Advanced Measurement Techniques in Fluid Mechanics and Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
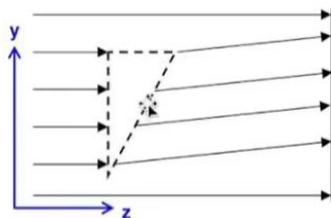**Indian Institute of Science, Bengaluru**
Week – 05
Lecture - 25
Schlieren and Shadowgraphy – 3

All right, so in this particular lecture, I think if you recall from the previous lecture, we talked about the sensitivity of the Schlieren, and we also talked about what contrast is, what differentiation is, what background is, and the focal length of the Schlieren, how everything is dependent, and what the knife edge does, how it basically displaces. So are all the things I kind of did in the previous class. Today, we are going to take this moment to talk a little bit about shadowgraphy mathematics, show a couple of slides, and explain why it is dependent on the second gradient. So, that is what we are going to cover in this class. So this is also from Sally Bain's PowerPoint presentation, part of Galaxy Tech. So if you look at the shadography in general, as we talked about before, if this was the Y and the Z, there would also be an X.
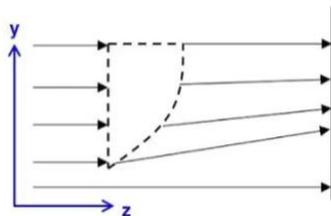

Now what happens is that here you have a gradient of the refractive index, but this gradient is constant. So this is exactly how it looks. So the refractive index basically changes linearly with y. This is exactly what is shown over here.

So this is the kind of change in the refractive index. So what happens is that all the rays are entering this particular cross section of interest. They are all kind of shifted by the same amounts. So we still get a uniform shift in the illumination. We see some sad shadows, but we don't get the outline of the Schlieren.

As we move down the path, all the rays shift in the same way because of this constant gradient. That is what you get here, all right? However, if you have a situation where n is no longer a linear function of y, in other words, this is an arbitrary profile in which n is definitely not a linear function of y, which means that let's assume that the second derivative of n is actually a constant. The curvature of the curve is actually a constant. So, this is what you get. The gradient of the gradient is basically a constant.

So what you actually see over here is that this is a constant, but this is not. As a result of that, the rays get deflected, but you get a non-uniform illumination. So the variation of these gradients therefore becomes critical if you want to outline the Schlieren, which is the purpose of the stratigraphy. So you can understand why, if you have a uniform gradient or a constant gradient, the rays are all shifted in a very similar fashion. The outline is not very clear.

You don't get an outline at all, but when you actually have the second derivative of the refractive index to be constant, or in other words, the $\frac{d\eta}{dy}$ or $\frac{dN}{dy}$ is not a constant, you get the rays which are non-uniformly distributed and non-uniformly shifted. As a result, you get non-uniformity. Now this was from Dr. Who's work at Aerospace Iowa State; this is from his presentation. This provides an idea of how the math actually works ultimately.

Figure 7.12 Displacement of light beam for shadowgraph evaluation

$$I_{sc} = \frac{\Delta y}{\Delta y_{sc}} I_0$$

$$\Delta y_{sc} = \Delta y + Z_{sc} \cdot d\alpha$$

$$\frac{\Delta I}{I_0} = \frac{I_{sc} - I_0}{I_0} = \frac{\Delta y}{\Delta y_{sc}} - 1$$

$$= -Z_{sc} \cdot \frac{d\alpha}{\Delta y_{sc}} \approx -Z_{sc} \cdot \frac{d\alpha}{dy}$$

$$\Rightarrow \frac{\Delta I}{I_0} = \approx -Z_{sc} \cdot \frac{d\alpha}{dy}$$

since $\alpha = \frac{1}{n_a} \int \frac{dn}{dy} dz$

$$\Rightarrow \frac{\Delta I}{I_0} = \frac{-Z_{sc}}{n_a} \cdot \int \frac{d^2 n}{dy^2} dz$$

- Sensitivity is proportional to index of refraction 1/n, and screen distance Z_{sc}

Maths of Shadowgraphy

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So if I take you through it, there is this test section where the light is coming in like a parallel beam here, and say this—uh, the... The light pipe has a diameter of Δy, and as it goes through this particular test section, it has a width of L. And from the test section, from the middle of the test section to the end of the screen, the length is Z_{SC}.

