Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer

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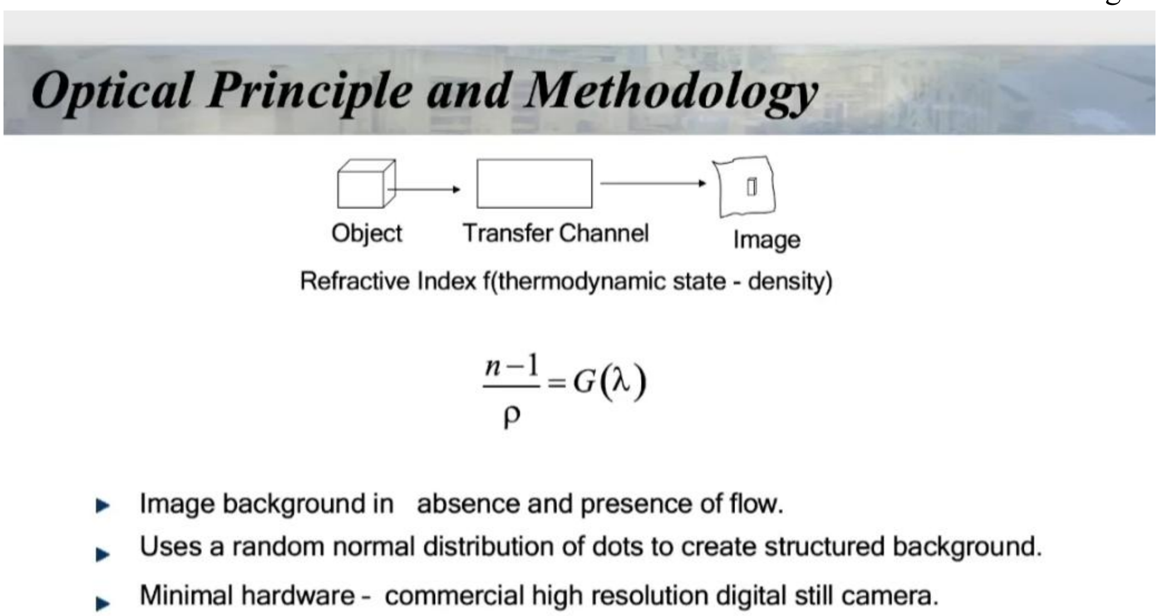
Week – 06

Lecture - 26

Schlieren and Shadowgraphy – 4

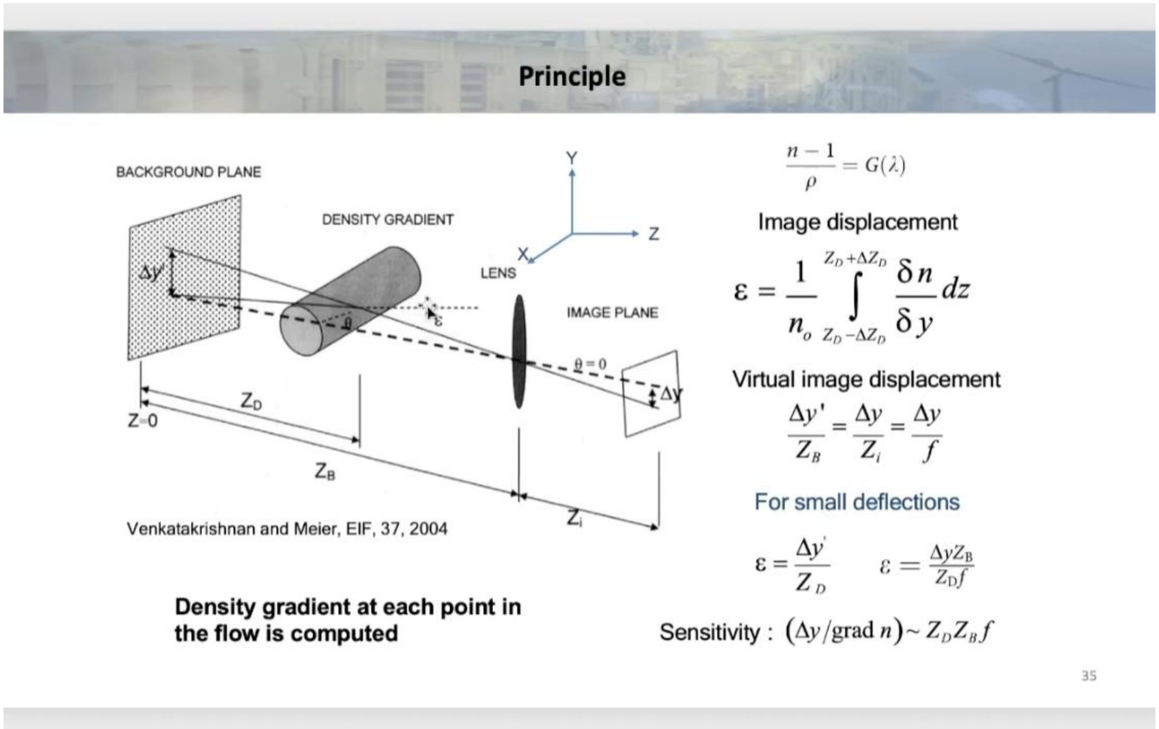
Okay, so in this particular lecture, we look into what we call background-oriented Schlieren. So the principle is essentially, you know, whatever we have learned so far, is like this. So, say you have a room. Or you create a background. So what happens is that you take an image in the absence of the flow or in the presence of the flow. So you take two images.



So what we do essentially is use a random normal distribution of dots to create a structured background, and then you use any cell phone or normal camera to image that particular flow. What we do is, basically, you have a screen and a camera. First, take an image of the background. The background, which is the screen, can have different textures.

It is usually textured, so it could be like a dotted wall or something similar. And then you introduce the object of interest, which, in this case, is a flow. So it could be like a wind tunnel; it could be a cup of tea; it could be anything. So the object of interest is where there is a density gradient present. As a result of that, you will have some distortion of the dots essentially.

And by using some image convolution, we try to find out what the actual flow situation will be. So this is essentially what the crux of background-oriented Schlieren is. So what we have here, if you look at this, is the principle. So, you have a density gradient. This is the object of interest.



This is the background plane. And this is the image plane, and these are the x, y, z coordinates. What happens, and this is taken from Venkat Krishnan and Meyer's highly cited paper from their experiments in 2004, is noteworthy. So what happens is that this is a checkered surface. So, in the same form where n-1 by ρ = g/ λ, this is still valid.

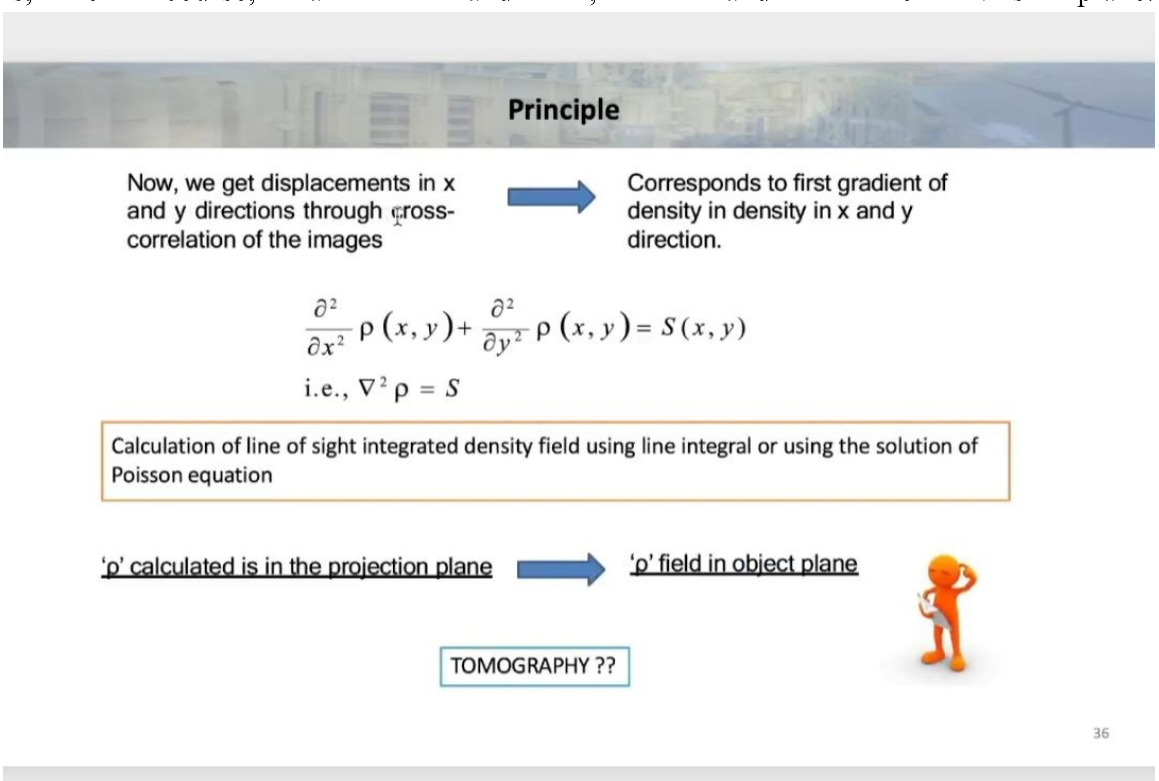
So, first, these are the equations that you have already seen. So, because of the presence of this density gradient, we get an image displacement, which is ε. Now, this image displacement, which is $1/n_o$ multiplied by the integral of the variation of n in the y direction, is. And then you are integrating it across z, which is in this particular direction. And it is integrated from z, d minus Δ z, and then a plus Δ z.

So if there is an image displacement of the background plane, okay, that translates to a virtual image displacement here, as you can see, and they are related by the corresponding distances, right? So Δ y'/ z_b is equal to Δ y by z_i, okay? That I = Δ y / f, which is the focal length. Now, for small deflections, your ε is given by Δ y'/Δz_D, which in other words translates to Δ y into z_B /z_DF. Or in other words, the sensitivity means the Δ y/ Δ z_B. => z_D z_Bf. So, if you look at this now, okay, this is what you are going to get.

