement is over, and in the next lecture, we will see what other kinds of DFD techniques can be developed or implemented.