And this is where the Z starts. And this is where Y starts. These are. This is where I have fixed my axis. As a result of that, what happens is that now let's assume that these rays now   gets   deflected.

So, this is basically the deflected rays. And this is the screen, and this would have been the path of the light if there had been no test section, if there had been no density variation, and no refractive index change. So this gets deflected, and you can see that there is this angle α by which this displacement happened, and it has illuminated the length. On the screen, which corresponds to this length, Δy is Δy_{SC}. So this was the light beam.

Now this is where the light beam ends up. And this would have been the original path of the light beam. If you now look at the illumination of SC, it is equal to Δy divided by Δy_c. So this is Δy, and this is Δy_c multiplied by the initial or the incident illumination on this object, which is I0. So you can readily see that, okay, ΔI_{SC} multiplied by Δy_c is equal   to $I_o \Delta$ Y.

It's almost like, you know, you're putting in a certain amount of mass flow rate into a pipe, and this is what has to come out the other end, similar to that. So what we can see here is what this Δy_{SC} is about? So Δy_{SC} is actually your Δy, and there is this z_{SC}, which is the distance multiplied by this D α. D α is this. Can you imagine? So this would be the additional length that is actually illuminated on the screen because of the presence of the desktop. So this is what you get when ZSC is measured from the center of the test section all   the   way   up   to   this   point.

Now, if you take ΔI divided by I naught, what is ΔI? ΔI is $(I_{SC} - I_o)/ I_o$. Which turns out to be ΔY divided by ΔY is SC minus one. All right. Now you substitute, you know, a little bit of substitution now over here that what is your final value you get; this almost turns out to be equal to minus z_{SC} into $d\alpha$ by dy, which basically sections as a function. I basically is a gradient of reflection or the spatial variation of the deflection with y, and z_sc   is   the   distance   at   which   the   screen   is   placed.

This is equal to the relative change in the illumination compared to the in. So this is what you get; this is the expression Δi by I_o. Now, this we already know: α, if it is the deflection, is the same as ε. If that is the deflection we are talking about, this is $\frac{zdn}{ndy}$.

This is what we do. Now, remember this is $\frac{d\alpha}{dy}$ now. So α is the first derivative of dn with respect to dy. And this illumination change is the first derivative of α, so in other words, if you plug this in, $\Delta I / I_o = -z_{SC} / \frac{-Zsc}{n}$ n; n is basically the index of refraction multiplied by, now it is d²n/d²y into dz. All right, so um. As you can see, this total deflection is therefore a function of Z, where the screen is placed, and it is dependent on the second derivative of the refractive index with respect to pi.

So the sensitivity is therefore proportional to the index of refraction, which is $\frac{1}{n}$, obviously, and the screen distance z_{SC}, and the integration and the variation of the second derivative of n with respect to y. So this is, in general, the math. behind the shadowgraphy, why it is dependent on the second derivative. And I think in these two slides, we have kind of explained that it is not just a function of the variation of n with respect to y only, but it is the second derivative with respect to y. So that is, I think that's all.

Buttons here. And that is essentially the mathematics of shadowgraphy. We already did some of the math of Schlieren, as well. know that how these two methods now kind of work. Now, if I look at other Schlieren systems, so far we have seen that we are using lenses mainly. So there are Z-shaped systems also.

Other Schlieren systems

Z-shaped system

Double-path systems

- Z type : Most commonly used technique. Especially in wind tunnels.
- Double path: higher sensitivity. Used in large scale, full scale applications

This is the most commonly used technique, mostly in wind tunnels and stuff like that. And what happens is that you have a light source, and then you have a condenser lens,

pass it through a pinhole or a slit, and then it goes and hits a parabolic mirror, which basically transforms this into a collimated, uh, again, a parallel beam of light, and it passes through the test area right here, which is meant to measure, and then you have another parabolic mirror that basically focuses it. to a knife edge, which we already did earlier, and you have the camera, which records the image. So, this is the Z-shaped system. It is most commonly used in wind tunnels, and it uses mirrors, which are much easier to manufacture, so to speak, and lenses, as we saw from historical references.