Okay, so where this is now, this background-oriented Schillerian technique is therefore based on the same relationship between the refractive index of a fluid given by the Gladstone-Dale equation, especially for gaseous media. And this is also sometimes compared with laser speckle density photography. Okay, but if we see that here, the main purpose is that the reference image is generated by recording the background. which is observed through air at rest. And the second step is an additional exposure, which introduces the flow under investigation.

Now the deflection of a single beam contains information, therefore, about the spatial gradient of the refractive index that is integrated along the line of sight. So, in other words, for small angle deflections, this is what you are ultimately going to get. So the image displacements are all given by this set of equations. These are the sets of equations that actually tell the whole story. So, in other words, you can do that.

The principle is that we can now get the displacement in the X, Y, and Z directions. To the cross-correlation of the images. And this corresponds to the first gradient of density in the X and Y directions. This is what we are stating: each point is deflected. And so there is, of course, an X and Y, X and Y of this plane.



And we record all of them. And then if we deconvolute, these are going to correspond to the first gradient of density in the X and Y directions. I mean, that is essentially what you are getting, so this is, for example, in the y direction. All right, so this is shown in one particular direction only. Okay, but it is prudent to convert this to a field equation now.

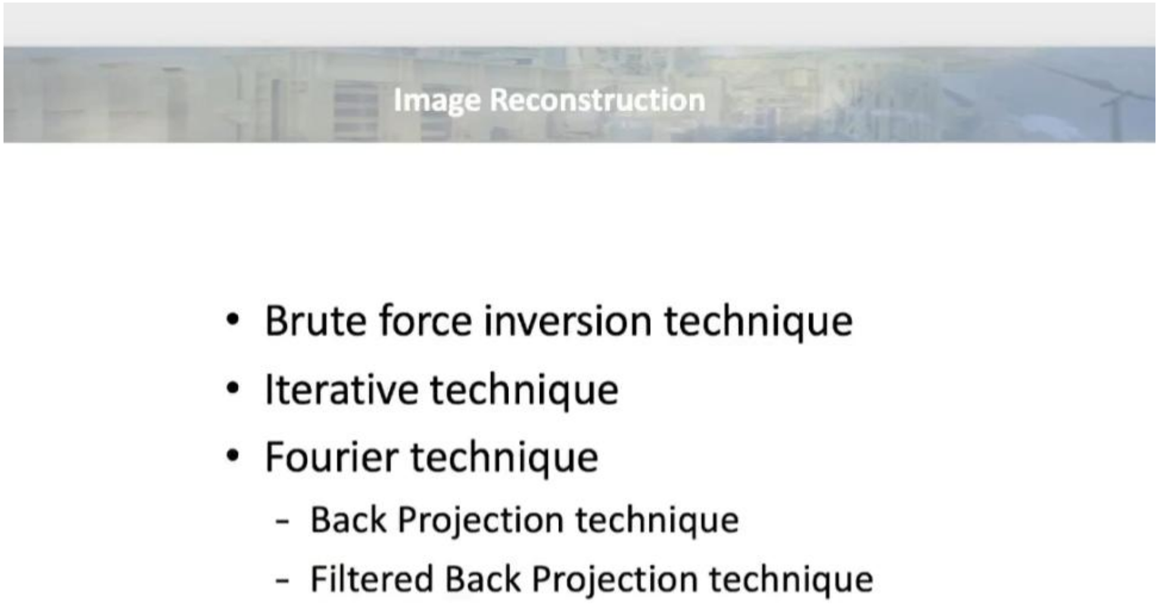
So, if you convert this to a field equation, you get a Poisson equation. Now this Poisson's equation essentially tells us that if you take the Laplace of the density field, which is x and y, that is equal to some source function in this particular case. So this is the net equation that you are going to solve. The calculation of the line of sight integrated density field can be done using line integrals or using the solution of the Poisson equation. These are the two ways by which you can calculate it.

One is basically the line of sight integrated density field using the line integral that we just covered. The other one is using the solution of Poisson's equation, where you solve for the density field. And once you solve the density field, you can calculate the row that is calculated in the projection plane, and that leads to the density field in the object plane. All right. So this is a relatively easier way to calculate.

So the background-oriented Schlieren gives us two interesting insights. So this particular technique actually allows us to calculate, get a field measurement. and get a more what we call quantitative measurement of the flow field in question because we are dealing with flow fields here. So, in other words, what this background-oriented technique does, how it is different, is that it uses correlation techniques on a background dot pattern to quantitatively characterize any mildly compressible or thermal flows with good spatial and temporal resolutions. The main advantages are experimental simplicity and the robustness of the correlation-based digital analysis.

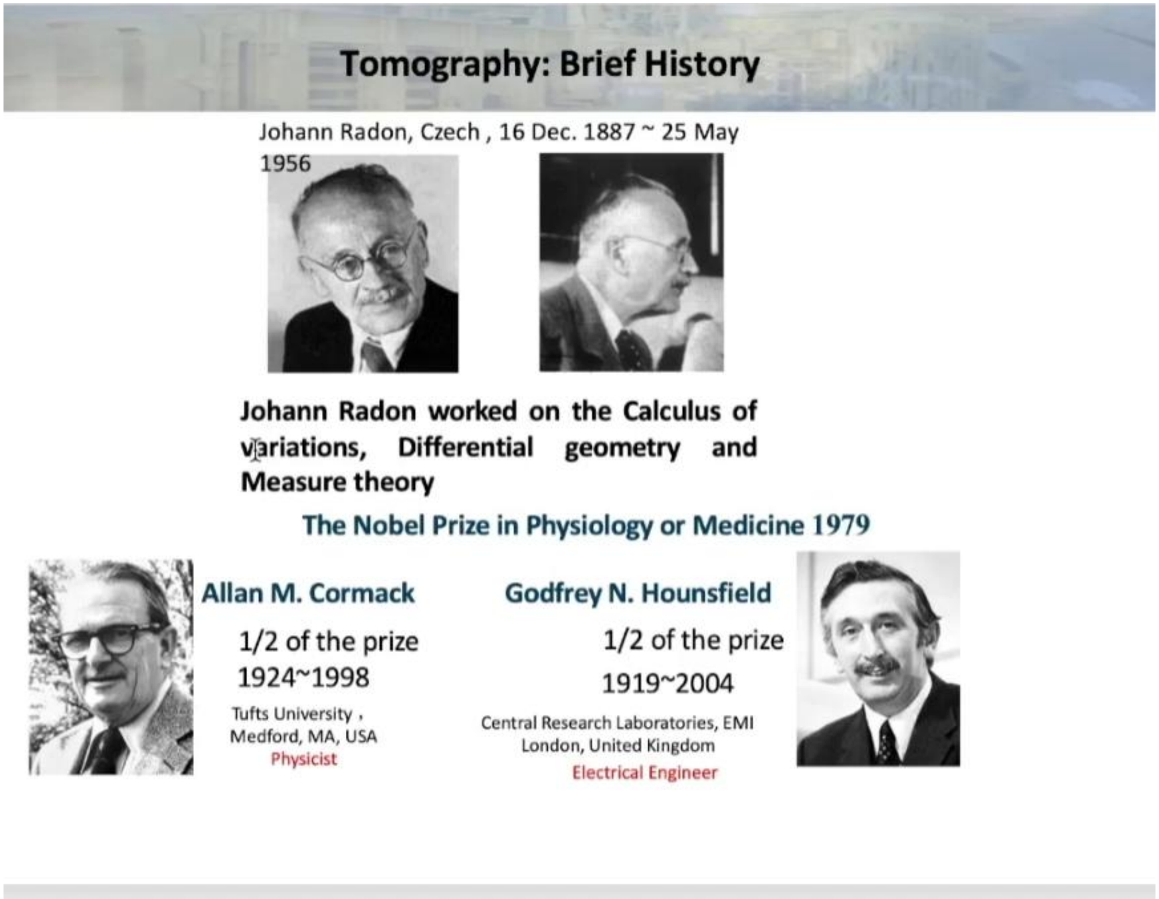
That means this is routinely used in many of the application contexts. So that is what it is. But, you know, because of the line of sight integrated density field, the natural question is: can we extract, like, you know, some kind of tomography, which I would say involves extracting three-dimensional features? Okay, so three-dimensional density fields. If we use multi-camera recording and do tomographic evaluation, it would generally be possible to perform some kind of tomography.

for this. As we mentioned, in the case of determining 2D flows, such as what we have shown here, we use, for example, a Poisson solver to integrate the density gradient and determine the relative density of the field. So, however, this non-two-dimensional nature of the flow field distorts the results because BOS is a line-integrating technique. Therefore, you need more tomographic techniques for the three-dimensional flow or tomographic algorithms to actually do that. So this leads to how you can do it. You can do it through image construction.

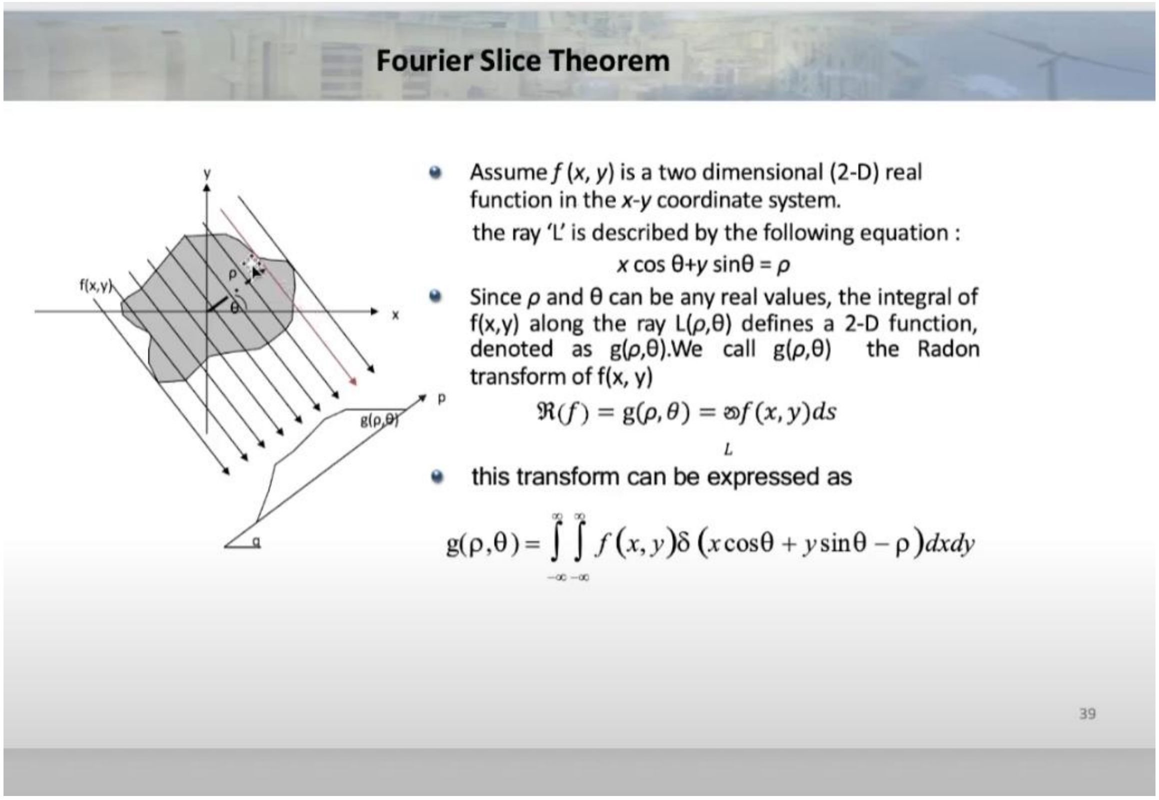


So there are plenty of ways to do that. You can use the brute force inversion technique. You can use iterative techniques or Fourier techniques, which can include a background projection technique or a filtered back projection technique. So, there are a lot of ways in which you can actually do that. In the case of axisymmetric flows, this becomes a bit easier.

But in the case of highly three-dimensional flows, it's a little bit more complicated than that. So the main people in the tomographic brief history were Johan Radden, who worked on the calculus of variations, differential geometry, and measurement theory. And the Nobel Prize in Medicine for the CAT scan essentially went to Alan McCormack and Gottfried Hounsfield. They were physicists and an electrical engineer. They were awarded the Nobel Prize in Physiology or Medicine.

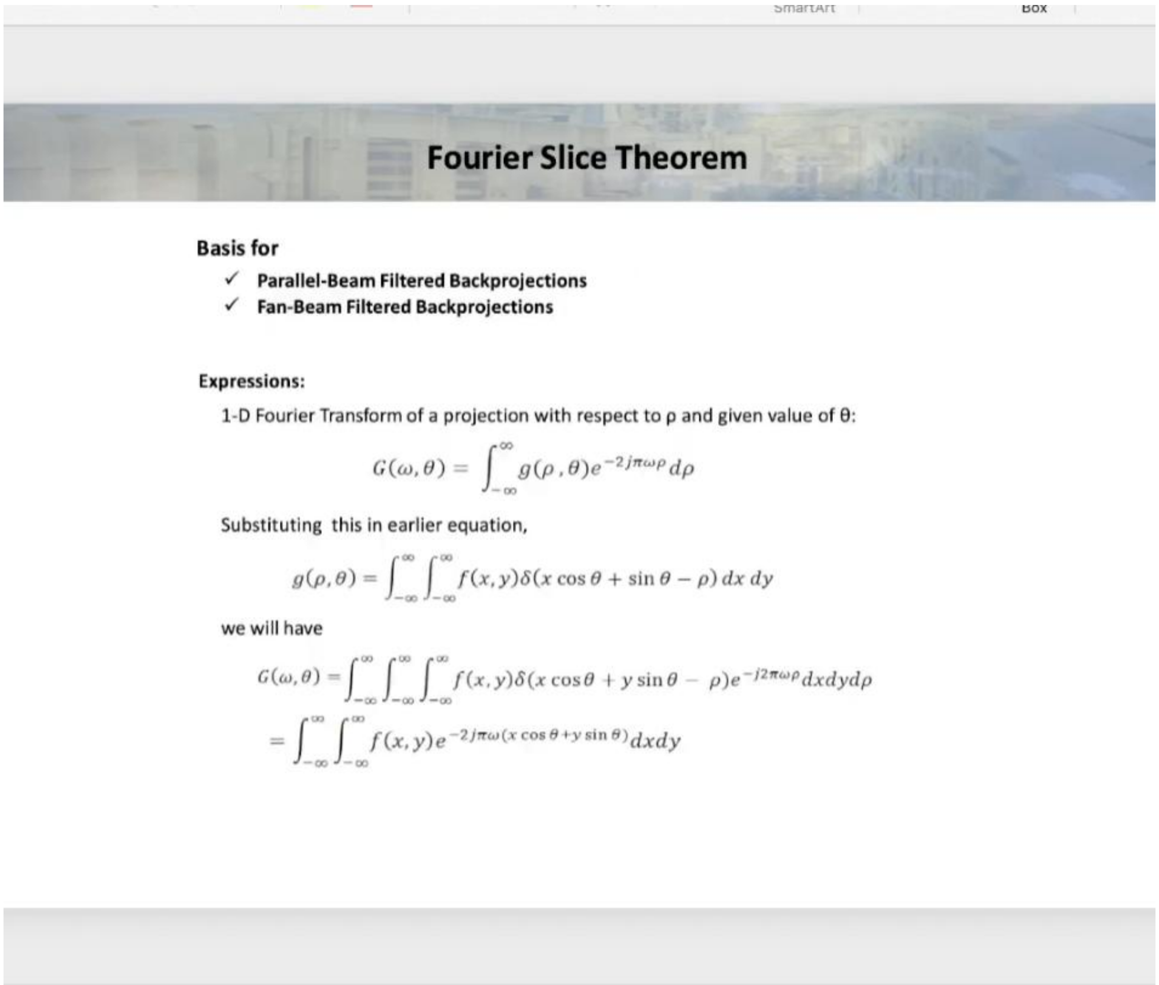


So, this is a brief history. So this is used even in X-ray tomography and things like that. So the essential part of this is the Fourier slice theorem. So what happens is that if you assume that f is a two-dimensional real field or a function in the xy coordinate, now this can be your flow of interest, and say l, these are the l's which are at some angle θ with respect to this, so this is like a projection plane, right? A directional lady is. Introduced. So if this ray actually passes through it, now in some cases, for example, in X-ray, what will be the absorption profile that you are going to get in this particular projection plane? In this case, of course, what is the distortion of the rays or the deflection of the rays because of this? And the ray is therefore described by a simple function like $x\cos\theta + y\sin\theta = \rho$.



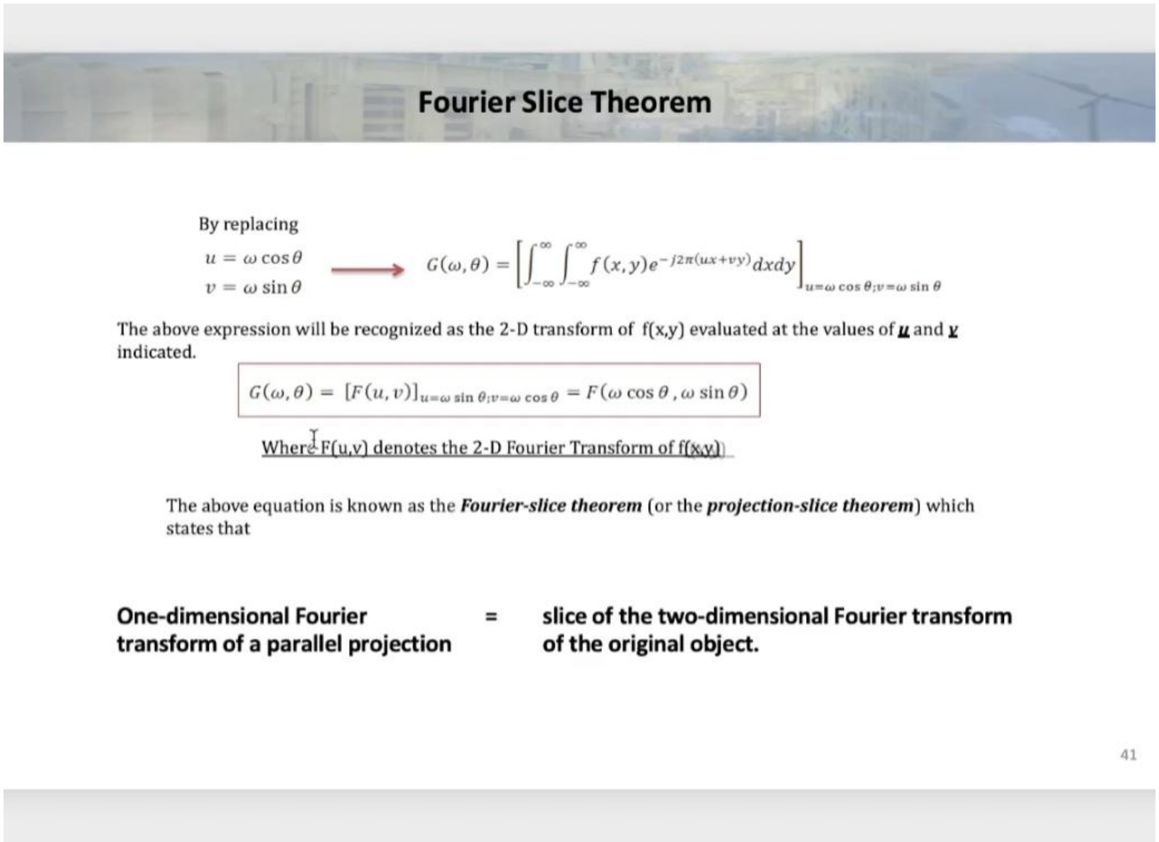
Now, what happens is that since ρ and θ can be any real values, the integral of f_{XY} along the ray, which is L(ρ,θ), now given as L as a function of ρ and θ, describes and defines a 2D function, which is denoted by g(ρ,θ).g(ρ,θ),. We call that g(ρ, θ), the Radon transformation of f(x, y), so the Radon transformation is basically what gives you g(ρ, θ) equal to the Radon transform of f(x, y). Now, this transform, if you open it up, is basically nothing but f(x, y) multiplied by Δ into the whole term into dxdy. Okay, this ultimately gives you the g(ρ, θ), which is the intensity of the deflection at this end.

So you basically take a slice along the ray direction and then you kind of determine what this transform will be. So now that we have this, the Fourier slice theorem forms the basis of parallel beam filtered back projections and fan beam filtered back projections. The parallel beam is the one that is of most interest. So the expression is that the 1D Fourier transform of a projection with respect to ρ and a given value of θ is given by g(ω,θ) = g(ρ,θ) times this, right? So substituting this into the earlier equation, you now basically know what your g(ρ,θ) was. If you look at this, this is what your g(ρ,θ)is.