Also, there are other types of systems, like τble path systems. Where you use a prism, basically, and a knife edge. And this is basically a mirror-prism combination. And this is essentially done for off-axis measurements. And then you can also use coincident kinds of methods where you use a beam splitter, which is very similar to the retro-reflective type.

loose camera helmet. So these are different types of fullerene systems. These are some examples. This is flow through a truncated aerospike nozzle. The very complicated structures, you can see right here, and the very fine grain structures you can kind of see. This is, for example, the shock waves of some strength, a strength that is changing here.
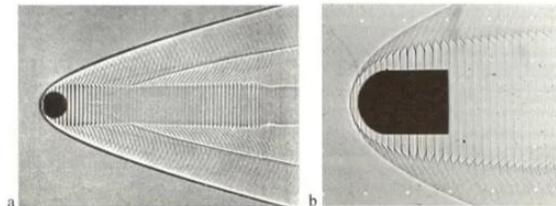

Examples : Schlieren
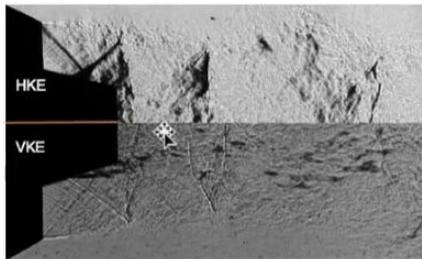
Flow through truncated aerospike nozzle
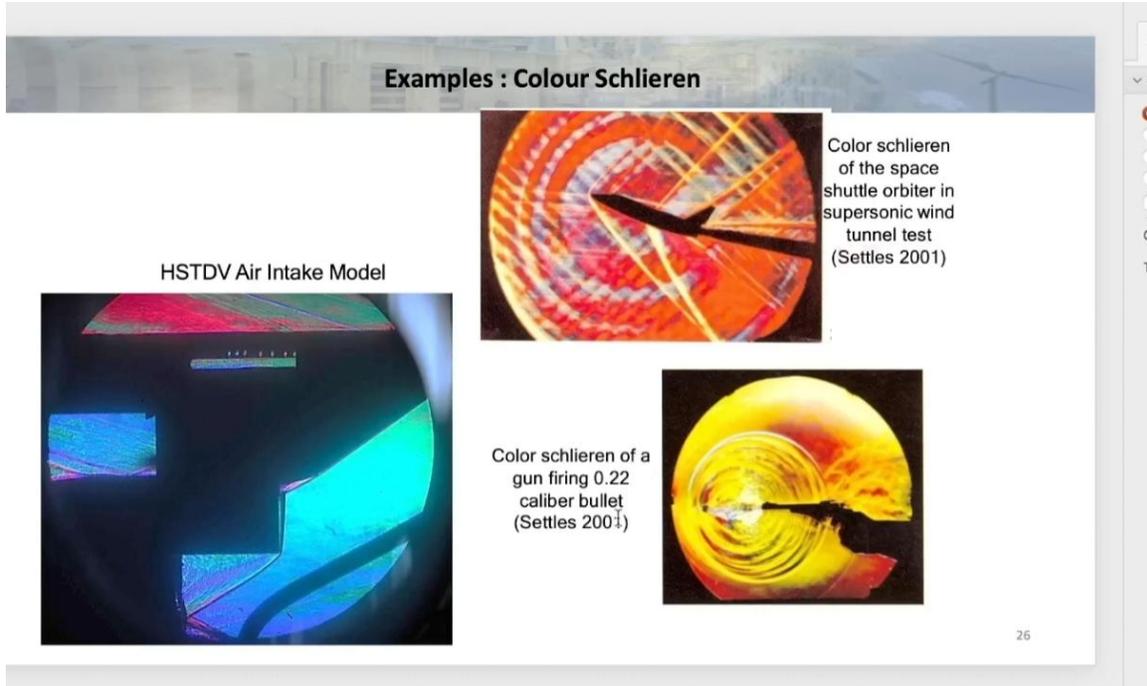
Shock waves of same strength
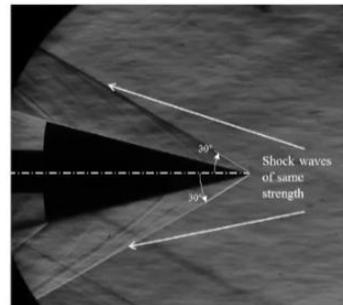
Projectile fired at Mach 4.75 in reactive H$_2$-air mixture - cyclic detonation behind the shock (Settles 2001)