And so, if you substitute it here, we will have the g(θ) given by the triple integral. And this is equal to this. So this g is the Fourier transform of g. The projection, so if this is the projection, this is the Fourier transform of the projection with respect to ρ and a given value of θ. For a given ρ, θ condition, this is what you are actually going to get, and you are basically substituting whatever was inside—remember the Radon transformation—and then you get this entire quantity.

Okay, so now by replacing u with ωcosθ and v with ωsineθ, you get this value of g(ω,θ). The above expression is basically recognized as a 2D transform of x and y, evaluated at certain values of u and v indicated. Okay, so in other words, g(ω, θ) is nothing but the 2D transformation of f evaluated at u and y. So that f, therefore, denotes the 2D Fourier transform of f. So this is what you get: the transformation of f. So you see that , g(ω, θ) is therefore F_{UV} evaluated at a certain value of uv, which is nothing but the 2D Fourier transformation. So the above equation is known as the Fourier slice theorem or the projection slice theorem, which states that the 1D, this is important, 1D Fourier transform of a parallel projection is equal to the slice of the 2D Fourier transform of the original object because you are evaluating it at certain u and v. So, to understand this, the FUV is the Fourier transform of XY. So F_{UV}, which is evaluated at a certain value of U and V, is basically a slice of the two-dimensional Fourier transform of the original object. Now this is equal to the one-dimensional Fourier transform of the parallel projection, which is essentially this.

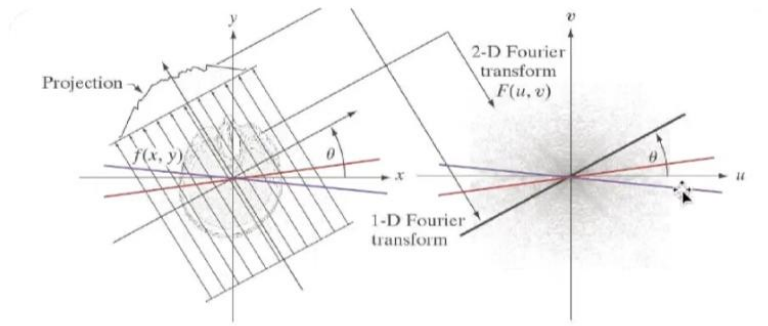


So this is nothing but the 1D Fourier transformation of the projection because that is what we did here. It's a one-dimensional projection. So this is the important part: the one-dimensional Fourier transform of a parallel projection is nothing but taking the two-dimensional Fourier transformation and extracting a slice at certain values of u and v. All right. So this actually implies that, you know, if you are going to take, for example, this is one projection, OK, which is evaluated at a certain angle.

You can have a projection that is evaluated at a different angle. And these are the corresponding 1D Fourier line transformations. Right. So in other words, if you now reconstruct the function. OK, so this is the inverse 2D Fourier transform of the 1D Fourier.

Back Projection

What does FST imply?



$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+uv)}\,dudv$$

Inverse 2D FT of the 1D FT of projection

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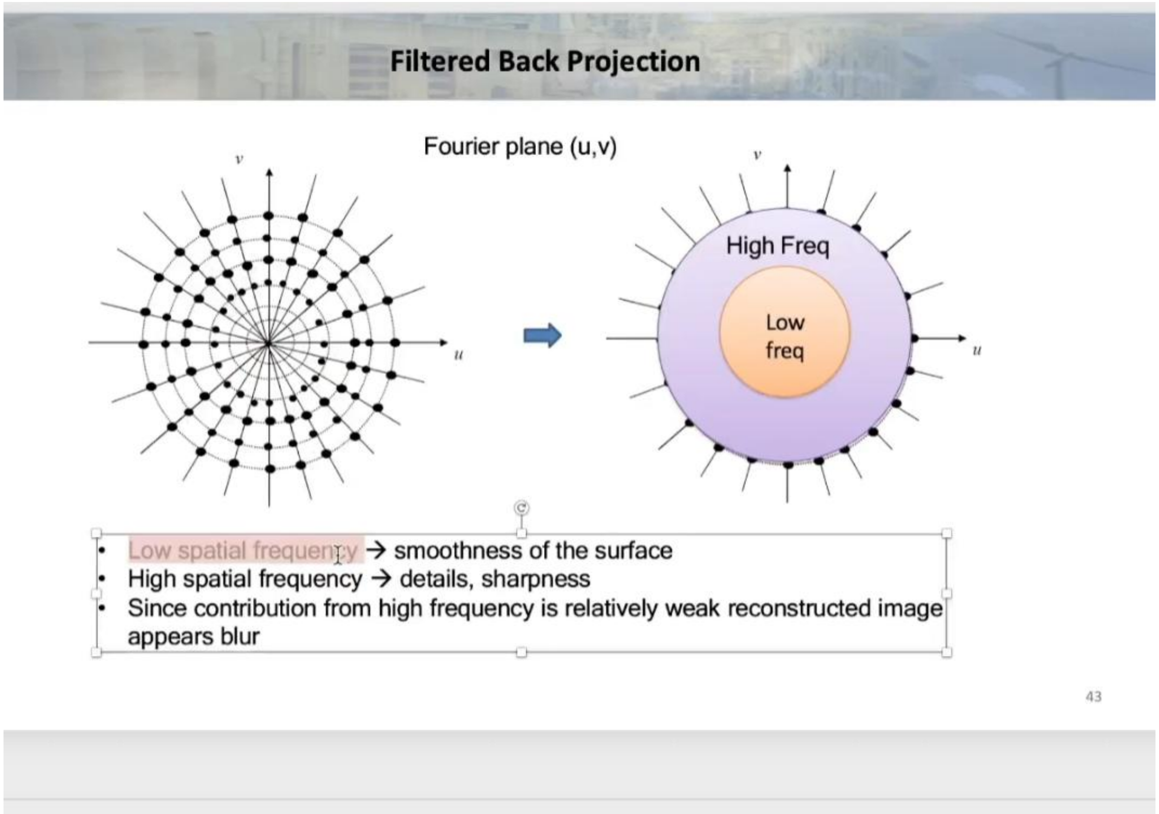
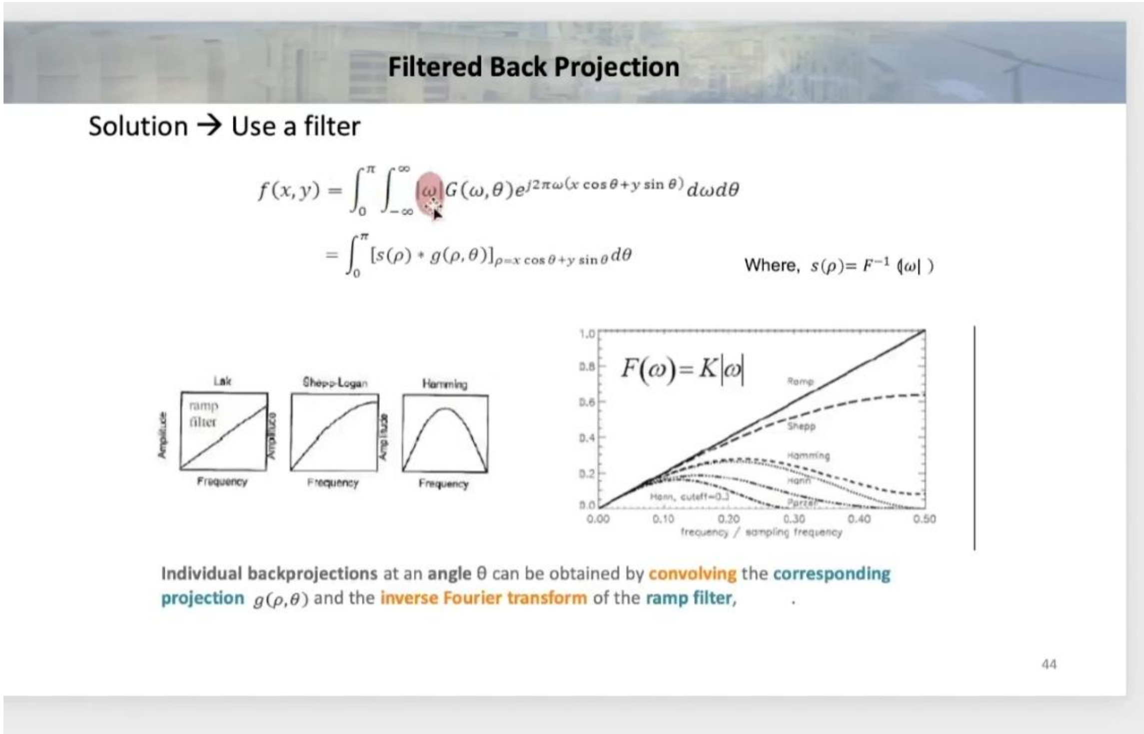
Fourier transformation of the projections. So this is essentially what you will get. So this is, for example, the 2D Fourier transformation of FUV, and then at a certain value of F and a certain value of UNV, you take out those slices. So this is almost like the 1D Fourier transform of the projection. So this is very simple. So these are the numbers of projections that you need.

So how many projections do you need to reconstruct this whole thing? So if you have, for example, in the Fourier plane, a low spatial frequency, you ensure that there is a smoothness of the surface, but you miss out on a lot of information.

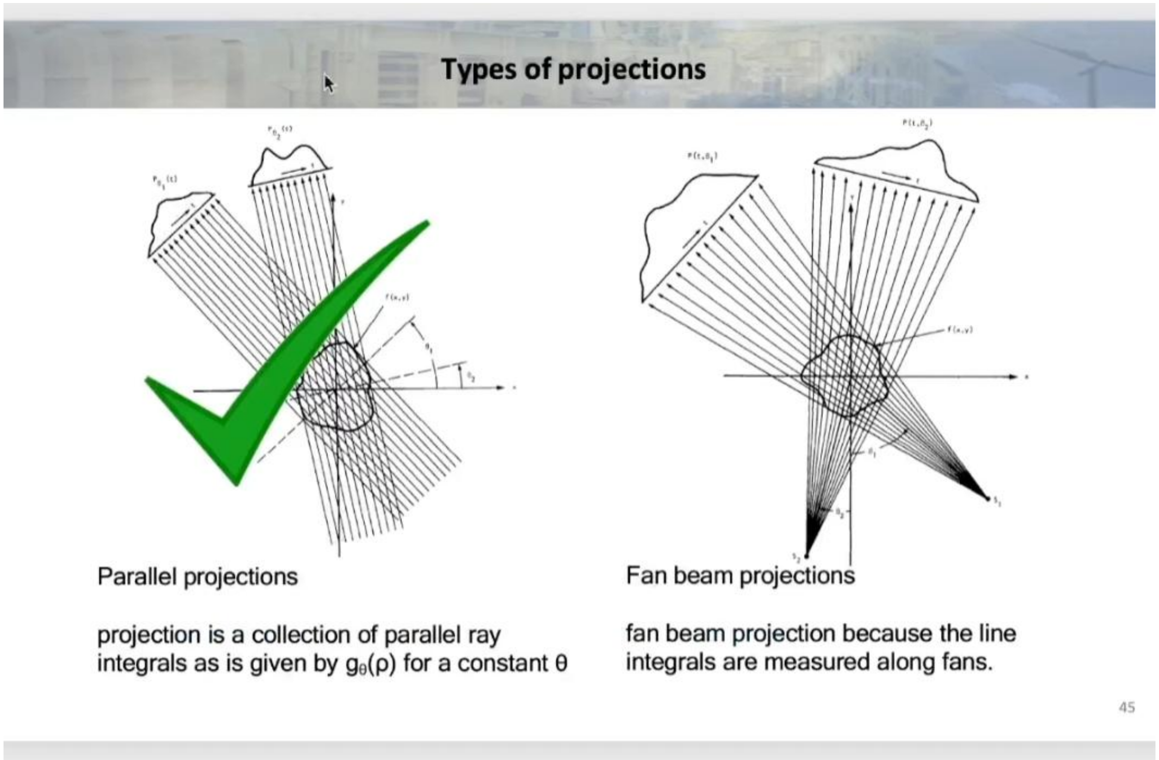


If you have high spatial frequencies, you get details and sharpness, but high spatial frequencies result in a relatively weak reconstructed image that appears to be blurred. In other words, low frequency means that you are using a smaller number of projections, while high spatial frequency means that you have more. A higher number of projections is okay, so the solution to this blurring and the weak reconstructed image is done using a filter. This is the filter; this is where you introduce the filter, okay? In this whole thing, these filters are of different kinds, like the lag filter, the Logan filter, and the Hamming filter.

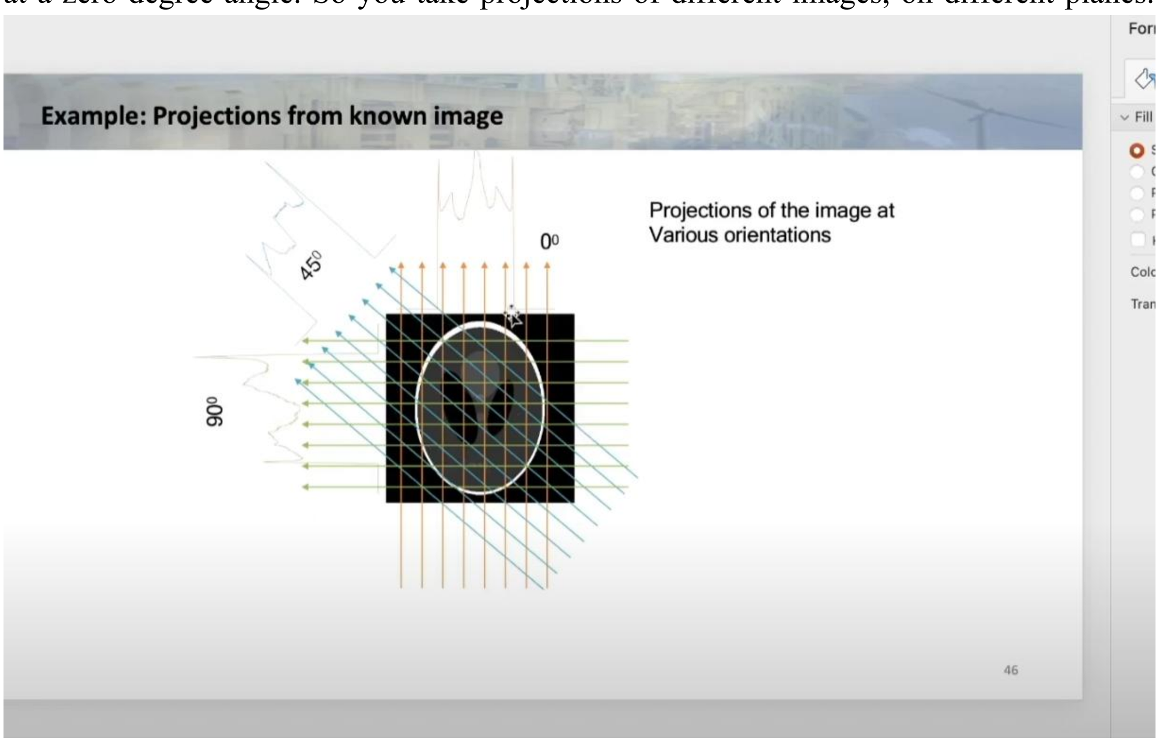


The individual back projections at an angle can be obtained by convolving the corresponding projection with the inverse Fourier transform of the ram filter. So this gives rise to the sharpness. Ascension, okay, so this is the important line: the back projections at an angle, which is what it is, can be obtained by convolving the corresponding projections, which is $g(\rho,\theta)$, multiplied by the inverse Fourier transformation of the Ram filter. So this is the Ram filter. OK, so this is what is done to achieve better imaging quality.

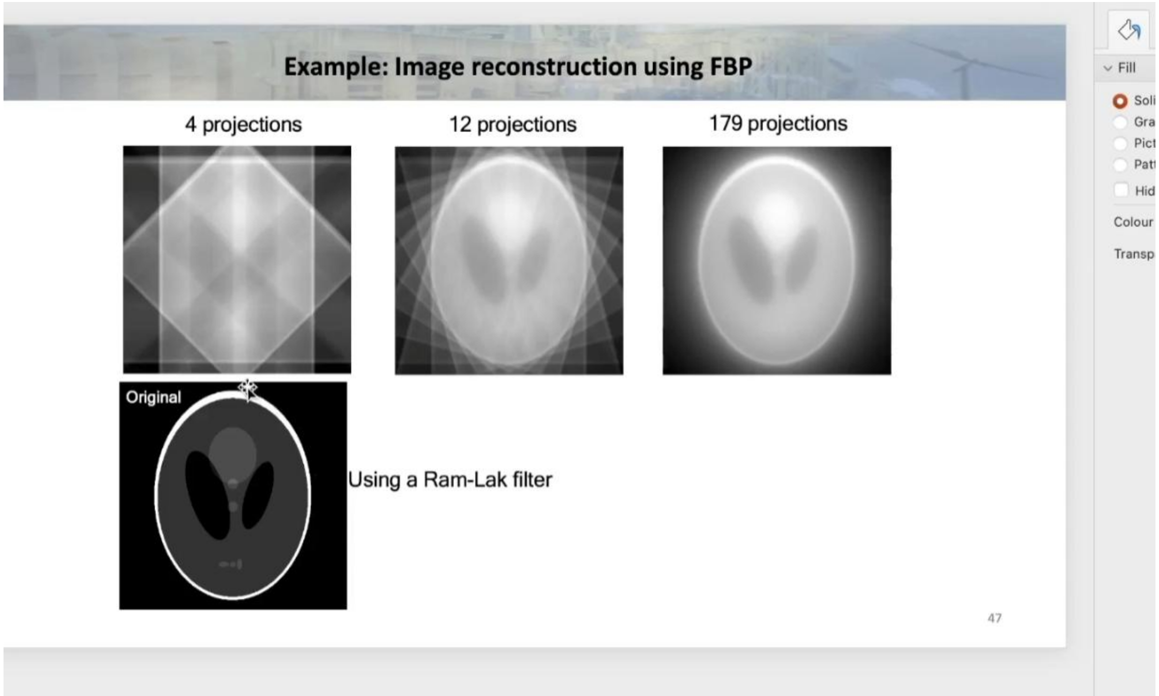
Now, this can be done with this type of projection. Now, this is what the parallel beam projection is. So these are at different angles, as you can see. OK, so different θ_1 and θ_2. So parallel projections are given as parallel integrals, as given by $g(\rho,\theta)$, for a constant θ. Then you can also do fan beam projections where they are kind of fanning out because the line integrals are measured along the fans.



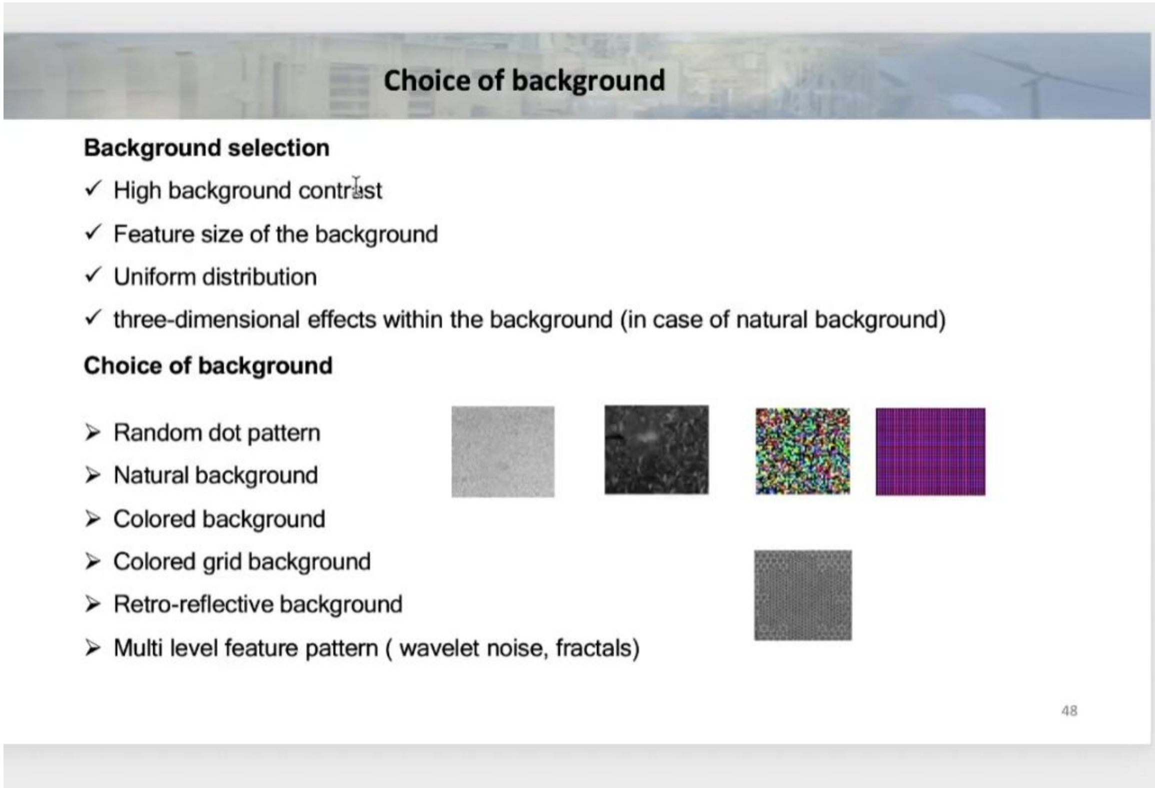
Now this can also be done, but it is preferable to use something like this in the cases concerned. So, examples of projections from known images. So, for example, if this is an image and then you take projections, say this is at a 45^0 angle, this is at a 90^0 angle, this is at a zero-degree angle. So you take projections of different images, on different planes.