And this is a projectile that is fired at Mach 4.75, which is hypersonic, in a reactive hydrogen-air mixture. The cyclic detonation behind the shock is coming from the work of Sittl. Now, you can see Sittl's books on, you know, tellurine and shadowgraphy, which are almost like a bible, so you should kind of consult them. If you do not have the book,

you   should   get   it.

and study in more detail if you want. So you can see a variety of very fine-scale to very large-scale structures. The idea is the sensitivity of the system; even though we say that the deflection is very small, n is only a weak function of density and all these things, even then you can get these very striking features. And with the advent of high-speed cameras and numerous other things, this has become much easier to do. These are examples of Color   Schlieren.

Examples : Colour Schlieren

HSTDV Air Intake Model

Color schlieren of the space shuttle orbiter in supersonic wind tunnel test (Settles 2001)

Color schlieren of a gun firing 0.22 caliber bullet (Settles 2001)

So this is the color Schlieren of the Space Shuttle orbiter in supersonic wind tunnel testing. This is again from Sittel's work. This is the color Schlieren of a gun that is firing a .22 caliber bullet. You can see the bullet moving and then the shock.

And you can also see that this is the HSTTV air intake model. And you can also see the flow structures and the sharp discontinuities. Over here, these are some examples of the color Schlieren images from different sources. You know, these things have a kind of fidelity and the kinds of things you can do with them. So here, for example, you have a full-scale   Schlieren   where   it   is   very   large   scale.

This is the heat released from a gas grill. This is also reading a paper, and you can... The space heater is where you take an ice cream off from there or something like that; you
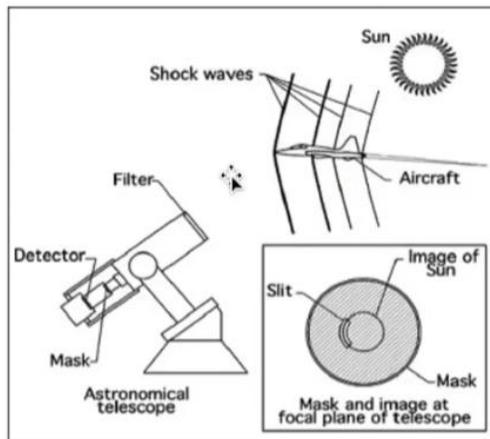
can see there are very large scales and small scales that are focused, which can be done.
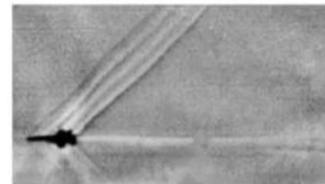
Examples : Full Scale Schlieren

Heat Released from Gas Grill   Heat from space heater, lamp& person   Cold Air Dragged From A Freezer

From http://www.mne.psu.edu/psgdl/FSSPhotoalbum/index1.htm

Large scale Schlieren

(a) Dryden F-18 #843 in flight.

(b) Schlieren of F-18.
Fig. 18. F-18 at M = 1.4 and 18.3 km range.

Image of a T-38 at Mach 1.1 (Leonard M. Weinstein, NASA Langley Research Center) – taken using a telescope, the sun, and a cutoff, field of view of 80 m!