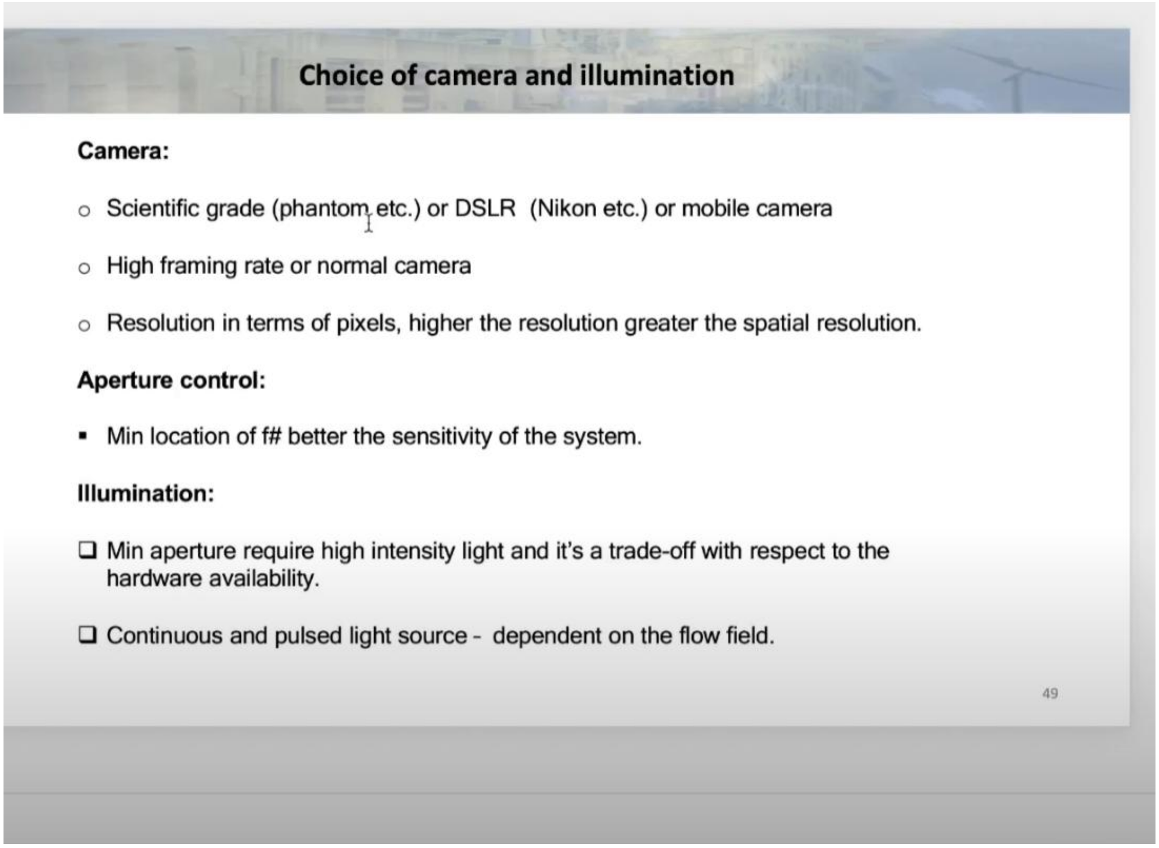
Right. Now, if we do four projections, So this is the original image. Original image. So if you do four projections, you get an image that is totally off. This is 12.



This is like 179 projections, which is basically a lot. If you use a RAMLAC filter, then you almost reconstruct the whole thing. So, as you can see, the higher the number of projections, the better it is, but there is incidental blurring and loss of image. So if you use a RAMLAC filter, you can get an image that is pretty much of the same quality. So this is exactly what is done. Okay, in the choice of background, you should choose a high contrast background.

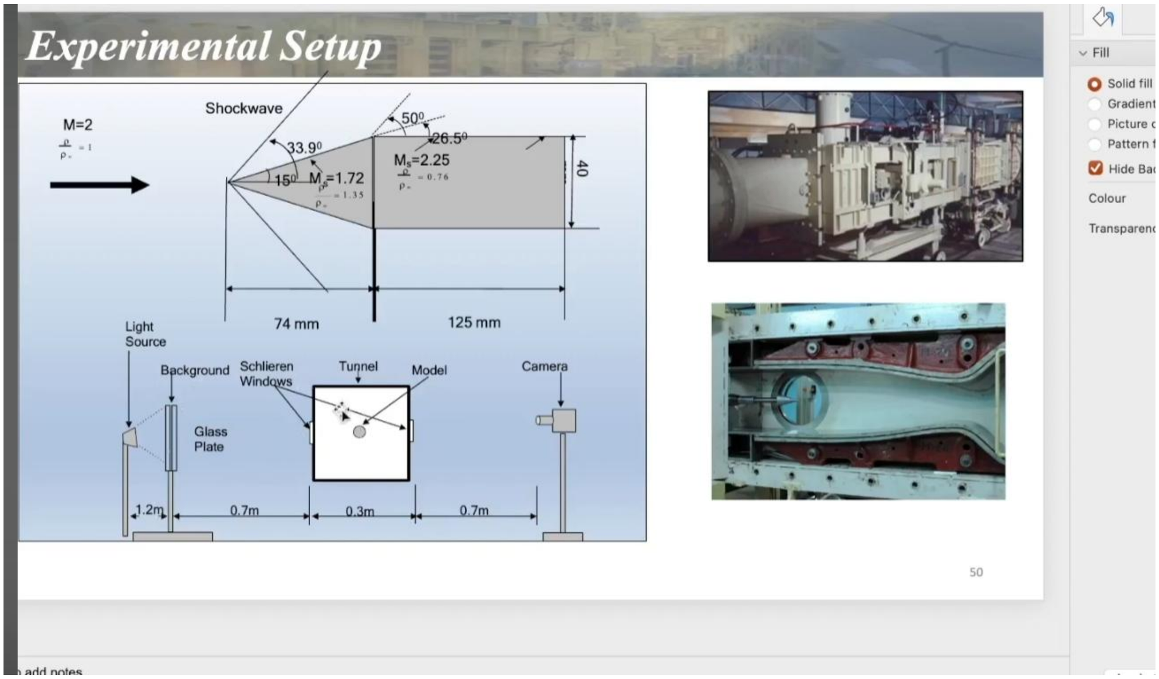


The feature size of the background is of interest. You should have a uniform distribution, and you should have three-dimensional effects within the background in the case of a natural background. So this can be different things. This can be random dots or a natural background, which can be like a textured wall, for example. If you have a colored background, a retro-reflective background, or a multi-level feature background, all of these are choices of backgrounds that you need for the BOS. The camera is usually a scientific-grade camera, like Phantom and Fortron, or DSLR cameras, which are also okay.

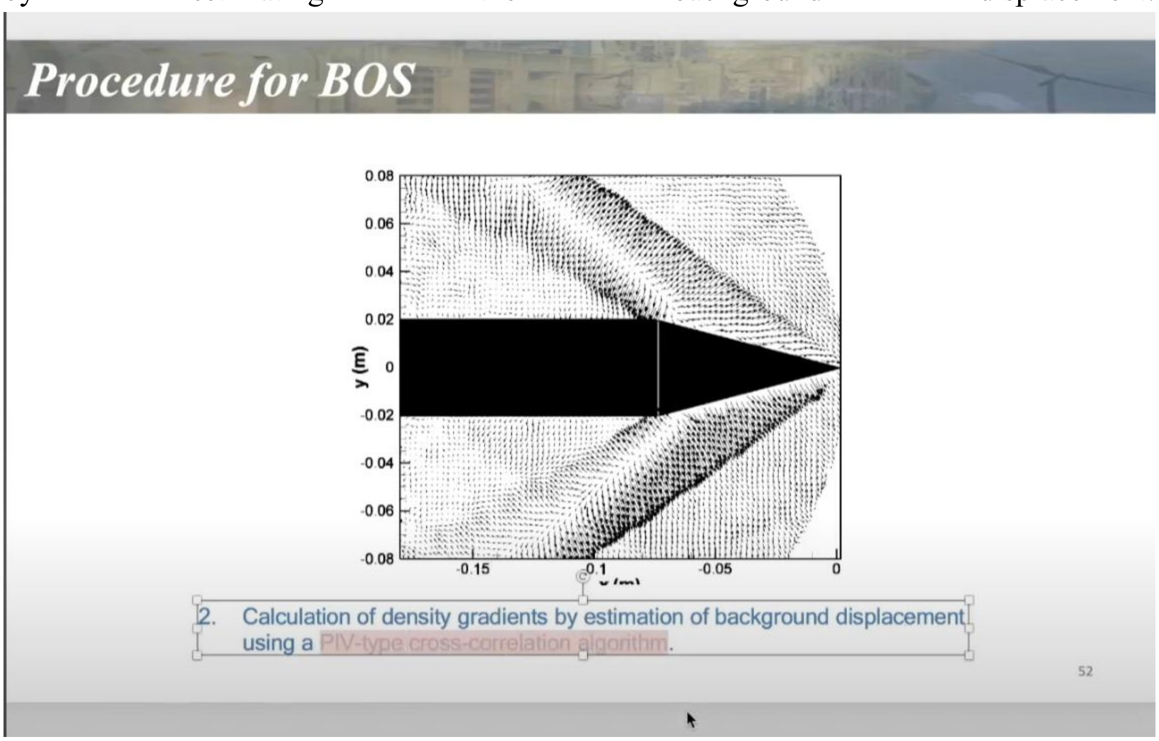


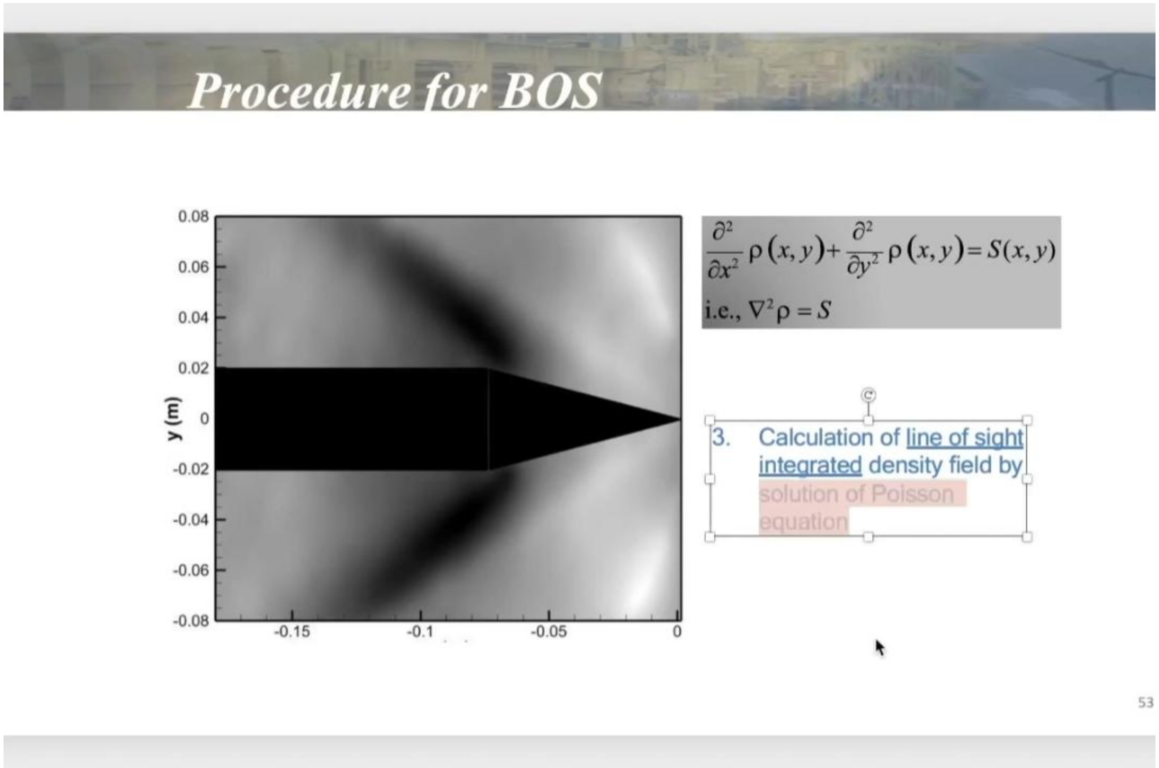
You can have a high frame rate or a normal camera depending on your requirements. The higher the resolution in terms of pixels, the better is a special resolution that you can achieve. Aperture control is better for the sensitivity, the minimum location of f, which gives the better sensitivity of the system. Now, if you look at this, you will just recall that f was here, so the sensitivity can be tailored according to your application. And then, of course, the minimum aperture is required for high light intensity, and it's a trade-off with respect to hardware availability.

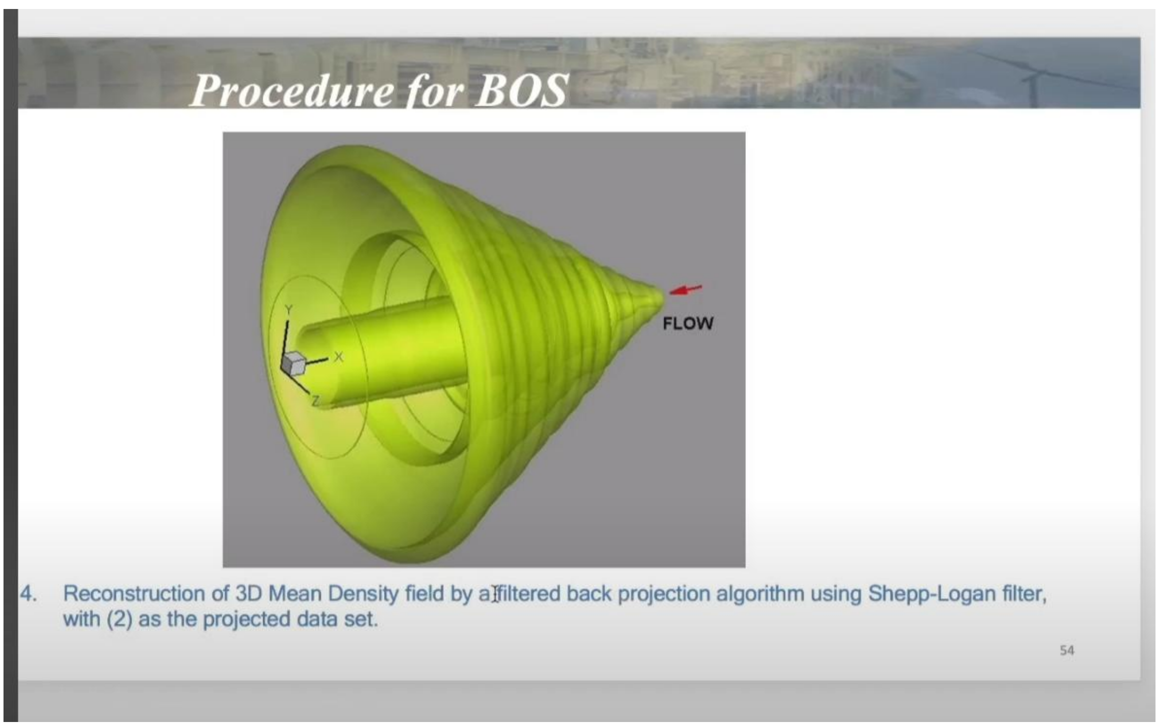
You could have a continuous and pulsed light source depending on the flow field that you are concerned with. So there are a variety of things. For example, this is an experimental setup.



This is the background. These are the schlieren windows. This is the camera. This is the light source. So this is the background image with and without the flow. So this is also taken from Venkat Krishnan's work. And this is the calculation of the density gradients by estimating the background displacement.





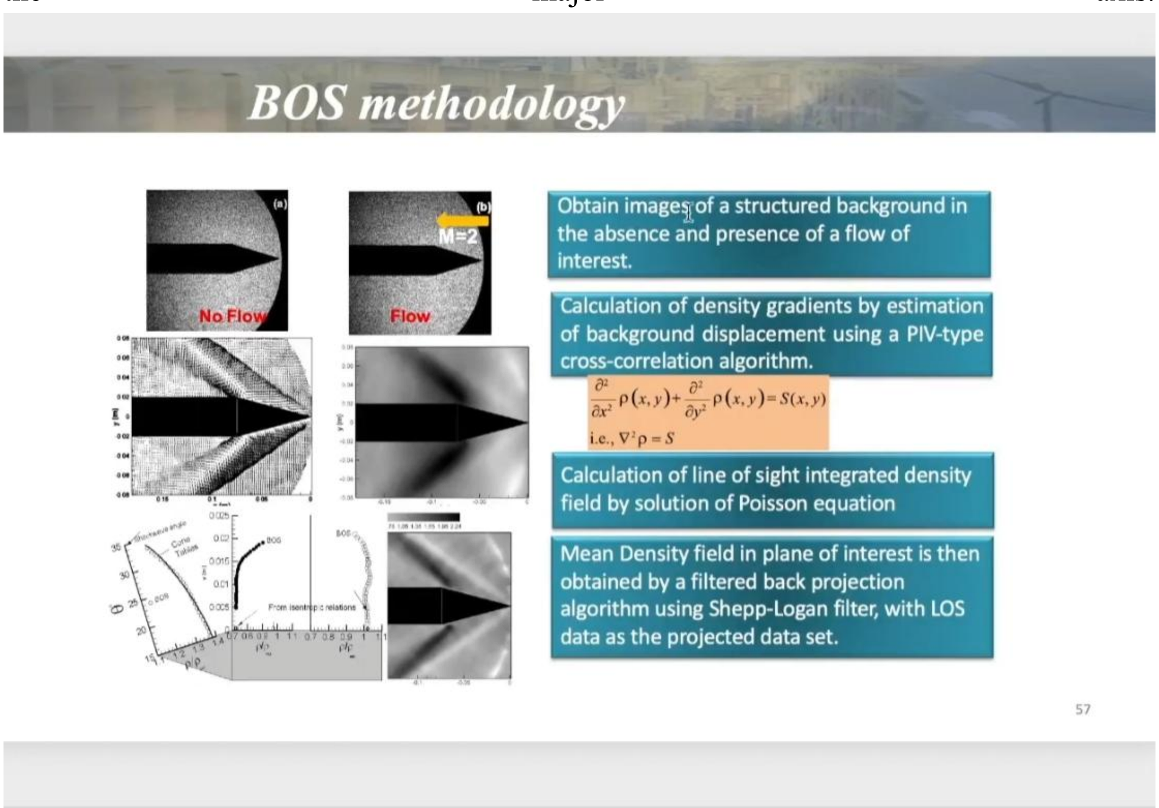


Here a PIV type cross-correlation algorithm has been used. So you can see the quantification now starts to happen here, which was the requirement, as we said. And the procedure, as we said, is the calculation of the line of sight integrity by the solution of