This is also an example of an extra-large scale Schlieren that uses, you know, an astronomical telescope and uses the sun, and you know this is aircraft and this. And you use the sun as the light source. Very interesting. So it is at Mach 1.1 using a telescope,

which takes in the sun, with a cutoff field of view of about 80m.

 You can imagine 80m. That's almost like, you know, the distance that Usain Bolt runs. hundred fractionally below it. And you can see the kind of rock structures on this aircraft. So these are the kinds of very high, very large scales, 80m in scale, down to scales that are like this. that you can resolve using Schlieren, large-scale, medium-scale, and also somewhere in the middle.

 These are the kinds of medium-scale Schlieren that you can do. And even you can do colored Schlieren, which can give you different aspects of the measurement. So the difference between Schlieren and shadowgraphy, let us read it once more. The Schlieren is used by a focused image formed by lenses.

 And shadowgraphy is not an image but a shadow. In some cases, with shadowgraphy, you don't even need any optics to screen. And this requires cutting off the refracted light by using the knife edge. Here, there is no cutoff of the refracted light; you can see there is a focusing here. You need not do any focusing at all.

Schlieren Vs Shadowgraph

Schlieren Imaging	Shadowgraphy
Focused optical image formed by a lens	Not an image but a shadow
Requires cutoff of the refracted light	No cutoff of refracted light
Illuminance level responds to $\partial\rho/\partial x$ and $\partial\rho/\partial y$	Responds to second spatial derivative, $\partial^2\rho/\partial x^2$ and $\partial^2\rho/\partial y^2$
Schlieren image displays the deflection angle e	Shadowgraph displays ray displacement
More sensitive in general	Less sensitive except for special cases (e.g. shock waves)
More difficult to set up – use lamps, mirrors, lenses	Extremely easy to setup, occurs naturally

 This requires a cutoff. This does not require a cutoff. And the illuminance level actually depends on the first-order derivative of the density with respect to space, either with

respect to x, y, or both. This responds to the second spatial derivative, which is the $\tau^2\rho$ / τx^2 or $\tau^2\rho$ / τy^2. The Schlieren image displaces the deflection angle, which is e.

The shadow graph will displace the ray's displacement. This is usually more; the shivering is usually more sensitive in general. Shadowgraphy is less sensitive, except for very special cases like shock waves, because, as I told you earlier, it is dependent on the second derivative, which requires the gradient of the refractive index to be very, very sharp, as is the case with shocks. Therefore, it is more sensitive to those kinds of features rather than where the density is more gradual, and therefore the second-order change is much less sharp or more gradual in that case. Shadowgraphy does a very good job in those cases. It is more difficult to set up because you need lamps, mirrors, lenses, and other   items.

This is extremely easy to set up. As I said, it need not even have optics. And we also saw the math. Fairly routine. So shadowgraphy can give you a preliminary idea of what is going on. And in case of shocks and other situations, it can give you some idea about what   is   going   on.

But Shittering is something that you should use if you want to get more fine-scale, detailed features of the flow. So that is what it actually is. A shock can use both. Okay, so some of the important equations come from Sally Bain's work. These are the things that you   do   when   you   want   to   set   up   a   Schlieren.

Important Equations

Object (FOV) — Lens 1 — Knife-edge — Inverted object image — Lens 2 — Object image

y_0, x_1, f_1, y_{I1}, x_2, x_3, f_2, x_4, y_{I2}

Equations:

Gaussian Lens Equation: $\dfrac{1}{x_1} + \dfrac{1}{x_2} = \dfrac{1}{f_1}$ and $\dfrac{1}{x_3} + \dfrac{1}{x_4} = \dfrac{1}{f_2}$

Magnification: $\dfrac{y_{I1}}{y_0} = \dfrac{-x_2}{x_1}$ and $\dfrac{y_{I2}}{y_{I1}} = \dfrac{-x_4}{x_3}$

Total Magnification $\Rightarrow$ $M = \dfrac{y_{I2}}{y_0} = \dfrac{x_4 x_2}{x_3 x_1}$

Constraints:

For Real Image: $x_2, x_3 > 0$

$\Rightarrow$ $x_1 > f_1$ and $x_3 > f_2$

Table Size:

$x_1 + x_2 + x_3 + x_4 < L$

where L is limited by the size of the optics table

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So, for example, you know, here is lens one, here is a knife edge, this is lens two, and this is the object. And this is the object. Light that's coming in. So if you use a Gaussian
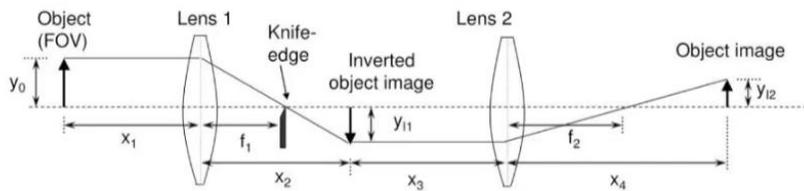
lens equation, then one over X_1 plus one over X_2 is equal to one over F. And one over X_3 plus one over X_4 is equal to $\frac{1}{n}$, the second lens.

So now, if you take magnification into consideration, you know, first magnification here, y L_1/ y_o,. Right. And similarly, $\frac{Yl2}{Yl1}$, because now this is the magnification that's happening, is $-X_4$/- X_3. A fairly simple equation. So the total magnification, which is y_{l2}, is the final image divided by the original image, which is given as X_4 X_2 /X_3X_1.

So this is the total magnification. Now, for a real image, x_2 and $x_3 > 0$. So this leads to x1 > f1 and x3 > f2. So the table size, when you are setting up this experiment, has to be such that X_4 X_2 X_3X_1 < 1. L is the size of your optical table where you are going to set this entire thing, right? So the equations continued.

So therefore you, in order to set up this Schelerin setup, you have to have obey these relationships. One over X_1+1 / X_2 is equal to $\frac{1}{F1}$. And $1/X_3$ +1/ X_4 is $1/F_2$, where X_1 >F1. and X_3 >F2.

Important Equations

Object (FOV) Lens 1 Knife-edge Lens 2 Object image

y_0 y_{l1} y_{l2} Inverted object image

x_1 f_1 x_2 x_3 f_2 x_4

SUMMARY

Must Satisfy:

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{1}{f_1}$$

$$\frac{1}{x_3} + \frac{1}{x_4} = \frac{1}{f_2}$$

$$M = \frac{x_4 x_2}{x_3 x_1}$$

Under the Constraints:

$$x_1 > f_1$$

$$x_3 > f_2$$

$$\sum_{i=1}^{4} x_i < L$$

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And magnification is given as (X_4 X_2 /X_3X_1)F1. All the details are labeled here again, from Talley-Bain's work. The sum total of all these Xi's has to be greater than L, the total. So these are the important equations when you remember that the knife edge is placed at F_1, which is the focal point of the first lens. And then you form an inverted image here at X, and then you form the final image at I_{l2}. Now, here are some simple setup instructions that one can follow, again from Sally Bain's own work.

Step 1: Calculate the required distances between he object, schlieren lens, focusing lens, and camera based on the equations on the previous slide and the focal lengths of your lenses

Step 2: Set up the light source, any flat mirrors, and test section with windows in place if applicable

Step 3: Set up a laser in the place where the camera will go

Step 4: Turn on the laser and ensure that the beam is straight in both the vertical and horizontal directions along the optical axis (line to next mirror)

Step 5: Adjust any mirrors on this side of the set-up to direct the laser to the test section, ensuring that the beam stays the same height the whole way (use a ruler or a height gauge to test this at every mirror)

Tip 1: Try to keep the laser dot as close to the center of the mirrors as possible