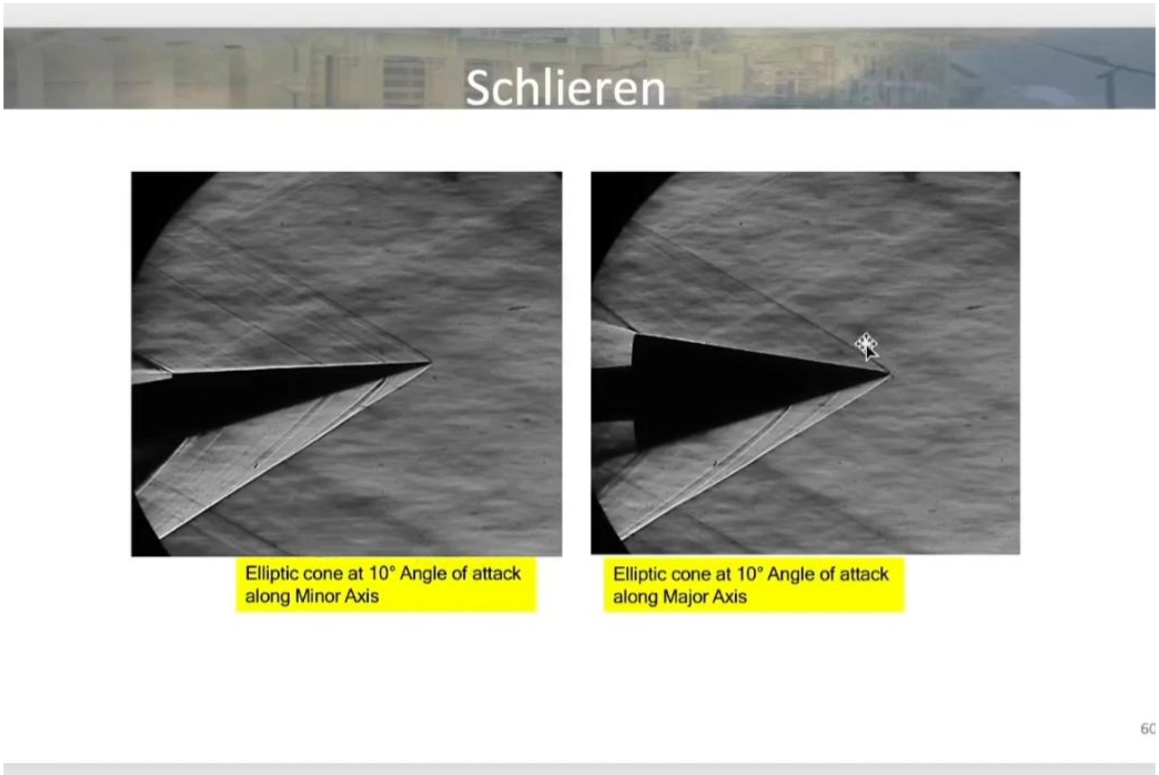
Poisson's equation. In this case, it's a 2D flow field, so you solve the line-of-sight integrated Poisson's equation. So you're seeing into the depths of the board, essentially.

So that is what you are trying to see. And, you know, the reconstruction of a 3D field by filtered back projection, which we kind of did using the Shepp-Logan method. Filter with two as a projected dataset. So this is the reconstruction of the 3D field. This is the 2D line-of-sight integrated density field from the solution of the same equation that we covered earlier. And okay, so this is the mean density field in the area of interest that is extracted from a particular plane.

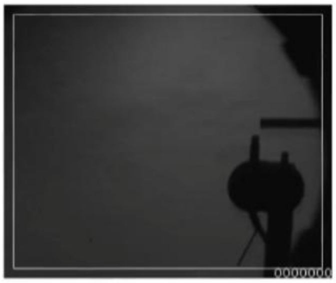
So the BOS methods, therefore, in a nutshell, obtain images of a structured background in the absence and presence of a flow field and calculate the density gradients by estimating background displacement using a PIV-type cross-correlation algorithm. That we use the calculation of the line of sight integrated density field by the solution of Poisson's equation. The mean density field in the plane of interest is obtained by a filtered back projection of the algorithm using the Shepp-Logan filter with LOS data as the background set. So there are various other ways to measure this. This is, for example, schlieren, which shows the arc angle and angle of attack along the minor axis and along the major axis.

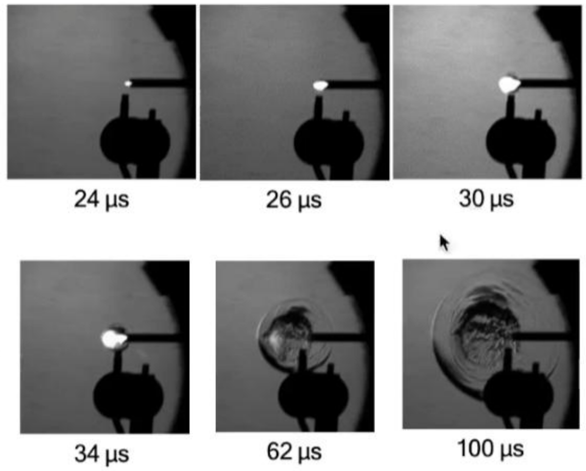


So you can get highly constructed three-dimensional data. And this is, of course, the quantitative density field, which is also calculated in a very similar fashion. These are some of the schlieren experiments where you can see that it is very high-speed, but it shows the micro-blast detonation using detonation mechanisms. This is very commonly used. This is from Venkat Krishnan's work, which is the density field of micro-explosion using a background-oriented schlieren in this particular case. This is another work, which is discrete time BOS results, showing this, the mock disk, the barrel shock, and a lot of other cool stuff over here.

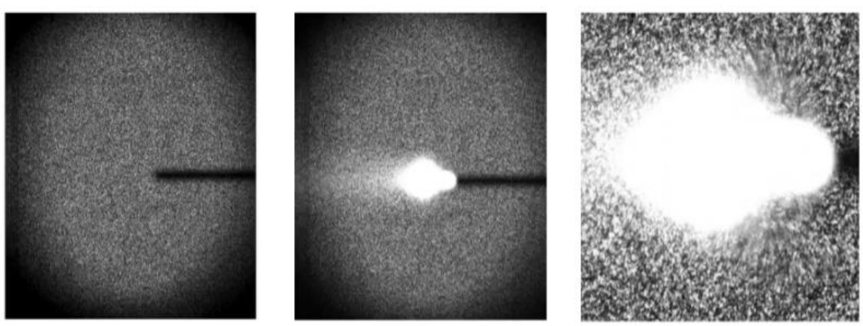


Schlieren Experiments





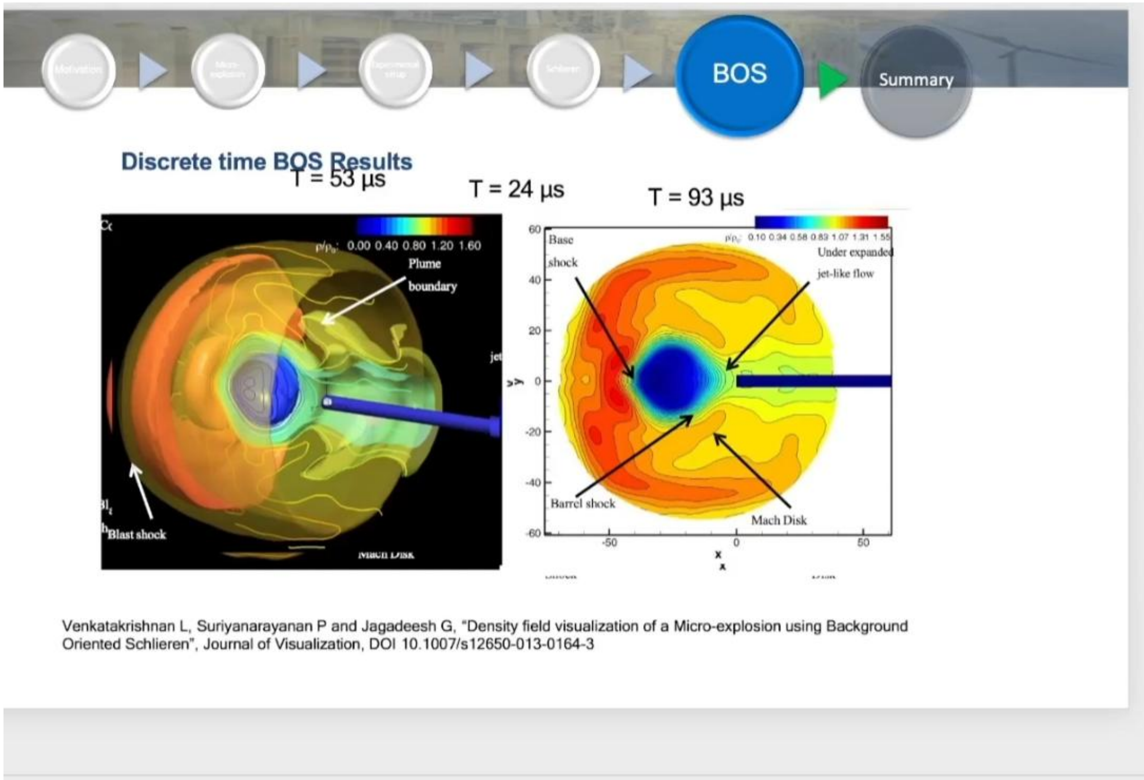
Obed SI, Jagadeesh G, and Kontis K, "Micro-Blast Waves Using Detonation Transmission Tubing", Shock Waves, DOI 10.1007/s00193-012-0416-5, 2012



(a) no-flow (b) flow condition (c) close-up

at t=24µs

** Venkatakrishnan L, Suriyanarayanan P and Jagadeesh G, "Density field visualization of a Micro-explosion using Background Oriented Schlieren", Journal of Visualization, DOI 10.1007/s12650-013-0164-3*



So all these things, as you can see, are kind of well understood. In the future, of course, bos can also use machine learning because this makes for a test case for machine learning. This is from Jiang's data, for example, which is the reconstructed density field. You can see that bos can be used for a variety of purposes; it's an extremely robust method and it's simple compared to other types of imaging techniques. So one can also use BOS data for machine learning, for flow reconstruction or flow prediction purposes, given the way things are going. I think this is one of the major areas of improvement or one of the major areas of research that will happen in due course. Thank you so much.