Tip 2: The laser light corresponds to approximately the center of the ultimate light beam, so locate the laser beam through the test section where you want the center of the light beam
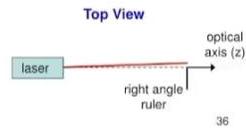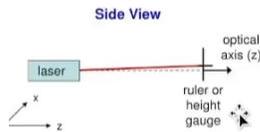
Step 6: If there are windows on the test section, check for reflections to ensure the laser is perpendicular to the windows
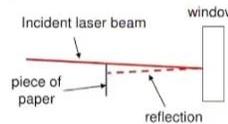
Tip 1: Use a piece of paper to probe all around the incident beam – any reflections will show up on the paper

Tip 2: When it is properly aligned, when you look through the windows all the laser dots will appear in a straight line through the glass

Side View

optical axis (z)

laser

ruler or height gauge

x

z

Top View

optical axis (z)

laser

right angle ruler

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Incident laser beam

window

piece of paper

reflection

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First, you need to calculate the required distance between the object, the Schrödinger lens, the focusing lens, and the camera, based on the equations that we just did. And then you set up the light source and mirrors that are needed for the test sections with windows and have it in place. Then step three is to set up the laser in a place where the camera will go. Then turn on the laser and ensure that the beam is straight in both the vertical and horizontal directions along the optical axis. Then step five is to adjust any mirrors on this side of the setup to direct the laser to the test section.

And you try to keep the laser dot as close to the center of the mirror as possible. If there are windows in the test section, check for reflection. Make sure that the laser is perpendicular. And there are certain tips that you can read.

This is all important and good information. So you are setting up; this is the side view, this is the top view, and how you are going to set up the experiment, and so on and so forth. So after this, you adjust the mirrors on the light source to direct the laser beam to the light source, ensuring that the beam stays at the same height and is centered on the mirror. And then you adjust the height of the light source. Then you remove the cover of the light source and make sure it's unplugged so that you can get the filament. Then use the controls on the light source to move the filament of the bulb up.

Laser light hits the center of the element. All then you check for protection. And once all the alignments are done, cure all the optics. Basically, you have to screw them onto the optical breadboard and, one by one, add lenses to the setup. And once the alignments are very important, as we say, a little bit, a few degrees off here and there, or a little bit of

distance here and there is going to ruin the entire measurement because it's highly sensitive due to the weak function that ρ has with respect to refractive index. And then replace the laser with the camera, place the knife edge, approximate the rotation, and turn on the light source.

And then you make some in-situ adjustments so that things are in place. And the Taliban says that after this, work begins. And my students also say the same thing: that the real work begins after this, where now you have a task on your hands where you are actually ready to get the measurements and try to fine-tune all these measurements to be perfectly aligned with the whole thing. So I think in this particular lecture, what we have covered is that we have gone through some of the details; man, just a quick recap. We stated that this is how the Schlieren principle works. What is the gain, what is the contrast, and what is the sensitivity? We also saw what the math of the shadography setup is.

And then we also saw other Schlieren systems, the kind of systems that are routinely used. We saw some very cool examples of small, large, and very large kinds of Schlieren systems. Especially this one takes the shocks for, say, 80 meters. We saw the differences between the systems, and then we saw the important equations that are needed to set it up, along with certain tips that come from the GALSIT on how the system should be set up. Honestly speaking, these are still functions of, you know, Trial and error and a little bit of hands-on experience are important, but there are certain protocols that you should follow.

It requires very careful alignment because, without alignment, things are not going to work. So, it is very sensitive. So you have to have a very good, you know, lot of patience.

And then only you can record images like this, which show structures of different lengths.

Okay, so we end this lecture here. In the next lecture, we are going to start looking at how Schlieren can be used for tomography as well. So these are called the background-oriented Schlieren technique, which is basically pioneered by many people, but among them, Dr. L. Venkat Krishnan and Meher from DLR are notable. So they are the ones who proposed some of these methods, and these are exemplary methods.

And they use some of the concepts of the PIV and use it here. It's very interesting. So I think now you have a reasonable idea of the math and a reasonable idea of how to do Schlieren. Now we will see how you can use background-oriented Schlieren to measure the density fluctuations. And so that would be, and it can also lead to tomographic measurement as well. Okay, so with this particular note, we end this lecture here, and we will see you in the next lecture, which will probably be the last lecture on the Schiller technique.